

Earth Observing Satellite Orbit Design Via Particle Swarm Optimization

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Designing the orbit of an Earth observing satellite is generally tedious work. Typically, a large number of numerical coverage simulations are run to understand the impact of orbital parameters on sensor performance and coverage characteristics. There is little guarantee of determining the optimal orbit for a given mission. Some work has been done to wrap an optimization loop around traditional numerical coverage simulations; however, the computational resources necessary for this can be prohibitive. A methodology of determining optimal orbital characteristics without requiring a large amount of computing is presented. Several existing methods are used to calculate an approximation to total pass time and average number of passes per day given a satellite's orbital altitude and inclination. These are used along with particle swarm optimization to determine optimal orbit parameters. This methodology only pertains to a single satellite in a circular orbit.

I. Introduction

Designing the orbit of an Earth observing satellite is typically tedious work. The method generally involves running a number of numerical simulations to determine coverage and sensor performance for different orbit parameters.¹ This methodology does not guarantee an orbit that is optimal for performing a given mission, rather the orbit designer is choosing the orbit that performs the best out of the alternatives analyzed. This often constricts mission requirements because the full design space is not considered.

Some work has been done to introduce optimal estimation into orbit design. References 2 and 3 discuss using Genetic Algorithms (GA) and traditional numerical coverage simulations to determine optimal orbit parameters. Difficulties arise with this process since traditional coverage analysis can be computational intensive and, thus, does not lend itself particularly well to be used within an optimization process (since this analysis would be run for each of the many evaluations of the objective function).

Much of this work was developed based off work in References 4, 5, 6 and 7.

An optimization methodology was developed that uses existing approximations to total pass time and average passes per day as functions of orbital altitude and inclination (as found in References 5 and 8). These formulas are not complex, computationally simple, so they are well suited to use within a meta-heuristic optimization method such as the Particle Swarm Optimizer (PSO). This method seeks to find the optimal set of parameters to minimize an objective function by using many evaluations of that objective function instead of making use of derivative information. The computational resources necessary for a meta-heuristic optimization method is particularly sensitive to the complexity of the objective function as it is evaluated many times.

When designing the orbit for a given satellite mission, it is often preferable to choose an orbit that maximizes the satellite's visibility for a given set of ground targets. If the design space is restricted to a single satellite in a circular orbit, the only free parameters are the satellite's altitude and inclination. Even in this case, one may consider the Right Ascension of the Ascending Node (RAAN) or initial true anomaly as free parameters; however, for missions of sufficiently long time span these parameters do not appreciably effect satellite visibility characteristics (as long as a repeating ground track (or sun-sync) orbit is not used).

This paper compares two approximation methodologies as cost functions, each optimized through PSO. The methods are compared using a world-wide set of ground targets. Additionally, the same satellite sensor characteristics are applied such as altitude resolution and elevation restrictions. This optimized analysis demonstrates applying mission requirements into a single, fast optimization process as opposed to a labor

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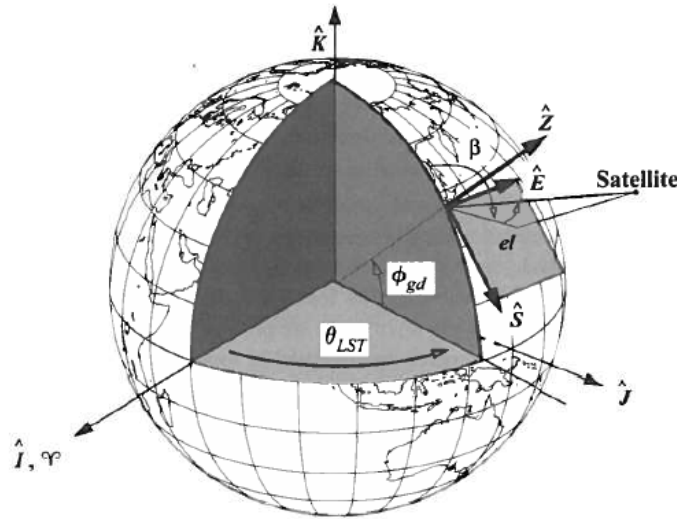
intensive manual process of analyze, change a parameter (or two), observe results and repeat until satisfied. The entire design space may be optimized, or the process could be further narrowed by constraining the free parameters to adjust for other mission considerations.

II. A Rapid Numerical Method Approximation of Total Pass Time Per Day

The first of the approximate methods includes a cost function expression based on total pass time developed in Reference 5. This approach uses a rapid numerical method for determining rise and set times for a satellite as seen from a ground site location. While the Alfano method includes flexibility to limit the ground site range, azimuth, and elevation, only elevation is considered for the approach developed here. Also, the orbit is constrained to be near circular since the second method does not accommodate elliptical orbits.

The approach uses an elevation function in the topocentric-horizon coordinate system (SEZ) shown in Figure 1.⁹ The method answers the simple question, "Can the site see the satellite at a given time?" The elevation is first constrained to be positive, above the SEZ horizon. Additionally the minimum elevation is constrained based on a Ground Sample Distance (GSD) which also keeps the semi-major axis in check. Without a GSD penalty, the semi-major axis would optimize towards the highest altitude allowed.

Figure 1. Topocentric-Horizon Coordinate System (SEZ)



The rapid numerical method is propagated on the order of a few minutes as opposed to seconds as required in a more traditional orbit propagation method thereby greatly reducing the amount of computation time needed to complete the rise/set algorithm. For a given free parameter set of semi-major axis and inclination, the algorithm propagates through a given time period (one day for this analysis.) At each propagated time step, the elevation limit function provided in Reference 5

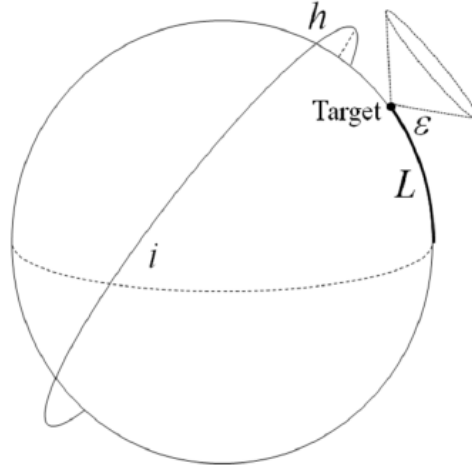
$$f_{\beta_{LIM}}(t) = \rho_E(t) - \rho_S(t) \tan(\beta_{LIM}) \quad (1)$$

is evaluated for sign changes to indicate a rise or set for the satellite. For this analysis only the lower elevation limit as determined by the GSD constraint is determined. The mathematics involved is found in References 5 and 9. Once the rise/set times are determined, the total pass time per day is simply the total summation of the valid pass times. This total pass time is provided to the PSO as a cost function value.

III. An Analytical Approximation of Passes Per Day

The second approximate expression is an analytical method developed in Reference 8. A geometrical approach is given to develop an analytical function for the average passes per day of a ground site given a circular orbit's altitude and inclination. The calculations are based on the ground site latitude L , minimum elevation ϵ , inclination i , and satellite altitude h as seen in Figure 2.⁸ The passes per day PPD are a function of the time duration D (one day for this analysis), orbital period P , and fraction of revolutions f

Figure 2. Arguments of Pass Per Day



that include a pass over the target.⁸

$$PPD = \left(\frac{D}{P} - \cos(i) \right) f \quad (2)$$

The PPD is provided to the PSO as a cost function value.

Reference 8 results indicate that the analytical model is accurate to within about %1 in the vast majority of cases tested. The function derived is closed-form, continuous, and piecewise differentiable. Please refer to Reference 8 for the full formula and derivation.

IV. Particle Swarm Optimization

In order to select the orbit parameters that maximize the collection opportunities over a given set of ground targets, an optimal state estimation method is employed. The Particle Swarm Optimizer (PSO) is used since designed to estimate globally optimal states in nonlinear systems.

The PSO was first formulated in References 10 and 11. Additional information is contained in References 12 and 13.

The PSO solves for optimal state estimates by first population the solution state space with a number of uniformly, randomly distributed particles. The cost function is then evaluated for each of the particles. The particles are then stochastically accelerated towards the current global best solution and the current, per-particle best solution.

A cost function was constructed so that the optimal orbit parameters may be determined. First, for a given target location, satellite altitude, h , and satellite inclination, i , both the approximate total pass time per day⁵ or the approximate passes per day⁸ can be calculated. Either of these quantities will be referred to as $S(\phi, h, i)$. Additionally, each target will have an associated weight, w , so that high priority targets may influence the orbital solution more than lower priority targets (so that higher weight values indicate higher priority targets). Last, satellite sensors often suffer from poorer performance as altitude increases. For example, satellite imagers will have an increased GSD, the projected distance on the ground between pixels, as altitude increase. As the satellite's altitude increase, the ability to distinguish fine detail decrease. Thus, each target will have a penalty function $p(h)$ to capture this trade-off between altitude and performance. The following cost function is then constructed for n targets.

$$J = \sum_{l=1}^n w_l p_l(h) S(\phi_l, h, i) \quad (3)$$

Now, an initial population of particles can be created. This is done by uniformly, randomly populating the search space with N number of initial states. This is accomplished by invoking the following equation

for the j^{th} particle.

$$\mathbf{X}_j = \mathbf{X}_{min} + r(0, 1) \mathbf{d} \quad (4)$$

$$\mathbf{d} = \mathbf{X}_{max} - \mathbf{X}_{min} \quad (5)$$

$\mathbf{X}_j$ is the state of the j^{th} particle, $\mathbf{X}_{max}$ is the upper bound of the search space, $\mathbf{X}_{min}$ is the lower bound for the search space and $r(0, 1)$ signifies a random number with uniform distribution between 0 and 1.

The following process is then iterated until a suitable stopping condition is reached. First, evaluate the objective function for each particle.

$$\Upsilon_j = J(\mathbf{X}_j) \quad (6)$$

Next, check to see if this current state represents either an individual best solution for the j^{th} particle, denoted by ψ_j , or a global best solution, represented by $\mathbf{G}$. Here, only one global best solution will be tracked. If these conditions are satisfied then the following are implemented.

$$\psi_j = \mathbf{X}_j \quad \text{if } \Upsilon_j < J(\psi_j) \quad (7)$$

$$\mathbf{G} = \mathbf{X}_j \quad \text{if } \Upsilon_j < J(\mathbf{G}) \quad (8)$$

For M particles, the algorithm tracks M individual particle best solutions and typically one global best solution. The individual and global best solutions are not necessarily updated at every iteration; rather, they are only updated when a better individual particle solution or global solution is found. This is known as a global best topology. A formulation can be made where each particle is only aware of the best solution within its group of neighbors as defined in state space, as opposed to each particle having knowledge of best solution globally. This formulation is generally regarded as allowing the PSO to be more resistant to premature convergence; however, it requires more computation and was not deemed necessary for this work. Further discussion is available in Reference.¹⁴

Now that all particles have been evaluated and checked for the individual or global best solutions, the particles must be moved to a new position for the next iteration. This is accomplished in the following manner. First, the velocity of each particle is set in the following manner.

$$\begin{aligned} \mathbf{V}_j^k = & c_I \mathbf{V}_j^{k-1} \\ & + c_C (\psi_j^k - \mathbf{X}_j^{k-1}) \\ & + c_S (\mathbf{G} - \mathbf{X}_j^{k-1}) \end{aligned} \quad (9)$$

Here, j refers to the j_{th} particle and k refers to the k_{th} iteration. The initial particle velocity can be assigned arbitrarily. In this implementation, the velocity direction was randomly assigned while the magnitude was set to be arbitrarily small. As can be seen, the particle's velocity is impacted by three main factors: the particle's previous velocity, the vector difference between its individual best state and its current state, and the vector difference between the globally best solution and its current state. These three quantities are scaled by inertial, c_I , cognitive, c_C and social, c_S weights, which are defined in the following manner.

$$\begin{aligned} c_I &= \frac{1 + r_1(0, 1)}{2} \\ c_C &= 1.49445 r_2(0, 1) \\ c_S &= 1.49445 r_3(0, 1) \end{aligned} \quad (10)$$

Notice each of these three constants involve an independent uniform random number, given by $r_i(0, 1)$. Each of these stochastic scale factors were chosen from suggested values in the literature and may be adjusted in a number of ways. Both References¹⁵ and¹⁶ discuss the effects of altering these parameters. Once the particle velocity is calculated, the particle is propagated to its new location in the following manner.

$$\mathbf{X}_{j+1}^i = \mathbf{X}_j^i + \mathbf{V}_j^i \quad (11)$$

This then concludes one iteration. This process is then repeated until an appropriate stopping condition is met. This approach is similar to the approach presented in Reference 17.

Thus, the PSO is used to determine the parameters h and i that maximize J for a given set of n targets.

V. Testing Results

Results are given using both the approximate total pass time and approximate passes per day methods. An arbitrary set of twenty targets and target weights are chosen. A simple penalty function is chosen so the performance loss of higher altitudes is captured. The PSO is run to determine the optimal set of orbit parameters to observe those targets.

The location and weights for the test ground target set is given in Table V. Results for the PSO analysis using each approximation is given in Table V.

Table 1. Ground Target Set

Country	Capital	Target Latitude	Target Longitude	Weight
Argentina	Buenos Aires	-34.5875	-58.6725	1
Australia	Canberra	-35.283333	149.216667	4
Brazil	Brasilia	-15.783333	-47.916667	2
Canada	Ottawa	45.416667	-75.7	5
China	Beijing	39.928889	116.388333	1
Costa Rica	San Jose	9.93333	-84.0833	2
Cote d Ivoire	Yamoussoukro	6.816667	-5.283333	3
Egypt	Cairo	30.05	31.25	2
Germany	Berlin	52.516667	13.4	2
Iceland	Reykjavk	64.15	-21.95	1
India	New Delhi	28.6	77.2	3
Indonesia	Jakarta	-6.174444	106.829444	3
Japan	Tokyo	35.685	139.751389	2
Mexico	Ciudad de Mexico	19.434167	-99.138611	3
Morocco	Rabat	34.02	-6.83	4
New Zealand	Wellington	-41.3	174.783333	2
Qatar	Doha	25.286667	51.533333	3
South Africa	Pretoria	-25.706944	28.229444	2
The Bahamas	Nassau	25.083333	-77.35	3
United Kingdom	London	51.5	-0.116667	1

Table 2. Resulting Orbit Parameters from PSO Analysis

Method	Altitude	Inclination
Passes Per Day	300.0km	36.9°
Time Per Day	391.1km	52.9°

The results show that the orbital altitude that balances length of pass (which occurs at higher altitudes) versus better GSD (which occurs at lower altitudes) is found to be 300.0km for passes per day versus 391.1km for total pass time per day. Interestingly, each method identifies a different orbital inclination. The passes per day method identifies an inclination of 36.9°, whereas the total pass time method identifies an inclination of 52.9°. This may be due to how weighting factors affect the analysis. The inclination that maximizes the coverage over an individual point is approximately equal to the target's latitude. So, the first method may favor increased coverage over the higher weighted targets (at mid latitudes), whereas the second method may favor increased coverage over the targets at higher latitudes.

The difference in average daily coverage of the two methods are shown in Figures 3 and 4. The target locations and their field of view to the orbit are also plotted. As can be seen, the orbit resulting from the passes per day method favors coverage over the mid latitudes, while the orbit from the total time per day solution favors coverage over the higher latitude targets.

To note, the total pass time method employed by the PSO does not provide a unique solution. It does however, with repeated runs, tend towards a set of solutions found around an approximate altitude of 390km and inclination between 50° and 60° . Using the passes per day cost function, the PSO does converge to a unique solution.

The run times for this analysis were greatly reduced in comparison to a traditional numerical coverage cost function analysis. The average run time for the pass per day method was under a second while the run time for the total pass time was within a few minutes. This shows much improvement over the needed run times of hours for the traditional method.²

Both methods show varied results based on the desired GSD and target weights. This, however, will allow the orbit designer to quickly explore the decision space for a given set of parameters.

VI. Conclusions

Two methods to design orbits for Earth observing satellites are developed for use with an optimization process. These methods attempt to find an orbit to allow for the best coverage of a given set of targets. A test case was constructed and global coverage results are compared.

These results show the feasibility of using a computationally intensive optimization algorithm, such as the PSO, with a simplified satellite access model to quickly explore the orbit design space.

Future work includes optimizing the orbit design around more complex mission parameters, such as target revisit time. Further work is also needed to better characterize the interaction between target weighting and coverage results.

Ultimately, the methodology may allow a mission planner to quickly design an orbit that yields better mission performance.

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Figure 3. Coverage for Passes Per Day Solution

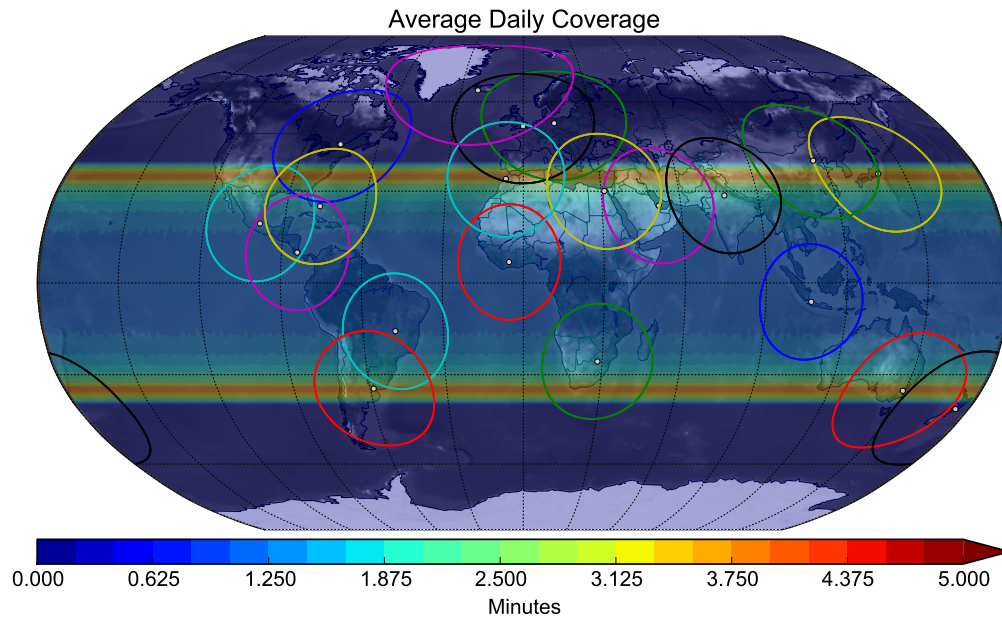


Figure 4. Coverage for Total Pass Time Per Day Solution

