



**A COMPARATIVE STUDY OF LEARNING CURVE MODELS IN DEFENSE
AIRFRAME COST ESTIMATING**

THESIS
MARCH 2015

Justin R. Moore, Captain, USAF

AFIT-ENV-MS-15-M-182

**DEPARTMENT OF THE AIR FORCE
AIR UNIVERSITY**

AIR FORCE INSTITUTE OF TECHNOLOGY

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THESIS

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Air Education and Training Command

In Partial Fulfillment of the Requirements for the

Degree of Master of Science in Cost Analysis

Justin R. Moore, BS

Captain, USAF

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Abstract

The goal of this research was to identify which learning curve model is most accurate when applied to Defense acquisition programs. Wright's original learning curve model is widely accepted and used within Defense acquisitions, but the 75+ year old model may be outdated. This study compares Wright's model against three alternative learning curve models using total lot costs for the F-15 C/D & E programs: the Stanford-B model, the DeJong learning formula, and the S-Curve model. However, the results of the study are inconclusive. Two of the three alternative models, the DeJong and S-Curve, rely on the use of an incompressibility factor between 0 and 1 that represents the percentage of the production process that is automated. A Bureau of Labor Statistics report identifies that percentage as very low but does not give an exact number. Therefore assumptions about that parameter were made. When the factor falls between 0.0 and 0.1 the DeJong and S-Curve models appear to be more accurate; when the number is 0.1 or greater, Wright's model is still the most accurate. Further research should be targeted at the exact value of this factor to validate this, or future, comparative studies.

Acknowledgments

I would like to express my sincere appreciation to my faculty advisor, Dr. John Elshaw, for his guidance and support throughout the course of this thesis effort. The insight and experience was certainly appreciated. I would like to thank Dr. Adeji Badiru for his insight and expertise into the multiple learning curve publications he has produced, which were the foundation for this research. I would, also, like to thank my program director, Lt Col Dan Ritschel, for both the support and guidance provided to me in this endeavor. Lastly, I would like to thank my sponsor, Mr. Mike Seibel, from Air Force Life Cycle Management Center for his relentless assistance in data collection and application throughout this process.

Capt Justin R. Moore, USAF

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A COMPARATIVE STUDY OF LEARNING CURVE MODELS IN DEFENSE AIRFRAME COST ESTIMATING

I. Introduction

General Issue

In 2008, the United States' economy took a plunge that affected every industry from the real-estate market to automobile manufacturers. This crash led to tightened budgets throughout the country and many companies looked to operate more efficiently with less capital. That economic turmoil is reflected in the Department of Defense (DoD) through funding cuts and shrinking budgets at every level. The ten year sequestration period approved by Congress with the Budget Control Act of 2011 places emphasis on commanders and managers using funds efficiently. On a micro level, the scrutiny of program cost estimates places more pressure on estimators than ever before. Due to the fact that sequestration effects and cuts will continue for nearly a decade, cost estimators and the accuracy of acquisition cost estimates play a more pivotal role than ever before in acquisition programs. Cost estimates are no longer just a box to check at milestone reviews; they now provide leverage for managers and valuable information in balancing budgets. One way to assist cost estimators is to provide them with the most current and appropriate tools in order to calculate the most accurate and reliable estimate; however, conventional learning curve methodology has been in practice since the pre-WWII build up in the in1930s, but those historical methods may be outdated in today's fast-paced, technological environment.

Over the past two decades, a new methodology rooted in the concept of forgetting curves has emerged, and may provide a more accurate tool for assessing learning curves.

Forgetting is becoming more widely accepted, but its application to learning curves in manufacturing is scarce. This thesis will examine the question of whether more accurate learning curve models exist that could be applied to cost estimates within large acquisition programs. Chapter I of the thesis will provide a background of modern learning curve methodology followed by an explanation of forgetting and a description of the problem to be investigated. Chapter I will also include a discussion of the assumptions made in this study and a review of the research methodology that will be used to test the theory followed by a description of the data sources collected. The conclusion will provide a synopsis of the points covered in this chapter as well as a blueprint for the subsequent chapters of this thesis.

Background

The concept of learning and the application of learning curves in manufacturing has been in practical use since the height of the pre-WWII build up in the late 1930s. From industrial manufacturing, to avionics software, the footprint of the learning phenomenon has been witnessed throughout both the public and private business sectors. Early applications of learning curves in aircraft date back to T.P. Wright in 1936 and his report while at Curtiss-Wright Corporation (Badiru, Elshaw, & Mack, 2013). Learning curve methodology has undergone an evolution over the seventy plus years since Thomas Wright's report, and it has adopted other names along the way such as cost improvement curve or experience curve; however, the theory has remained relatively unchanged despite drastic changes in manufacturing and technology. The learning concept itself is based on the theory that as a worker performs a task multiple times, he or she will require

less and less time to complete the same task due to familiarity with the process. A learning curve is a mathematical representation of this theory which states that as the quantity doubles the worker's performance will improve at a constant rate, and is represented in Equation 1.1 (Wright 1936). Wright's model has many different forms, but the basic architecture remains the same:

$$y = ax^b \quad (1)$$

In this model, y represents the estimated production hours (or cost) for the x^{th} unit produced where a is the production hours (or cost) of the theoretical first unit produced, and b is a factor of the learning rate which will be explained in greater detail in the Literature Review.

Wright's model shown above has been widely accepted and used in manufacturing for years; however, in recent years a contradicting phenomenon known as forgetting has been recognized. A 2013 *Journal of Aviation and Aerospace Perspective* article titled "Half-Life Learning Curve Computations for Airframe Life-cycle Costing of Composite Manufacturing" explains the concept of forgetting in learning curves. Throughout the article, Badiru et al. introduce forgetting and identify learning curve models that account for forgetting by varying the rate of learning. The authors state, "It has been shown that workers experience forgetfulness or decline in performance even while they are making progress along a learning curve (Badiru et al, 2013)." The article continues to add, "contemporary learning curves have attempted to incorporate forgetful components into learning curves (Badiru et al, 2013)." The forgetting concept and the possible use of these models are the groundwork for this research and leads to the question of whether contemporary learning curve models that ignore this phenomenon

are outdated. This thesis will attempt to demonstrate that modern learning curve models which account for forgetting are more accurate in predicting actual manufacturing hours (or relative costs) than conventional models. Subsequent chapters of this thesis will examine such questions in an effort to identify possible areas of improvement for learning curve estimation.

Learning curves are widely-used and even expected throughout DoD cost estimates. This thesis does not intend to discredit the use of learning curves, but rather determine if the commonly-used models can be improved upon throughout acquisition programs. Air Force guidance on learning curve theory and application primarily originates from the *Air Force Cost Analysis Handbook* (AFCAH) Chapter 8 and the DoD *Basic Cost Estimating Guidebook* (BCE) Chapter 17. These two resources primarily focus on two learning curve theories: unit theory and cumulative average theory. Unit theory focuses on the cost of a given unit and is expressed with the same equation shown in Equation 1; “The unit theory states that as the quantity of units doubles, the unit cost decreases by a constant percentage” (BCE, 2007).

Conversely, the cumulative average theory focuses on the average cost of all units produced up to a certain point in production. Cumulative average theory is often attributed to Wright himself and his 1936 article “Factors Affecting the Cost of Airplanes” in which he states, “as the total quantity of units produced doubles, the cumulative average cost decreases by a constant percentage” (Wright, 1936). This equation is essentially the same equation as the unit theory equation, but it differs in that y and x represents cumulative average costs and unit values respectively. These are the two primary methods currently accepted in DoD acquisition programs.

As an example, assume an avionics manufacturer wants to produce eight units of given aircraft component. The company believes the first unit will cost \$100,000 and the plant will experience an 80% learning curve. The chart below in Table 1 provides estimates of both the unit and cumulative average (Cumm Avg) theories. The table shows that the estimate for a given unit will always be higher with the cumulative average theory because it takes into account all of the previous units produced at a higher cost. In DoD cost estimating, cumulative average theory is considered conservative, but it can also provide more consistent analysis of the data due to the fact that actual costs are often reported in annual lot totals rather than individual unit costs.

Table 1: 80% Learning Curve Estimates (in \$K)

Unit	Unit Theory	Cumm Avg Theory
1	\$ 100.00	\$ 100.00
2	\$ 80.00	\$ 90.00
3	\$ 70.21	\$ 83.40
4	\$ 64.00	\$ 78.55
5	\$ 59.56	\$ 74.75
6	\$ 56.17	\$ 71.66
7	\$ 53.45	\$ 69.06
8	\$ 51.20	\$ 66.82

Problem Statement/Research Objectives

Both unit and cumulative average theories are used by cost estimators to better forecast total system costs, but in this fiscally constrained economic period, it may be time for the DoD to examine more modern methods in its forecasting techniques. This thesis will attempt to answer the question of whether DoD cost estimates can be significantly improved upon with the application of alternative learning curve models. Current DoD models assume a constant rate of learning, while many of the alternative

models incorporate some aspects of forgetting and thus a declining learning rate. With that research focus, the following investigative questions are presented:

- 1- Can any of the modern learning curve models be applied to current DoD aircraft cost estimating procedures? If so, which ones?
- 2- Are learning curve models that account for forgetting more accurate than the conventional learning curve model commonly used today? If so, which ones?
- 3- Which learning curve model is most accurate at predicting the actual cost of an acquisition system?

Subsequent chapters of this thesis will attempt to answer these questions as well as outline the research findings that apply to each. These results could prove to be paramount in an ongoing attempt to increase estimate accuracy and improve the efficiency of DoD acquisition spending.

Methodology

Once the data are collected and standardized for this research, the analysis should be straightforward for readers to follow. Each of the three models identified in the screening process for this study will be used to predict total airframe lot costs for the F-15 C/D & E. The three models and their formulations will be explained in-depth in Chapters II and III. Each of the predicted airframe lot costs for the three alternative models will then be compared to Wright's model and the actual lot costs to calculate the error, also known as the residual. The percent error for each of the models will be compared to the other models using an Analysis of Variance (ANOVA) and Dunnett means test, which will each be explained in Chapter III. A significance value or alpha (α) of .05 will be

used to determine whether at least one of the models has a mean residual value different from the rest.

Implications

If significant results are discovered as stated above, the final piece of analysis will be to determine which model is the best predictor of actual production costs. One simple way to compare the models will be to compare which model has the least amount of standard error expressed as a percentage. The smallest percent error will reflect the most accurate model. As a result, if it is supported that one of the modern learning curve models is a more accurate predictor than the conventional method used today, then those results could be presented for further analysis and potentially enacted into future Air Force and DoD guidance, or at a minimum provide a proxy for further research.

Assumptions/Scope

One of the greatest challenges of this research will be the application of variables used for the more modern learning curve formulas. Several of these formulas use constants or other learning factors that allow the models to compensate for the loss of learning. Variables such as previous experience units and incompressibility factor, which will be explained in Chapter II, must be correctly predicted in order for the models to be accurate. However, many of those factors will be estimated based on certain criteria that is extracted from the data set or calculated given other values in the formula. Constants and factors used in the models will be included based on the data provided and on reasonable assumptions rooted in expert opinion. A further description of these factors

and the assumptions made to apply the formulas can be found in Chapter III of this report.

This research contains a fairly narrow scope and focus solely on fighter aircraft costs within the Air Force, specifically the F-15. Analysis will focus on the airframe costs of the Air Force F-15 A-E spread over a 17 year period. This scope was narrowed by the availability and applicability of data, which will be detailed in Chapter III. Application to additional platform types such as cargo aircraft or bombers and even different system types such as ships, ground vehicles, or satellites is an area for potential follow on research.

Conclusion

The primary goal of this thesis is to address the research question of whether the application of modern learning curve models that account for performance decay predict actual production costs more accurately than the conventional models often used today. The data analysis involved will statistically compare the accuracy of three selected learning curve models against the conventional model used throughout DoD. Significant results and the identification of the most accurate model will provide a stepping-stone to possible methodological changes within the Air Force and DoD and provide increased accuracy of acquisition costs estimates.

The next chapter will provide a more in-depth look into the literature surrounding the concepts of learning and unlearning in manufacturing both inside and outside the government. Chapter II will also examine current DoD and Air Force guidance on learning curve methodology and application of learning curves in cost estimates, as well

as provide in-depth descriptions of the three models presented. Chapter III will step through the methodology used to test the investigative questions as well as provide details into the data sets collected for the study. Chapter III will also provide analysis of the data set needed for the application of the alternative learning models. Chapter IV will contain the data results compiled from the methods described in Chapter III including relevant charts and graphs from the analysis. The thesis will conclude with Chapter V, which will contain a discussion of the significance of the results as well as the potential impact of the findings on learning curve methodology both inside and outside of DoD. Chapter V will also include areas that require additional research, limitations to this study, and possible follow-on thesis topics.

II. Literature Review

Introduction

Very few things in business are constant; performance is no exception to that uncertainty. Performance varies externally from worker to worker, division to division, and internally from day to day, season to season, or year to year. Take for instance the production of an automobile. While the process and parts are always the same, a savvy car buyer may want to avoid cars that were built on a Monday or Friday. The worker and even the entire assembly line may suffer a loss in performance due to working at the beginning or end of the week. This concept of uneven and even degrading performance over time is the root of forgetting theory and the foundation for this research.

The Budget Control Act of 2011, which calls for a \$1.5 trillion deficit reduction over the next 10 years, has created a fiscally constrained environment in which competition for congressional funding is higher than ever before. On an organizational level, DoD acquisition programs have seen budget cuts up to ten percent, changes in acquisition schedule, reduction in the number of systems purchased, and an increased scrutiny over cost estimates. Adopting models and theories that potentially increase cost estimating accuracy can prove beneficial to organizations and provide leverage for leaders defending their budget position.

Learning curve theory has been debated and modified for decades; however, the theory and its application to Department of Defense (DoD) cost estimating has remained relatively unchanged and has not readily adapted to current industrial theories or trends. While many unanimously agree with the psychological effects associated with learning

and process improvement, the application of learning toward manufacturing and production is debated. In recent years, several learning curve models have attempted to capture the recently-identified phenomenon of forgetting, in which a worker's performance begins to decrease over time.

This chapter will deliver an in-depth review of present day learning theories and modern forgetting curve methodology including the models that attempt to relate the two together. The theory and methodology will be followed by a description of the issue and provide a look into current DoD learning methodology and application. This chapter will examine any prior research in the area, look at similar approaches found in the literature, as well as provide a description of other appropriate methodologies and applications adopted over the past two decades, and conclude with obstacles and limitations to the literature and research.

Theory Review

Learning Curves

Learning curves started being used by practitioners in the manufacturing world in the late 1930s. At the height of the pre-World War II build-up, the importance of aircraft production costs was realized to be equally as important as developing and producing the aircraft themselves. T. P. Wright (1936) first identified the existence of the learning relationship. He correctly theorized that as a worker performs the same task multiple times, the time required to complete that task will decrease at a constant rate. The workers are *learning* from previous experience and thus becoming more efficient in completing the task. Wright also identified the *80 percent learning* effect in aircraft

production. He believed that organizations would observe a learning rate of 80%, or a 20% production improvement, as the number of units produced doubled (Wright, 1936). This rule would serve as a suggested standard, but has been changed and modified over time to fit different industries. A graphical representation of Wright's 80% learning curve where the first unit costs \$100,000 can be seen below in Figure 1. As you can see in the graph, when the number of units produced doubles (from 1 to 2, 2 to 4, 4 to 8 and so on) the average cost to produce the unit is reduced by approximately 20%.

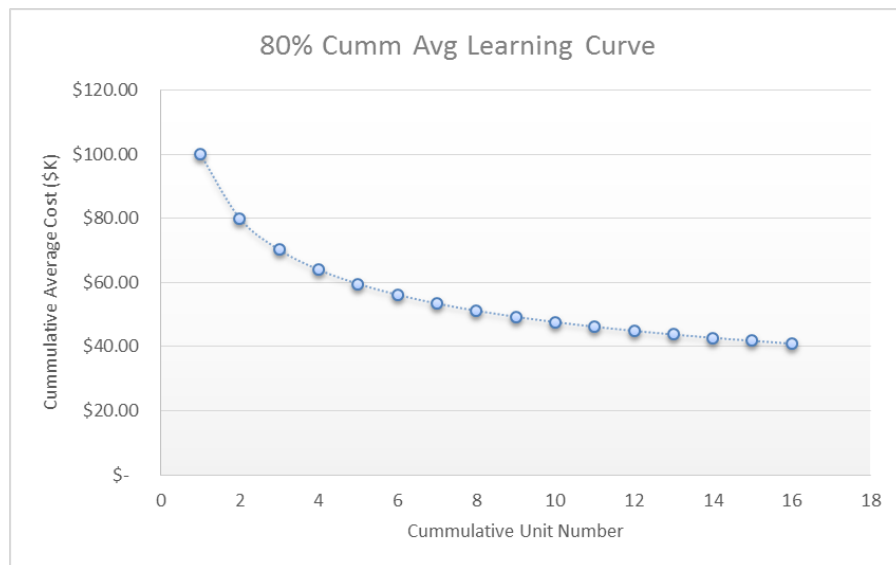


Figure 1: Wright's 80% Learning Curve Example

This classical learning curve model, often referred to as Wright's Learning Model, gives mathematical representations of Wright's basic learning theory. The model, shown in Equation 1 below, follows the assumption that as the quantity produced doubles, the cost will decrease at a constant rate.

$$y = ax^b \quad (1)$$

Where

y = the cumulative average time (or related cost) after producing x units

a = hours required to product (theoretical) first unit

x = cumulative unit number

$b = \log R / \log 2 = \text{learning index}$

R = learning rate (a decimal)

For the remaining sections of this chapter, Wright's model will be referred to in its more modern form of $T_x = T_1 x^{-b}$. This model can also be expressed linearly by transforming the equation through simple algebra. This transformation to a linear relationship becomes useful in regression analysis, in which practitioners attempt to fit a straight line to the transformed data. The log-linear form of Wright's equation, seen in Equation 2, can be derived through simple logarithmic algebra:

$$\ln y = \ln a + b \ln x \quad (2)$$

Using the log-linear form of the equation, the constant learning curve rate can be seen in linear form:

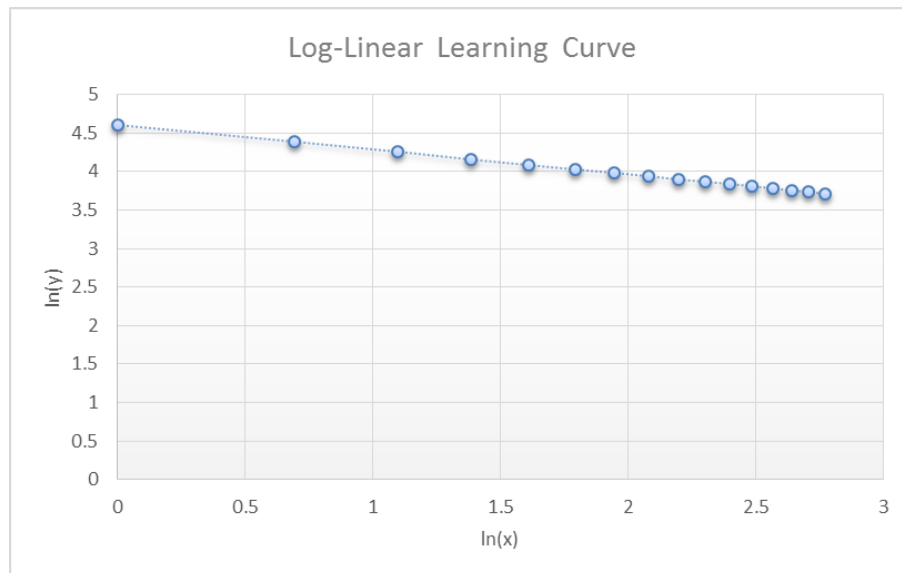


Figure 2: Log-Linear Learning Curve Example

The graph shows that Wright's Learning Curve assumes a constant learning rate over time illustrated by the straight line. At any point in production, the learning rate, and thus performance, are constant.

J. R. Crawford (1944) adopted a similar learning curve approach in the *individual unit* model that he introduced in a training manual at Lockheed Martin. Crawford's model uses the same basic formula as Wright's model, but attempts to estimate *individual* times (or related cost) to produce a given unit by changing which variables are input into the model. An example of this model can be seen in Figure 3 below. This model proved to be beneficial because it can be applied to individual workers or projects rather than to the organization as a whole (Jaber, 2011).

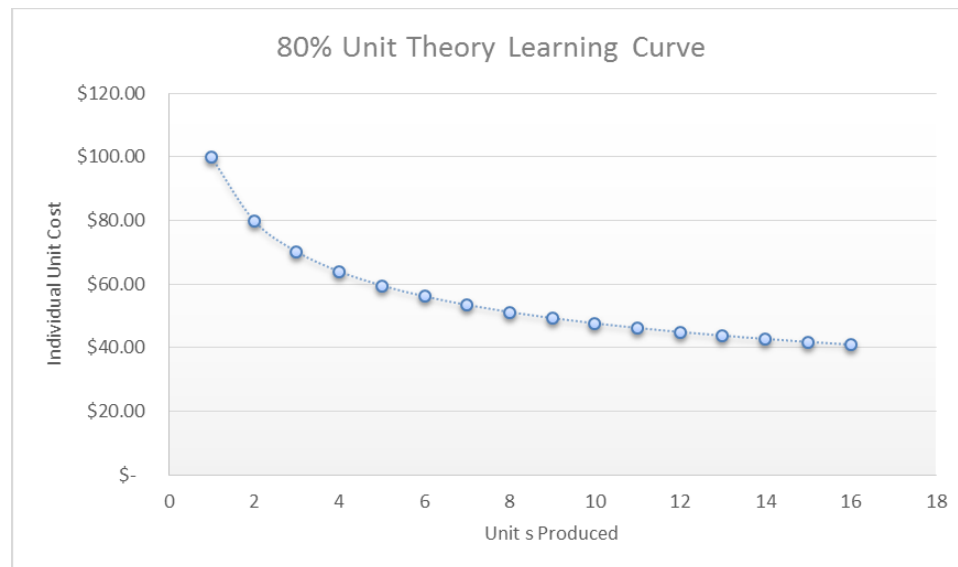


Figure 3: Unit Theory Learning Curve Example

Both unit theory and cumulative average approaches are used in acquisition cost estimating depending on the amount and validity of historical program data. However, contractor reports often come in the form of lots. This form of data is usually more advantageous to using a cumulative average learning curve. The *AFCAH* illustrates how

such data can be used as a lot average in the cumulative average learning curve theory rather than finding a theoretical lot midpoint as with the unit theory.

[A]pply the Cum Avg formulation to contractor lot information, add the hours/costs for a given lot to the hours/costs of all previous lots. The hour/cost plot value (Y axis) of a given lot is the total hours/costs through that lot divided by the last unit number of that lot, while the unit plot point (X axis) is the last unit number of that lot. Lot midpoints are not used with the Cum Avg formulation (AFCAH, 2007).

Furthermore, Hu and Smith (2013) identify a method for plotting and predicting learning curves using lot data. “If the cumulative average costs for all consecutive lots are present, then the direct approach can be applied to the lot data with the last unit in the lot as the lot plot point (LPP).” This LPP is the same as unit plot point described in the *AFCAH* and provides a means for plotting lot data against individual units (on the X axis) in order to determine the learning parameters. Hu and Smith describe this process saying, “ T_1 , b , and other exponents can be obtained directly from the ordinary least squares (OLS) method by regressing [cumulative average costs] vs. cumulative quantities” (Hu & Smith, 2013). The application of this process to the F-15 data will be described in greater detail in Chapter III.

Since Wright’s initial theory, several other models have been adopted in learning curve literature. One of the earliest modifications to the learning curve model came along with introduction of the Stanford-B model shown below in Equation 3.

$$T_i = T_1(x + B)^{-b} \quad (3)$$

Where:

T_i = the cumulative average time (or related cost) after producing x units

T_1 = hours required to product (theoretical) first unit

x = cumulative unit number

$b = \log R / \log 2$ = learning index

B = equivalent experience units (a constant); slope of the asymptote of the curve.

(Yelle 1979)

This model is first attributed to Louis E. Yelle (1979) during a government funded research initiative at Stanford. It introduces the equivalent experience unit parameter to Wright's original equation. This parameter, represented by B , is a constant from zero to ten accounting for the number of units produced prior to start of production of the first unit and is the slope of the asymptote of the learning curve. If this factor is zero, the model reverts back to Wright's original learning model shown earlier in Figure 1 (Badiru 2012). Conversely, if the factor is ten, the effects of learning will begin at the eleventh unit and the decrease in performance will occur much sooner causing the learning curve slope to flatten quickly. The effect of a high B constant on the same data set used earlier can be seen below is Figure 4, which assumes that 10 units have been produced on a previous contract. The prior experience parameter allows the formula to account for prior learning and essentially continue learning from some previous point in time rather than starting the learning process over from zero. Chapter III will address the use of the equivalent experience unit parameter in this study and how those values were determined for each of the models.

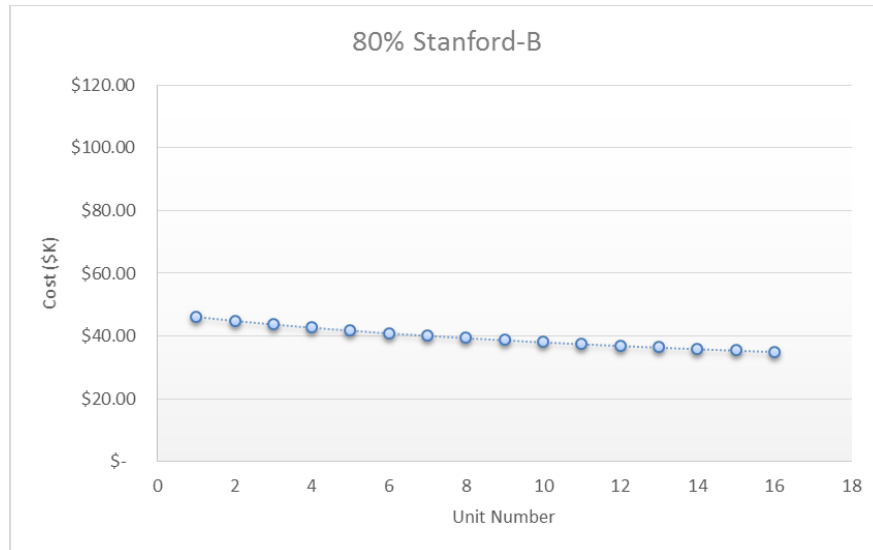


Figure 4: Stanford-B Model Example with B=10

When the Stanford-B model is graphed in log-linear form as shown in Figure 5, one can see a slow build up in performance that is attributed to the production of prior experience units.

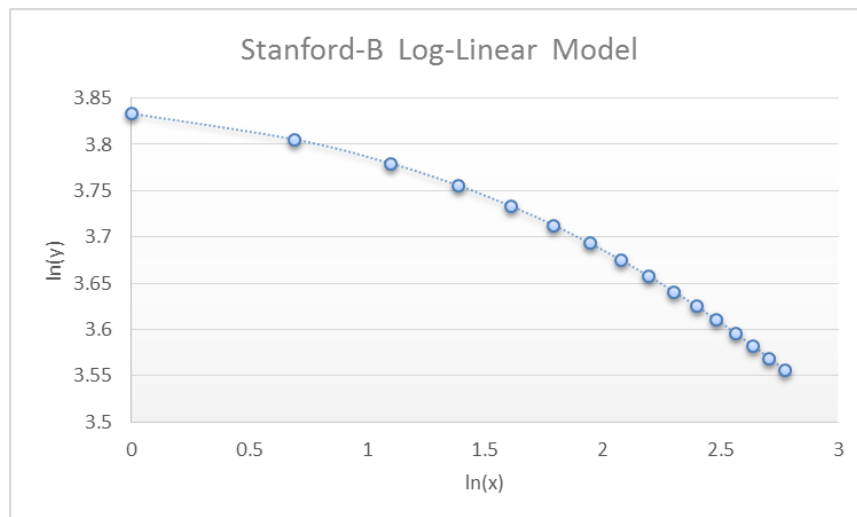


Figure 5: Stanford-B Model Example in Log-Linear Form

Another variation of learning curve models is DeJong's Learning Formula.

DeJong's model, seen below in Equation 4, is another derivation from Wright's original

function in which the incompressibility factor is introduced. Represented by the constant M , this factor represents the relationship between manual processes and machine-dominated processes. Incompressibility factor is a constant between zero and one in which a value of zero implies a fully manual operation and a value of one denotes a completely machine dominated operation (Badiru et. al, 2013).

$$T_x = T_1[M + (1 - M)x^{-b}] \quad (4)$$

Where:

T_i = the cumulative average time (or related cost) after producing x units

T_1 = hours required to product (theoretical) first unit

x = cumulative unit number

$b = \log R / \log 2 = \text{learning index}$

M = incompressibility factor (a constant)

Wright's original model, which inherently assumes an incompressibility factor of zero, fails to account for the advances in manufacturing technology that drive a major percentage of the production industry. A graph with an incompressibility factor of 0.70 is shown in Figure 6 to illustrate the difference in the models.

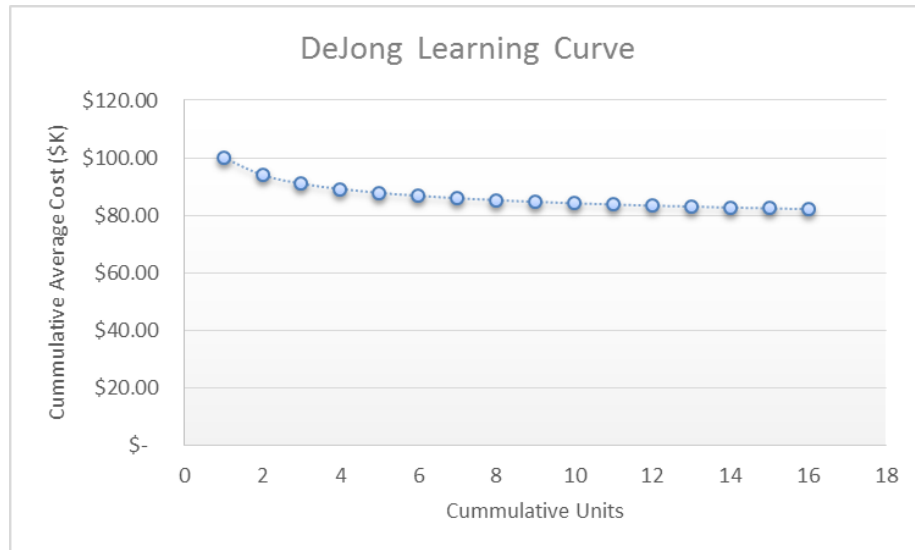


Figure 6: DeJong Learning Curve Example with $M = 0.70$

As the graph demonstrates, a high incompressibility factor reduces the effects of learning and causes a much quicker flattening of the curve. Figure 7 below shows the log-linear graph from the model, in which the loss of learning and decrease in performance can be seen over time.

Production of something as complex as a military aircraft, and a fighter aircraft in particular, will likely fall much closer to zero than one on that scale due to the specialization needed in the production process similar to that of a high end sports car. However, there is no literature on the exact value of that figure for aircraft production and may vary from company to company. Therefore, this research will assume a highly manual process and look at a range of incompressibility factors (from 0.0 to 0.2) to see if changes in M has an effect on the results. Explanation of how the factors for this study were determined can be found in the methodology section of Chapter III.

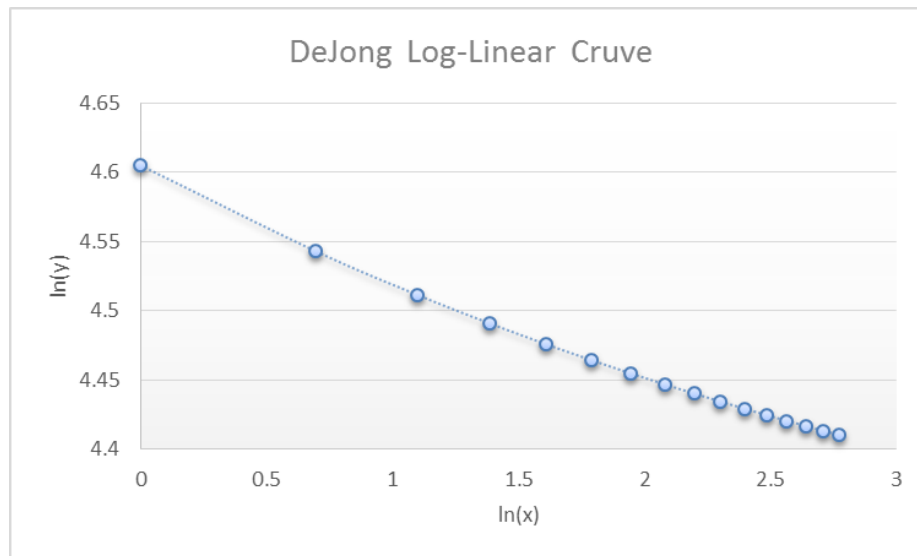


Figure 7: DeJong Model Example in Log-Linear Form

One of the potential weaknesses of the two previous models is that the Stanford-B model does not account for incompressibility, and DeJong's model does not account for previous units produced.

The S-Curve model, however, accounts for both of these factors together. Carr (1946) believed that there was an error in Wright's constant learning assumption and hypothesized that the effects of learning and thus performance followed the S-Curve shape seen below in Figure 8.

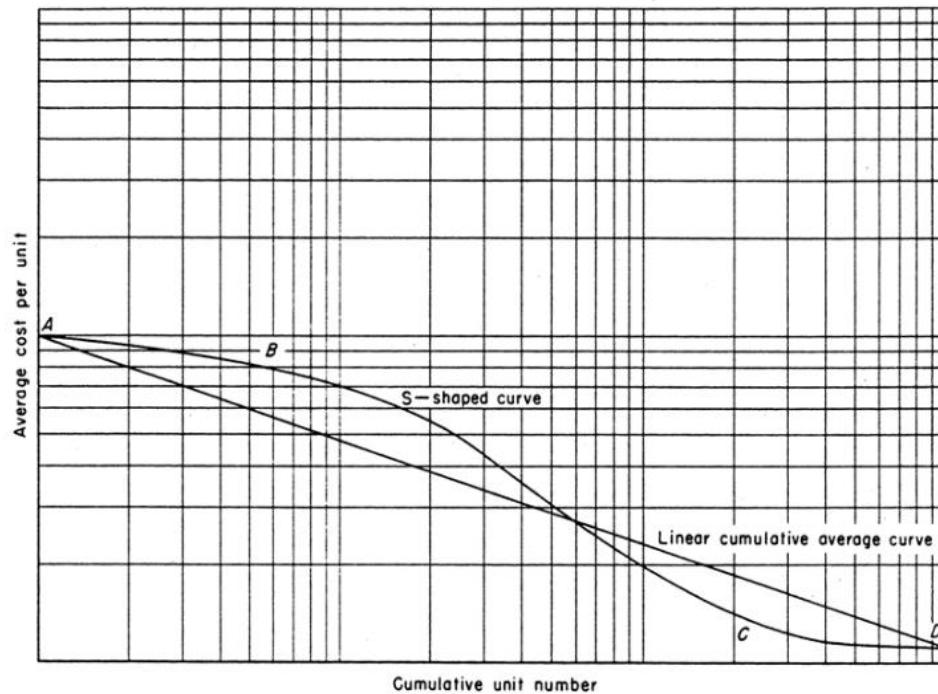


Figure 8: Carr's (1946) S-shaped Learning Curve

The S-Curve model assumes a gradual build up in the early stages of production followed by a period of peak performance. This build up is typically attributed to personnel and procedural changes as well as time needed for new machinery set-ups that occur early in the production process. Using the theory hypothesized by Carr, Towell and Cherrington (1994) developed a model that followed the S shaped pattern. The S-Curve model,

shown below in Equation 5, assumes that learning takes the S-shaped curve often seen in a cumulative normal distribution.

At the top of the curve, from points A to B, there is a slow build up period before the worker/ organization can be fully proficient in accomplishing the task. Then, from points B to C, there is a gradual improvement in production time due to repetition of the process. The trailing off effect, from points C to D, is referred to as the *slope of diminishing returns* and is similar to the trends seen on the tail of the log-linear form of the DeJong Model; after a worker or organization has reached maximum efficiency, he or she will experience forgetting and other inefficiencies in their production

$$T_x = T_1 + M(x + B)^{-b} \quad (5)$$

Where:

T_i = the cumulative average time (or related cost) after producing x units
 T_1 = hours required to product (theoretical) first unit
 x = cumulative unit number
 $b = \log R / \log 2$ = learning index
 M = incompressibility factor (a constant)
 B = equivalent experience units (a constant)

Badiru et al describe the slope of diminishing returns with the following scenario:

[C]onsider when a worker begins learning a new task. The individual is slow initially at the tail end of the S-Curve, but the rate of learning increases as time goes on, with additional repetitions. This helps the worker to climb the steep-slope segment of the S-Curve very rapidly. At the top of the slope, the worker is classified as being proficient with the learned task. From then on, even if the worker puts much effort into improving upon the task, the resultant learning will not be proportional to the effort expended. (Badiru et al, 2013)

This concept captures the impact of forgetting. Even as the worker is progressing along the learning curve, forgetting will eventually take place. Use of this model in research may provide a more accurate look at the actual learning and forgetting that occurs over a production life-cycle.

Several other learning models have been identified in other literature. Models such as Levy's adaptation function which uses a k constant to level off the learning curve, Knecht's upturn model that uses a c constant to reverse the direction of the learning curve at higher cumulative volumes, Glover's learning formula which applies individual learning results at an organizational level,

Pagel's Exponential Function which uses parameters based on empirical analysis, and the Cobb-Douglas model which applies independent variables to the learning function have all been used and applied in other areas of research (Kar 2007). The three models that will be used in this research will be the Stanford-B Model, DeJong's Learning Formula, and the S-Curve Model. A graphical comparison of these models is shown below in Figure 10. Several of the other models require additional information and data that is not available. Also, the three models listed have similar parameters that can be easily identified or assumed making them more useful to cost estimators who put them to practical use. The goal is to make the estimator's job easier, not complicate it with a series of equations that cannot easily be explained to decision makers. The following section will investigate some of the literature regarding forgetting theory and some of the modern forgetting models and how they are used.

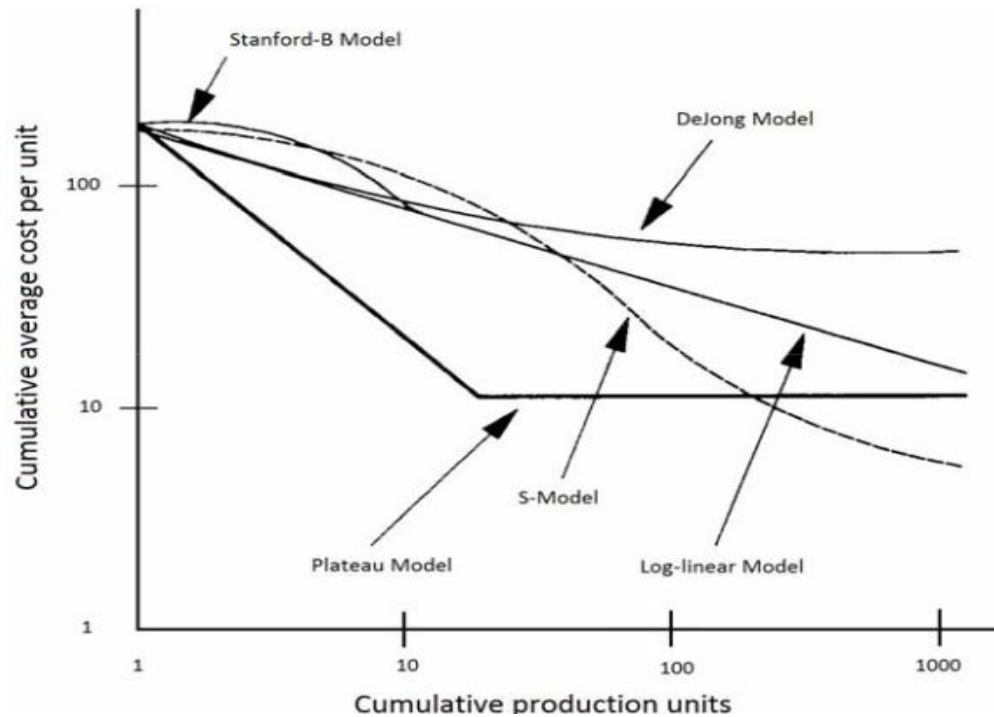


Figure 9: Learning Curve Models (Badiru 1992)

Forgetting and Forgetting Curve Models

Learning and unlearning often take place simultaneously in manufacturing and production environments. Learning has been recognized and modeled in these environments, but the unlearning, or forgetting, aspect is often neglected. Forgetting simply refers to the concept that workers will inevitably see a decline in performance (from many potential sources) while still theoretically moving along the learning curve (Badiru 1995). Badiru (2012) also expresses this concept visually in a chart that displays a worker's performance over time shown below in Figure 10 below. Unlike the constant rate of learning first proposed in Wright's original model, this figure illustrates that a worker or organization will experience intermittent periods of forgetting that cause the performance to be lower than anticipated

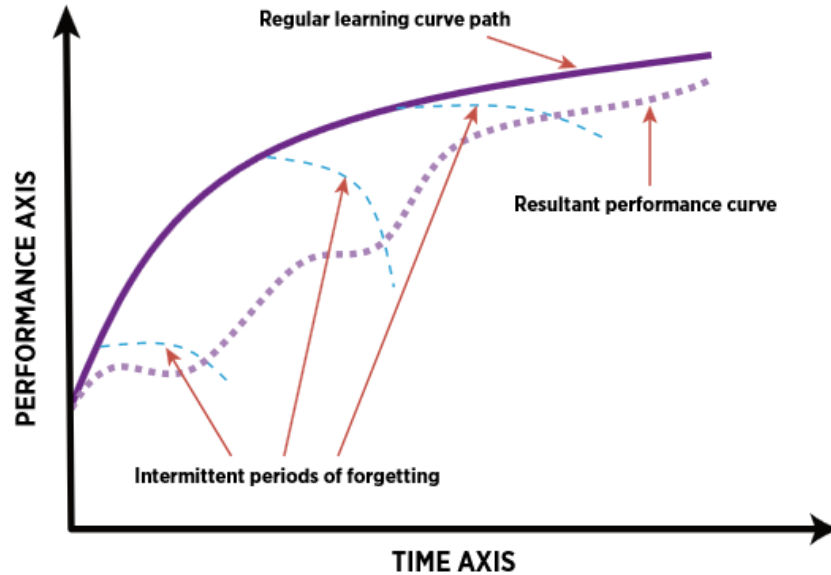


Figure 10: Effects of Forgetting on Performance

. This decline in performance leads to longer production times and thus higher costs than estimated. This assumption may be one of many reasons that DoD cost estimates have been inaccurate in the past. Understanding the forgetting phenomenon and successfully applying it to Air Force and DoD acquisition programs can be an integral step in improved estimate accuracy.

In recent decades, several learning curve models have been applied to a number of manufacturing and production settings. Increasingly, contemporary models have attempted to incorporate the forgetting concept to measure the impact of forgetting on overall performance. Jaber and Sikstrom (2004) identify the potential for forgetting curve research.

Learning and forgetting processes have received increasing attention by researchers and practitioners in the field of production and operations management for the last two decades. A handful of theoretical, experimental and

empirical mathematical forgetting models have been developed, with no unanimous agreement among researchers and practitioners on the form of the forgetting curve.

One potential cause for forgetting is production breaks. Nembhard and Osothslip (2001) performed a comparative study of 14 different forgetting curve models designed to account for production breaks. The study tested the models against the three pre-determined criteria of efficiency, stability and parsimony. The study showed that the Recency Model produced the best results and had the ability to capture multiple production breaks along the same learning curve (Nembhard and Osothslip 2001). However, the limitations of this model were scrutinized by Svikstrom and Jaber who argued that the findings were not consistent with fundamental memory literature and there is still no consensus today on the best forgetting model.

Many forgetting models have useful aspects from an internal perspective in the private sector, but their use may be limited for the government. These models are used to predict starting costs after production breaks or evaluate individual performance. One argument against the use of forgetting curves in military production is that while military budgets are turbulent, military production is fairly constant and spans over several years. While production numbers may change and production schedules may slip and cause programs to extend the life of their contract, production breaks are very rare. Benkard (2000) explains, “Because of the regularity in military programs, organizational forgetting and spillovers of production experience are less apparent.” This makes the application of forgetting models difficult and at times inappropriate within the DoD. However, this research applies the concept of forgetting over time even while progressing

along a learning curve rather than forgetting due to production breaks. The theory at work in this research is that learning rates are not constant (due to forgetting) and models that do not assume a constant learning rate may be more applicable to DoD estimating.

There is some DoD literature regarding learning lost due to production breaks despite how rarely they occur. DoD guidance references the Anderlohr method as a way to determine the amount of learning lost during a production break. George Anderlohr (1969) identifies five factors that influence the amount of learning lost: personnel learning, supervisory learning, continuity of production, methods, and special tooling. Personnel learning refers to the physical loss of personnel due to regular movement or lay-offs, and supervisory learning refers to supervisory personnel lost due to regular movement. Continuity refers to the production line itself, and how closely integrated the workers and stations are. The methods of production are typically recorded and documented, so there is very little if any learning lost in this area. Special tooling refers to wear and physical damage of tooling and the possible need of newer and better equipment.

These five factors are weighted as a percentage summing up to 100% and then those weights are multiplied by the percentages of learning lost in each category. The sum of all of the percentages reflects the total learning lost within the organization. Once this percentage is calculated, it is added to the production cost of the last unit produced to estimate the cost of the first unit after production break. The programs used in this analysis do not have any production breaks and therefore calculating learning lost using the above methods is not required. However, this is significant because it begins the progression towards accepting a learning rate that is not constant and accepts the

principle behind forgetting within the DoD. Conversely, up to this point, that methodology has not been applied to the learning curve models used. This research will look to build upon that progress and assess if modern models can be applied to DoD cost estimates. The next section will address this issue and the purpose of this research.

Problem Statement

Learning curve literature and theory have evolved over the decades and the negative effects of forgetting are widely accepted by researchers and practitioners alike. Technology in both aircraft design and manufacturing has also continued to improve over the years since Wright first identified the relationship between learning and production costs. However, some learning curve methodology has failed to keep pace with this improvement. DoD guidance in both the *AFCAH* and *BCE* refer to Wright's model as the appropriate learning curve application for cost estimators. While the validity of the Wright's original theory has long been accepted, the need to integrate the impact of forgetting into learning curves to improve accuracy cannot be ignored.

Badiru et al address the issue saying, "In defense-contractor manufacturing of airframes, where a mix of contract employees, government civilians, and military coordinators can exist, the issues of overall learning, unlearning, or half-learning can become very significant" (Badiru et al, 2013). In a time of such financial turmoil and uncertainty amid government furloughs and sequestration, exercising every tool and method available to improve estimating accuracy should be paramount. Badiru et al also address the need for forgetting curves within defense cost estimating by adding, "With life-cycle costing that stretches over generations of airframes, breaks in production are

not the exception, but rather, the rule. Coping with these production gaps and properly estimating the associated costs is of primary concern.” This paper will address that very issue of forgetting curves in DoD aircraft production. Later chapters investigate whether defense cost estimators should incorporate more modern learning curve models into their estimate and which model is the best predictor.

The Air Force initiated the *Better Buying Power (BBP)* Initiative in 2010. This initiative, currently under its third iteration, sets forth a group of core acquisition principles aimed at increasing affordability and making the DoD acquisition process more efficient. BBP encourages innovation and elimination of wasteful practices. BBP consists of seven core focus areas: Achieve affordable programs, control costs throughout product lifecycle, incentivize productivity and innovation in industry and government, eliminate unproductive processes and bureaucracy, promote effective competition, improve tradecraft in acquisition of services, and improve professionalism of the total acquisition workforce.

One possible application from the findings of this research is in *should-cost* estimates. The should-cost initiative falls within the cost control focus of BBP and is focused around setting cost savings goals. Should-cost is the concept of setting cost targets that are below those figured from independent and internal program cost estimates (Better Buying Power 3.0, 2013). These targets are achieved through efficiencies and changes in DoD practices and culture that center around driving down program costs. Finding a more accurate tool for predicting the effects of learning may be a way of setting and achieving these targets.

Towill and Cherrington (1994) identify three primary sources for estimating error. The first of which being errors due to inevitable fluctuations in performance that occur naturally. Estimators have little if any control over this source. The second is psychological, physiological or environmental cause that affect deterministic errors. These can be accounted for by estimators, but again this lays largely outside of their control. The final source for prediction error is modelling error, meaning that the form of the model used may be inappropriate and therefore not fit the trend line of the data. This thesis will address the third issue and determine the model form which is most appropriate to fit Defense aircraft over a production life.

Addressing the issue identified by Towill and Cherrington led to the necessity for this research. This thesis will focus around a comparison of three modern learning curve models (Stanford-B, DeJong, and S-Curve) to Wright's learning curve model which is still used in DoD cost estimating today. This comparison has led to research questions mentioned in Chapter I and the following hypotheses:

H1: One or more of the four models compared will have Mean Average Percent Error (MAPE) significantly different from the others.

H2: One of more of the modern learning curve models will be significantly more accurate than Wright's learning model in predicting aircraft costs.

H3: The S-Curve model will have the lowest MAPE and prove to be the most accurate predictor of aircraft costs over time.

Conclusion

This chapter serves as the foundation for the rest of this paper by providing readers with a basic understanding of some of the primary concepts that lead to the research. Learning and forgetting are both evident in aircraft manufacturing and failing to incorporate both into cost estimating can be detrimental to the accuracy of future cost estimates. The following chapter will give a detailed description of the dataset used, the methods applied to compare the four models and any assumptions or ranges of values that were used in each of the models.

III. Methods

Introduction

The primary theory behind this research is that modern learning curve models, which do not assume a constant learning rate, provide a more accurate estimate of annual aircraft production costs than the conventional learning curve models used by estimators today. There is a growing interest in finding ways to improve the accuracy of cost estimates within the DoD; one way of doing so may be improving the accuracy of learning curves, which are used in a large majority of estimates, especially those extending over long life-cycles (sometimes over 30 years). If finding a more accurate forecasting model is possible, then finding which model is best will be of great value. Part of that theory is to test whether the results of these models are significantly different, and if so, which one is the best predictor. Current Department of Defense (DoD) methodology institutes Wright's basic learning curve equation of $T_i = T_1 x^b$, which is described in detail in Chapter II. While Wright's model has long been used successfully, it neglects to include the effects of forgetting, or a decline in performance over time. Forgetting theory has several applications that can be applied in multiple learning curve models that do not assume a constant rate of learning.

The initial task is to determine which of the models should be used in comparison to conventional learning curves, and how to improve upon conventional learning curve application. Several learning and forgetting curve models were identified for application in this study, but three models were selected for analysis based on expert opinion from cost analysts who confirmed the three models used were applicable to cost estimators and

the relevance to the available data from Life Cycle Management Center Cost Staff (AFLCMC/FCZ) at Wright-Patterson AFB, OH (WPAFB) and other on-line repositories: the Stanford-B model, DeJong's Learning Formula, and the S-Curve model. The conventional model lacks the application of key factors that affect learning: prior experience and incompressibility. Accounting for these factors can reduce the amount of estimating error for airframe costs, and even an error reduction of up to 5% could save millions of dollars in cost overruns over the life of a program. The three models above account for one or more of these un-learning factors, which can be easily determined by cost estimators and quickly applied to their models. That applicability and ease of use is the another driver behind using the three afore mentioned models in this study. Providing a model that takes hours or days of secondary analysis and data collection is of little practical value to estimators, even if it is more accurate. This chapter explains how those models will be applied to the data in this study, which methods will be used to compare them, the data analyzed in this research, and limitations in the data that will need to be addressed.

Data Collection

Having identified the three models for analysis, a key step in the process is collecting the data needed to complete a meaningful and useful comparison. When initially approached by the members of AFLCMC/FCZ to find a more accurate way of predicting the effects of learning, they were confident that they had a great deal of relevant data to assist with the task. AFLCMC/FCZ provided learning curve data for 17 Major Acquisition Programs (MDAPs). These data files consisted of Learning Curve

Reports of Annual Unit Cost (AUC) averages as well as the Special Program Office's (SPO) estimate methods using the conventional learning curve model. Many of the programs were already completed and only those with ten or more years of data had enough information to be useful. However, those costs were the unit flyaway cost, for which learning curves have very little practical use. A flyaway cost for aircraft consists of prime mission equipment such as basic structure, propulsion and electronic systems, systems engineering and program management (SE/PM), allowances for engineering changes (ECO) and warranties (AFSC Cost Estimating Handbook Series, 1986). Areas such as SE/PM, ECO and warranties do not experience learning in the way the learning models depict and therefore make the use flyaway costs in this analysis irrelevant.

Airframe costs were chosen for this analysis for a number of reasons. First, using airframe costs allows for the assumption of homogeneity over multiple model types. It is safe to assume that the F-15 A/B, C/D & E all have similar if not identical airframes making it easier to possible to compare the costs and continue the assumption of learning. Also, in foreign military sales (FMS) to the allies of the U.S., the airframe of the aircraft will likely not change despite changes to avionics or electronics systems. Also, Badiru et al (2013) state, "as rapid emergence of new technology necessitates that airframe designs and manufacturing processes be upgraded frequently... the opportunity for forgetting clearly increases." Therefore, the application of airframe costs to this study will provide results consistent with that theory.

After some initial research, fighter aircraft became the primary platform-type for this analysis for a multitude of reasons. The first reason being that several years of production data exist and hundreds of units were produced for these aircraft; over 1150

aircraft were produced in a twenty year span for the F-15 alone. Bailey (1989) stated that forgetting is a function of both the amount of learning and the passage of time. This makes the analysis of aircraft production cycles spanning over several years a prime candidate to exhibit the declining performance rate attributed to forgetting. The second reason is that there are several models of fighters (F-15 A-E and F-18 A-F to name a few) all of which are variants of the same basic airframe making the assumption for comparison of airframe costs from model to model possible. The final reason for choosing fighters was the ability to work face to face with cost estimators from the program offices who are at Wright-Patterson AFB, OH. This makes collection and interpretation of data much easier than a long-distance dialogue.

The initial pool of aircraft considered for analysis consisted of five fighters: the Air Force F-15, F-16, and F-22; the Navy F/A-18; and the joint (Air Force, Navy and Marines) F-35. The F-35 was eliminated from analysis due to having too few data. The F-22 had two factors which eliminated it because the program had two primary contractors, Lockheed Martin Aeronautics and Boeing Defense, Space & Security, both of whom contributed components to the airframe production making it difficult to measure the effects of learning by one against the other. For this reason, it would not provide a suitable comparison to other aircraft being tested. The F-16 was a prime candidate for analysis given the long production life and model upgrade, but relevant airframe data were incomplete or missing completely in some cases. The F/A-18 had sufficient available data, but the program switched primary contractors making it difficult to homogenously compare the costs over that transition. This left the F-15 as the primary

platform for analysis based on production history and availability of relevant airframe costs.

F-15 airframe costs were discovered in two data bases. The F-15 A-D airframe lot averages were acquired from the *Cost Estimating System Volume 2 Aircraft Cost Handbook* published in 1987 by the Delta Research Corporation. This handbook includes all 19 lot purchases from 1970-1985 and details the quantity produced as well as the total airframe costs (minus administrative costs). This data was presented in Base Year 1987 dollars (BY\$87), meaning that the values for each year are set at a fixed price as if all of the funds were expended in 1987 (*AFCAH*, 2007). Summarized, this statement means that each of the values were initially represented as their equivalent purchasing power in the year 1987.

The F-15E data was taken directly from the Joint Cost Analysis Research Database (JCARD) system. This data was much more detailed and included five of the six lot purchases with Lot 1 data missing. The system had data broken out into each cost element (including airframe) and the total quantity produced. The JCARD data was in Then Year dollars (TY\$) which are BY\$ inflated/deflated to represent the purchasing power of the funds if they were expended in that given year (*AFCAH*, 2007). Both the F-15 A-D BY\$87 values and the F-15E TY\$ values are standardized in this research to a Base Year 2014 (BY\$14) value using the 2014 OSD Inflation Tables. The OSD inflation tables are published every year, and this research was begun in 2014 so those tables have been used to avoid crossing over to and from inflation tables. This step ensures that all dollar amounts are compared on a level plane in and also represent a dollar value that is relevant to today's economy.

The unit theory data of the entire F-15 A-E data set is shown below in Figure 11. The data indicate that there are clear signs of forgetting in the later stages of the production cycle. The average unit cost is actually increasing towards the end of production rather than decreasing as would be the case with learning theory.

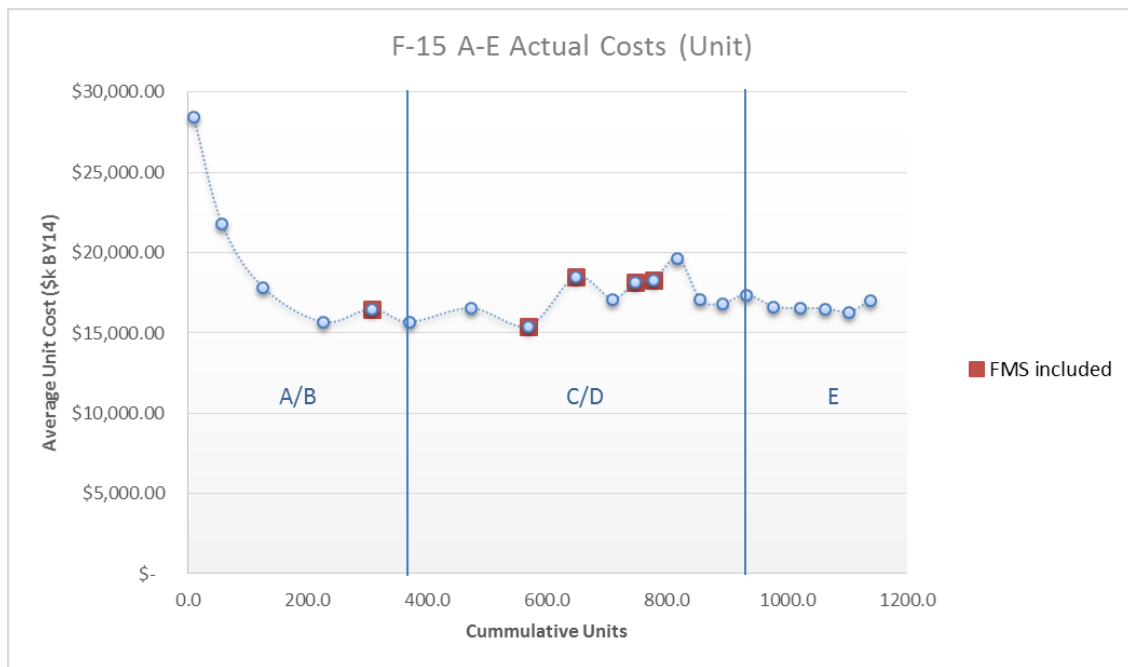


Figure 11: F-15 Actual Costs (Unit)

The F-15 data appears to show significant signs of declining performance over the program's life cycle. Figure 12 below shows the cumulative actual average flyaway cost plotted against the cumulative unit number. Clear signs of forgetting over time and a decline in performance can be seen from sharp flattening trend in the data. After the production of around 600 units, the effects of learning nearly come to a complete stop and in some cases, the costs actually increase over time.

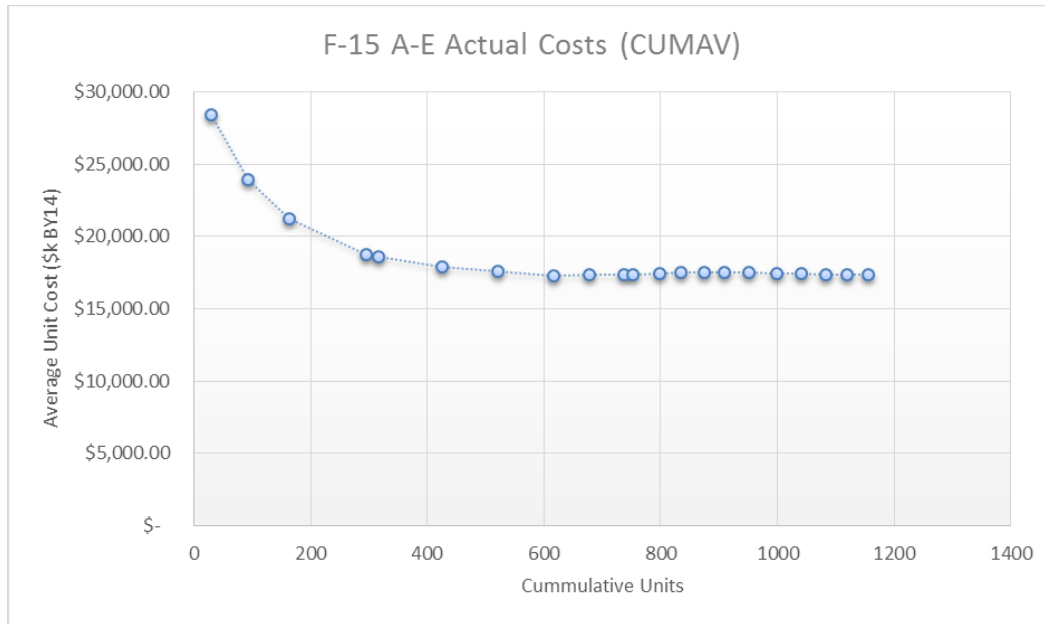


Figure 12: F-15 A-E Actual Costs

When the F-15 cumulative average unit costs are plotted on a log-log graph another significant trend becomes evident. Figure 13 below shows the log-log graph with a linear regression line to provide a frame of reference. A clear S-shaped curve can be seen from the data with a flattening tail towards the bottom of the curve. This indicated that there are diminishing returns at the end of the production cycle and the rate of improvement is not constant over the life of the program.

The goal of this study is to identify a model, or models, which more accurately predicts the decline in performance over time and provides more accurate estimates for airframe costs than Wright's contemporary model. For this research, the F-15 A/B lots will be treated as historical data and each of the models will be used to estimate the costs for the C/D and E lots based on that data. This scenario allows for the simulation of a *real-world* cost estimating scenario rather than a controlled study where the data are treated in a way that is beneficial to the researcher.

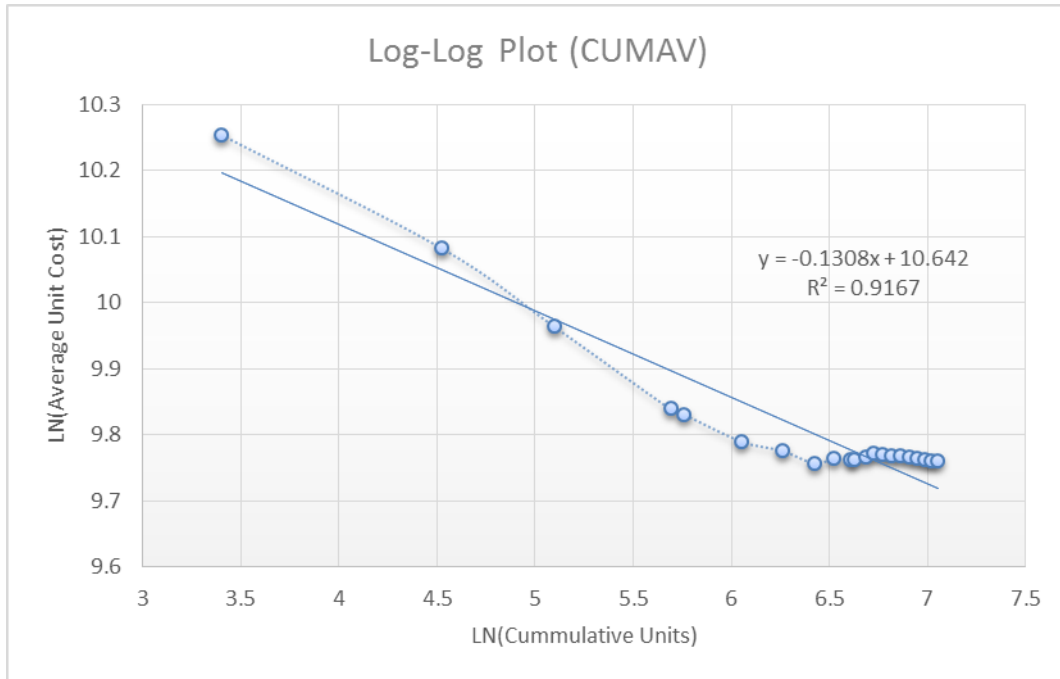


Figure 13: F-15 Actuals Log-Log Plot

Learning Curve Models

Wright's Learning Curve

The status quo for the learning curve models is Wright's model which take the form $T_x = T_1 x^{-b}$. The parameters of the model are detailed in Chapter II. The two parameters that must be determined to perform an estimate are T_1 and b . In common cost estimating practices, b and T_1 are determined through a linear regression on a plot of the natural log of cumulative unit number $[\ln(x)]$ against the natural log of the actual reported costs $[\ln(y)]$. This regression will determine whether the cumulative average or unit learning curve theory should be applied to the data. The regression providing the most accurate fit as according to the R^2 value will determine whether unit theory or cumulative average theory will be used for the duration of the study and the regression equation from that method will determine the parameters for the model. R^2 is a simple goodness of fit

measure that represents the amount of variance between the independent and dependent variables explained as a percentage. In other words, it represents the amount of variability that can be explained by the model (McClave, Benson, and Sincich 2011). From the linear regression b is simply the slope of the line and T_1 is derived by taking the natural log of the y-intercept. Once these two parameters are determined for the Wright model, they remain constant for the other 3 models used in this analysis.

Stanford-B Model

The first model selected for comparison was the Stanford B-model. The Stanford B-model is a relatively older application of the learning curve using the equation $T_i = T_1(x + B)^{-b}$. The parameters of the model are described in Chapter II, but the point of interest in the equation is the equivalent experience unit constant represented by the constant B . The B constant falls between 0 and 10, and represents the equivalent units of previous experience at the start of the production process. If more than 10 units have been produced, then the constant remains at 10. This parameter accounts for how many times the process has already been completed and adjusts the learning curve based on that number. The Stanford-B model is only a slight derivation from Wright's traditional learning curve model, and when B is equal to the first unit produced then the models are identical (Badiru et. al, 2013). Properly applying previous experience into the model is the key to using this equation and for this study B is represented by the number of previous units produced. This can be in the form of prototypes, test aircraft, or any other relevant production unit that was not part of the F-15 A/B production lines. There were 20 test units produced beginning in 1970 which will be counted for prior experience and therefore the factor B will be ten. This prior experience unit constant of ten will remain

consistent when used in the S-Curve model described below. With B determined, the data is incorporated into the model to estimate the total lot costs for the 15 remaining F-15 C/D and E. The residuals from these estimates when compared to the actual lot costs are then compared to each of the other three models. Methods for the comparisons will be covered later in this chapter.

DeJong's Model

The second model considered for comparison was the DeJong Learning Formula. DeJong's model is essentially a simple power function, similar to Wright's model, which accounts for the percentage of the task that requires mechanical activity to the amount that is touch labor. The effects of learning are typically only seen in touch, or human, labor because there are often very little improvements in machine efficiency over time. The basic form of this learning curve is $T_i = T_1 + Mx^{-b}$. Unlike previous models, DeJong's model incorporates the incompressibility factor (M); however, there is no equivalent experience constant. The incompressibility factor, M , is a constant between 0 and 1 where 0 represents a fully manual process and 1 represents a machine-dominated process (Badiru et. al, 2013). Aircraft production falls somewhere in between the two, but there is no precedent set for application to aircraft production. A U.S. Bureau of Labor Statistics report from June 1993 gives the following description of the industry; "[A]lthough the industry assembles a high-tech product, its assembly process is fairly labor intensive, with relatively little reliance on high-tech production techniques" (Kronemer and Henneberger, 1993). This report indicates that the highly specialized process of aircraft production, similar to that of high-end performance automobiles, supports a proper application of M closer to 0 than 1. Where exactly that number falls is

undefined and leads to some subjectivity. In order to avoid any biases that may skew the results and apply robustness to the analysis, the application of the constant will start at 0.0 and move to 0.2 in increments of 0.05 resulting in 5 sets of analysis. This range incompressibility factors will remain consistent in the application of the S-Curve model as well.

S-Curve Model

The third and final model that will be used for comparison in this study is the S-Curve Model, which was developed by Towill and Cherrington in 1994. The S-Curve model is a combination of the Stanford-B model and DeJong's model. As mentioned in Chapter II, this model is based on the assumption of gradual build-up early on in production, a period of steady learning, and flattened portion at the top of the S-curve called the slope of diminishing returns often attributed to forgetting. The basic S-Curve model, $T_i = T_1 + M(x + B)^{-b}$, uses the same previous experience unit constant, B , and incompressibility factor, M , as the Stanford-B and DeJong models respectively. Three of the four variables on the right side of the equation (T_i , b , M and B) must be known to make an assumption about the fourth (Badiru et. al, 2013). In this study, we will use the same known T_i , b , and B used in the prior equations to make an educated assumption about M as described in the DeJong model above. The S-Curve model is a very strong representation of how forgetting will affect the rate of learning and is a sound model to use in testing the theory.

Research Hypotheses

As previously mentioned, the primary theory for this study is that at least one of these alternative learning curve models are more accurate predictors of actual production costs than traditional learning models. This theory is founded on the belief that forgetting occurs in airframe production and models that do not assume a constant rate of learning will provide a more accurate estimate. The research hypothesis for this theory is that there is a significant difference between the mean average percent error (MAPE) of the predicted lot costs between four models. MAPE is a measure of variation that takes the average of the absolute values from the error of each prediction. The absolute value is taken to avoid any cancelling out of positive and negative error values. The smaller the MAPE, the more accurate and reliable the estimates. This theory led to the following research hypothesis:

H1: One or more of the alternative learning curve models has a MAPE statistically different from the conventional DoD model.

H2: One or more of the alternative learning curve models is more accurate than the conventional DoD model.

H3: The S-Curve model, accounting for both prior experience and incompressibility, will be the most accurate predictor of airframe costs.

The null hypothesis (H_0) for the first hypothesis in this study is that $\mu_1 = \mu_2 = \mu_3 = \mu_4$, meaning all of the MAPEs are the same, against the alternative hypothesis (H_a) that at least one of the models has a mean that is different. If the null hypothesis can be rejected and there is evidence to support significant difference, then it will be necessary to test each of the new learning models against the conventional model. The second null

hypothesis mathematically states that $\mu_1 = \mu_i$ where $i = 2, 3, 4$ to be tested against the H_a : $\mu_1 > \mu_i$. These individual hypotheses test whether each of the modern learning curve models have a MAPE significantly lower than the conventional model. One final test will be to investigate the third hypothesis and determine which of these models that have displayed significantly smaller mean errors from the conventional model is the best predictor. The third null hypothesis states that $\mu_i = \mu_j$, where i and j are both significantly lower than μ_1 , to be tested against the H_a : $\mu_i < \mu_j$. That analysis will provide an answer to the initial inquiry of this thesis of determining if there is an alternative *best fit* model that is more accurate than Wright's model.

Analysis Methods

Once the data is standardized to BY\$14 averages, the estimates from each of the models will be placed in a spreadsheet seen below in Table 2, with a column for the actual lot costs, as well as a column for each of the predicted lot costs using one of the four models described above. There will also be a column for cumulative units and lot number. The error column is the difference between the actual and predicted (Unit or Cumulative Average Theory) values. Absolute error (Abs Error) is simply the absolute value of the error, and absolute percent error (Abs PE) is the absolute error divided by the actual cost.

Once the tables have been populated, the next step is to perform the analysis of data and test the hypotheses. For the overall research hypothesis $\mu_1 = \mu_2 = \mu_3 = \mu_4$, the set of percent errors will be compared using either an ANOVA or Kruskal-Wallis test with IBM® SPSS statistics software. These tests produce an F-statistic falling within a

Chi-distribution and a resulting p-value that can reject or fail to reject the null hypothesis based on the given confidence level that will be addressed later in this section. The null hypothesis in this case is that all of the sample means are the same, being tested against the alternative hypothesis that at least one of the sample means is different.

Table 2: Example of Data Table (Predicted vs. Actual)

Wright Learning Curve							
Lot	Units	Cumm Units	Actual Lot Cost	Predicted Lot Cost	Error	Abs Error	Abs PE
1	30	30	\$ 852,826.86				
2	62	92	\$ 1,350,530.04				
3	72	164	\$ 1,282,332.16				
4	132	296	\$ 2,067,667.84				
5	21	317	\$ 346,113.07				
6	108	425	\$ 1,691,696.11				
7	97	522	\$ 1,603,356.89	\$ 1,691,386.31	\$ (88,029.42)	88029.42303	0.054903199
8	94	616	\$ 1,450,706.71	\$ 1,585,219.83	\$ (134,513.12)	134513.1182	0.092722476
9	62	678	\$ 1,145,759.72	\$ 1,021,354.05	\$ 124,405.67	124405.6656	0.108579193
10	60	738	\$ 1,026,855.12	\$ 972,387.31	\$ 54,467.81	54467.80998	0.053043325
11	15	753	\$ 272,791.52	\$ 240,819.76	\$ 31,971.76	31971.76115	0.117202181
12	46	799	\$ 840,106.01	\$ 733,188.48	\$ 106,917.53	106917.5318	0.127266715
13	36	835	\$ 706,890.46	\$ 568,463.82	\$ 138,426.64	138426.642	0.195824742
14	39	874	\$ 665,194.35	\$ 610,849.27	\$ 54,345.08	54345.0799	0.081698048
15	36	910	\$ 605,830.39	\$ 559,487.49	\$ 46,342.90	46342.90206	0.076494846
16	42	952	\$ 729,328.62	\$ 647,695.92	\$ 81,632.70	81632.70346	0.111928561
17	48	1000	\$ 798,870.89	\$ 733,921.93	\$ 64,948.96	64948.95581	0.081300942
18	42	1042	\$ 694,080.06	\$ 636,953.56	\$ 57,126.50	57126.49757	0.082305344
19	42	1084	\$ 693,381.43	\$ 632,316.73	\$ 61,064.70	61064.70419	0.088067983
20	36	1120	\$ 586,856.87	\$ 538,456.02	\$ 48,400.85	48400.84906	0.082474708
21	36	1156	\$ 613,192.10	\$ 535,328.10	\$ 77,864.00	77863.99559	0.126981408
						MAPE =	9.87%

ANOVA requires three conditions for valid results: the samples must be randomly selected from the population; the samples have distributions that are approximately normal, and the population variances must be equal (McClave, Benson, Sincich 2011). The samples are random in the sense that there was no selection process from the data samples collected. The normality of the data will be addressed in Chapter IV through a group of histograms using Microsoft Excel. A histogram can be used to display the frequency of measurements and will thus provide insight into the shape of the distribution

(McClave et al, 2011). The equality of the variances will be tested by dividing the largest sample standard deviation by the smallest standard deviation. As a *rule of thumb*, if that value is two or less, then the variances can be assumed equal. If these conditions are not met, the analysis will use a non-parametric test to investigate the first hypothesis; non-parametric tests, unlike ANOVA, do not require an assumption of normal distribution. The Kruskal-Wallis test can be used to determine if multiple samples arise from the same distribution and have the same parameters (Kruskal & Wallis, 1952). F-test from the initial ANOVA or Kruskal-Wallis test, both performed in SPSS, will provide insight into the first hypothesis. If the F-statistic is significant, then the data rejects the null hypothesis and at least one of the sample means is different.

To test the second hypothesis that at least one of the models is more accurate this research will use Dunnett's test performed in SPSS. Dunnett's test is used to compare multiple sample means to one value held as the control (Everett & Schron dal, 2010). Wright's learning curve model, the status quo, will be used as the control for this study and the significance will be used to test if any of the other model's MAPE values are less than ($<$) the control. If the assumption for equal variance is not met, Dunnett's T3 test will be used for comparing the sample means. The T3 is similar to Dunnett's test described above, but it uses each sample as a control individually to compare against the other values.

The final analysis will be to test which model is most accurate given significant results for more than one model from the second hypothesis. This analysis will be conducted through a simple paired difference t-test again performed in SPSS. A paired difference experiment uses a probability distribution when comparing two sample means

and produces a t-statistic that falls within a student-t distribution that can either reject or fail to reject the null hypothesis depending on the desired confidence level (McClave et al, 2011). If the assumption for equal variances is not met and the T3 test is used, information regarding which models are significantly different will be found in the T3 test and there will be no need for paired t-tests.

For this study, an α of 0.05 will be used, meaning that the results will produce results with 95% confidence. For purposes of this analysis, this α value means that F-statistic (or t-statistic) with a resulting p-value < 0.05 will reject the null hypotheses and support the alternative hypothesis that the mean values between the models are different. A p-value, or observed significance level, is defined as “the probability (assuming H_0 is true) of observing a value of the test statistic that is at least as contradictory to the null hypothesis, and supportive of the alternative hypothesis, as the actual one computed from the sample data” (McClave et. al, 2011). In other words the p-value is the chance of having an actual result that is contradictory to the sample result. By rejecting the null hypothesis, the data is essentially demonstrating that there is a 95% chance the means of the two populations are different.

Conclusion

Assuming that all H_0 are rejected in favor of the H_a and production rate does not have a significant effect on the accuracy of the models, the results of this study can provide a valuable proxy into future research and application. If it can be shown that one of the models is significantly more accurate than the others, then those results can be presented for further analysis and possibly be enacted into DoD policy. At minimum the

results can provide analysts with a methodology cross-check, which will be explained in greater detail in Chapter 5. The following section will show detailed results from the analysis. Each of the tables and a description of the data as well as the final results from each of the t-tests will be included. Chapter IV will not include the interpretation and meaning of the results, that discussion and potential impacts of the findings will be included in Chapter V.

IV. Results

Introduction

The following section contains the results from the tests and methods described in Chapter III. Chapter IV attempts to answer the three primary research questions proposed earlier in this research: first, is one or more of the alternative learning curve models statistically different from Wright's conventional model; second, is one or more of the alternative learning curve models statistically more accurate than Wright's conventional model, and third, which model is the most accurate. The following graphs and charts will attempt to answer these questions, and will be accompanied by a brief description of the results shown within. This analysis will begin by investigating the F-15 C/D & E models using the A/B model as historical data. Discussion on the implications of the findings, limitations of the study, and possible areas for further research within the area will be reserved for Chapter V.

F-15 C-E Analysis

Unit Theory & Cumulative Average Theory

The first step of the analysis was to identify which learning theory was most appropriate for the given data. For the F-15 data using an M value of 0.20, a log-log regression was run against the A/B model data for using both the unit theory and cumulative average theory to predict the learning parameters for the C/D and E models used in the analysis. Figure 13 below shows the regression using the cumulative average theory which produced an R^2 value of 0.9951. Using the entire data set (shown previously in Figure 12) produced a much lower R^2 value of .9167, and the parameters

from the A/B model regression were used because they better explained the learning taking place. The cumulative average R^2 value for the A/B model was slightly higher than the 0.9735 value produced using the unit theory data (regression graph can be seen in Appendix A). This indicates that the cumulative average theory should be used for estimating the C-E model costs and the lot-plot point assumption holds for the data.

These results also provide the basic parameters for all four learning models used in the study. The learning rate factor, b , is the slope of the linear regression line, which in this case is -0.1813. This value indicates a learning curve slope of 88.19% ($LCS = 2^b$). Figure 13 also provides information into the T_1 value that will be used in the analysis. The intercept of the linear regression equation is the natural log of the theoretical unit 1, T_1 , value. By raising the mathematical constant e to the value of the intercept (10.883), one can determine the average cost of the theoretical first unit; in this case, that value is \$53,263K.

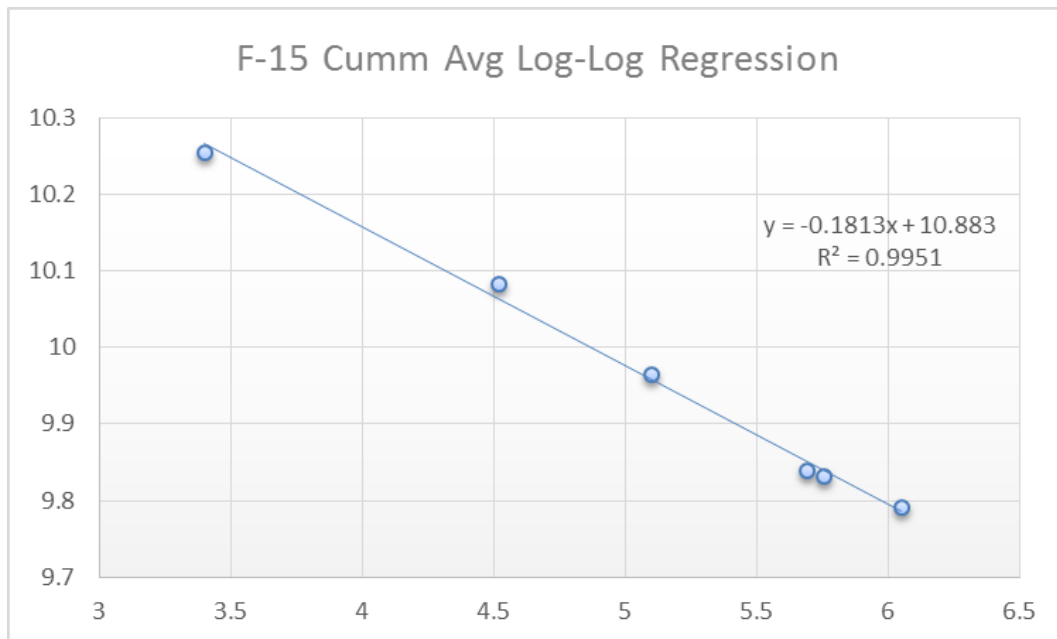


Figure 14: F-15A/B Log-Log regression

Assumption Parameters

The next step was populating the data tables so that the comparative analysis could be run. Table 3 below shows the APE values for all 15 lots calculated using each of the four learning models with an incompressibility factor of 0.1. As the table shows, Wright's Curve and the Stanford-B models initially has the lowest MAPE of the four models, but analysis must be conducted to determine if there is a significant difference in the data. Then that analysis will be applied to a range of incompressibility factors to determine how sensitive the results are to a change in that factor.

Table 3: F-15 APE Values for Each Model

<i>M</i> =0.1				
Lot	WLC	Stanford-B	DeJong	S-Curve
7	0.0549032	0.0509017	0.2716447	0.2680433
8	0.0927225	0.0892703	0.3285742	0.3254672
9	0.1085792	0.1085792	0.0904993	0.0882712
10	0.0530433	0.0554482	0.1634820	0.1613176
11	0.1172022	0.1193309	0.0873964	0.0854805
12	0.1272667	0.1292897	0.0771023	0.0752816
13	0.1958247	0.1975958	0.0049876	0.0065815
14	0.0816980	0.0836323	0.1387508	0.1370100
15	0.0764948	0.0783588	0.1476580	0.1459804
16	0.1119286	0.1136465	0.1059919	0.1044458
17	0.0813009	0.0829968	0.1468597	0.1453335
18	0.0823053	0.0839250	0.1482298	0.1467721
19	0.0880680	0.0896143	0.1433682	0.1419766
20	0.0824747	0.0839757	0.1525089	0.1511580
21	0.1269814	0.1283646	0.0984203	0.0971754
AVG	0.0987196	0.0996620	0.1403649	0.1386863

In order to test the samples, certain assumptions must be tested. The assumption of normality was not met, meaning that non-parametric tests must be used for comparing the means. Table 4 below shows the skewness and kurtosis values for each of the

samples with an M value of 0.1. Kurtosis, is a measure of the *peakedness* of the distribution.

Table 4: F-15 Descriptive Statistics (M=0.1)

	N	Mean	Std. Deviation	Skewness		Kurtosis	
	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic	Std. Error
WLC	15	.0987	.03529	1.426	.580	3.247	1.121
Stan_B	15	.0997	.03584	1.378	.580	3.134	1.121
DeJong	15	.1404	.07749	1.031	.580	2.090	1.121
S_Curve	15	.1387	.07663	1.052	.580	2.086	1.121
Valid N (listwise)	15						

High kurtosis values are assumed to be non-normal and result in a sharply peaked distribution. Histograms for each of the samples are provided in Appendix B, and the effects of the kurtosis are displayed visually. All of the samples also have a skewness greater than one, so normality cannot be assumed. The KW test must be used to determine if the sample distributions are significantly different and if at least one sample has a median different from the others.

The assumption for equal variances must also be tested by dividing the largest sample standard deviation by the smallest standard deviation (σ). The DeJong model had the highest σ with a value of 0.07749 and the Wright (WLC) Model had the smallest σ with a value of 0.03529. Dividing the WLC σ by the S-curve σ equates to a value of 2.19, which is much larger than two meaning that the variances are assumed to be unequal. This value indicates that the Dunnett T3 test must be used to compare the means for this analysis.

Means Comparison

Since the samples are not normally distributed, the KW test is used to test if the samples are significantly different. The KW test will analyze the null hypothesis that the distribution of the APE value is the same regardless of model type. Table 5 below shows the KW test results for an M value of 0.1. As the table shows, the p-value of 0.028 is significant and therefore rejects the null hypothesis indicating that at least one of the sample distributions is significantly different from the others. This result, that the distributions are significantly different, indicates that there is a chance that the means of the samples are different. This process was repeated using the full range of M values from 0.0 to 0.2. The results were consistent across the range except for 0.0 which had no statistical difference. The results of these Kruskal-Wallis tests can be seen in Appendix C.

Table 5: F-15 Kruskal-Wallis Test Results ($M = 0.1$)

	Null Hypothesis	Test	Sig.	Decision
1	The distribution of APE is the same across categories of Model.	Independent-Samples Kruskal-Wallis Test	.028	Reject the null hypothesis.

Asymptotic significances are displayed. The significance level is .05.

The following step was to determine if the means are statistically different and which models are accounting for that difference. The Dunnett T3 test was used as a post-hoc ANOVA analysis because the variances are assumed to be unequal. Table 6 below illustrates the results of the post-hoc analysis. For the purposes of the analysis in SPSS, the models were each assigned numbers: Wright's Learning Curve is Model 1, the Stanford-B is Model 2, the DeJong Formula is Model 3, and the S-Curve is Model 4. For

this test however, the means are not significantly different. All of the p-values (represented by the sig. column) are much greater than 0.05 indicating that although the distributions are different, the means of those distributions are not.

The final step was to test which model was the *most* accurate. However, none of the models are statistically different and therefore the results are inconclusive for which model is most accurate if the incompressibility factor is assumed to be 0.1. In the following sections, this means comparison will be repeated for the full range of M values from 0.0-0.2.

Table 6: F-15 Dunnett T3 Test (M=0.1)

(I) Model	(J) Model	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
1.00	2.00	-.00094	.01299	1.000	-.0375	.0356
	3.00	-.04165	.02199	.343	-.1055	.0222
	4.00	-.03997	.02178	.376	-.1032	.0232
2.00	1.00	.00094	.01299	1.000	-.0356	.0375
	3.00	-.04070	.02204	.369	-.1047	.0233
	4.00	-.03902	.02184	.404	-.1024	.0243
3.00	1.00	.04165	.02199	.343	-.0222	.1055
	2.00	.04070	.02204	.369	-.0233	.1047
	4.00	.00168	.02814	1.000	-.0776	.0810
4.00	1.00	.03997	.02178	.376	-.0232	.1032
	2.00	.03902	.02184	.404	-.0243	.1024
	3.00	-.00168	.02814	1.000	-.0810	.0776

Sensitivity Analysis

As mentioned above, the means comparison process was repeated for the F-15 using an M value of 0.0, 0.05, 0.15 and 0.20. When using a value of 0.00 the results (shown in Table 7 below) did not change. In fact, the models had similar distributions as

well as means. All of the p-values from the Dunnett T3 test were 1.000 and indicate that none of the means are significantly different. This should not be surprising because when $M=0$, the DeJong model essentially turns into Wright's model and the S-Curve model turns into the Stanford-B model.

Table 7: F-15 Dunnett T3 Test ($M=0.0$)

(I) Model	(J) Model	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
1.00	2.00	-.00094	.01299	1.000	-.0375	.0356
	3.00	.00000	.01288	1.000	-.0363	.0363
	4.00	-.00111	.01299	1.000	-.0377	.0355
2.00	1.00	.00094	.01299	1.000	-.0356	.0375
	3.00	.00094	.01299	1.000	-.0356	.0375
	4.00	-.00017	.01309	1.000	-.0371	.0367
3.00	1.00	.00000	.01288	1.000	-.0363	.0363
	2.00	-.00094	.01299	1.000	-.0375	.0356
	4.00	-.00111	.01299	1.000	-.0377	.0355
4.00	1.00	.00111	.01299	1.000	-.0355	.0377
	2.00	.00017	.01309	1.000	-.0367	.0371
	3.00	.00111	.01299	1.000	-.0355	.0377

Using an incompressibility factor of 0.05 provided slightly differing results. The Kruskal-Wallis test (shown in Appendix C) yields a p-value of 0.000 indicating that the distributions of the models are different and presents the possibility that the means may be different. When comparing the descriptive statistics shown below in Table 8, the results for standard deviation display that the variances can be assumed equal. The largest σ over smallest σ yields a value of 1.69 which is less than two; therefore, the original Dunnett test can be used.

The results of the Dunnett test holding Model 1 (WLC) as the control are shown below in Table 9. Assuming an incompressibility factor of 0.05 both the DeJong and S-Curve models are significantly more accurate with low p-values of 0.033 and 0.030 respectively.

Table 8: F-15 Descriptive Statistics (M=0.05)

	N	Mean	Std. Deviation	Skewness		Kurtosis	
	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic	Std. Error
WLC	15	.0987	.03529	1.426	.580	3.247	1.121
Stan_B	15	.0997	.03584	1.378	.580	3.134	1.121
DeJong	15	.0526	.05983	1.936	.580	3.057	1.121
S_Curve	15	.0520	.05862	1.952	.580	3.070	1.121
Valid N (listwise)	15						

The DeJong Model had a MAPE value of 5.26% and the S-Curve model had a value of 5.20%, both of which were the two smallest MAPE values from the entire study.

Table 9: F-15 Dunnett Test (M=0.05)

(I) Model	(J) Model	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
2.00	1.00	.00094	.01784	1.000	-.0421	.0440
3.00	1.00	-.04616 [*]	.01784	.033	-.0892	-.0031
4.00	1.00	-.04670 [*]	.01784	.030	-.0898	-.0036

a. Dunnett t-tests treat one group as a control, and compare all other groups against it.

*. The mean difference is significant at the 0.05 level.

The results for an incompressibility factor of 0.05 are shown graphically below in Figure 15. The graph shown the actual vs predicted values for the F-15E model, which accounts for the last 5 lots of the production process. The WLC and Stanford-B values essentially fell on top of each other, and the same was seen for the DeJong and S-Curve models; therefore the graph only shows the WLC and S-Curve models to illustrate how

the incompressibility factor changes the estimate. As the graph indicates, the S-Curve predicted values fall much closer to the actual costs resulting in a MAPE that is nearly 4.5% lower than WLC. A similar graph will also be shown for $M = 0.15$, to illustrate when large incompressibility values result in a less accurate estimate.

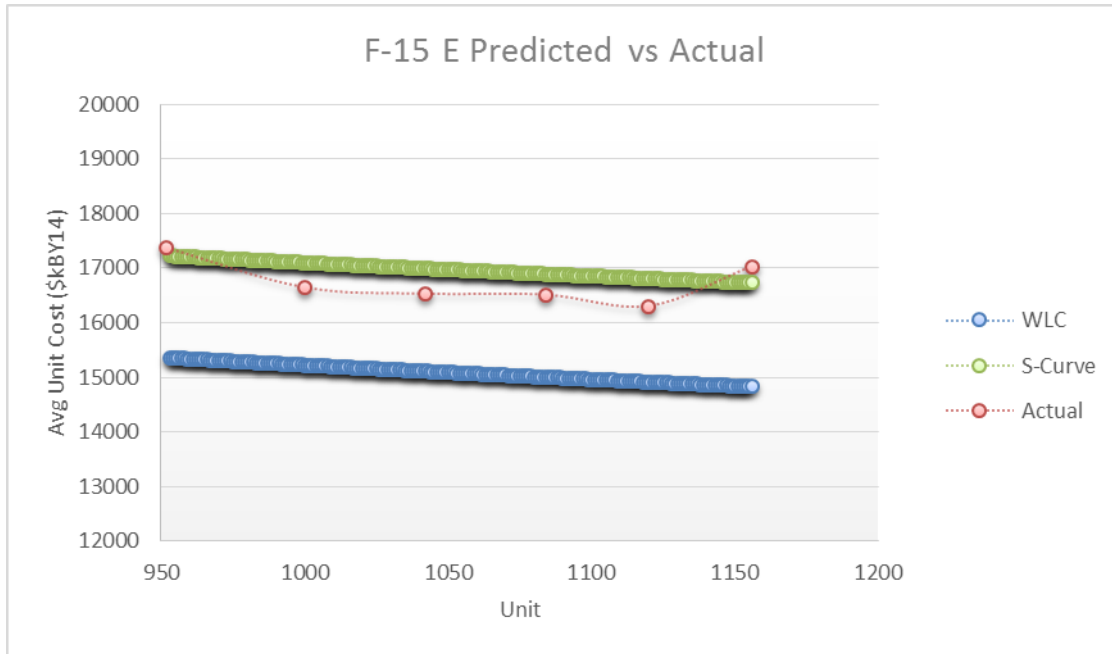


Figure 15: F-15E Predicted vs. Actual ($M=0.05$)

To test which model is the most accurate, a paired sample t-test test was used to determine if there was any significant difference between DeJong Model and the S-Curve model. Table 10 shows the results of the t-test.

Table 10: F-15 t-test DeJong-S-Curve

		Paired Differences					t	df	Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
					Lower	Upper			
Pair 1	DeJong - S_Curve	.00054	.00211	.00054	-.00063	.00171	.991	14	.339

The high p-value of 0.339 indicates that there is no difference between the two models although they are both more accurate than the other two models.

Repeating the process for an M value of 0.15 again produces a low p-value for the Kraskal-Wallis test of 0.000 meaning that the sample distributions are different (Shown in Appendix C). The next step was to determine if any of the means were different and if so, which ones. The descriptive statistics shown below in Table 11 indicate that the variances are unequal with a value of 2.36 when comparing the largest σ over smallest σ . Therefore, the Dunnett T3 test must be used to compare the means.

Table 11: F-15 Descriptive Statistics (M=0.15)

	N	Mean	Std. Deviation	Skewness		Kurtosis	
	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic	Std. Error
WLC	15	.0987	.03529	1.426	.580	3.247	1.121
Stan_B	15	.0997	.03584	1.378	.580	3.134	1.121
DeJong	15	.2491	.08336	.729	.580	1.917	1.121
S_Curve	15	.2473	.08295	.713	.580	1.906	1.121
Valid N (listwise)	15						

The results of the Dunnett T3 test are shown below in Table 12. The results verify that at least one of the models has a significantly different mean from the others with two p-values of 0.00

In this case, the S-Curve and DeJong models are significantly different with p-values of 0.000; however, they were less accurate than the WLC with MAPE values of 24.7% and 24.9% respectively. The results also indicate that there is no difference between the Stanford-B and WLC models. Figure 16 below details the actual and predicted costs. Unlike Figure 15 above, in this case the larger incompressibility factor

cuts out too much learning and the S-Curve estimate rises far above the actual values while the WLC estimates remain the same.

Table 12: 12: F-15 Dunnett T3 Test (M=0.15)

(I) Model	(J) Model	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
1.00	2.00	-.00094	.01299	1.000	-.0375	.0356
	3.00	-.15035*	.02337	.000	-.2185	-.0822
	4.00	-.14856*	.02328	.000	-.2164	-.0807
2.00	1.00	.00094	.01299	1.000	-.0356	.0375
	3.00	-.14941*	.02343	.000	-.2176	-.0812
	4.00	-.14762*	.02333	.000	-.2156	-.0797
3.00	1.00	.15035*	.02337	.000	.0822	.2185
	2.00	.14941*	.02343	.000	.0812	.2176
	4.00	.00179	.03037	1.000	-.0838	.0874
4.00	1.00	.14856*	.02328	.000	.0807	.2164
	2.00	.14762*	.02333	.000	.0797	.2156
	3.00	-.00179	.03037	1.000	-.0874	.0838

*. The mean difference is significant at the 0.05 level.

The final portion of the sensitivity analysis was to test the means assuming an incompressibility factor of 0.20. The results for these tests were the same as assuming an M value of 0.15 and the MAPE values for the DeJong and S-Curve models were even higher at 35.8% and 35.7% respectively. These results are shown in Appendix D and not in the body of this thesis due to the redundancy from the earlier results.

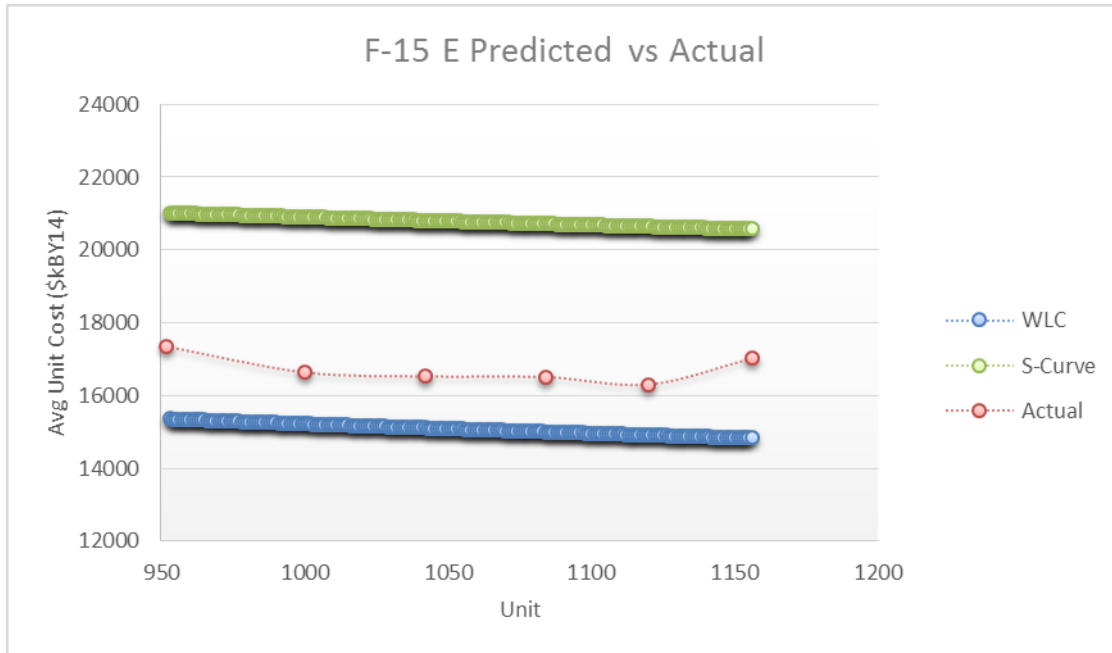


Figure 16: F-15E Predicted vs. Actual (M=0.15)

Conclusion

The purpose of this chapter was to provide the analytical results from the methods described in Chapter III. The tables and charts above describe test results for both the F-15 using a range of incompressibility assumptions from 0.0 to 0.20. The results varied as the value of the assumed incompressibility factor changed. A summary chart is shown below in Table 13.

Table 13: F-15 Analysis Summary

	$M=0.0$	$M=0.05$	$M=0.10$	$M=0.15$	$M=0.20$
WLC	N/A	N/A	N/A	N/A	N/A
Stanford-B	X	X	X	X	X
DeJong	X	-	X	+	+
S-Curve	X	-	X	+	+

X indicates model is not significantly different from WLC

(+) indicates model is statistically less accurate than WLC (Higher MAPE)

(-) indicates model is statistically more accurate than WLC (Lower MAPE)

When the factor was held at 0.0 or 0.1, there was no statistical difference between the models and these results reject all of the hypothesis. On the contrary, when the factor is held at 0.05, the DeJong and S-Curve models are more accurate and these findings support all three of the hypothesis. Chapter V will delve into the implications of the finding above; it will also give a brief description of the assumptions and limitations of the study and areas for improvement. Chapter V will conclude with the significance of these results as well as areas of future research and possible follow-on research topics.

V. Conclusions and Recommendations

Introduction

The purpose of this thesis was to determine if there are more accurate learning curve models than the conventional models currently used in Defense cost estimating. Four models were investigated through a series comparative tests: Wright's learning model (used as the status quo), the Stanford-B model, DeJong's learning formula, and the S-Curve model. The raw results from the hypotheses tests are shown in Chapter IV and Appendices. Chapter V will address the impacts of the findings and the effects they have on the research questions. The following section will examine what the test results indicate about each of the four models and if any conclusions can be drawn from the F-15 with regards to the research questions. There is also a section detailing the possible implications at the Air Force and DoD level and how the results may indicate a way forward in DoD methodology as a whole. The limitations of the study will also be addressed in this chapter and it will conclude with a discussion of possible follow-on research recommendations moving forward.

Conclusions of Research

The results of this research are inconclusive in regards to answering in the overarching research question of whether there is a more accurate learning curve model available for DoD use than Wright original formulation. However, the results do provide some insight into the effects of learning and where to go from here. The findings also emphasize the importance of incompressibility in the learning process. Slight changes in

the assumed incompressibility of the process lead to drastically different results as to which model is most accurate. This significance will be addressed later in the chapter.

The first hypothesis from this thesis was that at least one of the models would have a MAPE value statistically different from the others. This was not the case when the incompressibility factor was assumed to be 0.0 or 0.1, but the hypothesis holds for values of 0.05, 0.15 and 0.20. These results indicate that, although not uniformly, there does appear to be evidence that there is a statistical difference between at least two of the models. This result is important because it sets up the framework to be able to test the other hypotheses in the study.

The second hypothesis was that at least one model would have a MAPE value statistically lower than Wright's model. This hypothesis only held when the incompressibility factor was assumed to be 0.05 and in all of the other cases; there was no statistical difference at 0.1, and the models were actually less accurate than Wright's model when $M = 0.15$ and 0.20 . This finding indicates that as the process is assumed to be more automated, Wright's curve actually performs best. These results clearly do not fully support the second hypothesis, but do illustrate potential for learning curve improvement if an actual, universal incompressibility factor is found to be somewhere between 0.0 and 0.1. Post hoc analysis found that the S-Curve and DeJong models switch from being statistically more accurate to having no significant difference in MAPE value somewhere between 0.05 and 0.06. These results can be seen in Appendix E. The follow-on research section will provide potential impacts of a statistically supported incompressibility factor and how that factor could potentially support the findings from these results.

The final part of this analysis was to test which model was the most accurate between the four. The third hypothesis from this research was that the S-Curve model would be the most accurate because it accounts for the slow decline in performance over time due to forgetting. As with the second hypothesis, this hypothesis is only partially supported when the incompressibility factor is assumed to be 0.05, and rejected by the other results. At 0.05 both the DeJong and S-Curve models are more accurate than Wright's model, but there is no statistical difference between the two. These results lead to inconclusive outcomes about which model is best, but again point to a potential area of improvement in learning curve estimating and the importance of incompressibility.

The findings of this study lead to two additional theoretical questions: why were the results extremely sensitive to the incompressibility, and what conclusions can be drawn about the application of modern learning models in DoD acquisitions. While the second question will be addressed at the end of this chapter, the first question may be due to the data itself. The incompressibility factor essentially represents the amount of potential learning that is lost for each unit due to automated production processes. If an incompressibility factor is .3, then only 70% of the potential learning can be achieved. When compounded over several lots and units (over 1000 units for the F-15 A-E), a small shift in that percentage can result in a massive change in the cost of the units at the end of the production process.

This sensitivity affirms the need for additional research into incompressibility factors within the DoD and defense contractors in general. As mentioned earlier, the production of an aircraft is not that unlike the production of a high end sports car. The level of precision and craftsmanship required eliminates the use for certain automated

processes that may be present in an assembly line at Ford or Toyota. Given this dynamic, assuming the *real* incompressibility factor is somewhere between 0.0 and 0.1 is not farfetched. Follow up investigation involving inquiries to top practitioners in the learning curve field, including Dr. Badiru, support the belief that the percentage of automation is very, very small. Additionally, different defense contractors may use different production processes that result in different incompressibility factors and thus increase the sensitivity of the costs to those factors. This is yet another reason for future incompressibility research that will be described later in the chapter.

These results also indicate that learning is affected much more by incompressibility than prior experience units. The prior experience units parameter (B) was the differentiating parameter between the WLC and Stanford-B model, as well as the difference between DeJong's learning formula and the S-Curve model. One explanation for this result may be the large number of units produced for the F-15. When examining over 1100 units, a change to a mere ten of the units will have a very limited impact on the outcome. However, if the same prior experience units factor were applied to a smaller production line such as the B-2 bomber, the difference may become very significant. In all five cases, there was no statistical difference between the model and its close relative, meaning that the maximum change in B of 10 had no impact on the long term estimates of the models. Therefore, it is safe to assume that simply adding a prior experience units factor alone provides no value to the estimate if the production number is high, but the interaction between prior units and incompressibility could be very significant.

Significance of Research

The results above indicate that there is potential for a more accurate model in predicting the effects of learning within DoD acquisitions. This study was unique in two primary areas. First, it investigated Defense aircraft costs where past studies had primarily investigated commercial aircraft or component parts, and second, due to the nature of DoD cost estimating, it examines costs from an external perspective rather than internal and therefore the availability and accuracy of data may lead to more assumptions than prior studies.

Despite these intricacies, a few major conclusions can be drawn from the results. The first is that there is potential with two of the alternative learning curve models to increase estimate accuracy using learning curves by up to 5% over the entire production cycle based upon the results for an incompressibility factor of 0.05. Post hoc analysis indicated that the largest difference between the Wright and S-Curve models, just over 5.2%, was seen at 0.04 (these results can also be seen in Appendix E). While this percentage may seem small, for the \$20B+ production cycle of the F-15 A-E airframes, this percentage could result in a savings of over \$1B just by changing one estimating tool. This thesis does not go so far as to say current cost estimating methodology is wrong; cost estimates are just that, estimates. This research suggests and hopes to provide the foundation for ways to improve current learning curve methodology. Which model should be used is an area that requires more analysis. Thus far, the S-Curve and DeJong models appear to be worthy candidates. Further analysis incorporating incompressibility could reveal more information related to the application of the S-Curve and DeJong models and consequently, the theory of forgetting within DoD methodology.

While the findings of this study do not support all of the hypotheses of this research or indicate which model is the best predictor of future costs, they do open up a dialogue for future change in DoD acquisition methodology. These results stress the importance of incompressibility in learning and the potential for improvement based on that significance. Future research into incompressibility in aircraft production and comparative research into additional airframes as well as any of the dozens of other learning models available may help provide decision makers with additional information and hopefully increase the accuracy of cost estimates as a whole.

Assumptions and Limitations

As always, there are limitations to this research and the methods used to test the hypotheses. In addition to the limitations, there were some threats to external validity identified. One of those threats is the type of aircraft used in the analysis. It may prove that different types of aircraft provide different results and that one model may be more accurate for fighters but provide results that are non-significant for cargo aircraft. This research began by applying the methods only to fighter aircraft and open up the door for other researchers to expand the theory into other platforms and domains. However, dividing aircraft data into categories may spread an already small sample size too thin.

One major limitation to this study was the amount of data that was available to analyze. While the results of the analysis prove to be inconclusive, the data presented in this analysis is only a small fraction of all aircraft programs and an even smaller portion of DoD programs as a whole. AFLCMC/FCZ only has access to programs under their control, and only data from those programs which reported on learning curves. These

factors will limit the number of aircraft available for future analysis. A larger data-set would have been preferred, but in this case the sample was limited to the data available and adding one or two additional aircraft did not improve the validity of the results given the inconclusive nature of the results. Follow on analysis of incompressibility and additional Air Force and DoD programs is necessary before generalization of the findings can be made.

Another limitation is the accuracy of the data reported as actual costs. The accuracy or lack thereof in updating actual values for estimates has long been an issue in DoD and has just recently been brought to light in an effort to clean up data repositories. However, the fact that many of the programs are under AFLCMC/FCZ local control and span over multiple decades should help to mitigate some of the uncertainty of the results. An additional assumption was using the lot plot point with the cumulative average theory. Lot data is often used in DoD cost estimates due to the nature of contractor reports, but that type of analysis has not been applied to the additional models used in this analysis. However, the methods used were backed up by the Air Force Cost Analysis Handbook as well as other studies into learning curves. This methodology in addition to the fact that lot data is widely used throughout the DoD, should reduce the effect the lot plot point assumption has on the results while at the same time may make them more generalizable to individual unit data.

Recommendations for Future Research

This research answered several questions about the effects of learning in DoD, but there are still more questions that need to be addressed. This research sought to

determine if any alternative learning models are more accurate than Wright's model, which is commonly used throughout Defense acquisition programs today. This study took steps toward accomplishing that goal and found that the S-Curve and DeJong models may be more accurate if the incompressibility factor for aircraft production is found to be between 0.0 and 0.5. However, the evidence is inconclusive as to which model is the most accurate and whether or not the incompressibility assumption above is valid. Future research should look to expand upon these findings to determine which of these models, or any additional models, is the most accurate.

Additional research into incompressibility factors would prove valuable to this learning curve analysis and paramount to any additional research using these models. As mentioned earlier, one of the major assumptions from this study was using an incompressibility range from 0.0 to 0.2. Future research into what incompressibility factor should be used for aircraft production would provide insight into which models may be more appropriate and also provide further insight into the validity of these results. Also, analysis into how incompressibility factors change with different Defense contractors or how different platform types affect the production process could provide even more accuracy in this and future findings. Clarifying these uncertainties will help produce more accurate and useful cost estimates using the models described above.

Once a defensible and accurate incompressibility factor can be found, future research should also look to broaden the scope of the programs used in the analysis. This research focused on fighter aircraft and the initial pool of six was trimmed down to one aircraft. Follow on studies should attempt to incorporate the findings to additional platforms such as bombers, cargo/tanker, and unmanned aircraft. Also, the use of

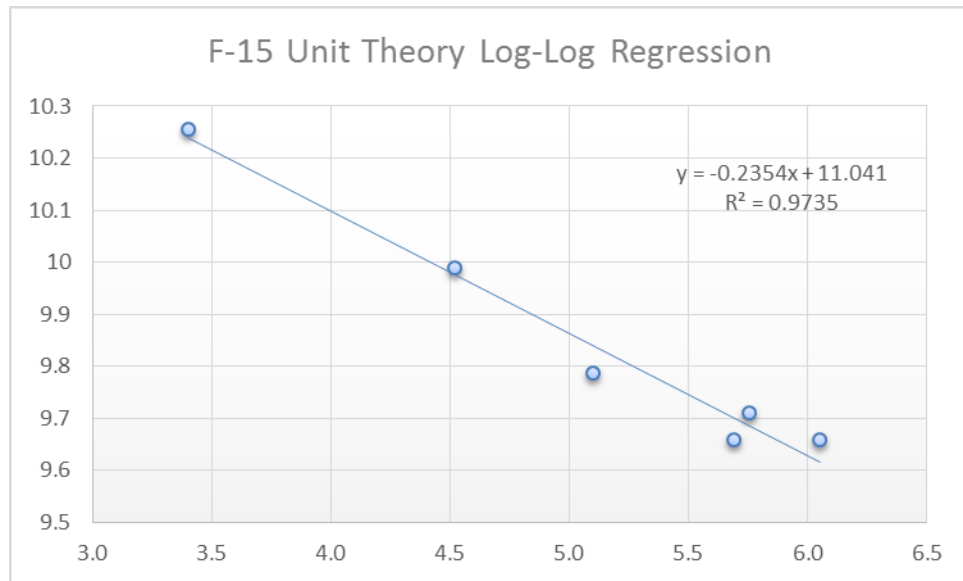
additional models that do not rely on the incompressibility factor would provide more robust results. Results from the analysis of the F-15 should not necessarily be generalized to all aircraft as a whole. Further analysis may shed light into which models perform best on which aircraft or if there is a single model that can be generalized to all platforms.

Summary

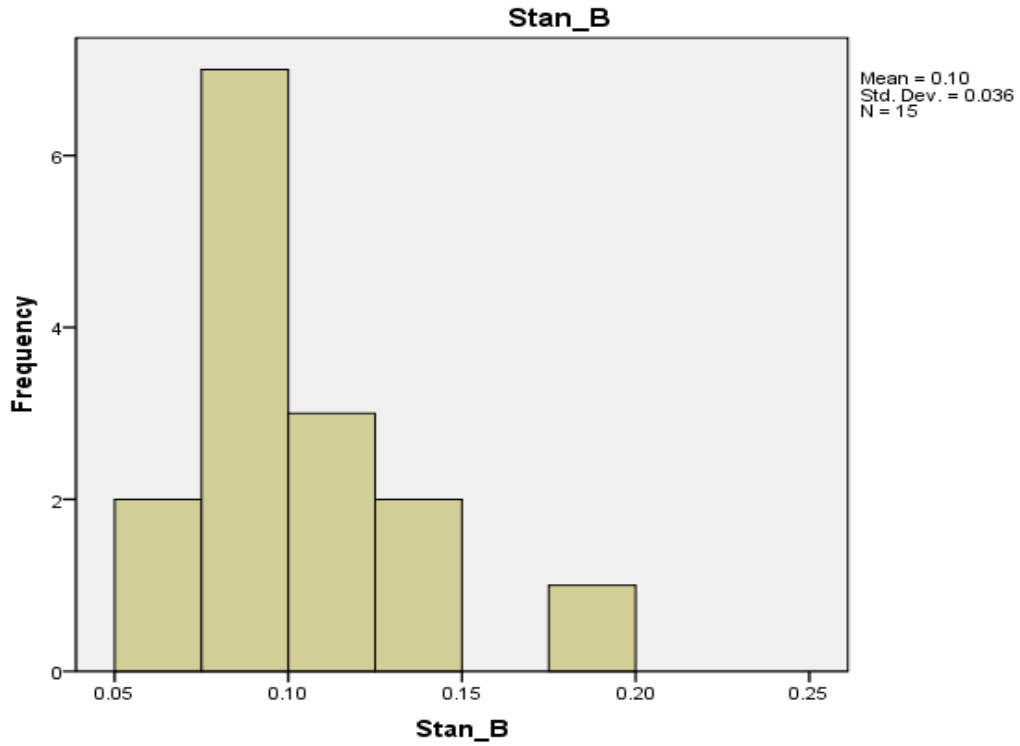
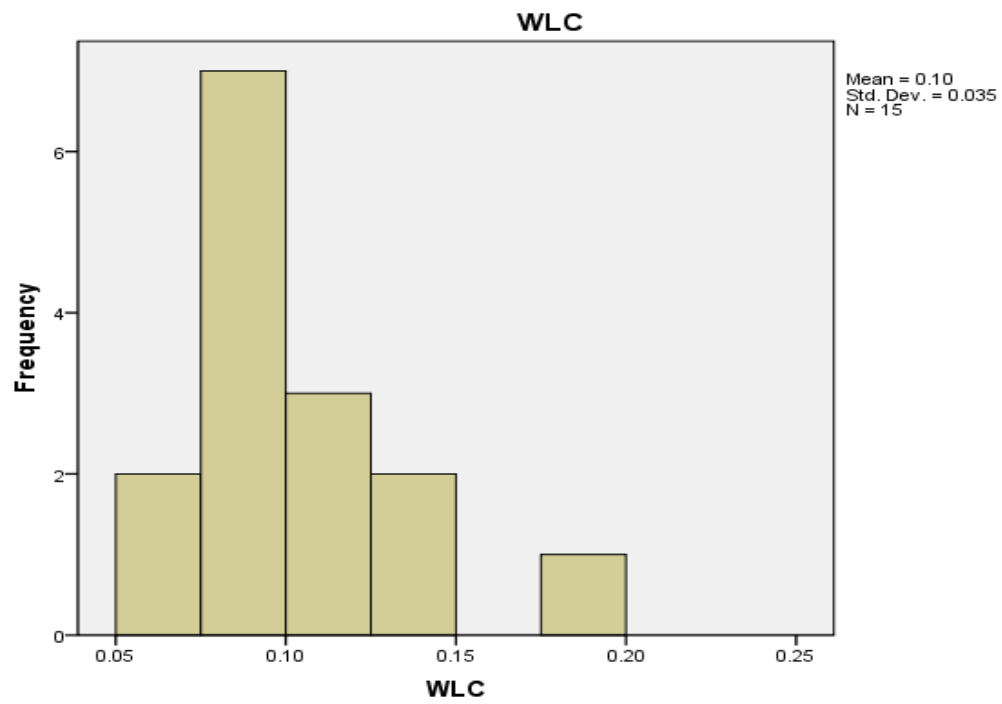
When this research began, the goal was to find out if a more accurate learning curve model than what is currently used in DoD exists. The AFLCMC cost staff supported the effort to find a way to improve current learning curve methodology in Defense acquisitions. Through the efforts of this thesis and the findings entailed within, there is evidence to support the hypothesis that at least one of the models may be more accurate than Wright's original model. This research found that both the DeJong and S-Curve models are statistically more accurate than the status quo given the incompressibility factor is somewhere between 0.0 and 0.5. However, if the factor is assumed to be .01 or higher, then Wright's model is the most accurate and the additional models do not improve on the current methodology. The results as to which model is the most accurate are inconclusive and do not support nor disprove the hypothesis that the S-Curve model is the most accurate of the four. At a minimum, this thesis provides the foundation for further research into additional types of aircraft as well as an applicable incompressibility factor that may indicate which model is the most accurate and then the alternative models can be considered for DoD methodology.

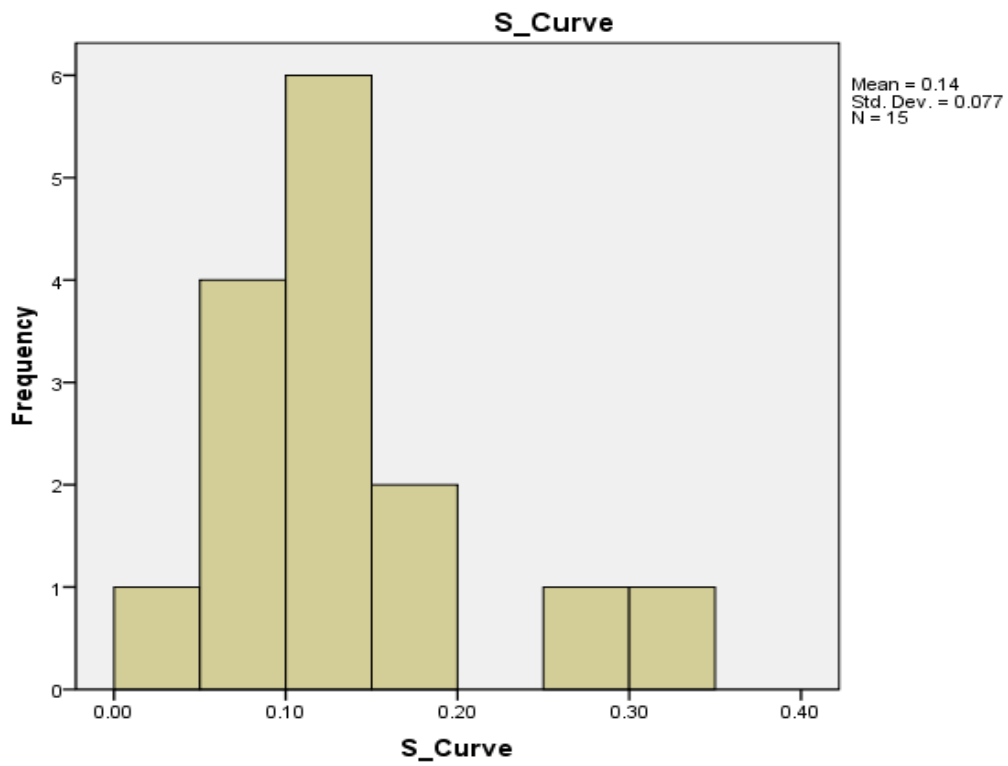
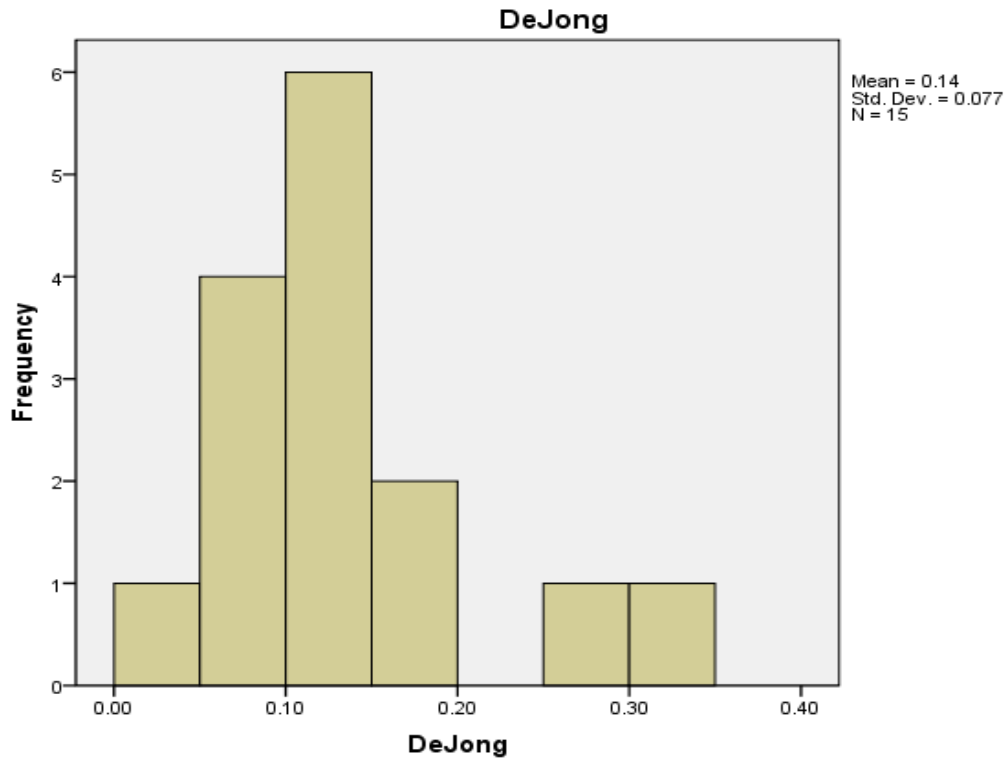
The argument behind this thesis is that the current DoD learning curve methodology using Wright's 75+ year old model should not be accepted as the status quo for the sake of simplicity or nostalgia. If a more accurate learning model exists that can be applied to cost estimating within the Defense department, it should be investigated and analyzed. While the results of this thesis are inconclusive in regards to which model may be the best, they do illustrate the point that there are additional models available that are more accurate in certain cases as well as provide the foundation for future research in Defense Acquisitions, which can hopefully increase the accuracy and reliability of cost estimates and create a more efficient use of government funding.

Appendix A



Appendix B





Appendix C

$$M = 0.0$$

Hypothesis Test Summary

	Null Hypothesis	Test	Sig.	Decision
1	The distribution of APE is the same across categories of Model.	Independent-Samples Kruskal-Wallis Test	.911	Retain the null hypothesis.

Asymptotic significances are displayed. The significance level is .05.

$$M = 0.05$$

Hypothesis Test Summary

	Null Hypothesis	Test	Sig.	Decision
1	The distribution of APE is the same across categories of Model.	Independent-Samples Kruskal-Wallis Test	.000	Reject the null hypothesis.

Asymptotic significances are displayed. The significance level is .05.

$$M = 0.15$$

Hypothesis Test Summary

	Null Hypothesis	Test	Sig.	Decision
1	The distribution of APE is the same across categories of Model.	Independent-Samples Kruskal-Wallis Test	.000	Reject the null hypothesis.

Asymptotic significances are displayed. The significance level is .05.

$$M = 0.20$$

Hypothesis Test Summary

	Null Hypothesis	Test	Sig.	Decision
1	The distribution of APE is the same across categories of Model.	Independent-Samples Kruskal-Wallis Test	.000	Reject the null hypothesis.

Asymptotic significances are displayed. The significance level is .05.

Appendix D

F-15 Descriptive Statistics ($M = 0.20$)

	N	Mean	Std. Deviation	Skewness		Kurtosis	
	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic	Std. Error
WLC	15	.0987	.03529	1.426	.580	3.247	1.121
Stan_B	15	.0997	.03584	1.378	.580	3.134	1.121
DeJong	15	.3584	.08824	.563	.580	1.765	1.121
S_Curve	15	.3568	.08788	.547	.580	1.754	1.121
Valid N (listwise)	15						

F-15 Dunnett T3 Test ($M = 0.20$)

(I) Model	(J) Model	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
1.00	2.00	-.00094	.01299	1.000	-.0375	.0356
	3.00	-.25972*	.02454	.000	-.3314	-.1880
	4.00	-.25804*	.02445	.000	-.3295	-.1866
2.00	1.00	.00094	.01299	1.000	-.0356	.0375
	3.00	-.25877*	.02459	.000	-.3306	-.1870
	4.00	-.25709*	.02451	.000	-.3286	-.1855
3.00	1.00	.25972*	.02454	.000	.1880	.3314
	2.00	.25877*	.02459	.000	.1870	.3306
	4.00	.00168	.03216	1.000	-.0889	.0923
4.00	1.00	.25804*	.02445	.000	.1866	.3295
	2.00	.25709*	.02451	.000	.1855	.3286
	3.00	-.00168	.03216	1.000	-.0923	.0889

*. The mean difference is significant at the 0.05 level.

Appendix E

Results for $M = 0.06$

Descriptive Statistics

	N	Minimum	Maximum	Mean	Std. Deviation	Skewness		Kurtosis	
	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic	Std. Error
	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic	Std. Error
WLC	15	.0530	.1958	.098720	.0352870	1.426	.580	3.247	1.121
StanB	15	.0509	.1976	.099662	.0358351	1.378	.580	3.134	1.121
DeJong	15	.0046	.2342	.063668	.0652121	1.765	.580	2.962	1.121
SCurve	15	.0036	.2310	.062168	.0645087	1.755	.580	2.933	1.121
Valid N (listwise)	15								

Multiple Comparisons

AbsPE

Dunnett t (2-sided)^a

(I) ModelType	(J) ModelType	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
2	1	.0009424	.0190991	1.000	-.045171	.047056
3	1	-.0350517	.0190991	.174	-.081165	.011061
4	1	-.0365514	.0190991	.149	-.082665	.009562

a. Dunnett t-tests treat one group as a control, and compare all other groups against it.

Results for $M = 0.04$

Descriptive Statistics

	N	Minimum	Maximum	Mean	Std. Deviation	Skewness		Kurtosis	
	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic	Std. Error
	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic	Std. Error
WLC	15	.0530	.1958	.098720	.0352870	1.426	.580	3.247	1.121
StanB	15	.0509	.1976	.099662	.0358351	1.378	.580	3.134	1.121
DeJong	15	.0045	.1871	.047245	.0558451	1.700	.580	1.903	1.121
SCurve	15	.0030	.1837	.046700	.0553423	1.635	.580	1.702	1.121
Valid N (listwise)	15								

Multiple Comparisons

AbsPE

Dunnett t (2-sided)^a

(I) ModelType	(J) ModelType	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
2	1	.0009424	.0170399	1.000	-.040199	.042084
3	1	-.0514745 [*]	.0170399	.011	-.092616	-.010333
4	1	-.0520198 [*]	.0170399	.010	-.093161	-.010879

a. Dunnett t-tests treat one group as a control, and compare all other groups against it.

*. The mean difference is significant at the 0.05 level.

Bibliography

- Andelohr, G. (1969). What Production Breaks Cost. *Industrial Engineering* , 34-36.
- Badiru, A. B. (1995). Multivariate Analysis of the Effects of Learning and Forgetting on Product Quantity. *International Journal of Production Research* , 777-794.
- Badiru, A. (2012). Half Life Learning Curves in the Defense Acquisition Life Cycle. *Defense ARJ* , 283-308.
- Badiru, A., Elshaw, J., & Mack, E. (2013). Half Life Learning Curve Computations for Airframe Life-cycle Costing of Composite Manufacturing. *Journal of Aviation and Aerospace Perspective* , 3, No. 2, 6-37.
- Bailey, C. D. (1989). Forgetting and the Learning Curve: A Laboratory Study. *Management Science* , 340-352.
- Benkard, C. L. (2000). Learning and Forgetting: The Dynamics of Aircraft Production. *The American Economic Review* , 1034-1054.
- Carr, G. W. (1946). Peacetime Cost Estimating Requires New Learning Curves. *Aviation* , 76-77.
- Cherrington, D. R. (1994). Learning Curve Models for Predicting the Performance of ATM. *The International Journal of Advanced Manufacturing Technology* , 195-203.
- Crawford, J. R. (1947). *Learning Curve, Ship Curve, Ratios, Related Data*. . Burbank, California: Lockheed Aircraft Corporation.
- Department of Defense (2007). *Basic Cost Estimating Guidebook*. Washington D.C.: U.S. Department of Defense.
- Department of the Air Force (2007). *Air Force Cost Estimating Handbook*. Washington D.C.: U.S. Department of the Air Force.
- Henneberger, A. K. (1993). *Productivity in Aircraft Manufacturing*. Washington, D. C.: U.S. Bureau of Labor Statistics.
- Hu, S.-P., & Smith, A. (2013). Accuracy Matters: Selecting a Lot-Based Cost Improvement Curve. *Journal of Cost Analysis and Parametrics*, 23-42.

- Jaber, M. Y. (2011). *Learning Curves: Theory, Models and Applications*. Boca Raton, FL: CRC Press.
- Jaber, M. Y., & Sikstrom, S. (2004). A Numerical Comparison of Three Potential Learning and Forgetting Models. *Internation Journal of Production Economics* , 281-294.
- James T. McClave, P. G. (2011). *Statistics for Business and Economics 11th Edition*. Boston, MA: Pearson Education Inc.
- Kar, A. M. (2007). *A Cost Modeling Approach Using Learning Curves to Study the Evolution of Technology*. Cambridge, MA: Massachusetts Institute of Technology.
- Nembhard, D. a. (2001). An Empirical Comparison of Forgetting Models. *IEEE Transactions of Engineering Management* , 283-291.
- Nembhard, D., & Uzumeri, M. (2000). Exponential Learning and Forgetting for Manual and Cognitive Tasks. *International Journal of Industrial Ergonomics* , 315-326.
- Office of the Under Secretary of the Air Force Acquisition, Technology and Logistics. (2014). *Better Buying Power 3.0*. Washington D.C.: Office of the Under Secretary of the Air Force Acquisition, Technology and Logistics.
- Wright, T. (1936). Factors Affecting the Cost of Airplanes. *Journal of the Aeronautical Sciences* , 122-128.
- Yelle, L. E. (1979). The Learning Curve: Historical Review and Comprehensive Study. *Decision Science* , 302-328.

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14. ABSTRACT The goal of this research was to identify which learning curve model is most accurate when applied to Defense acquisition programs. Wright's original learning curve model is widely accepted and used within Defense acquisitions, but the 75+ year old model may be outdated. This study compares Wright's model against three alternative learning curve models using total lot costs for the F-15 C/D & E programs: the Stanford-B model, the DeJong learning formula, and the S-Curve model. However, the results of the study are inconclusive. Two of the three alternative models, the DeJong and S-Curve, rely on the use of an incompressibility factor between 0 and 1 that represents the percentage of the production process that is automated. A Bureau of Labor Statistics report identifies that percentage as very low but does not give an exact number. Therefore assumptions about that parameter were made. When the factor falls between 0.0 and 0.1 the DeJong and S-Curve models appear to be more accurate; when the number is 0.1 or greater, Wright's model is still the most accurate. Further research should be targeted at the exact value of this factor to validate this, or future, comparative studies					
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