Comparison of three Approximate kinematic Models for Space Object Tracking

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***Abstract*—In this paper we compare the performance of three kinematic state models, i.e., the White Noise Acceleration (WNA), the Wiener process acceleration (WPA), and the Keplerian State (KPS) model, for the tracking of earth orbiting space objects (SOs). The three models considered are all simplified approximate models for the motion of Earth orbiting SOs and are not suitable for the prediction of target tracks for long time periods. However, for track updates with new measurements coming at a high rate, such simplified motion models can be effectively used with small or no loss in estimation accuracy. For the KPS model, we use a novel mixed-coordinate SO tacking (McSOT) filter, where the target state space is defined in the Cartesian, i.e., Earth-Central Inertial (ECI), coordinates for track representation and updates, while the track propagation is done in the Keplerian Coordinates. It is shown that when the measurement accuracy is high, the McSOT filter with the KPS model, which has the highest complexity among the three, is able to achieve significantly better estimation accuracy than the filters with the WNA and WPA models. The WPA model is able to achieve better tracking accuracy than the WNA model at the cost of moderate increase of algorithm complexity. On the other hand, when the measurement accuracy is low, the filters with the WNA and WPA models which operate solely in the Cartesian coordinates, i.e., the Earth-Central Inertial (ECI) coordinates, is more robust than the McSOT filter with the KPS model.**

Keywords*—*tracking; space situation awareness; kinematic model; filter design, Keplerian state model,

I. INTRODUCTION

Tracking of space objects (SOs) is an important task for space surveillance [16]. In addition to tacking accuracy [6, 8, 9], complexity of the tracking algorithm are just as important due to the large amount of SOs (over 20k) currently orbiting the Earth that need to be tracked. The selection of the tracking algorithm and its associated comparisons are important for sensor management [5, 7], collision avoidance [18, 21], and cooperative search [17]. In addition, the selection of the tracking algorithm and its results impact space situation awareness [3] and coordinated game-theoretical threat detection [4, 5, 18]

Under the effect of Earth gravity and multiple perturbing forces in the complex space environment, including third body gravity, atmospheric drag, and solar radiation pressure (SRP), orbital dynamics of the SOs are significantly nonlinear and highly complex, which is further complicated by many unknown characteristics of the SOs and the space environment [10,12]. Different from the tracking of ground and airborne targets where dedicated sensors are used to provide a constant flow of measurements, there is few dedicated sensor resources for space surveillance, and the SOs are only observed during a very small fraction of their orbiting period (with the short-arc observations) [10,20]. State estimates and orbit parameters of a SO will be obtained during the short observation period and, for the rest of orbiting period, the target states are obtained by propagating the estimated over time. For the propagation of SO states over a long unobserved time period, the use of high fidelity kinematic models for the SOs is crucial, which involve complex dynamics and require complex numerical integrations [11]. However for the state estimation during the short observation period, simplified motion models can be effectively used to drastically reduce complexity and delay of the tracking algorithm with small or no loss in estimation accuracy. This is mainly because that impact of the complex perturbation forces such as the SRP and third body gravity is negligible during the observation time interval and can be effectively accounted by a small level of process noises in the simplified kinematic models. In this paper we investigate three simplified kinematic models for the SO state estimation, which are the White Noise Acceleration (WNA), the Wiener process acceleration (WPA) , and the Keplerian State (KPS) model to demonstrate their performance in terms of tacking accuracy and robustness.

The paper is organized as follows. Section II introduces the three kinematic models considered and addresses the design issues for the SO tracking. Section III presents the simulation scenarios used in the paper, which involve the tracking of a Low Earth Orbit (LEO) satellite. The simulation results are presented in Section IV. Section V summarizes the paper with concluding remarks.

II. THREE SIMPLIFIED KINEMATIC MODELS FOR SPACE OBJECT TRACKING

A. The White Noise Acceleration (WNA) Model

The WNA model [1] is a simple linear kinematic model commonly used in tracking applications. It models (slight) changes in target velocity as a zero mean white Gaussian noise. For SO tracking the 3-Dimensional (3D) state space consists of position and velocity in 3D.

$$\bar{x} = [x \;\; \dot{x} \;\; y \;\; \dot{y} \;\; z \;\; \dot{z}]' \tag{1}$$

Assuming a sampling period of *T*, the discrete-time state equation is given by

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14. ABSTRACT

In this paper we compare the performance of three kinematic state models, i.e., the White Noise Acceleration (WNA) the Wiener process acceleration (WPA), and the Keplerian State (KPS) model, for the tracking of earth orbiting space objects (SOs). The three models considered are all simplified approximate models for the motion of Earth orbiting SOs and are not suitable for the prediction of target tracks for long time periods. However, for track updates with new measurements coming at a high rate, such simplified motion models can be effectively used with small or no loss in estimation accuracy. For the KPS model, we use a novel mixed-coordinate SO tacking (McSOT) filter, where the target state space is defined in the Cartesian, i.e., Earth-Central Inertial (ECI), coordinates for track representation and updates, while the track propagation is done in the Keplerian Coordinates. It is shown that when the measurement accuracy is high, the McSOT filter with the KPS model, which has the highest complexity among the three, is able to achieve significantly better estimation accuracy than the filters with the WNA and WPA models. The WPA model is able to achieve better tracking accuracy than the WNA model at the cost of moderate increase of algorithm complexity. On the other hand when the measurement accuracy is low, the filters with the WNA and WPA models which operate solely in the Cartesian coordinates, i.e., the Earth-Central Inertial (ECI) coordinates, is more robust than the McSOT filter with the KPS model.

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$$\vec{x}(k+1) = F_1\vec{x}(k) + v_1(k) \tag{2}$$

where

$$F_1 = \begin{bmatrix} F_{11} & 0 & 0 \\ 0 & F_{11} & 0 \\ 0 & 0 & F_{11} \end{bmatrix} \text{ with } F_{11} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix};$$

and $v_1(k)$ is a zero mean Gaussian process noise whose covariance is given by

$$Q_1 = \begin{bmatrix} Q_{11} & 0 & 0 \\ 0 & Q_{11} & 0 \\ 0 & 0 & Q_{11} \end{bmatrix} \text{with } Q_{11} = \begin{bmatrix} \frac{1}{3}T^3 & \frac{1}{2}T^2 \\ \frac{1}{2}T^2 & T \end{bmatrix} \tilde{q}_1$$

where $\tilde{q}_1$ is intensity of the corresponding continuous-time noise process, which has a physical unit of $[\text{length}]^2/[\text{time}]^3$. For consistent state propagation with the WNA kinematic model, the magnitude of change of velocity during the sampling time T should be of the same order: $\sqrt{Q_{11}(2,2)} = \sqrt{T\tilde{q}_1}$ [1]. According to this guideline, for the tracking of Earth orbiting SO, the change of velocity during T is mainly caused by the Earth gravity. One has approximately the magnitude of the acceleration as

$$g = G\frac{M_1}{r^2} \tag{3}$$

where $G = 6.6742 \times 10^{-11}$ is the gravitational constant, $M_1 = 5.9736 \times 10^{24}$ is the mass of the Earth in kilograms and r is the distance of the SO to the center of the Earth, i.e., the distance to the origin of the ECI coordinates. Using these parameters, one has

$$\begin{aligned} &\sqrt{T\tilde{q}_1} \propto gT \Rightarrow \tilde{q}_1 \propto g^2T \Rightarrow \tilde{q}_1 \propto \frac{T}{r^4} \\ &\Rightarrow \sqrt{Q_{11}(1,1)} \propto \frac{T^2}{r^2} \end{aligned} \tag{4}$$

Eq. 4 shows that the smaller the sampling interval T and the further away the SO is from the Earth, r, the smaller the required intensity of the process noise $\tilde{q}_1$ will be, which means the motion of the SO can be more accurately modified the WNA model and with less loss of tracking accuracy, i.e., smaller uncertainty in position $\sqrt{Q_{11}(1,1)}$, and velocity $\sqrt{Q_{11}(2,2)}$ to account for the model mismatch. In Section III, we will show the actual choice of $\tilde{q}_1$ for the tracking of a LEO satellite and compare it with the designs of the other two models.

B. The Wiener Process Acceleration (WPA) State Model

The WPA model [1] is a more complex model than the WNA model, where the acceleration is modeled as an Wiener process. For SO tracking the state space is given by

$$\vec{x} = [x \;\; \dot{x} \;\; \ddot{x} \; y \;\; \dot{y} \;\; \ddot{y} \;\; z \;\; \dot{z} \;\; \ddot{z}]' \tag{5}$$

The discrete-time state equation is

$$\vec{x}(k+1) = F_2\vec{x}(k) + v_2(k) \tag{6}$$

where

$$F_2 = \begin{bmatrix} F_{22} & 0 & 0 \\ 0 & F_{22} & 0 \\ 0 & 0 & F_{22} \end{bmatrix} \text{ with } F_{22} = \begin{bmatrix} 1 & T & \frac{1}{2}T^2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix}$$

The covariance matrix of $v_2(k)$ is given by

$$Q_2 = \begin{bmatrix} Q_{22} & 0 & 0 \\ 0 & Q_{22} & 0 \\ 0 & 0 & Q_{22} \end{bmatrix} \text{with}$$

$$Q_{22} = \begin{bmatrix} \frac{1}{20}T^5 & \frac{1}{8}T^4 & \frac{1}{6}T^3 \\ \frac{1}{8}T^4 & \frac{1}{3}T^3 & \frac{1}{2}T^2 \\ \frac{1}{6}T^3 & \frac{1}{2}T^2 & T \end{bmatrix} \tilde{q}_2$$

where $\tilde{q}_2$ is the process noise intensity, whose choice should follow the guideline that the changes in the acceleration over a sampling period T are of the order of $\tilde{q}_2 T$ [1]. For an Earth orbiting SO (assuming a circular motion for the sake of simplicity) the major change in acceleration is due to the direction change of the gravity during the time period. One has

$$\begin{aligned} &\sqrt{T\tilde{q}_2} \propto g\omega T \Rightarrow \tilde{q}_2 \propto g^2\omega^2 T \\ &\Rightarrow \tilde{q}_2 \propto g^3 rT \Rightarrow \tilde{q}_2 \propto \frac{T}{r^5} \\ &\Rightarrow Q_{22}(1,1) \propto \frac{T^3}{r^{2.5}} \end{aligned} \tag{7}$$

Similar to $\tilde{q}_1$ for the WNA model the required $\tilde{q}_2$ decreases with smaller sampling interval and larger range. However, accuracy of the WPA model changes more rapidly w.r.t. the sampling interval T and range r. In addition, for Earth orbiting SOs, the change of acceleration during the sampling period is much smaller compared to the velocity change, which leads to smaller covariance and better accuracy than the WNA model with reasonably selected sampling time interval (see Section III and IV).

C. The Keplerian State (KPS) Model

The motion of an Earth orbiting SO follows Newton's law of universal gravitation and is nonlinear in the ECI (Cartesian) Coordinates. The orbit information is better described with the ***Keplerian elements*** [22]. In the Keplerian space (with six Keplerian elements), the state of an orbital motion is defined as

$$S=[a,e,i,\Omega,w,M_0]' \tag{8}$$

where ' is the transpose operator; *e*, the eccentricity, and *a*, the semi-major axis of the orbit, determine the shape and size of the orbit (as illustrated in Fig.1); the inclination *i* and the longitude of the ascending node Ω define the orientation of the orbit plane to a reference plane (for Earth-orbiting satellites, the reference plane is usually the Earth's equatorial plane); the argument of periapsis *w* which specifies the orientation of the ellipsis in the orbit plane, and the mean anomaly at epoch (M_0) that defines the position of the orbiting body along the ellipse at the "epoch"; it is not a real geometric angle, instead the true anomaly (ν) is shown in Fig.1.

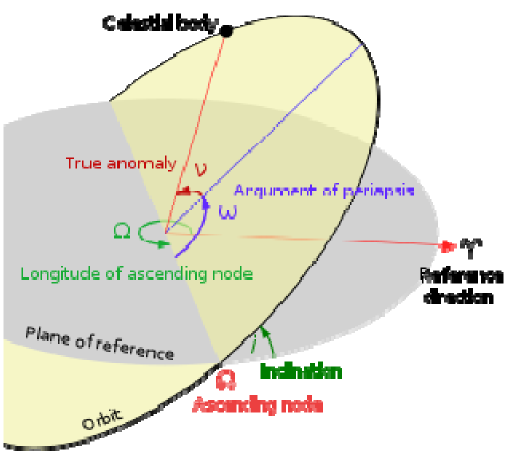

Figure 1. Illustration of Keplerian elements [22]

Under ideal conditions of a perfectly spherical central body, and zero perturbations, all orbital elements, with the exception of the mean anomaly are constants, and mean anomaly changes linearly with time.

$$S(t+\Delta t)=[e,a,i,\Omega,w,M+n\Delta t]'=S(t)+[0,0,0,0,0,n\Delta t]' \tag{9}$$

Where

$$n=\sqrt{\frac{G(M_1+m_2)}{a^3}}=\sqrt{\frac{\mu}{a^3}}=2\pi/P \tag{10}$$

is the mean motion. In (10) *G* is the gravitational constant, M_1 and m_2 are the masses of the orbiting bodies and *P* is the orbit period.

However, the tracking filter cannot operate in the Keplerian coordinates, since the distribution of the states would be far from Gaussian. To solve the problem we use a novel mixed-coordinate space object tacking (McSOT) approach, where the target state space is defined in the Cartesian (ECI) coordinates. For tracking, we define the state of an orbital object in the ECI Coordinates for track representation and update. The six dimensional state vector consists of position, velocity in 3D space, namely,

$$\bar{x}=[x\ \ \dot{x}\ \ y\ \ \dot{y}\ \ z\ \ \dot{z}]' \tag{11}$$

The six dimensional state (without acceleration) in (11) can be converted to the **Keplerian elements** that fully characterize the orbital movement at any given time instance. However the **track propagation** will be conducted in the Keplerian coordinates using the following procedure.

First the track in the ECI coordinates (consisting of the state estimate $\hat{x}(k\,|\,k)$ and $P(k\,|\,k)$ the covariance) is converted to a sigma-point representation [2, 14] as

$$x^i(k)=\hat{x}(k\,|\,k)+c\,\mathrm{sgn}(i)\left[P(k\,|\,k)\right]_i^{1/2},\ \ i=-6,\dots,6 \tag{12}$$

where $\left[P(k\,|\,k)\right]_i^{1/2}$ is the *i*-th column of the Cholesky factor (square root) of $P(k\,|\,k)$. The weights associated with the sigma points are

$$w_i=\begin{cases}\dfrac{1}{2c^2}, & |i|=1,\dots,6\\[2ex] \dfrac{c^2-6}{c^2}, & i=0\end{cases} \tag{13}$$

which sum up to unity for any *c*. Then the sigma points are converted to the Keplerian state space by

$$S^i(k)=f_{ECI2Kep}(x^i(k)),\ \ i=-6,\dots,6 \tag{14}$$

where $f_{ECI2Kep}()$ denotes the state mapping from the ECI coordinates to the Keplerian coordinates. Due to the limited space of the paper, the mapping algorithm [25] is not presented. The predicted state $S^i(k+1)$ can be easily performed using (9).

Then $S^i(k+1)$ are converted back to the ECI coordinates as

$$x^i(k+1)=f_{Kep2ECI}(S^i(k+1)),\ \ i=-6,\dots,6 \tag{15}$$

where $f_{Kep2ECI}()$ denotes the state mapping from the Keplerian coordinates to the ECI coordinates [23].

Finally the predicted track can be obtained by the weighted sum of the sigma points from (15).

$$\hat{x}(k+1\,|\,k)=\sum_{i=-6}^{6}w^i x^i(k+1) \tag{16}$$

with covariance

$$P(k+1\,|\,k)=$$
$$\sum_{i=-6}^{6}w^i\left\{[x^i(k+1)-\hat{x}(k+1\,|\,k)][x^i(k+1)-\hat{x}(k+1\,|\,k)]'\right\}+Q_3 \tag{17}$$

where Q_3 is the covariance of the cumulated process noise over the sampling interval T to account for perturbations, frictions and other non-ideal factors of the orbital movement. Here a similar covariance matrix as in WNA model is used.

$$Q_3 = \begin{bmatrix} Q_{33} & 0 & 0 \\ 0 & Q_{33} & 0 \\ 0 & 0 & Q_{33} \end{bmatrix} \text{with } Q_{33} = \begin{bmatrix} \frac{1}{3}T^3 & \frac{1}{2}T^2 \\ \frac{1}{2}T^2 & T \end{bmatrix} \tilde{q}_3$$

where $\tilde{q}_3$ is continuous time process noise intensity of the orbital motion. Since the levels of those cumulated process noises over the short sampling time interval T are close to zero, a very small intensity can be used. See section III for the practical choice of $\tilde{q}_3$.

It can be seen that track propagation with the KPS model is the most accurate one among the three. Arguably the model has practically no loss in estimation accuracy due to model mismatch for the very small choice of $\tilde{q}_3$ and the resulting covariance Q_3. On the other hand, it is much more complicated than the WNA and WPA models due to the use of the multiple sigma-points and back-and-forth coordinate conversions. (Note that it is still much more efficient than the more accurate Kinematic models that require numerical integration.) The performance question is: Is it always beneficial to use the KPS model? The next two sections will address this question with the assistance of a simulated Earth orbiting SO tracking scenario.

III. THE SIMULATION SCENARIO

The simulation scenario used is the tracking of a Low Earth Orbit (LEO satellite) with a sensor (e.g., radar station) on the ground. Fig.2 shows the radar station (denoted as the yellow circle) and the target trajectory (in light green) during an observation time period of approximately 10 minutes.

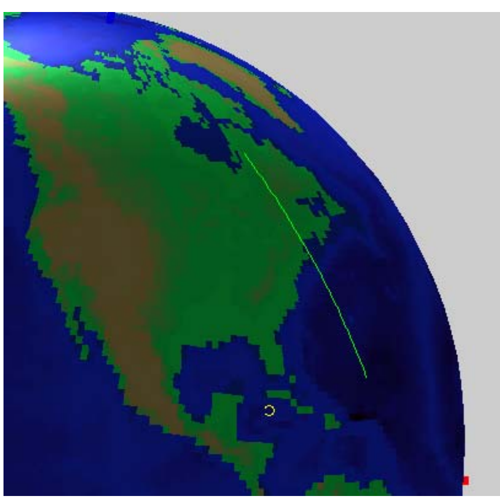

Figure 2. Scenario 1 - Keplerian Model

During the observation period, the radar obtains range and angle measurements (azimuth and elevation) of the satellite. For the sake of simplicity and with no change to the results of the paper, it is assumed that the sensor's local coordinates aligns to the ECI coordinates. The measurements are related to the target state by the following equations.

$$\begin{aligned} r_m &= \left((x-x_s)^2 + (y-y_s)^2 + (z-z_s)^2\right)^{\frac{1}{2}} + w_r \\ a_m &= \arctan((y-y_s)/(x-x_s)) + w_\mathrm{a} \\ \mathrm{e_m} &= \arctan((z-z_s)/((x-x_s)^2 + (y-y_s)^2)^{\frac{1}{2}}) + w_\mathrm{e} \end{aligned} \tag{17}$$

where w_r, w_a and w_e are the measurement noises on range, azimuth and elevation; respectively. They are assumed to be independent zero mean white Gaussian noises with standard deviation σ_r, σ_a and σ_e; respectively

Two cases were used to demonstrate the performance of the tracking filters with the three kinematic models considered. In the first case (as shown in Fig.2) the SO orbit was generated according to the following Keplerian elements values:

Eccentricity:	e=0 (circular orbit)
Semi-major axis:	*a*=8000km
Inclination:	*i*=70 deg
longitude of the ascending node:	Ω=70 deg
argument of periapsis:	*w*=0

In this case, the KPS model is an exact match to the true target motion dynamics.

In the second case, the target trajectory was generated based on the two line elements (TLE) [26] of a satellite in the Iridium constellation using the SGP4 model [27], which introduces the effect of perturbation forces to the SO motion in space environments. Fig.3 shows the target trajectory (in red) during the observation period.

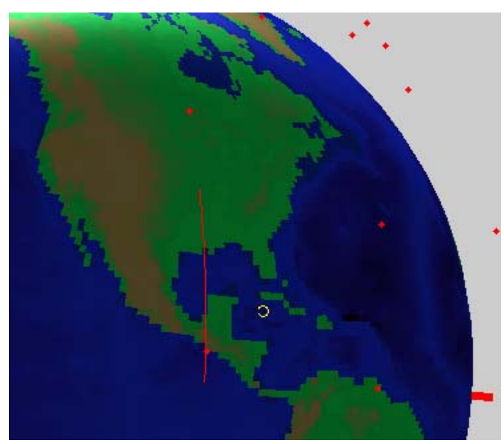

Figure 3. Scenario 2 - SGP4 model

IV. THE SIMULATIONS AND RESULTS

In **simulation 1** the Keplerian scenario case in Section III, is used with the simulation parameters set as: Sampling interval

T=5s, $\sigma_r = 30$ m, $\sigma_a = 0.01^o$, $\sigma_e = 0.01^o$. With the distance the center of the earth at about 8000km, and according to (3) the gravitational acceleration is about 6m/s^2. Based on the discussion in section II-A, $\sqrt{\tilde{q}_1 T}$ should has the same order of $gT \approx 30$. In the simulation, $\tilde{q}_1$ is chosen as 50 and the corresponding for the WNA model. For the corresponding process noise covariance, one has

$$Q_{11} = \begin{bmatrix} \frac{1}{3}T^3 & \frac{1}{2}T^2 \\ \frac{1}{2}T^2 & T \end{bmatrix} \tilde{q}_1 \approx \begin{bmatrix} 2000 & 600 \\ 600 & 250 \end{bmatrix} \quad (18)$$

For the WPA model, as discussed in section II-B, $\sqrt{T\tilde{q}_2}$ should be on the order of $g\omega T = 0.0275$. The choice of in the simulation is $\tilde{q}_2 = 5\times10^{-4}$ and correspondingly $\sqrt{T\tilde{q}_2} = 0.05$. One has

$$Q_{22} = \begin{bmatrix} 0.0781 & 0.0391 & 0.0104 \\ 0.0391 & 0.0208 & 0.0063 \\ 0.0104 & 0.0063 & 0.0025 \end{bmatrix} \quad (19)$$

For the KPS model, as discussed in section II-C, the process noise intensity is set as $\tilde{q}_3$=0.0025 in the simulation, which yields

$$Q_{33} = \begin{bmatrix} \frac{1}{3}T^3 & \frac{1}{2}T^2 \\ \frac{1}{2}T^2 & T \end{bmatrix} \tilde{q}_3 \approx \begin{bmatrix} 0.1042 & 0.0313 \\ 0.0313 & 0.0125 \end{bmatrix} \quad (20)$$

It can be seen that both Q_{22} and Q_{33} are very small. However Q_{11} is very large in order to deal with the very significant model mismatch of the WNA.

For **track initialization**, the 2-point differencing method in [1] is used for tracking filters with the WNA and KSP models, since their state vector contain only positions and velocities. For the WPA model, a modified 2-point differencing method was used, where the position and velocity states and their covariance are initialized using the standard 2-point differencing method, while the acceleration state components are initialized as

$$[\hat{\ddot{x}} \;\; \hat{\ddot{y}} \;\; \hat{\ddot{z}}]' = -[\frac{\hat{x}}{\hat{r}}\hat{g} \;\; \frac{\hat{y}}{\hat{r}}\hat{g} \;\; \frac{\hat{z}}{\hat{r}}\hat{g}]' \quad (21)$$

where the "hat" indicates estimated values of the coordinates, range and gravitational acceleration. A conservative covariance

$$Q_a = \begin{bmatrix} 0.005 & 0 & 0 \\ 0 & 0.005 & 0 \\ 0 & 0 & 0.005 \end{bmatrix} \quad (22)$$

is associated with the acceleration state components.

For **track updates**, the standard Extended Kalman filter (EKF) [2] was used for the three filters due to insignificant measurement nonlinearity for the levels of the measurement accuracies. The simulation results presented next were based on 100 Monte Carlo runs.

Fig.4 shows the consistency in terms of the Normalized Estimation Error Squared (NEES) [1] of the filters with the three kinematic models. Note that for the WPA model, only the position and velocity states and their covariance were used for the evaluation (acceleration components were not included).

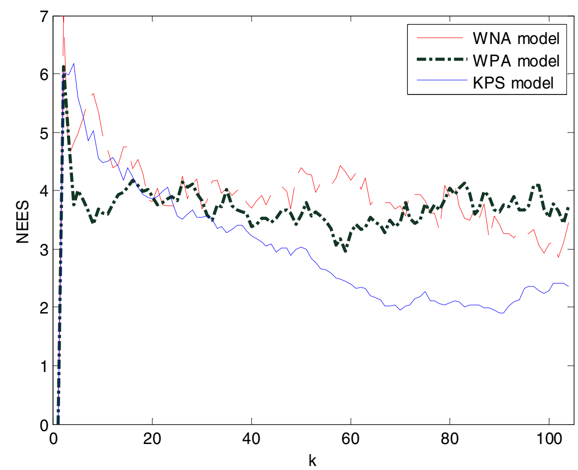

Figure 4. consistency test of the filters
(Keplerian scenario: Case 1)

It can be seen that, for all the three filters, the NEES are below 6 which means the estimated covariances of the states cover well the estimation errors. After the beginning stage, the NEES of the filters dropped to around 4, which means the choice of process noise is a bit conservative. However, verified through simulations, this will not cause visible impact on the achieved tracking accuracy.

Fig.5 compares the overall Root Mean Square Errors (RMSE) [1] in positions of the filters with the three models. It can be seen that the WNA model has significant loss in overall position accuracy compared to the other two. The filter with the WPA model has practically the same accuracy as the filter with the KPS model at the initial stage of the filtering when the overall position error is above 400m. As more measurements are fused, the most accurate KPS model does achieve significantly higher accuracy than the WPA model. Here the choices of Q_2 and Q_3 do not account for this significant performance difference which is instead due to the fact that the nonlinear KPS model is more accurate in the projection of the tracking uncertainties than the linear model of the WPA.

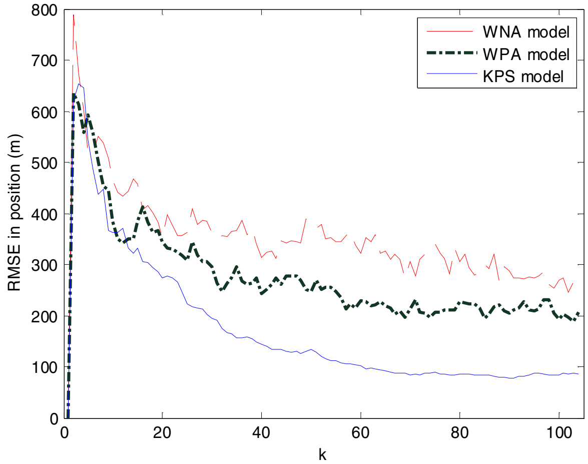

Figure 5. Root mean square error in position (scenario case 1)

In **simulation 2** the second case, SGP4 model, is used and the rest of the simulation configurations stay the same. Fig.6 shows that the three filters are consistent with the NEES at about 6. Fig.7 shows that, similar to the first case, the filter with the WNA model the minimum level of overall RMSE is about 400 meters. The minimum RMSE level with the WPA model is about 250 meter. And for the tracking scenario considered, the filter with the KPS model is able to achieve a minimum RMSE level of 150 meters. While the achievable tracking accuracy varies for different tracking scenarios and measurement accuracies, the results so far show significant advantage in terms of tracking accuracy of the KPS model over the other two models. However as shown next, the KPS model also has its limitations.

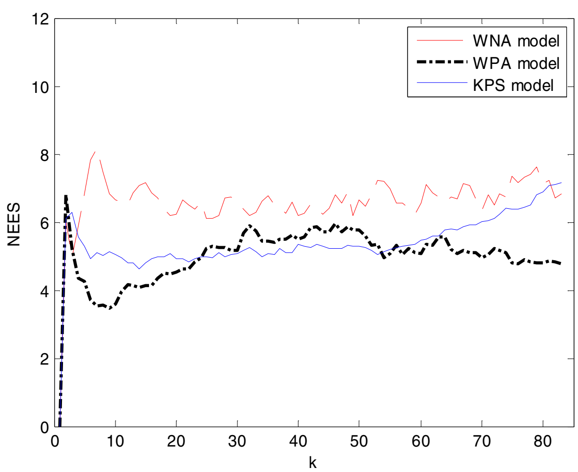

Figure 6. Consistency test of the filters (scenario case 2)

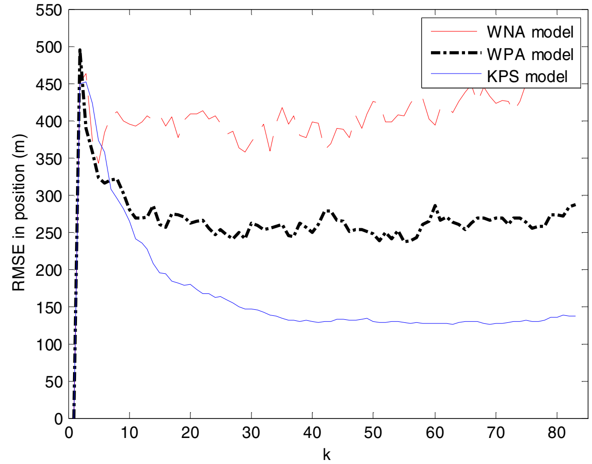

Figure 7. Root mean square error in position (scenario case 2)

In **simulation 3** we change the azimuth and elevation measurement noise standard deviations to $\sigma_a = 0.05^o$, $\sigma_e = 0.05^o$ with the rest of the simulation configurations unchanged. In such a case, the filter based on the KSP model became instable and failed to converge. However as shown in Figures 8-9, the filters with the WNA model and the WPA model (which operate purely in the ECI coordinates) are able to retain their good consistency and have decent estimation accuracy.

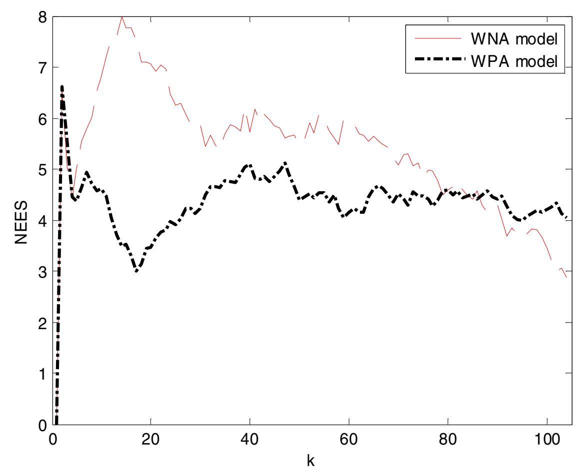

Figure 8. Consistency test of the filters (scenario case 3)

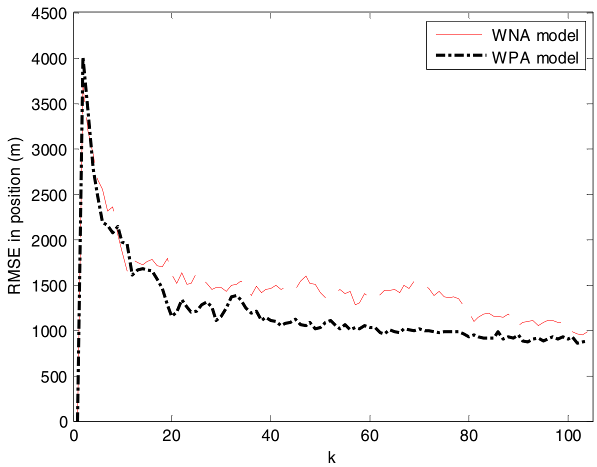

Figure 9. Root mean square error in position (scenario case 3)

The filter with the KPS model failed because the initial accuracy of the track estimates is very low due to the large measurement errors. As a result, the converted sigma-points of the state uncertainty in the Keplerian coordinates are too far away from the Keplerian elements of the true orbit, which leads to huge track propagation errors. There are several approaches to allow a filter with the KPS model to work in cases with very low track accuracies. First, Gaussian mixture filtering approaches [11,13,19] can be used to generate more accurate subtracks over which the KPS model can be effectively used. Second, filtering results from the WPA or WNA can be utilized for more accurate track initialization for the KPS model. These solutions will be investigated in future research.

V. Conclusions

In this paper we compared the use of three approximate kinematic models, i.e., the White Noise Acceleration (WNA) , the Wiener Process Acceleration (WPA), and a Keplerian State (KPS) model, for the tracking of Earth orbiting space objects (SOs). Mismatch of these models to the true SO motion (which is subject to multiple perturbation forces) is handled by the proper selection of process noise covariance matrix in the corresponding tracking filter. For SO target state estimation during a short observation period, such a simplification is expected to have small or negligible loss of estimation accuracy. It is shown that, when the accuracy of the initial track estimate is high (due to accurate measurements), all three filters are consistent, and the filter with the KPS model, which is the most complex one, yields significantly higher tracking accuracy. While the filter with the WPA model is significantly more accurate than the filter with the WNA model. However, when the initial track accuracy is low (due to low measurement accuracy), the KPS model will be instable and failed to converge. In comparison, the filters with the WNA and WPA model, which operate solely in the Cartesian (ECI) coordinates, are very consistent. Results in this paper are useful to develop efficient and effective state estimation algorithms for space surveillance applications.

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