On Processing Hexagonally Sampled Images

SOAR2 Review

12-15 JULY 2011

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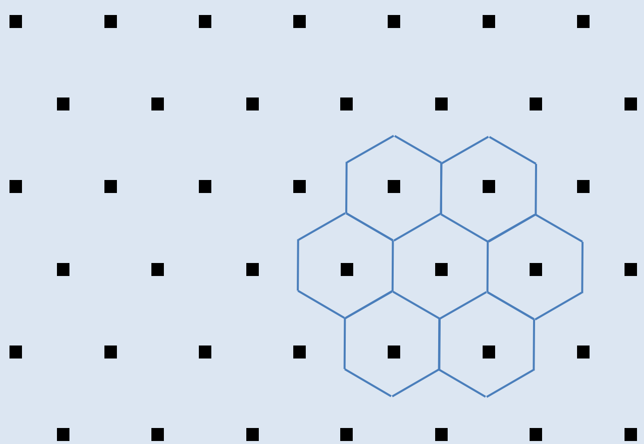
Outline

- Hexagonal sampling
- Array set addressing (ASA)
- Processing with ASA
  - Gradient estimation, convolution, downsampling, wavelet decomposition, and hexagonal DFT
  - Comparison with spiral addressing
- Hex-Rect sensor
- Fourier transform experiment
- Conclusion / questions

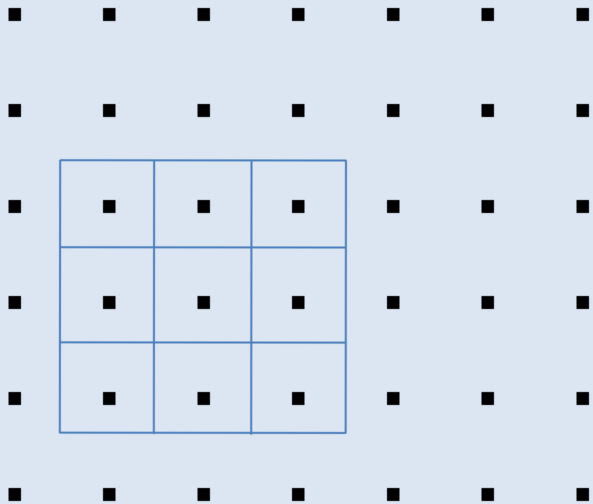
Hexagonal vs. Rectangular

Hexagonal Grid



- Optimal representation
- Consistent connectivity
- Angular resolution is 60 degrees
- Equidistant Spacing
- 6-fold symmetry
- Mimics nature

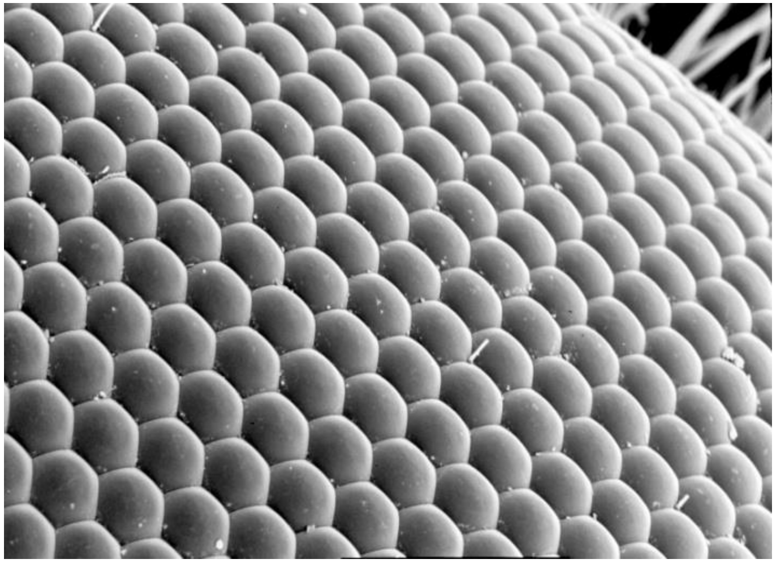
Rectangular Grid



- Non-optimal representation
- Connectivity ambiguity: 4-way vs. 8-way
- Angular resolution is 90 degrees
- Unequal spacing
- 4-fold symmetry
- Man-made

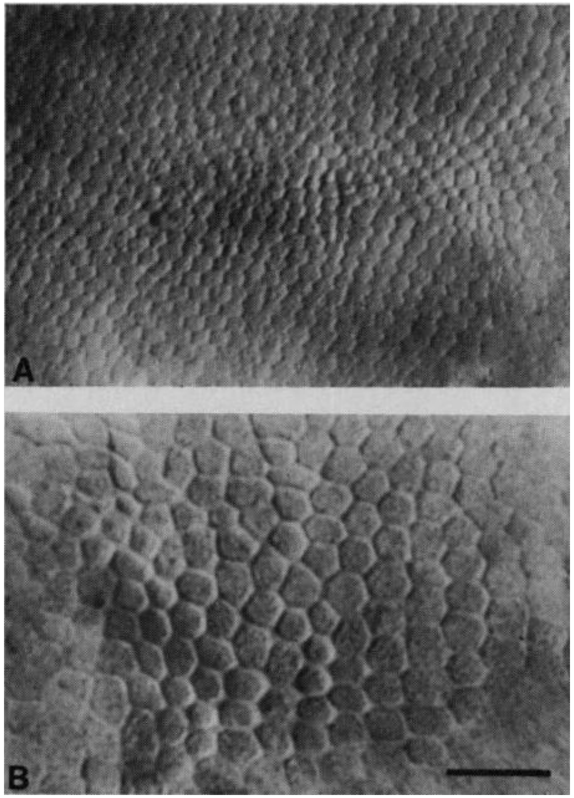
Natural Systems

Compound eye of the blowfly (*Calliphora Vomitoria*)



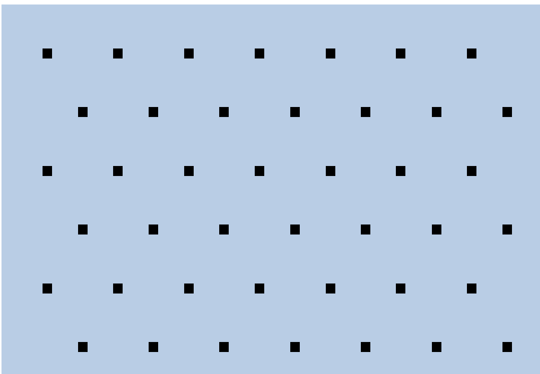
Reproduced from http://www.bath.ac.uk/ceos/Insects1.html

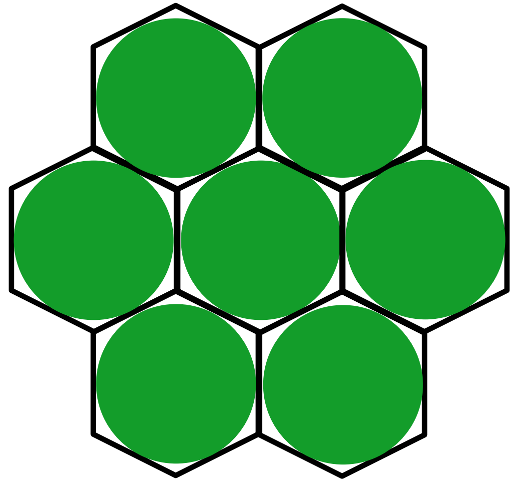
Distribution of cones in the fovea of a human retina showing high peak density (A) and low peak density (B) (bar is 10 microns).



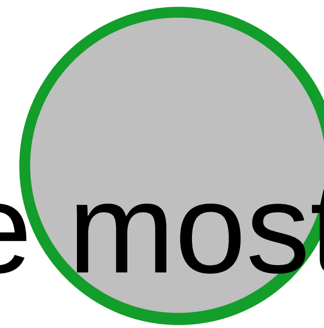
Reprinted from Curcio et al. (1987)

Why is Hex Optimal?





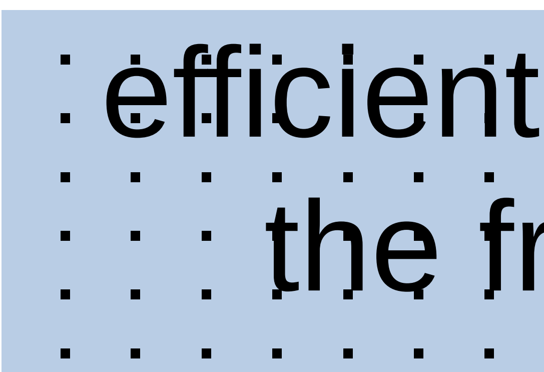
$$\frac{A_{gray}}{A_{green}} = \frac{\sqrt{3}}{2}$$

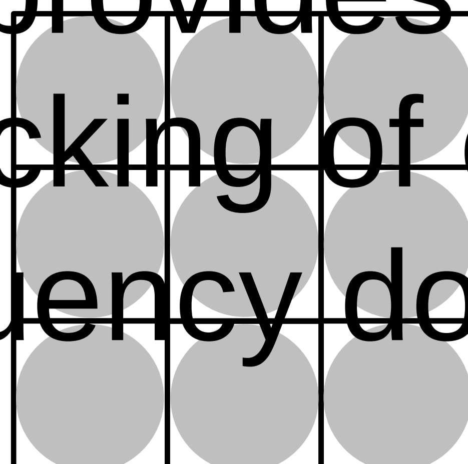


The spatial sampling geometry determines the spectral tiling, and the density of the spatial samples determines the area of the tile.

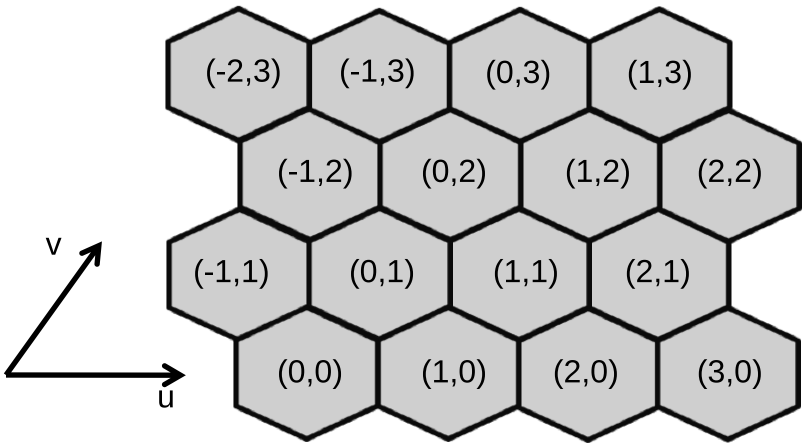
$$A_{hex} = A_{rect}$$

Because it provides the most efficient packing of circles in the frequency domain.

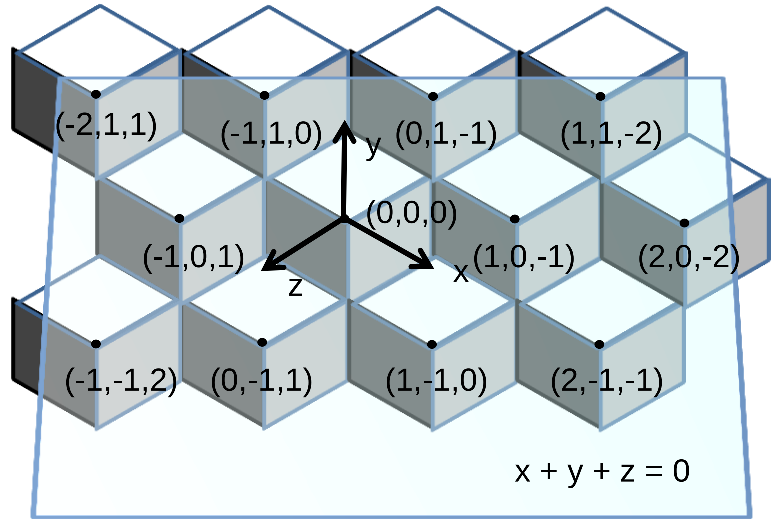




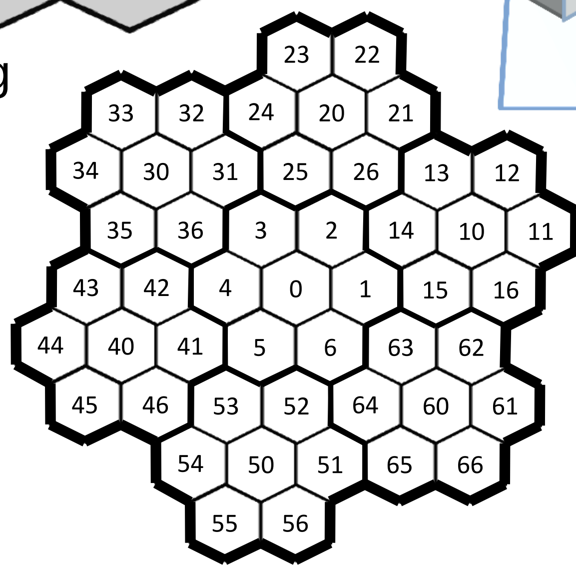
Addressing Schemes



Oblique Addressing

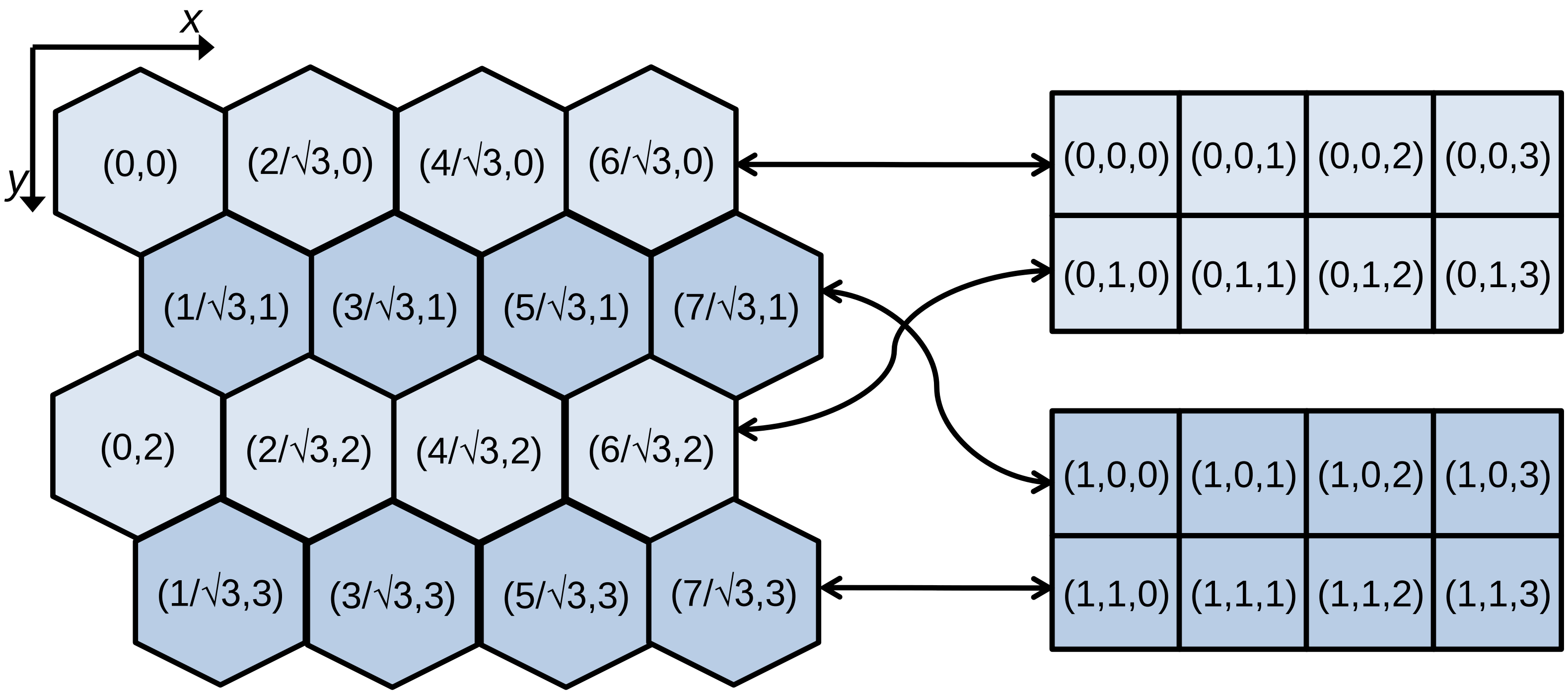


Her’s Approach



Spiral Addressing
(GBT, HIP, etc.)

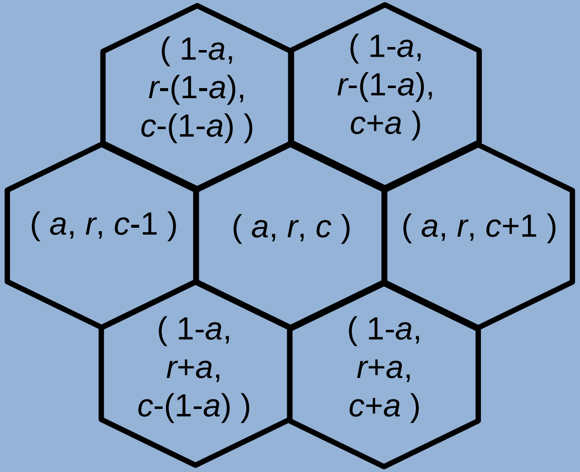
Array Set Addressing (ASA)



- ASA separates the hexagonal grid into two rectangular arrays
- A three coordinate system addresses the individual points on the grid – a binary array coordinate followed by the familiar row and column coordinates: $(a,r,c) \in \{0,1\} \times \mathbb{Z} \times \mathbb{Z}$

Hexagonal Neighbors

For any pixel (a,r,c), it's neighbors are:



- Finding a neighbor's address is an $O((\log N)^2)$ operation using spiral addressing
- No connectedness ambiguity – a neighbor is a neighbor

Distance Measures

Converting ASA to Cartesian is a simple matrix multiplication:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1/2 & 0 & 1 \\ \sqrt{3}/2 & \sqrt{3} & 0 \end{bmatrix} \begin{bmatrix} a \\ r \\ c \end{bmatrix} = \begin{bmatrix} (a/2 + c) \\ (\sqrt{3})(a/2 + r) \end{bmatrix}$$

Euclidean distance (on the image plane) between two points $\mathbf{p}_1 = (a_1, r_1, c_1)$ and $\mathbf{p}_2 = (a_2, r_2, c_2)$:

$$d(\mathbf{p}_1, \mathbf{p}_2) = \sqrt{\left(\left(\frac{a_1 - a_2}{2}\right) + (c_1 - c_2)\right)^2 + (3)\left(\left(\frac{a_1 - a_2}{2}\right) + (r_1 - r_2)\right)^2}$$

"City-Block" distance (on the image plane) between two points $\mathbf{p}_1 = (a_1, r_1, c_1)$ and $\mathbf{p}_2 = (a_2, r_2, c_2)$:

$$U = (c_1 - c_2) - (r_1 - r_2)$$

$$V = (a_1 - a_2) + (2)(r_1 - r_2)$$

$$d_6(\mathbf{p}_1, \mathbf{p}_2) = \begin{cases} |U| + |V| & \text{if U and V have the same sign} \\ \max(|U|, |V|) & \text{otherwise} \end{cases}$$

Vector Operations

$$\text{Let } \mathbf{p}_i = \begin{pmatrix} a_i \\ r_i \\ c_i \end{pmatrix} \in ASA$$

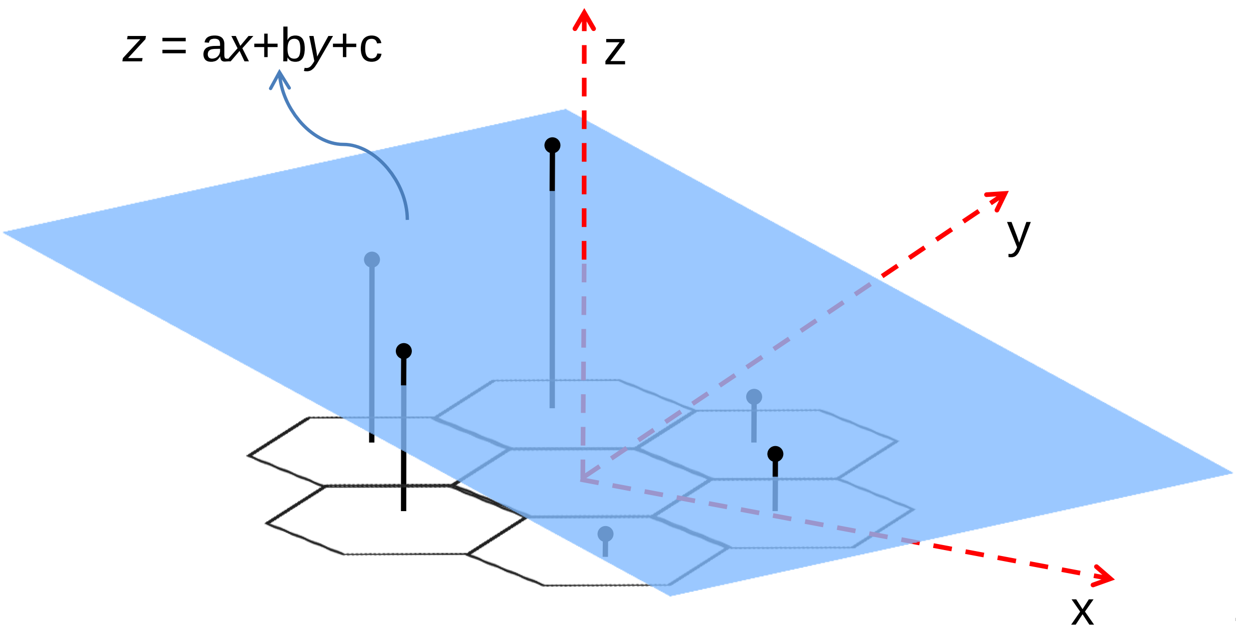
Operation	Definition
Addition	$\mathbf{p}_1 + \mathbf{p}_2 \equiv \begin{pmatrix} a_1 \oplus a_2 \\ r_1 + r_2 + (a_1 \wedge a_2) \\ c_1 + c_2 + (a_1 \wedge a_2) \end{pmatrix}$
Negation	$-\mathbf{p} \equiv \begin{pmatrix} a \\ -r - a \\ -c - a \end{pmatrix}$
Subtraction	$\mathbf{p}_1 - \mathbf{p}_2 \equiv \mathbf{p}_1 + (-\mathbf{p}_2)$
Scalar Multiplication	$k\mathbf{p} \equiv \begin{pmatrix} (ak) \bmod 2 \\ kr + (a)\lfloor k/2 \rfloor \\ kc + (a)\lfloor k/2 \rfloor \end{pmatrix}, \quad k \in \mathbb{N}$ and $-k\mathbf{p} \equiv k(-\mathbf{p})$

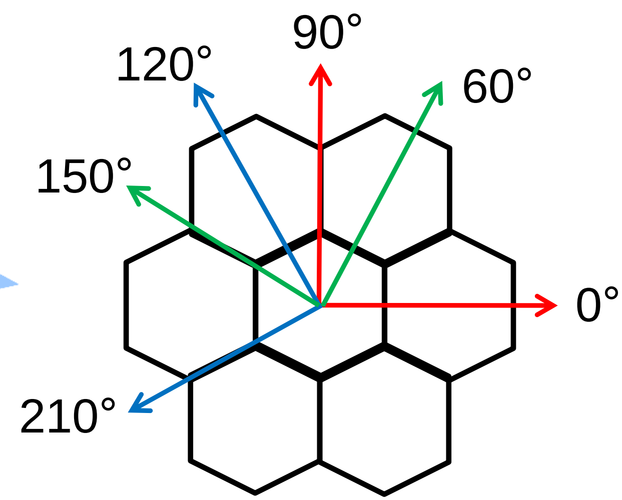
ASA is a Z-Module

ASA satisfies the 8 properties of a Z-module:

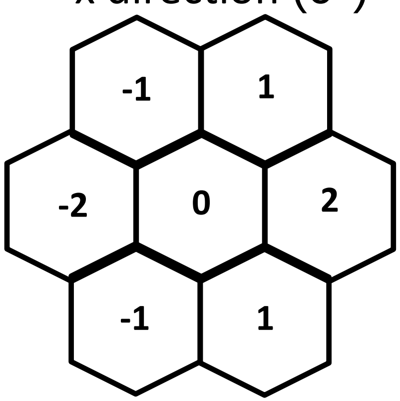
Property	Significance
Commutativity of addition	$\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{p}_2 + \mathbf{p}_1$
Associativity of addition	$\mathbf{p}_1 + (\mathbf{p}_2 + \mathbf{p}_3) = (\mathbf{p}_1 + \mathbf{p}_2) + \mathbf{p}_3$
Identity element of addition	$\exists \mathbf{0} \in \text{ASA}: \mathbf{p} + \mathbf{0} = \mathbf{p}, \forall \mathbf{p} \in \text{ASA}$
Inverse elements of addition	$\exists \mathbf{q} \in \text{ASA}: \mathbf{p} + \mathbf{q} = \mathbf{0}, \forall \mathbf{p} \in \text{ASA}$
Distributivity of scalar multiplication (wrt vector addition)	$k(\mathbf{p} + \mathbf{q}) = k\mathbf{p} + k\mathbf{q}$
Distributivity of scalar multiplication (wrt scalar addition)	$(k + j)\mathbf{p} = k\mathbf{p} + j\mathbf{p}$
Compatibility of scalar multiplication (with multiplication of scalars)	$k(j\mathbf{p}) = (kj)\mathbf{p}$
Identity element of scalar multiplication	$1\mathbf{p} = \mathbf{p}$

Gradient Estimation

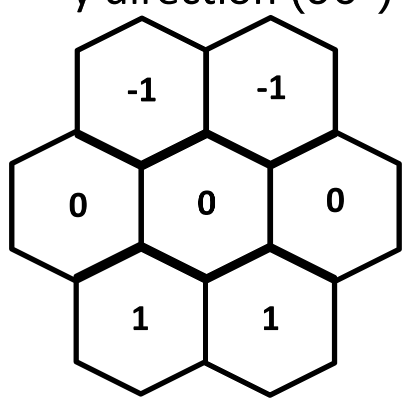




x direction (0°)

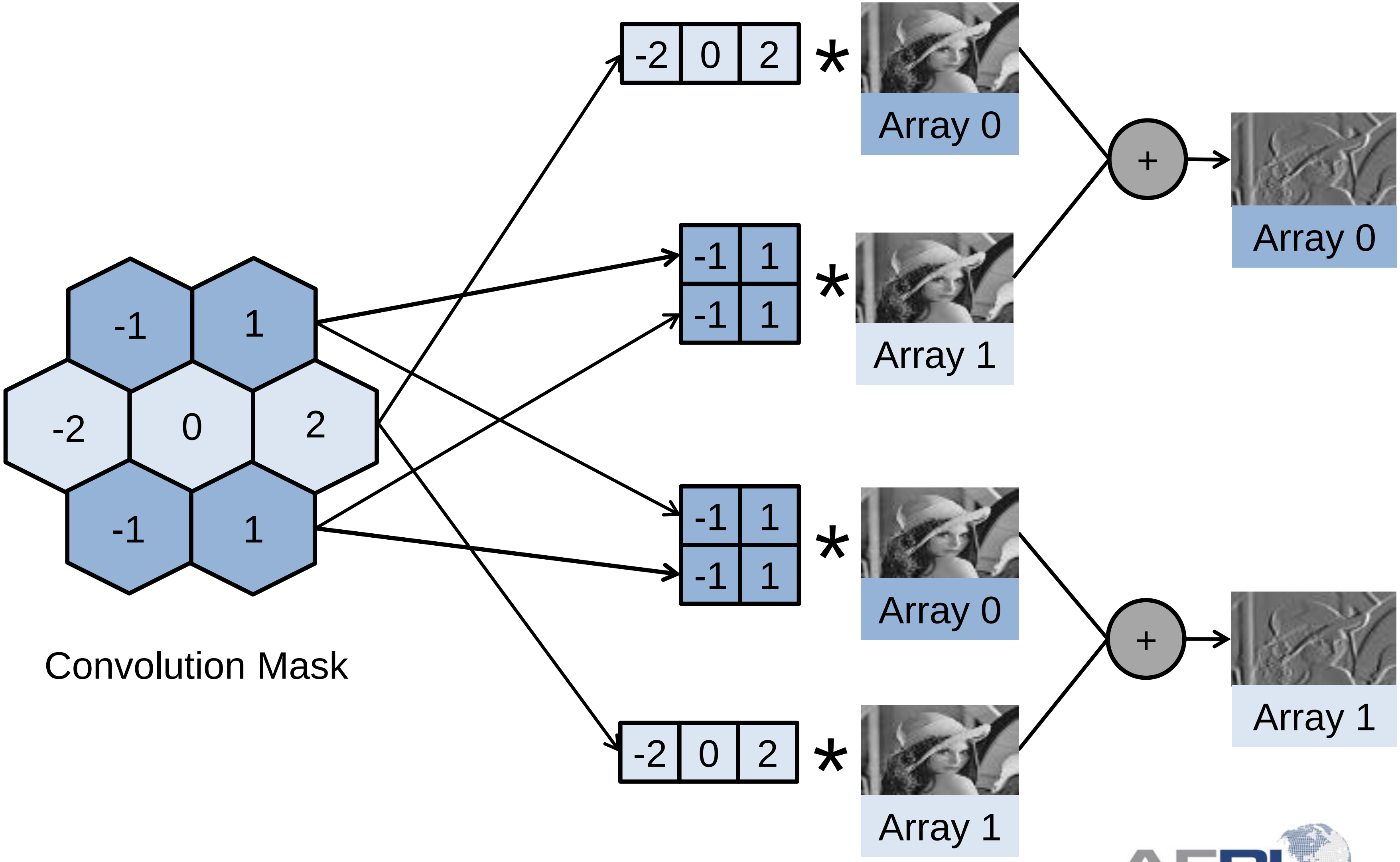


y direction (90°)



By symmetry, we can estimate gradients along 6 axes

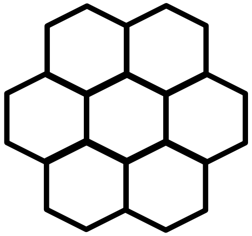
Performing Convolutions



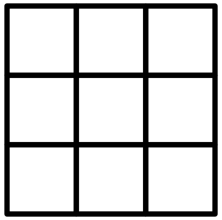
Convolution Complexity

Assumptions:

- Hexagonal and rectangular images are each M x N pixels
- Image borders are padded to allow each pixel to use the full convolution mask
- Let C_{ij} be the convolution of the i-array of the image with the j-array of the convolution mask



Hexagonal Neighborhood of 1st Nearest Neighbors (7 point mask)



Rectangular Neighborhood of 1st Nearest Neighbors (9 point mask)

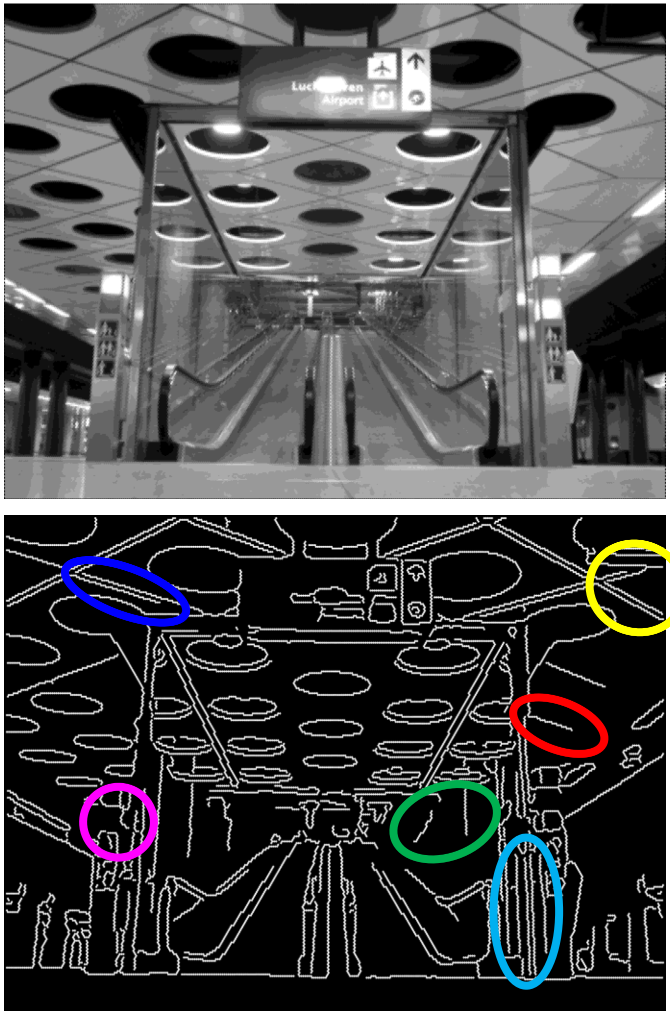
ASA convolution (7 point mask):

Step	Multiplications	Additions
Calculate C_{00}	(3)(M/2)(N)	(2)(M/2)(N)
Calculate C_{01}	(4)(M/2)(N)	(3)(M/2)(N)
Calculate C_{10}	(3)(M/2)(N)	(2)(M/2)(N)
Calculate C_{11}	(4)(M/2)(N)	(3)(M/2)(N)
Sum of C_{00} and C_{11}	0	(M/2)(N)
Sum of C_{01} and C_{10}	0	(M/2)(N)
TOTALS:	**7MN**	**6MN**

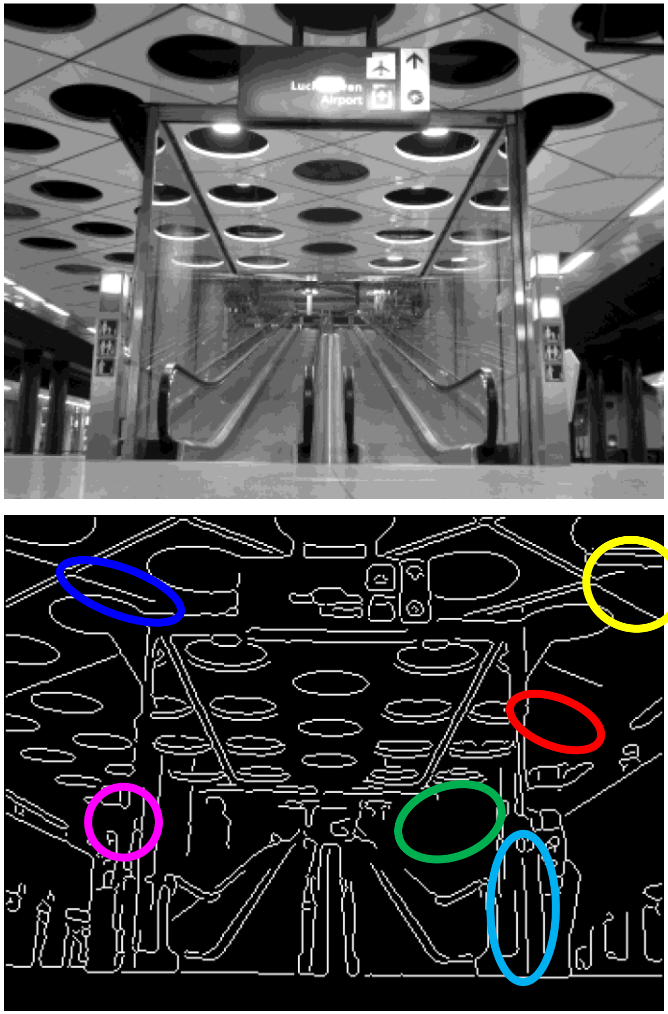
Rectangular convolution (9 point mask):

	Multiplications	Additions
TOTALS:	**9MN**	**8MN**

Canny Edge Detector

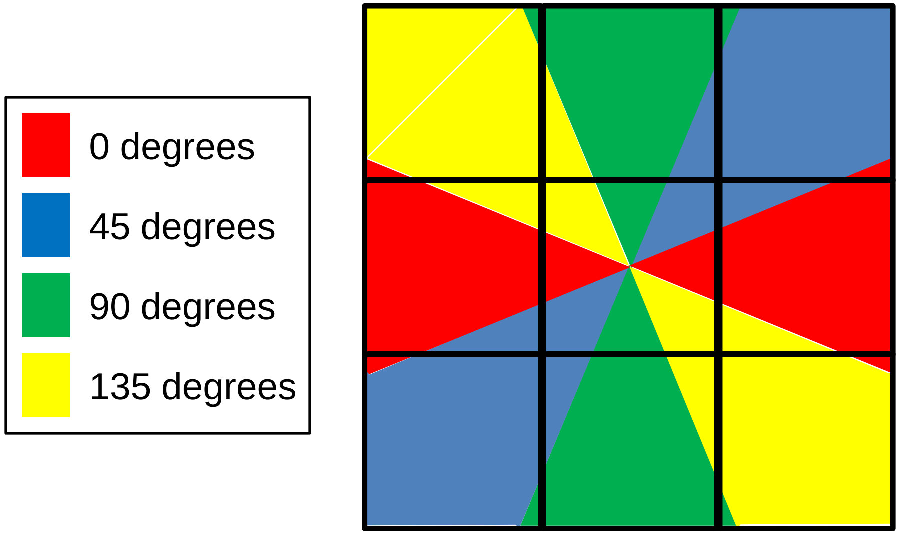


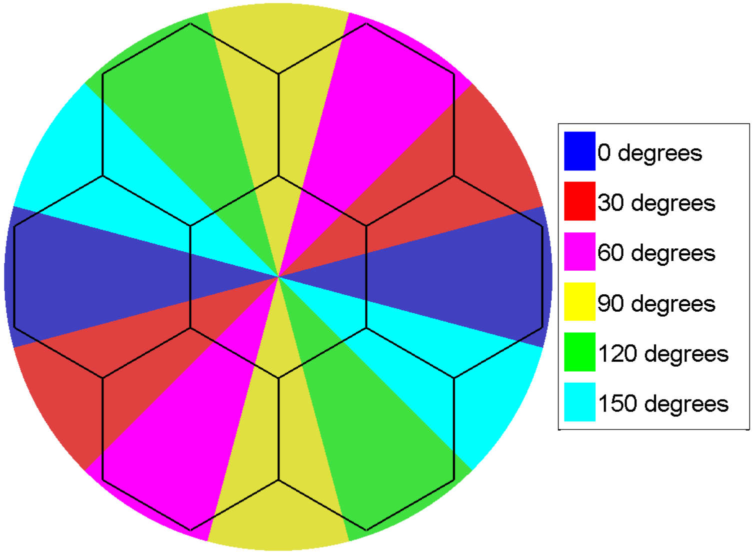
Hexagonally sampled



Rectangularly sampled

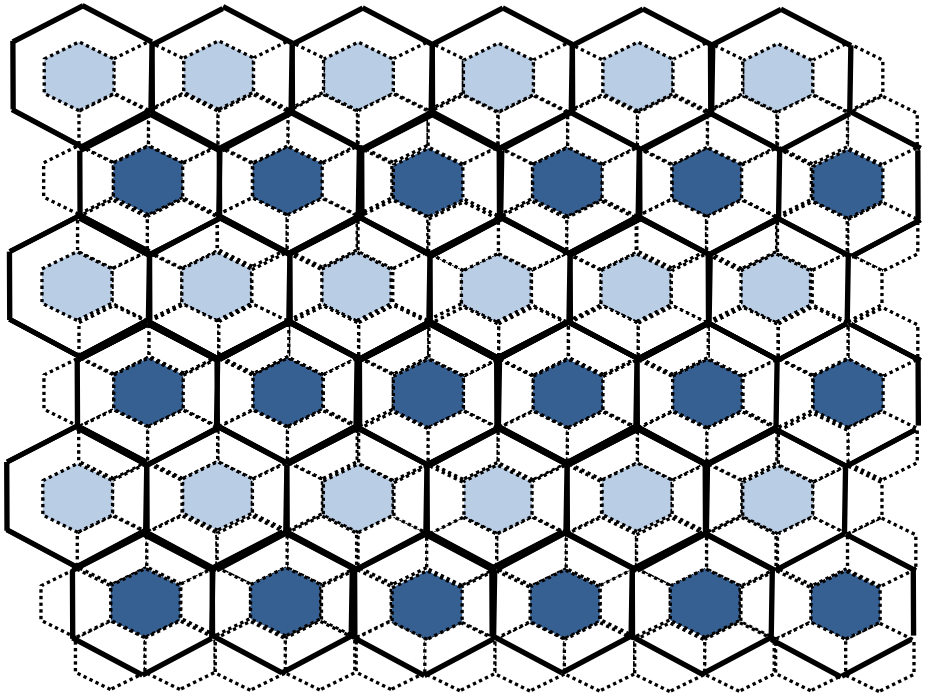
Angular Resolution



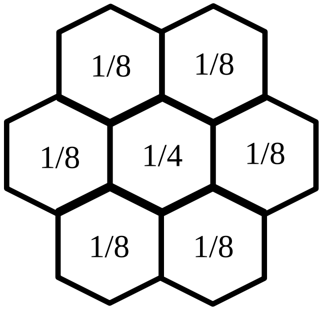


The increased angular resolution of the hexagonal grid may account for the increased performance of the Canny edge detector.

Downsampling



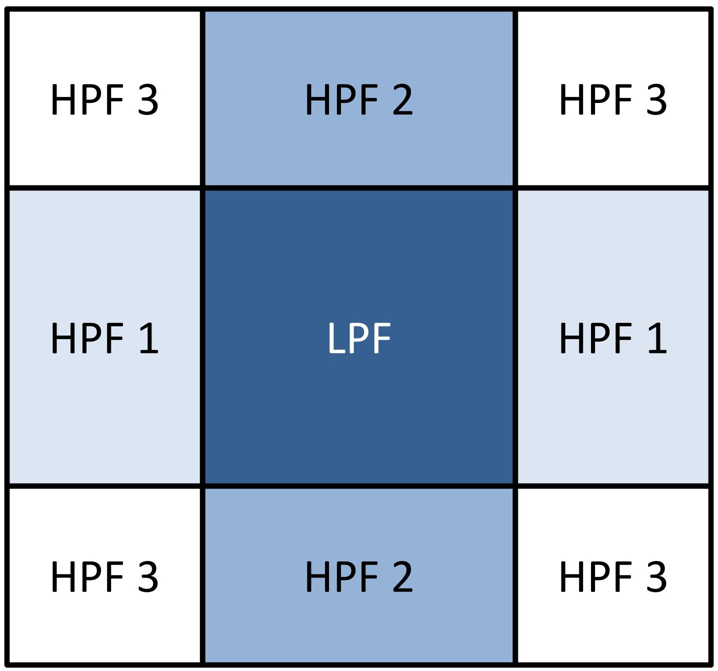
Anti-aliasing Mask



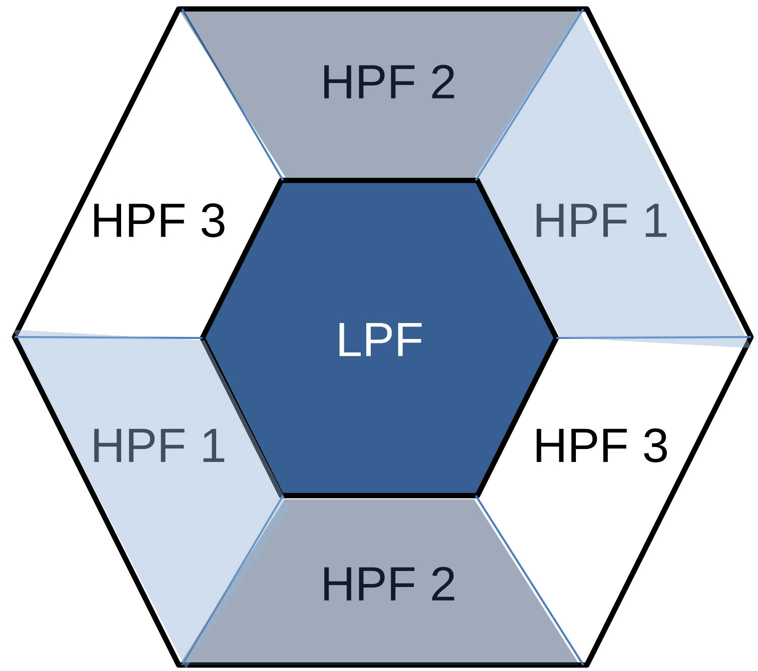
We want to use ½ of each of the neighboring pixels since they are shared with adjacent "superpixels". So we are averaging together (6)(1/2) + 1 = 4 pixels, resulting in the above averaging mask.

After convolving the image with the averaging mask, the light blue pixels form the downsampled 0-array and the dark blue pixels form the downsampled 1-array. The resulting arrays are 1/4 the size of the original arrays (i.e. (N/2) x N => (N/4) x (N/2)).

Wavelet HPF Advantage



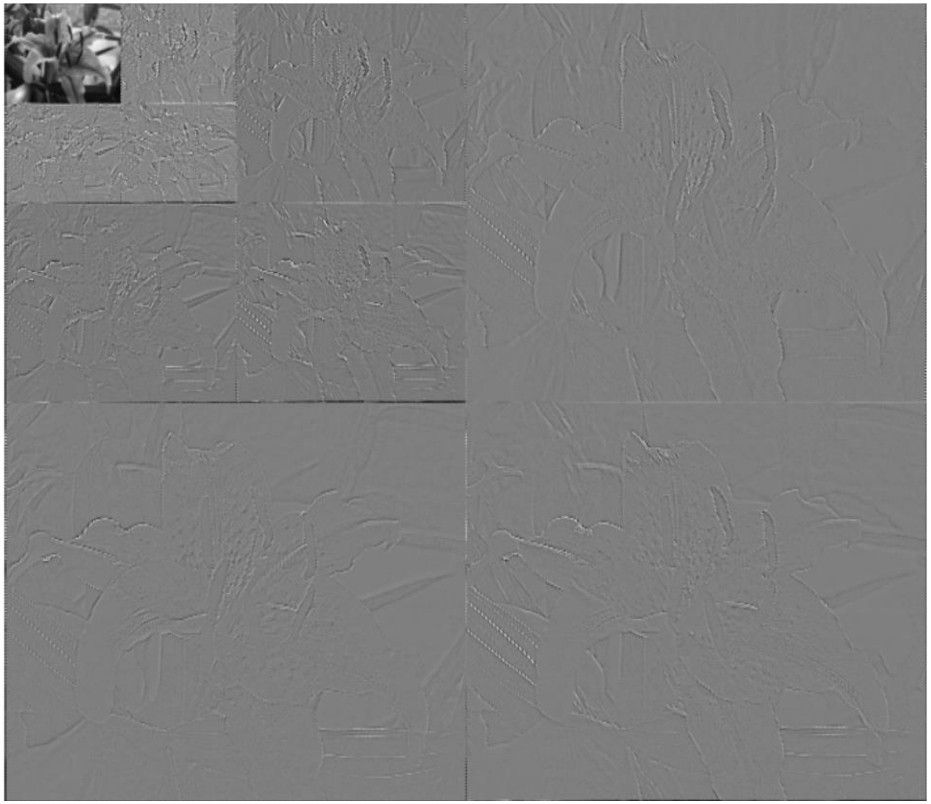
Rectangularly Sampled



Hexagonally Sampled

Idealized Frequency Domain Regions of Support

Perfect Reconstruction (PR) Example



ASA implementation of Allen PR wavelet, runtime = 0.5017 (0.0077) sec

Rect. implementation of CDF 9/7 wavelet, runtime = 0.5484 (0.008) sec

HDFT / HFFT

Mersereau's HDFT:

$$X(k_1,k_2) = \sum_{n_1}\sum_{n_2} x(n_1,n_2)\exp\left[-j\pi\left(\frac{1}{2N_1+N_2}(2n_1-n_2)(2k_1-k_2)+\frac{1}{N_2}(n_2k_2)\right)\right]$$

$$x(n_1,n_2) = \frac{1}{N_2(2N_1+N_2)}\sum_{k_1}\sum_{k_2} X(k_1,k_2)\exp\left[j\pi\left(\frac{1}{2N_1+N_2}(2n_1-n_2)(2k_1-k_2)+\frac{1}{N_2}(n_2k_2)\right)\right]$$

Mersereau encountered an "insurmountable difficulty" when attempting to develop a fast algorithm to compute the hexagonal DFT, due to the product of mixed coordinates in the exponential.

HDFT / HFFT (Cont.)

The HDFT in ASA becomes:

$$X(b,s,d)=\sum_a\sum_r\sum_c x(a,r,c)\exp\left[-j\pi\left(\frac{1}{2m}(a+2c)(b+2d)+\frac{1}{n}(a+2r)(b+2s)\right)\right]$$

$$x(a,r,c)=\frac{1}{2mn}\sum_b\sum_s\sum_d X(b,s,d)\exp\left[j\pi\left(\frac{1}{2m}(a+2c)(b+2d)+\frac{1}{n}(a+2r)(b+2s)\right)\right]$$

Column Coordinates	Row Coordinates

$$X(b,s,d)=\sum_a\sum_r\left[\sum_c x(a,r,c)\exp\left(\frac{-j\pi}{2m}(a+2c)(b+2d)\right)\right]\exp\left(\frac{-j\pi}{n}(a+2r)(b+2s)\right)$$

$$x(a,r,c)=\frac{1}{2mn}\sum_b\sum_s\left[\sum_d X(b,s,d)\exp\left(\frac{j\pi}{2m}(a+2c)(b+2d)\right)\right]\exp\left(\frac{j\pi}{n}(a+2r)(b+2s)\right)$$

The Fourier kernel is separable in ASA space!

Fourier Transform of Allen's Filter Bank

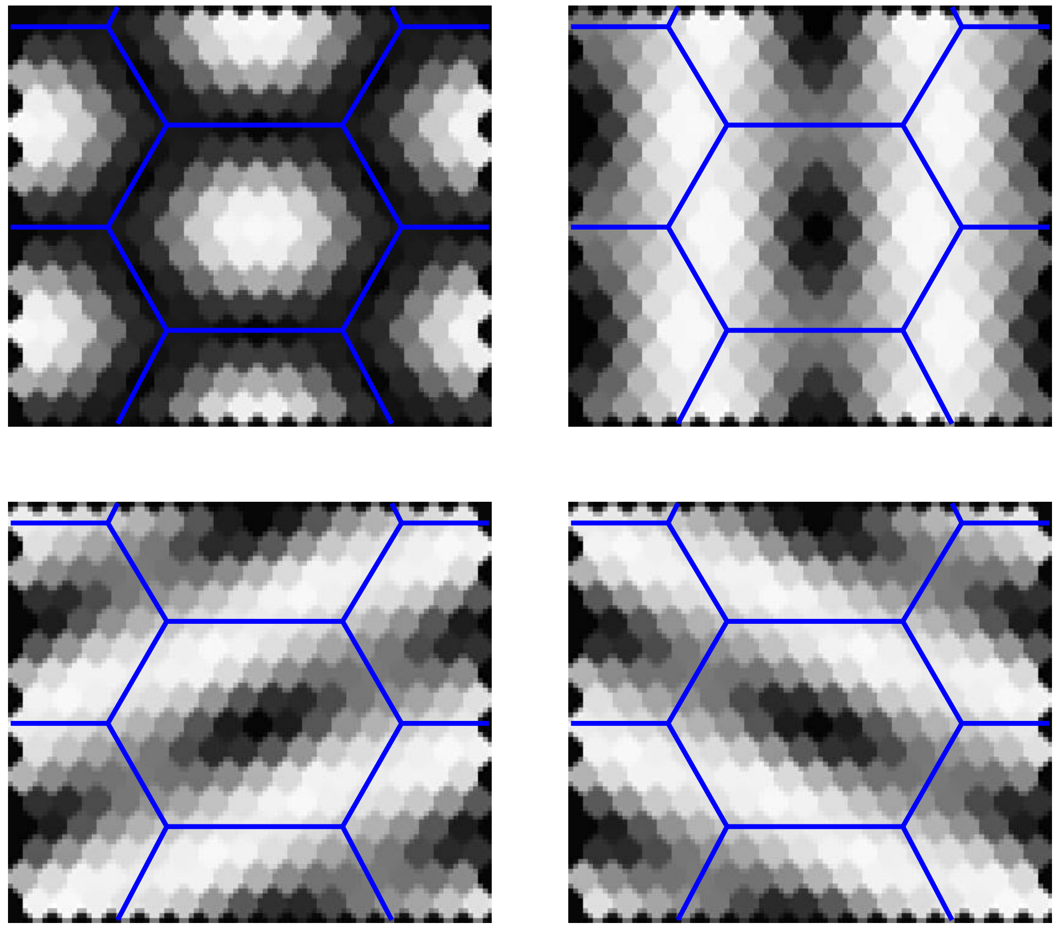
Low-Pass filter

$$
\begin{Bmatrix}
 & & & -14 & & & \\
 & & & & -84 & -14 & \\
-14 & & +276 & +259 & & & \\
 & -84 & +259 & +759 & +276 & & \\
-14 & & +276 & +259 & & & \\
 & & & & -84 & -14 & \\
 & & & -14 & & &
\end{Bmatrix}
$$

High-Pass filter

$$
\begin{Bmatrix}
 & & & +50 & & & \\
 & & & & +300 & +50 & \\
-2 & & +84 & -925 & & & \\
 & -12 & +37 & +231 & +84 & & \\
-2 & & +84 & +37 & & & \\
 & & & & -12 & -2 & \\
 & & & -2 & & &
\end{Bmatrix}
$$

The values given are *exact*. (They must be divided by 1014 to achieve normalization.) The other two filters can be visualized by rotating the High-pass filter 120° and 240°.



J. D. Allen, "Perfect reconstruction filter banks for the hexagonal grid," in *Proc. 5th Int. Conf. Information, Communications. and Signal Processing*, Dec. 2005, pp. 73–76.

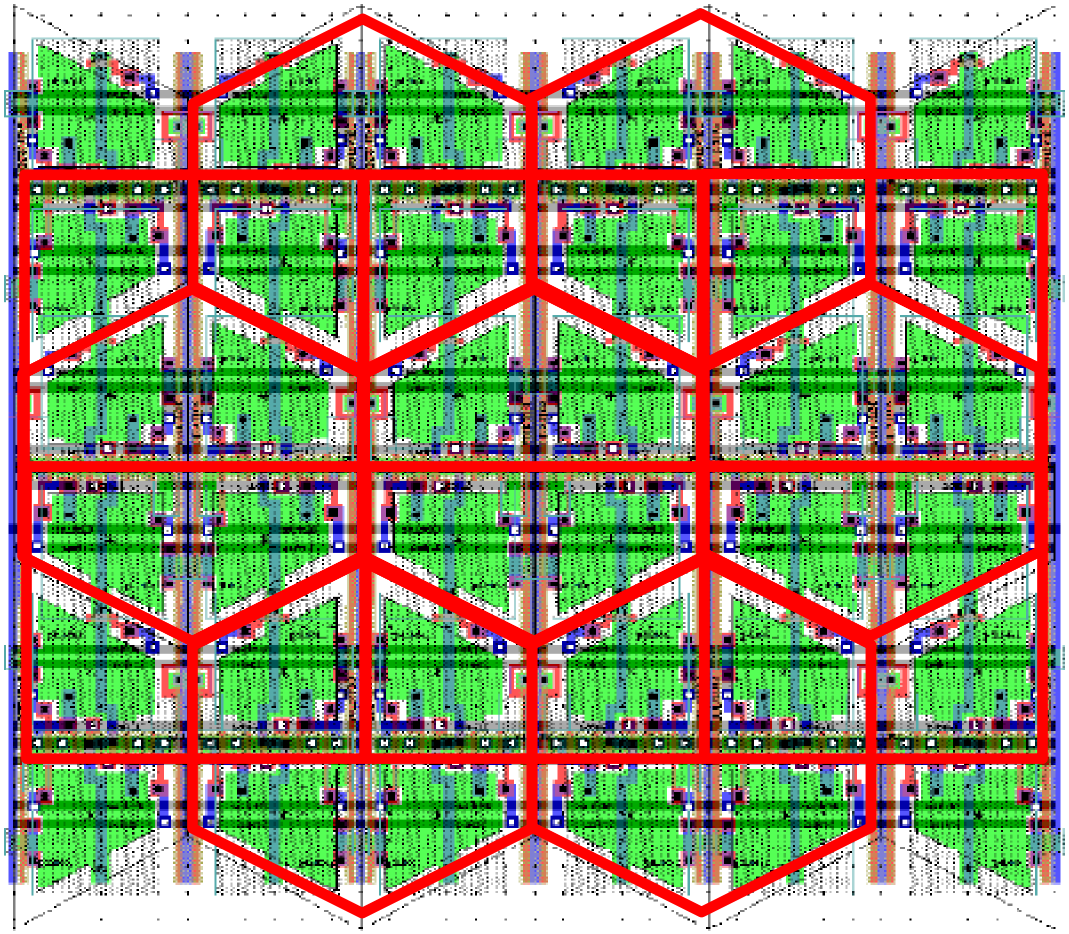
ASA vs. HIP

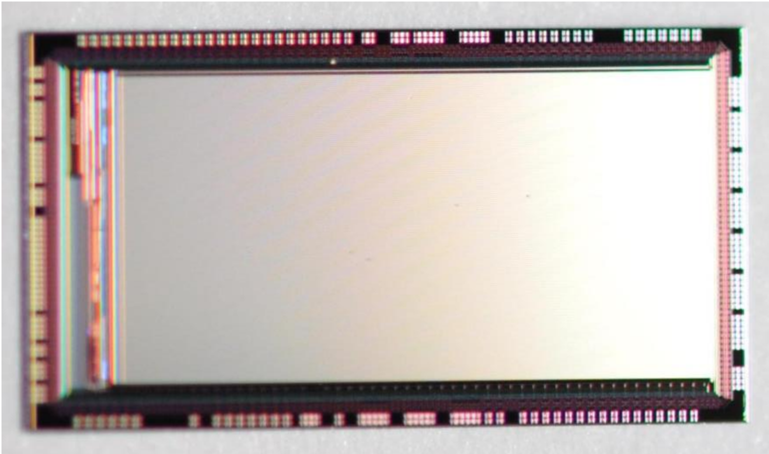
Operation	HIP	ASA	Ratio
Address (Vector) Addition	23.85 (3.15)	2.11 (0.97)	**11.28**
Address (Vector) Subtraction	33.98 (3.56)	2.56 (0.47)	**13.28**
Scalar Multiplication	6652.08 (4076.89)	3.73 (0.73)	**1782.20**
Calculate Euclidean Distance	15.83 (2.43)	2.73 (0.56)	**5.79**
Calculate 6 Nearest Neighbor Addresses	118.94 (10.49)	3.31 (0.75)	**35.89**
Convert From Cartesian	9189.68 (3784.79)	4.48 (1.13)	**2052.31**

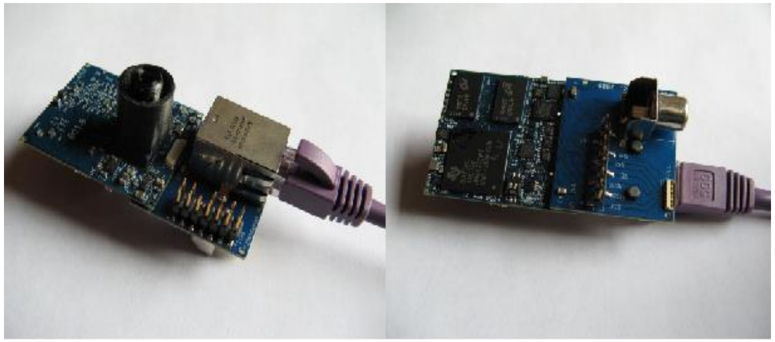
Each result is the mean of 10,000 operations on randomly selected addresses (μs, mean (std))

Operation	HIP	ASA
Address (Vector) Addition / Subtraction	$O((\log N)^2)$	O(1)
Scalar Multiplication	$O(N(\log N)^2)$	O(1)
Calculate Euclidean Distance	$O(\log N)$	O(1)
Calculate 6 Nearest Neighbor Addresses	$O((\log N)^2)$	O(1)
Convert From Cartesian	$O(N(\log N)^2)$	O(1)

Hex-Rect Imager

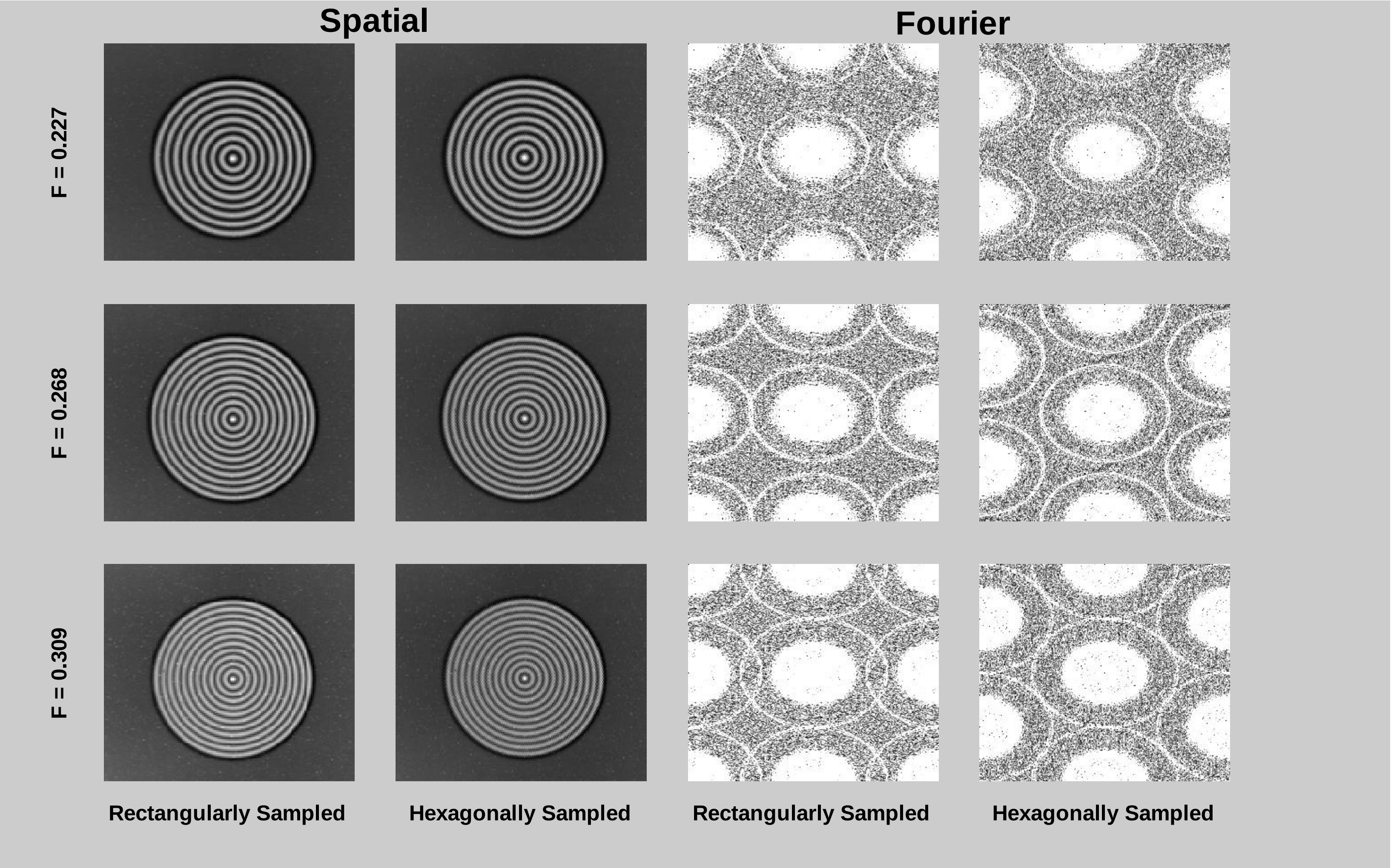






Developed by Centeye, Inc.

Experiment Results



$$0.268/0.309 \approx 0.867 \approx (\sqrt{3})/2 \approx 0.866$$

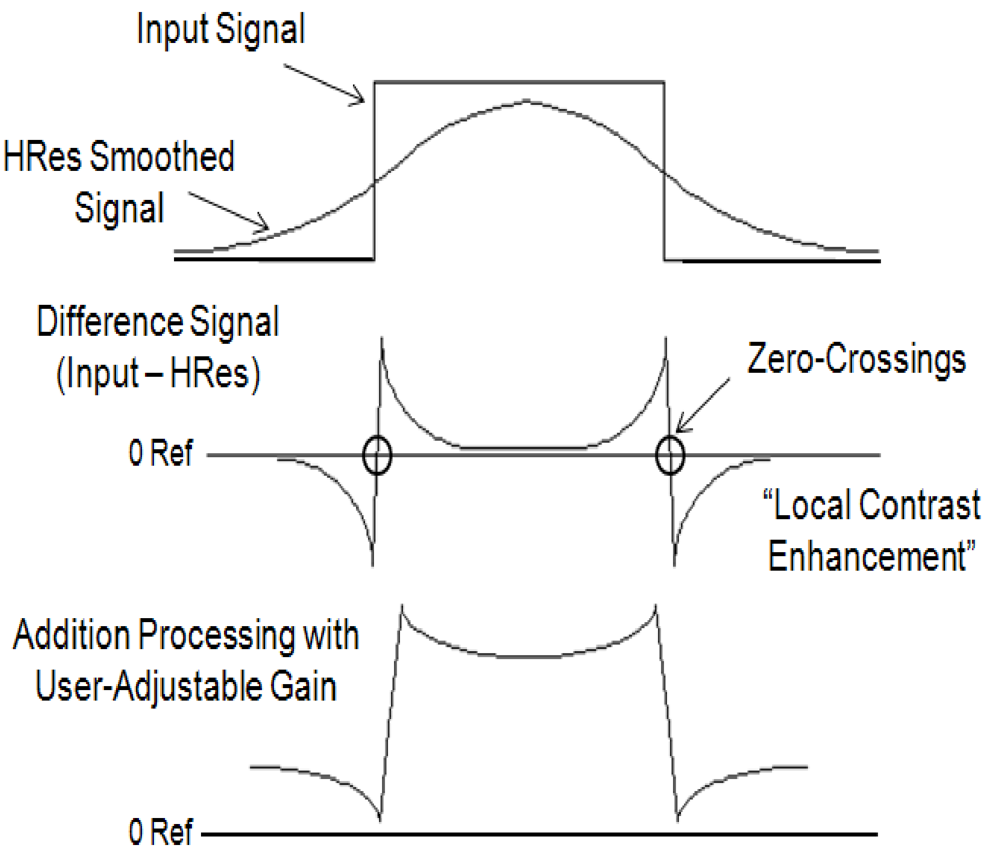
Conclusion

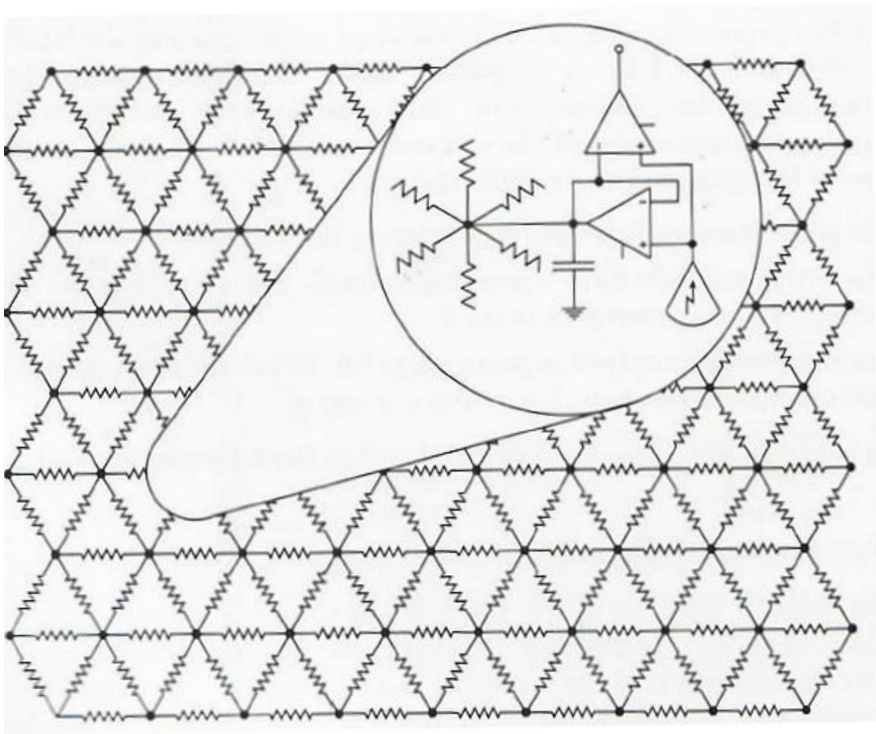
- There are several advantages to sampling digital images hexagonally rather than rectangularly
- ASA is tri-coordinate system for addressing a hexagonal grid that provides support for efficient image processing
- Efficient ASA methods were shown for gradient estimation, convolution, downsampling, wavelet decomposition, and hexagonal DFT
- The Hex-Rect imager can be used to quantitatively compare hexagonal and rectangular sampling

Questions?

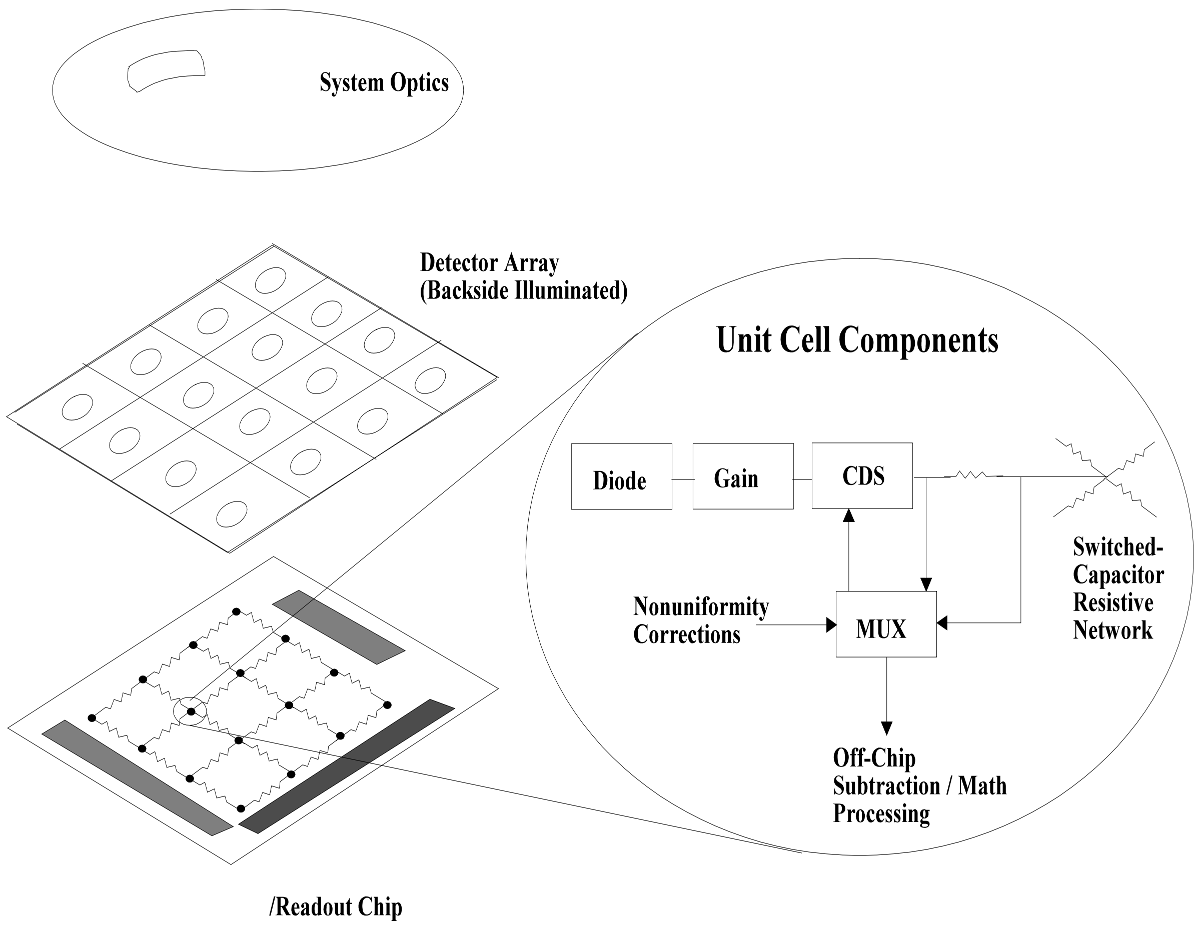
Backup Slides Follow

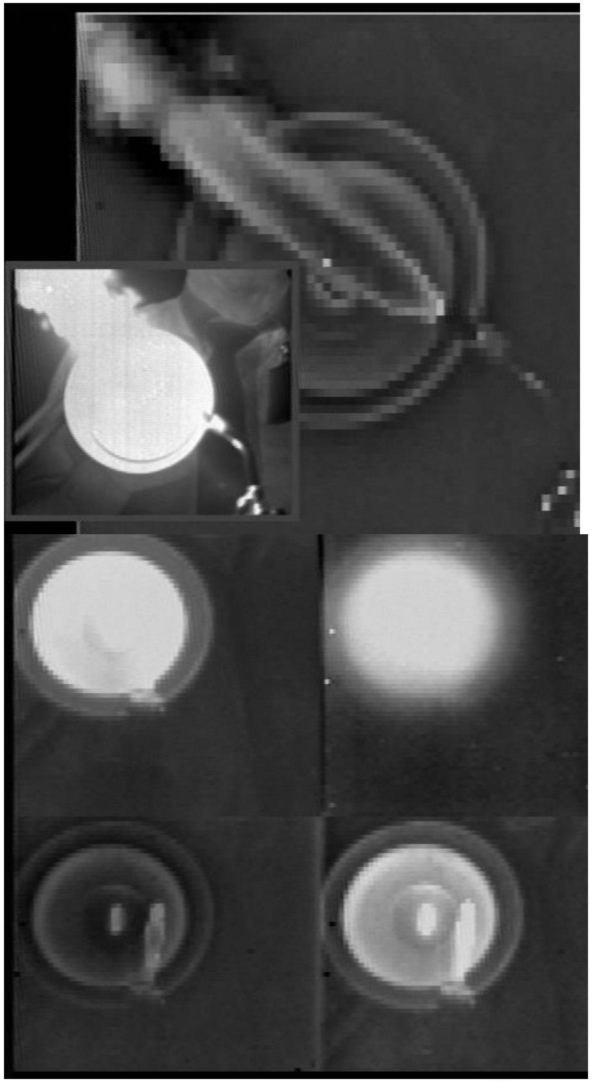
On-FPA Processing with Difference of Gaussians





Neuromorphic Infrared Sensor (NIFS)

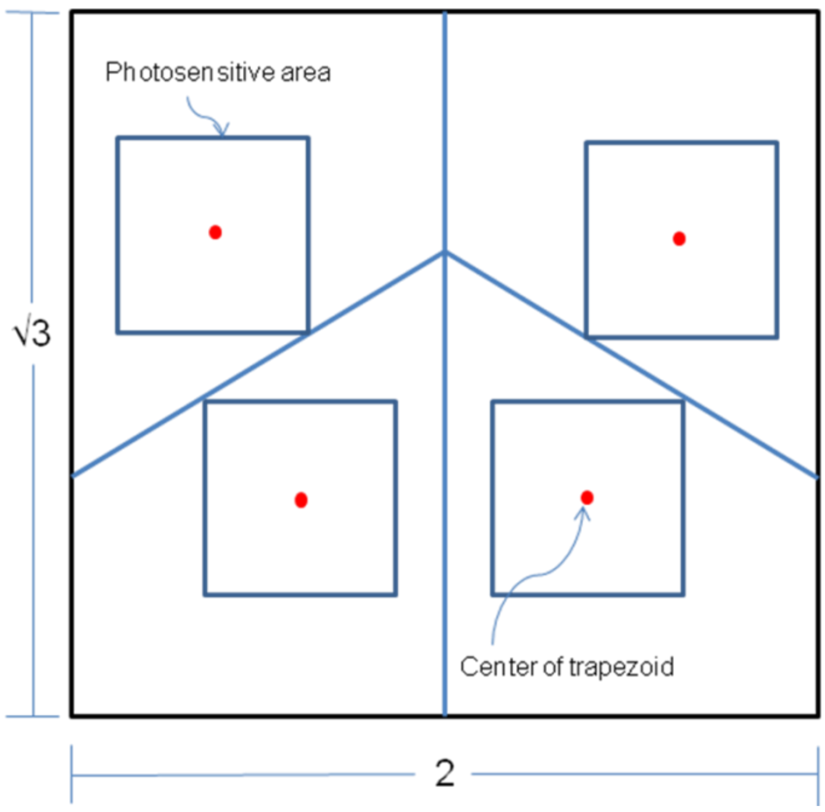


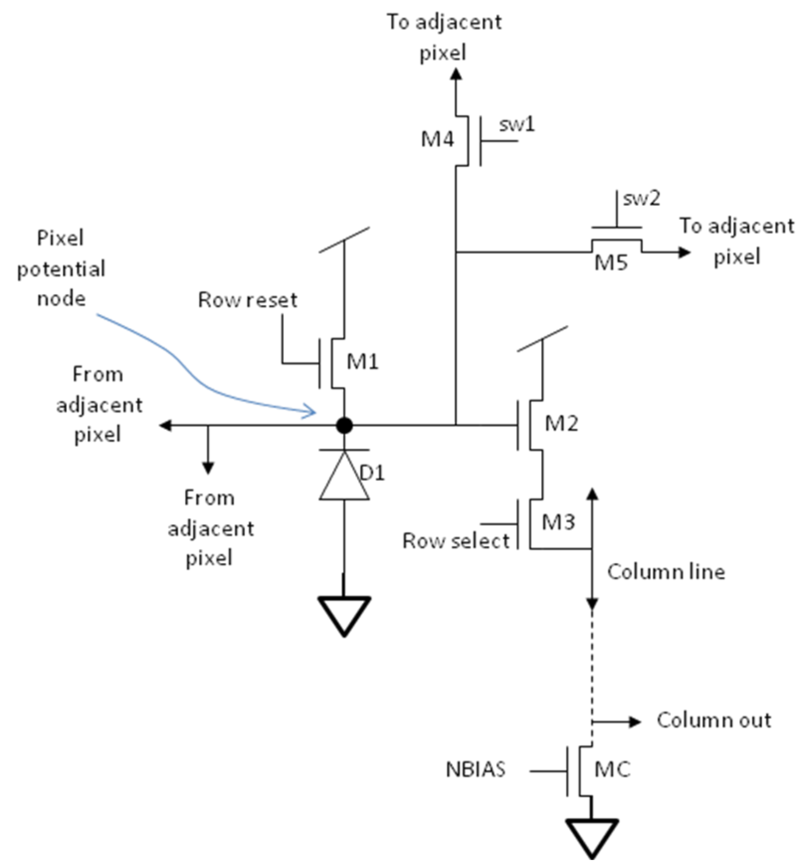


Hexagonal Imagers

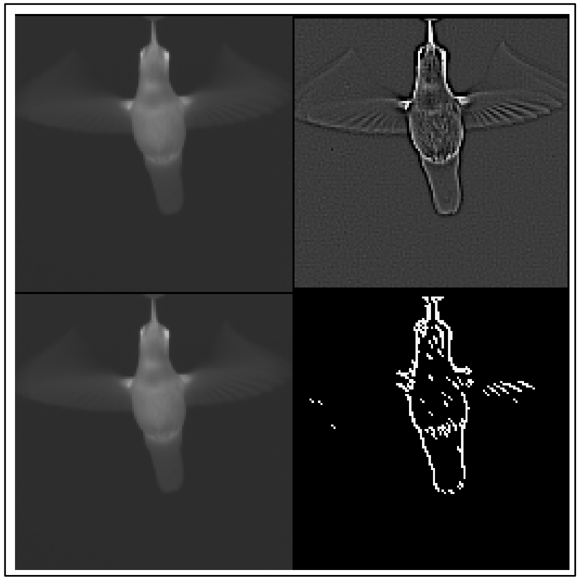
- Carver Mead’s Silicon Retina
- Hauschild’s Prototype
- Gaber’s Design
- Centeye’s Hex-Rect
- More to come…

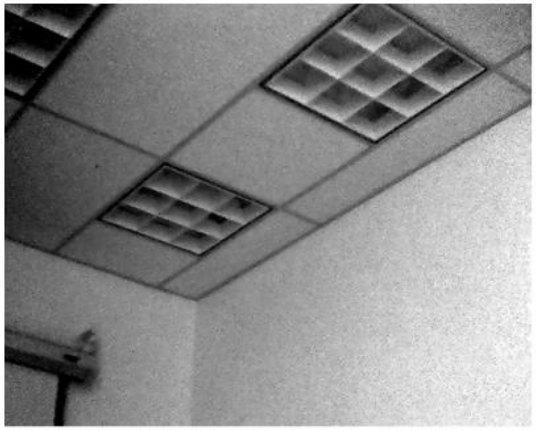
Hex-Rect Unit Cells





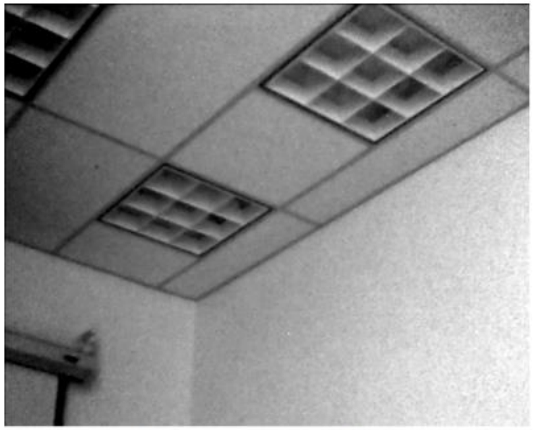
Examples





Input Data, Salt/Pepper Noise

After 3x3 Median Filtering



After Anisotropic Filtering

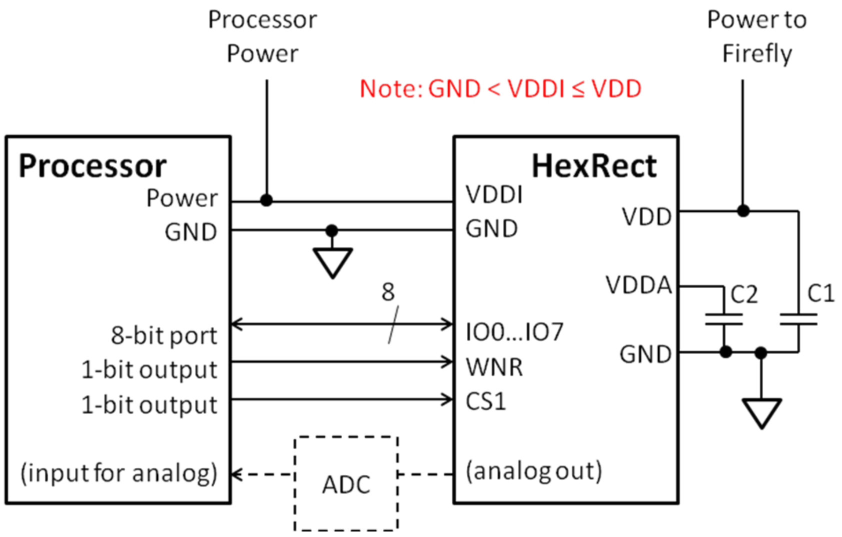
Input Scene

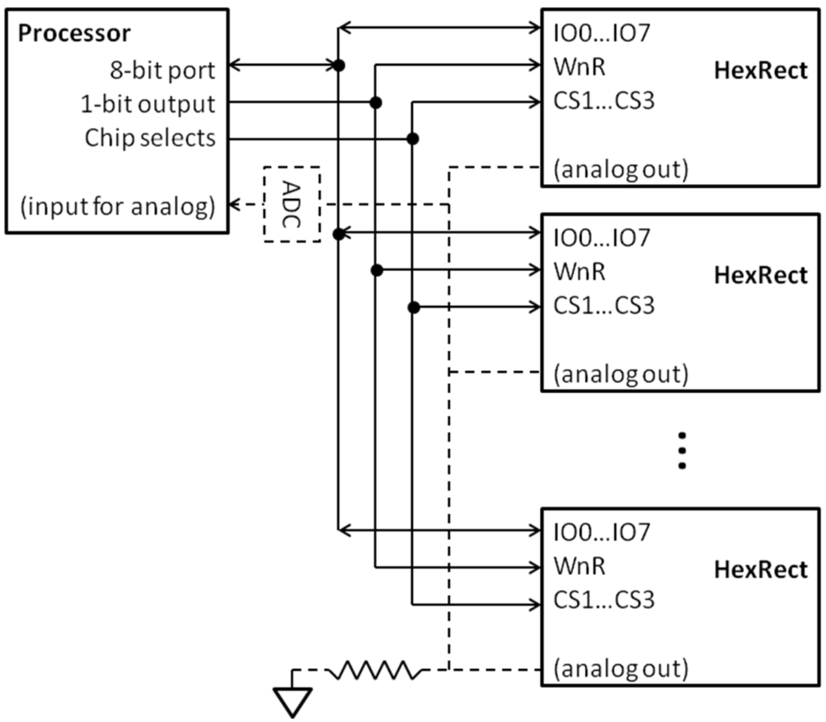
After DoG, zero-crossing

Hex-Rect Specs

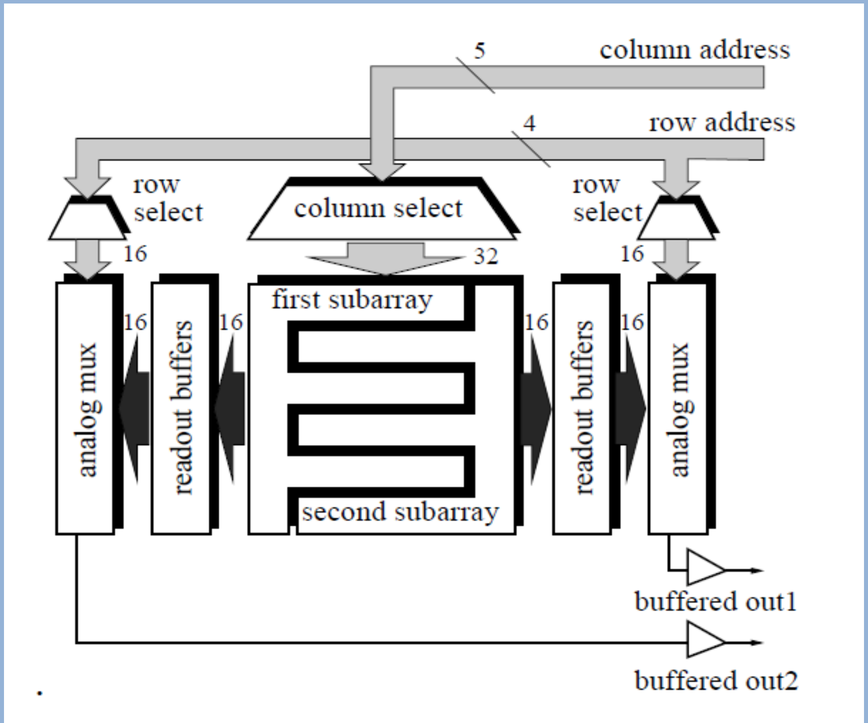
Drawn chip size	6.1mm x 11.1mm
Focal plane size	4.7mm x 9.2mm
Focal plane resolution	Raw trapezoid pixels: 304 x 512 Hexagonal array: 152 x 255 (even rows have 256 hex pixels) Rectangular array: 151 x 256
Pixel type	3-transistor active pixel, with support for both logarithmic response and linear response
Pixel pitch	18 microns wide by 15.6 microns high for raw pixels
Post-pixel circuitry	8-bit flash ADC
Interface	PIO12B parallel interface: 8 bidirectional digital, 2 digital in, 1 analog out 12-bit command bus in two 6-bit words 8-bit digital out Optional 3 input chip select Optional analog out Alternative 12 bit input / 8 bit output parallel interface
Process	ON-Semi C5N 3 metal 2 poly 0.5 micron process
Chip operating voltage	4V to 5V preferred
Digital input 0/1 threshold	About 0.95V
Voltage regulation	On-chip voltage regulator for analog circuits and bias generators

Hex-Rect Interface





IR Readout Considerations



From R. Hauschild et al., "A CMOS Optical Sensor System Performing Image Sampling on a Hexagonal Grid" in *Proc. 22nd European Solid-State Circuits Conf.*, 304-307, 1996.

- Typical readouts (ROICs) are designed to read out rectangular arrays
- Slight modifications should allow hexagonally sampled images to be read out into the ASA data structure
- Images from the prototype on the right could have been processed directly using ASA

IR Detector Materials and Bump Bonding Considerations

- Indium Gallium Arsenide (InGaAs)
  - NIR (0.4 – 1.6 um), Uncooled or slightly cooled
- Indium Antimonide (InSb)
  - MWIR (3-5 um), Cooled to 77K
- Mercury Cadmium Telluride (HgCdTe)
  - MWIR (3-5 um), Cooled to 77K or 120K+
  - LWIR (8-12 um), Cooled to 77K or 120K+
- QWIP
- Strained Layer Superlattice

Retesselating the Image

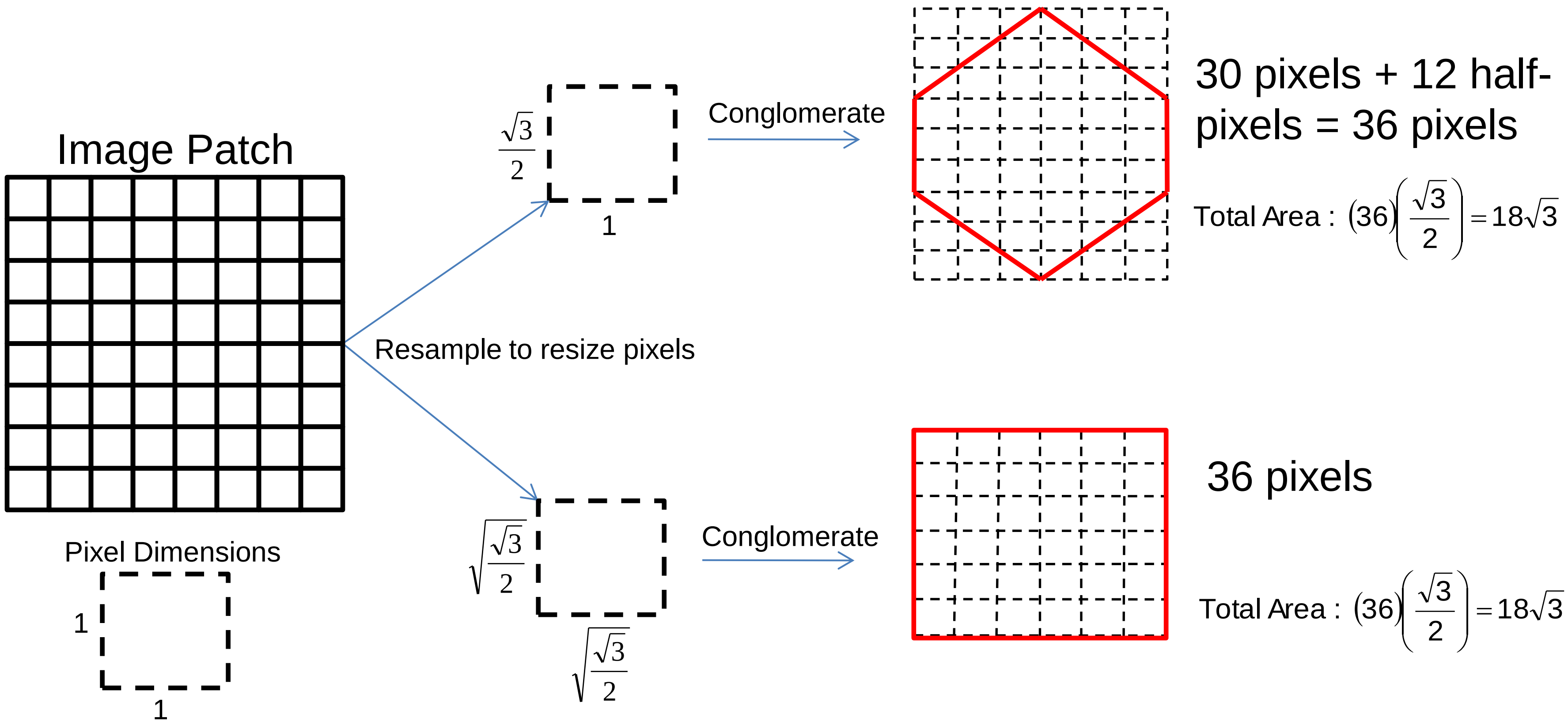


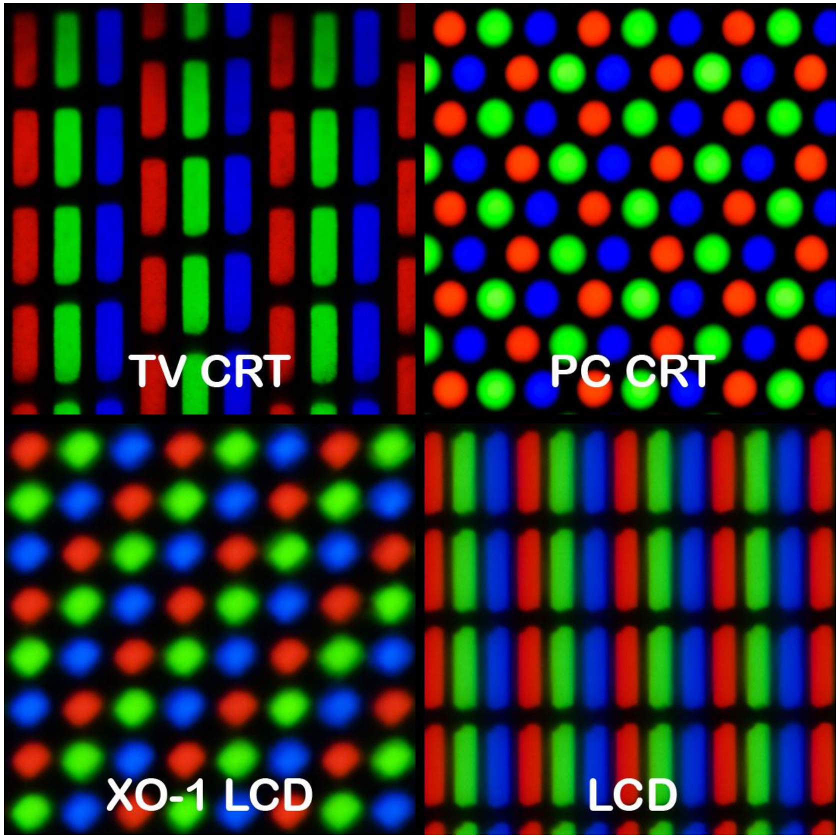
Image Formation Results

Original Image

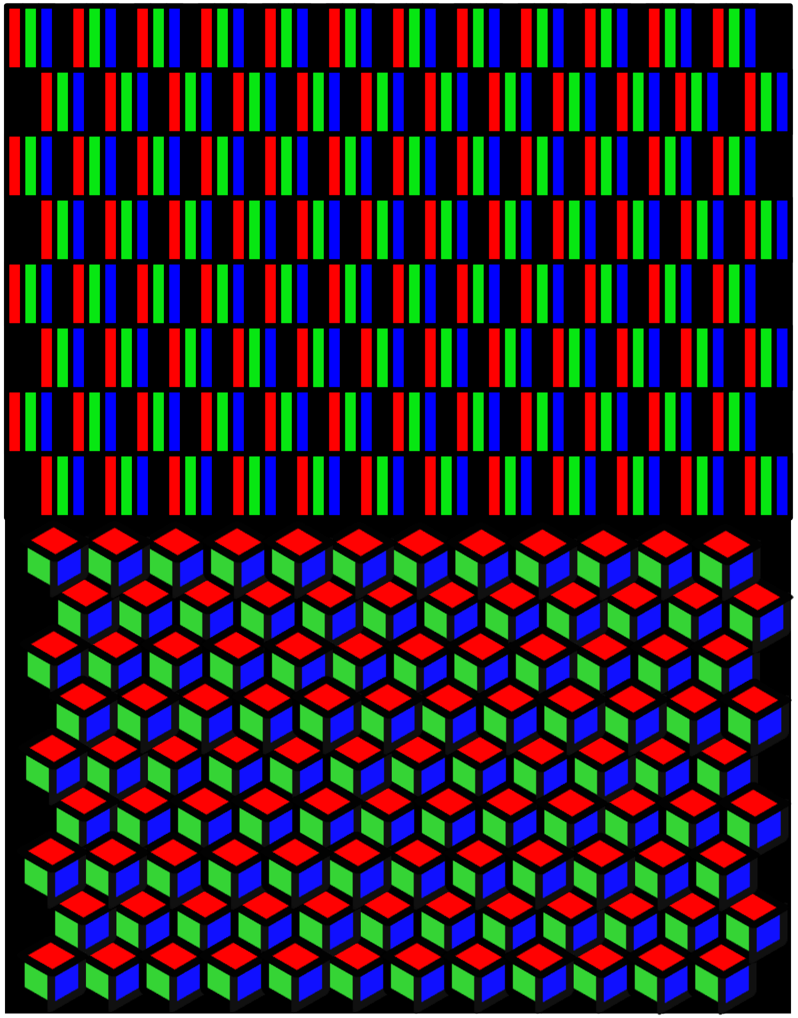
Hexagonally Sampled

Rectangularly Sampled

Pixel Geometries



"Pixel Geometries", P. Halasz
Reproduced from:
http://commons.wikimedia.org/wiki/File:Pixel_geometry_01_Pengo.jpg



Hexagonal RGB Designs

ASA Storage

Use memory addresses as indices:

Assume an N x 2^j ASA image and a 32 bit address space

Column index = j

Row index = $\text{ceil}(\log_2(N/2))$ bits = m

Array index = 1 bit

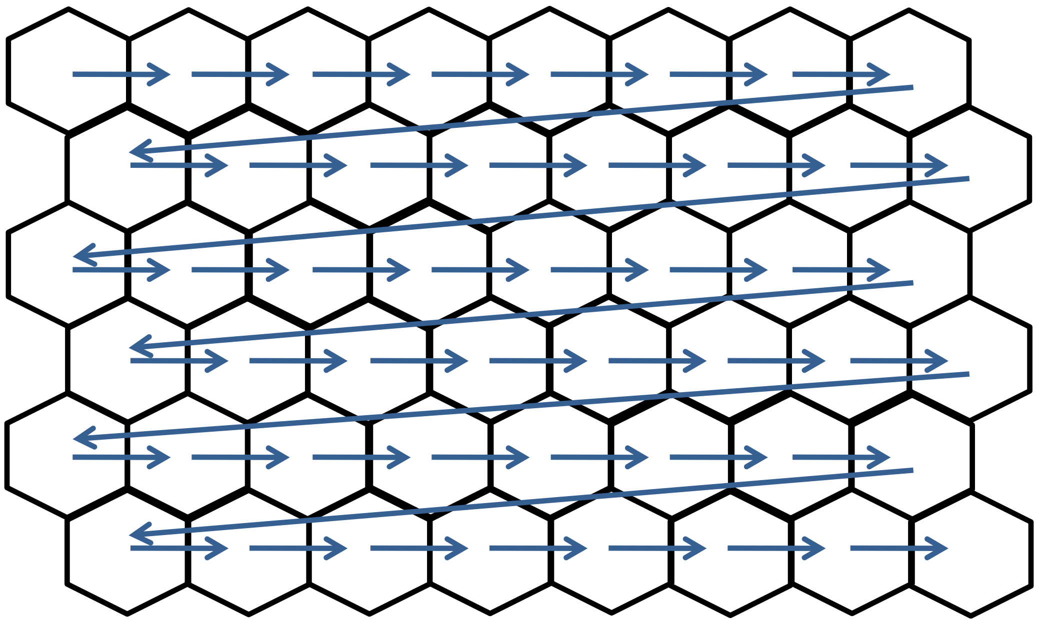
Base address = 32-(j+m+1)

For example, a 443x512 format provides a 4:3 aspect ratio (in Cartesian space) and the address format is

XXXXXXXXXXXXXXRRRRRRRACCCCCCCCC

Base Row Array Column

Yields row-major order storage



Converting ASA to Cartesian

For a regular hexagonal grid described by

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} d & d/2 \\ 0 & d\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

where x and y are Cartesian coordinates, n_1 and n_2 are integers (oblique coordinates), the conversion from ASA to Cartesian coordinates is a simple matrix multiplication:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} d/2 & 0 & d \\ d\sqrt{3}/2 & d\sqrt{3} & 0 \end{bmatrix} \begin{bmatrix} a \\ r \\ c \end{bmatrix} = \begin{bmatrix} (d)(a/2+c) \\ (d\sqrt{3})(a/2+r) \end{bmatrix}$$

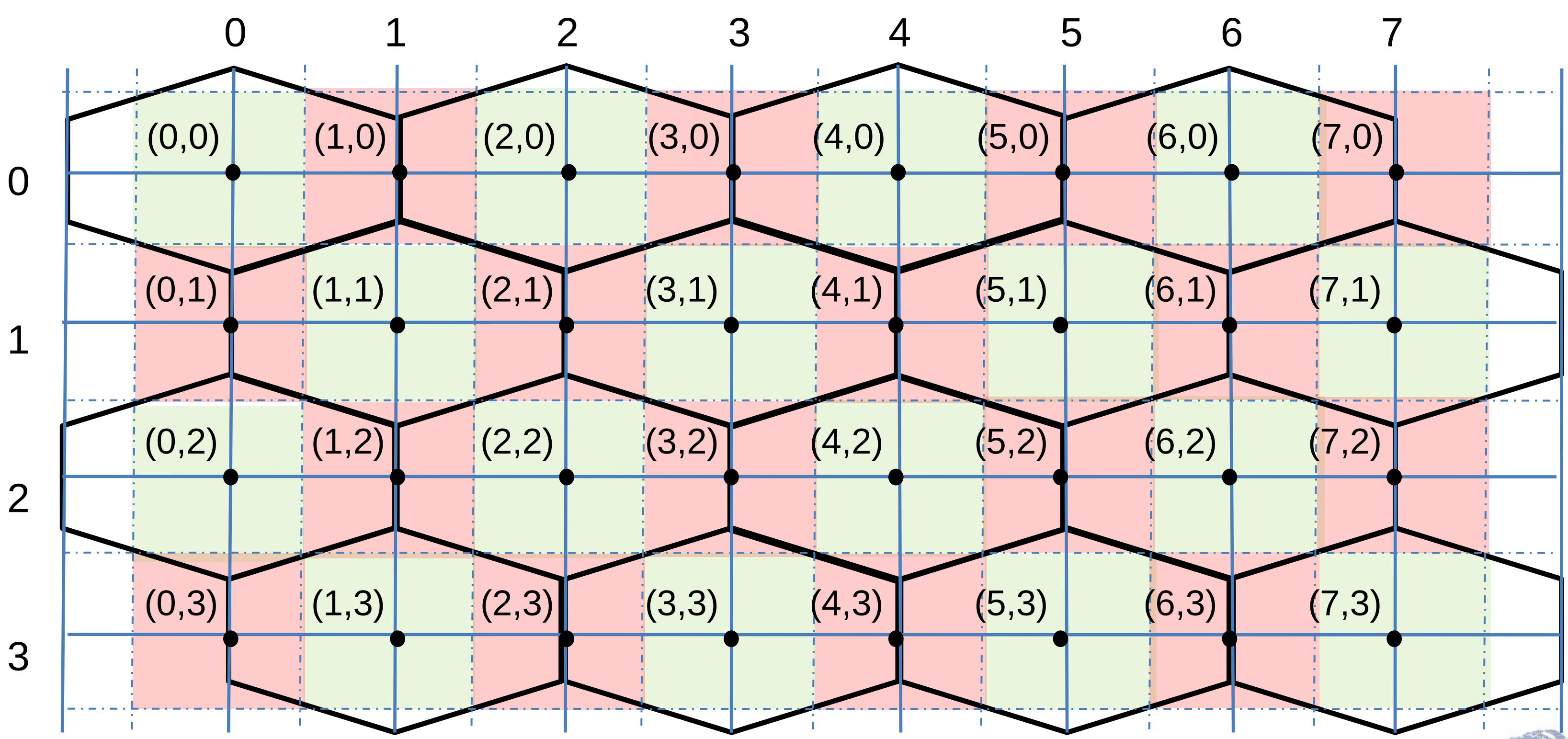
The parameter d is the distance between any two adjacent grid points. Assume that d=1 for the remainder of the presentation.

Converting Cartesian to ASA

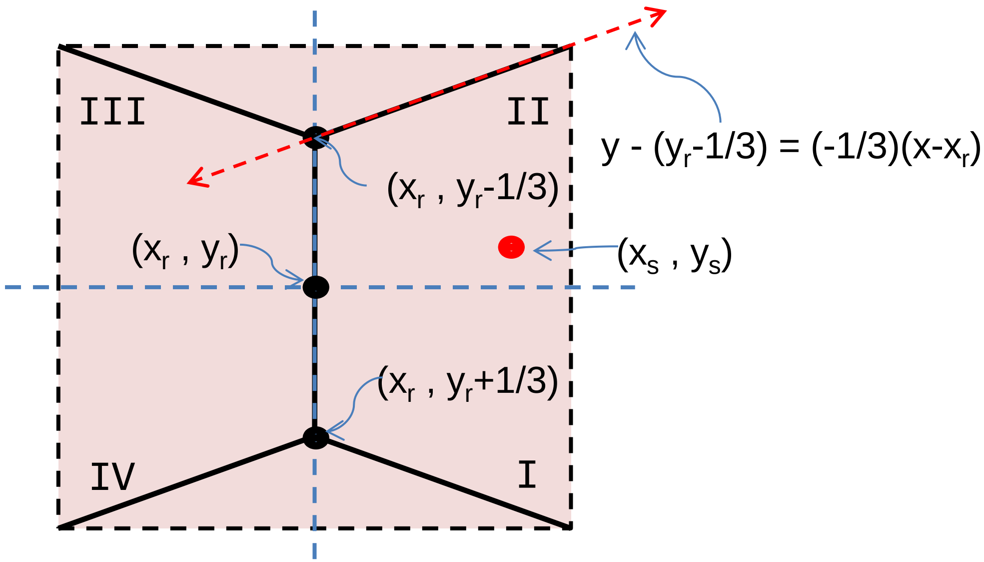
Convert the Cartesian coordinates (x,y) into integers (x_r,y_r) by first scaling each dimension, then rounding to the nearest integer:

$$x_s = 2x \qquad x_r = \text{round}(x_s)$$

$$y_s = \frac{2y}{\sqrt{3}} \qquad y_r = \text{round}(y_s)$$



Converting Cartesian to ASA (Cont.)



- Determine which quadrant (x_s,y_s) is in by comparing to (x_r,y_r)
- Using the known point and slope determine if (x_s,y_s) is above or below the line
- Adjust (x_r,y_r) to correct hexagon center
- Convert (x_r,y_r) to ASA using:

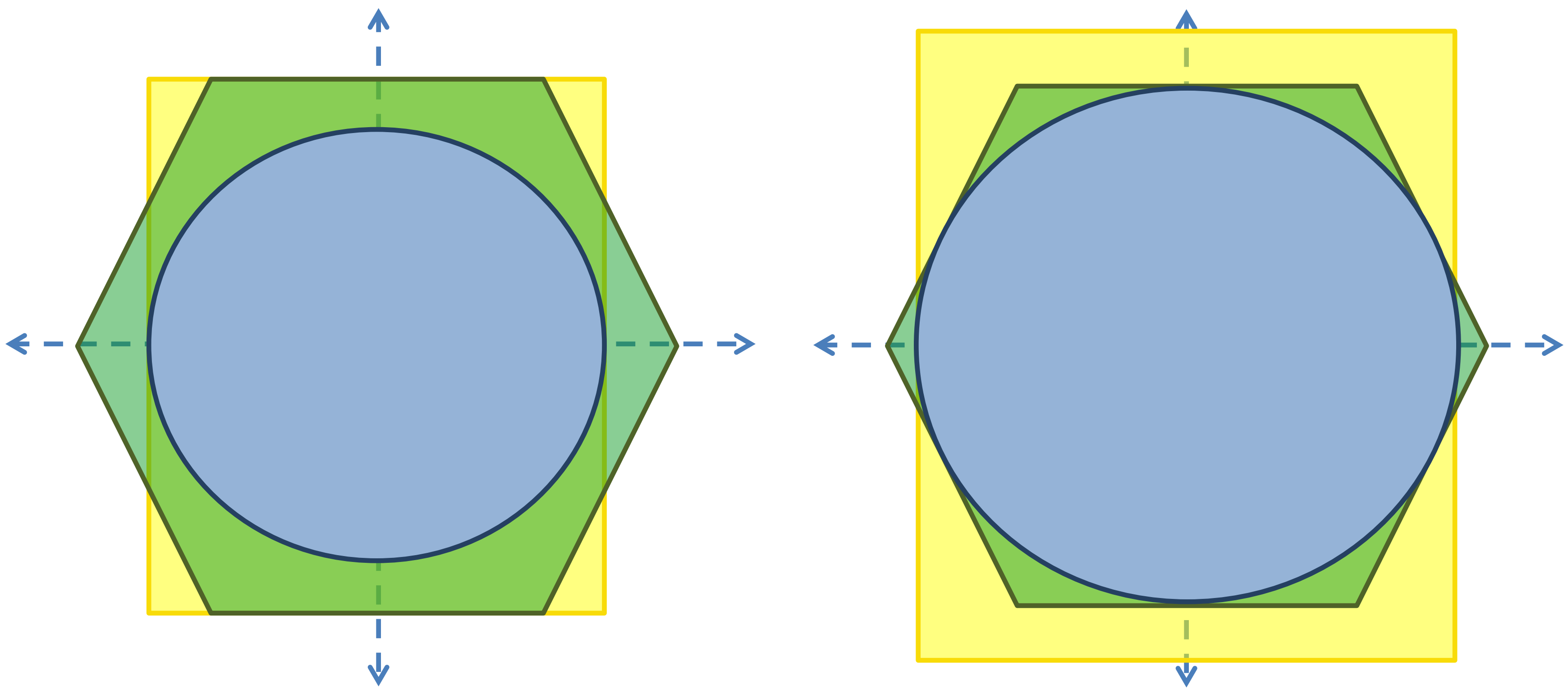
$$a = y_r \bmod 2$$

$$r = \frac{y_r - a}{2}$$

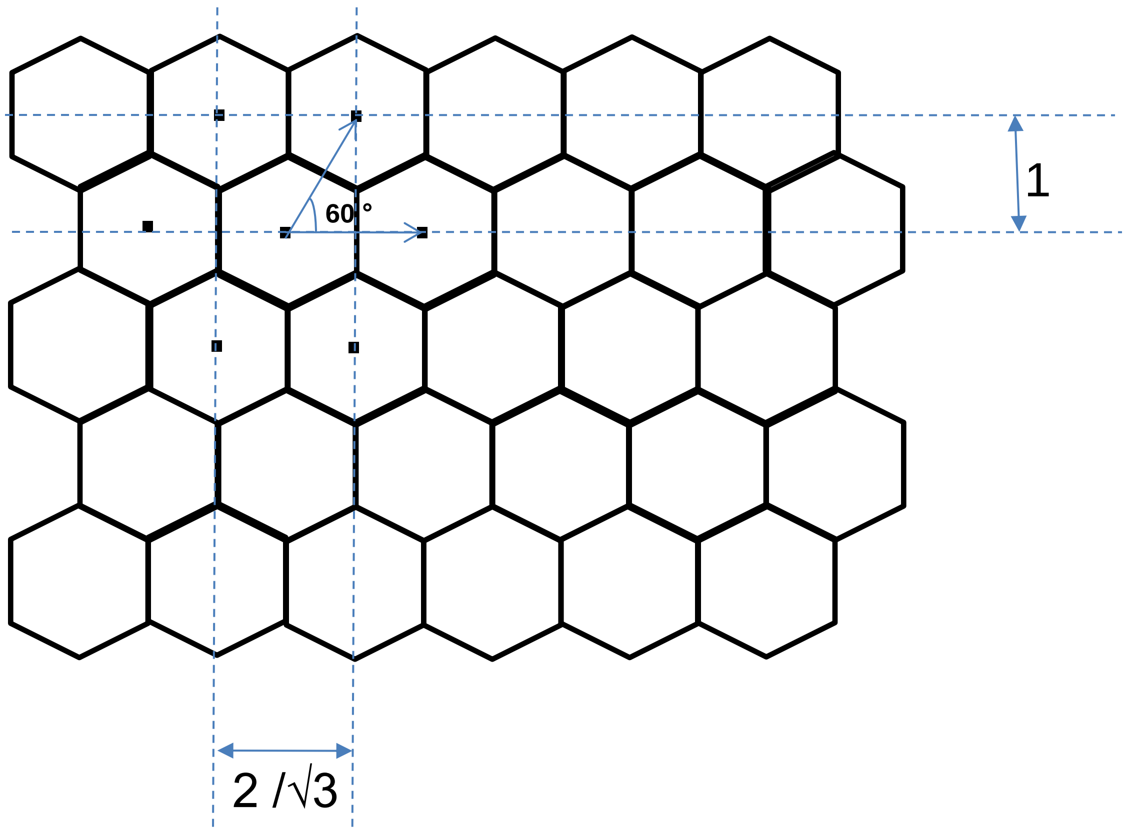
$$c = \frac{x_r - a}{2}$$

Downsampling Example

Sampling Densities



Hex Characteristics



The spacing is important to maintaining the natural symmetry of the hexagonal grid.