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The puzzle of a marble in a spinning pipe

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Abstract
What trajectory does a marble follow if it is held inside a rotating pipe and then suddenly released?

1. Problem statement

A marble is placed one-third of the length along a pipe (of slightly larger diameter than the marble) and held there by a catch. The pipe is glued to a rotating turntable having frictionless bearings, such that the midpoint of the pipe coincides with the centre of the disk. The turntable is spun up to angular speed ω. The catch is now released. Which of the four coloured paths indicated in figure 1 does the marble follow in the lab frame, starting from its point of release?

2. Solution

There is neither a velocity nor a force component in the initial radial direction in figure 1, so path A is incorrect. That answer would only be valid in the 'centrifugal' frame of reference of the pipe [1].

In the lab frame of reference, the marble follows path C because no forces act on it. At the initial instant shown in figure 1 (call it time 0), the marble is moving to the left at exactly the same speed as the portion of the pipe enclosing it. Thus there is no normal force or frictional force between the marble and pipe.

Now consider a later instant in time t, while the marble is still in the pipe but radially farther outward than it was initially, such that it is a distance r from the centre as shown in figure 2.

Initially the marble was a distance r_0 (equal to one-sixth of the length of the pipe) from the
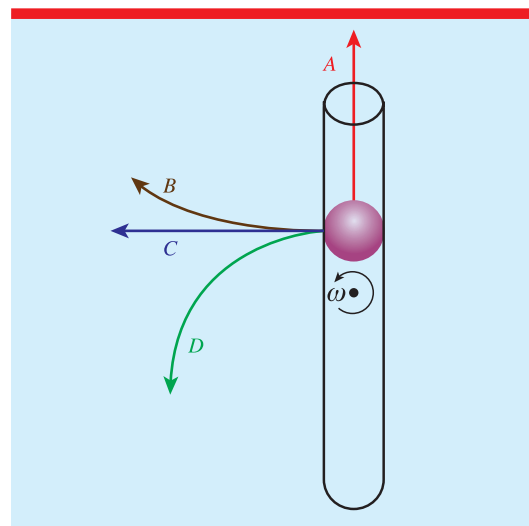

Figure 1. Top view of a marble initially held inside a pipe rotating counter-clockwise at angular speed ω.

centre and moving with the velocity vector indicated by the blue arrow, whose magnitude is ωr_0. If there are no forces between the marble and pipe, then the pipe will rotate at constant angular speed ω. In that case, the portion of the pipe enclosing the marble at the later time t will have the velocity vector indicated by the red arrow, whose magnitude is ωr. The component of the red velocity vector projected along the direction of the blue velocity vector is indicated by the green arrow.

However, the two angles indicated by the double parallel lines are equal; the cosine of that
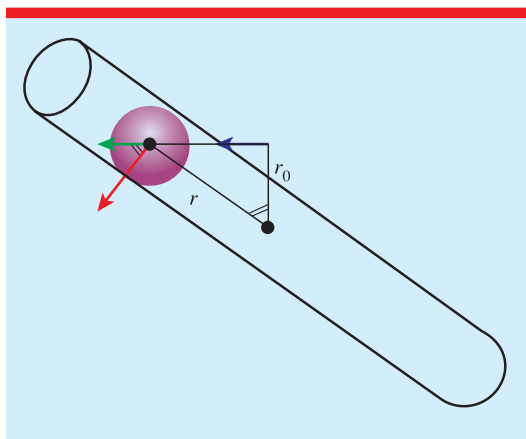
Figure 2. Top view of the marble inside the pipe after the catch holding it has been released.

angle is r_0/r. As a result, the green vector has a magnitude of $(\omega r)(r_0/r) = \omega r_0$, which is equal to the magnitude of the blue vector. The marble moves along path C at constant speed $v = \omega r_0$. There is never any contact (and hence any normal force) between the marble and pipe. If there were, one of the two objects would speed up angularly and (by Newton's third law) the other object would slow down angularly, so that they would fall out of rotational synchronicity with each other.

This solution depends critically on two details of how the system was set up. First, if the pipe is not rotationally isolated from the rest of the environment (but instead is rotating on a rough tabletop and gradually coming to rest), then the marble and pipe will start to fall out of rotational sync—contact will then occur to bring them back into sync and hence to deflect the marble's path from a straight line in the lab frame, so that it follows a trajectory similar to B. Second, if

the marble is not held by a catch and brought up to the pipe's angular speed prior to release, then again it will be necessary for there to be a (possibly complicated) interaction between the marble and pipe to force the marble to rotate with the pipe. One could imagine, for instance, the marble being pushed out of the pipe by the normal force along a trajectory slightly deflected below path C if the marble and pipe are oiled so that there is essentially no friction to produce the centripetal force required to make the marble follow a significantly curved path. On the other hand, if the marble tends to stick to the pipe, one could imagine a path similar to D, in which the marble rotates with the pipe while its progress radially outward along the length of the pipe is impeded.

In summary, the marble follows path C if its angular acceleration is zero (because it starts out with just the right azimuthal speed), path B if its angular acceleration is clockwise (when the pipe is slowing down and the marble needs to be rotationally slowed also), and path D if its angular acceleration is counter-clockwise (as it is rotated up to the pipe's angular speed).

Reference

[1] Taylor K 1974 Weight and centrifugal force *Phys. Educ.* **9** 357–60

Carl Mungan is an associate professor of physics, and enjoys the physics of everyday life such as spinning pipes and whitewater rafting.