

Towards a Comparative Measure of Legged Agility

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Abstract. We introduce an agility measure enabling the comparison of two very different leaping-from-rest transitions by two comparably powered but morphologically different legged robots. We use the measure to show that a flexible spine outperforms a rigid back in the leaping-from-rest task. The agility measure also sheds light on the source of this benefit: core actuation through a sufficiently powerful parallel elastic actuated spine outperforms a similar power budget applied either only to preload the spine or only to actuate the spine during the leap, as well as a rigid backed configuration of the identical machine.

Keywords: Legged locomotion, experimental metric, agile mobility

1 Introduction

The past decades' slow trickle of dynamical legged robots has grown to a stream of academic [1] and commercial [2] advances yielding an emerging set of design and control principles sufficient for steady-state locomotion [3–9]. In contrast, leaping, dodging, recovering and similar transitional mobility behaviors characteristic of animals' explosive agility — the intuitive motivation for legs — has received much less attention. Recent interest in such transitional legged behaviors [10–15] is impeded by the lack of a well-formulated theory alongside the absence of appropriate performance metrics.

In this paper we propose a pair of measures for nimble legged transitions that help organize a suite of experiments designed to test hypotheses about the comparative benefits of specific morphological features. In Section 2 we introduce a candidate measure of *specific agility*, counterposed with a measure of *endurance* with the goal of quantifying the transitional performance of legged platforms across different scales, morphologies, power resources, and operating points. We use these measures in Section 3 to compare the empirical performance of two comparably powered but morphologically different robots, Canid [16] and XRL [17], in a leaping-from-rest transition, and to reach the judgement in Section 4 that Canid's parallel elastic-actuated spine confers greater leaping agility. We review the main experimental insights in Section 5 and comment on future work.

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2 Technical Approach: Specific Agility and Endurance

Legged agility has not yet been formally defined in the robotics literature so for this paper we explore the implications of a well-cited definition within the sports science community holding that agility is “a rapid whole-body movement with change of velocity or direction in response to a stimulus” [18].

Notwithstanding the many informative and inspiring studies of legged animal performance, e.g. [19–25], we have not been able to find any formalization of this idea suitable for comparing robots of different morphologies and different sizes over different tasks. Perhaps the most common measure for acceleration and leaping used in the legged biology literature is specific power (watts per kilogram taken over a gait cycle of leg power output relative to leg muscle mass or body mass) [22, 26–28] but it is not scale invariant as we observe in Appendix 2. Specific work has been proposed as a measure for legged leaping with respect to muscle mass [22], and this seems closest to the body mass normalized measure we will introduce below. In contrast, characterizing directional aspects of agility performance seems trickier. Animal turning maneuvers have been studied in robotics [29] as well as biology [30] yielding a variety of useful associated performance measures such as turning radius at speed, leg effectiveness, linear maneuverability number [31], and usage of braking/acceleration forces [32]. But it is not clear to us how to generalize such measures for reasons we will discuss below as well.

Many intuitive measures for a legged platform involving, say, jumping height or the magnitude of linear acceleration, are equivalent to a change in kinetic and gravitational potential energy during the stance phase of locomotion. Thus, we focus our proposed measure on the change in what we term the extrinsic body energy, the sum of the mass center’s kinetic and gravitational potential energy, relative to the natural unit over which a legged platform can adjust it, a single, isolated stance. We use the qualifier “extrinsic” to distinguish this notion from the body energy introduced in [16] that is sensitive to the state of a platform’s internal mechanical springs. Catapult-like elastic energy storage used to augment muscle power in leaping from rest has been shown to occur in animals across widely different scales [33, 21] and, intuitively, we feel such use of initially stored spring energy should not count against the agility of a transition. We also avoid the notion of “stride” which connotes a regularity of stance and swing that may not prevail in sudden legged maneuvers characterized by combinatorial sequences of leg contacts [10]. Instead, we construe “stance” as the dimensionless event characterized by some number of legs in ground contact, punctuated either by a prior or subsequent aerial phase (or both).

Thus, for present purposes, we find it useful to introduce a working notion of *specific agility* during stance in terms of the mass-normalized change in extrinsic body energy:

$$\alpha := \frac{\Delta W}{m} / \text{stance event}, \quad (1)$$

where ΔW is the extrinsic body energy (the sum of the mass center’s kinetic and gravitational potential energy) at the end of stance minus the extrinsic body energy at the start stance¹ ², and m is the mass of the agent. The SI units of α are $(m/s)^2$ and can be interpreted as mass-specific work in the equivalent units of (J/kg) .

As we have tried to suggest in our brief survey of the extensive literature, and seems most carefully summarized in [34], it does not appear straightforward to find a single dimensionless group capable of capturing all relevant aspects of maneuverability and agility. We tolerate the lack of a dimensionless measure in our quantification of agility because mass-specific work seems to be the fundamental quantity of interest — at least for changes in velocity magnitude. For example, measuring work done on the body during stance is sensitive to accelerations along a velocity vector fixed in the inertial frame and takes into account the operating point, capturing the greater energetic cost of accelerating a given amount at higher relative to lower speeds (such energetic costs are consistent with biological observations of animal accelerating and braking [26]). However, it does not reward purely directional changes even though, intuitively, rapid turns ought to represent a similarly important concomitant of any comprehensive “agility” measure. Any attempt to reconcile nimble turning with energetic expressions of performance must address the fact that fixed rate circular motion entails no work since the direction of motion is orthogonal to the force.

The proposed measure (1) does appear to confer some scale invariance. In biology, this is predicted by arguments found in [35] and empirical observations of vertical jumping height known as Borelli’s law [36]. This ‘law’ is demonstrated in animals across eight orders of magnitude mass variation which are shown to have vertical jumping heights (proportional to specific agility if air resistance is neglected) within a factor of three — ranging from around 20 to 60 cm or a specific agility of around 2 to 6 m^2/s^2 . Similar arguments about the scale invariance of this measure with electromagnetic actuators in a robotic leg are detailed in Appendix 2.

The operational utility of an agile motion will generally depend on the number of times n it can be performed in succession — which we term *endurance*. Given resource constraints present in executing a movement, we expect endurance to decrease with increasing specific agility. For example a robot that heats its motors to its thermal limits in a single leap cannot immediately perform the same leap on the next step; it must wait until its motors cool before completing the action again, giving it an n of 1. A robot capable of performing an agile motion an infinite number of times (unlikely with current technology given

¹ Steady state motions such as running or hopping that can be approximated with Hamiltonian systems will have negligible agility according to our metric in accordance with biological observations that these motions require significantly less muscle power output as compared to leaping accelerations [28, 22].

² Likely it will be useful in later work to consider a notion of integrated specific agility accumulated over a sequence of stance events, such as when evaluating the agility of an accelerating bound containing a brief aerial phase between front and rear leg-ground contacts.

limited energy storage) would have an n of ∞ . Although “stance event” was introduced as taking integer values, we find it convenient to recast the measure as taking (extended) real values. Specifically, we outline in Appendix 1 our appeal to a motor thermal model as a means of estimating how much time our actuators might be able to sustain the maneuver under consideration, and thereby back out an equivalent real estimate of the predicted number of viable stance events. Thus we will consider the ordered real pair (α, n) when evaluating agile motions in an experimental setting.

3 Experiments and Results

We use this framework to compare the performance of Canid [16] and XRL [17], in the open-loop leaping-from-rest task, a transitional behavior of near ubiquitous value, e.g. in gap crossing or rapid preparation [37] of high energy steady-state gait basins [38]. We use this comparison to examine the relevant benefit of distal versus core actuation as the quadrupedal Canid uses two motors to actuate its spine while the hexapedal XRL uses these two motors to actuate a pair of additional legs. This comparison seems particularly apt because of the close relationship between the two machines described in [17]: both robots have the same electronics, use similar motors and gearing, and are capable of comparable (respecting speed and specific resistance) steady-state locomotion as suggested in the accompanying video and partially documented in [16]. Disregarding the spine, the platforms differ primarily in their mass — Canid weighs 11.3 kg while XRL weighs 7.3 kg — and leg actuation as Canid’s four hip actuators drive their C-legs through a four-bar linkage while XRL directly actuates its six C-legs. It is worth noting the enabling role the specific agility measure plays in allowing this comparison that requires somehow normalizing for the very different actuation strategies used by these two nominally similar machines during forward leaping. Canid only uses 3 of its 6 motors (actuating its rear 2 legs and the top spine cables), while XRL uses 4 of its 6 motors (2 are not used since they contribute little to leaping [10]).

Canid leapt 11 times under a motion capture system³, including 5 times across an 85cm gap which is close to the observed limit of its repeatable leaping ability from standstill (leaps over gaps up to 1m across have been achieved however not in a repeatable fashion). XRL leaping data for this paper was taken from [39] during which parameters for quadrupedal forward leaping were systematically varied to search for various high extrinsic body energy forward leaps. The best XRL forward leap crossed a 50cm gap, which is likely very close to the limit of its leaping ability from standstill.

³ Vicon motion capture data is used to back out the kinetic and potential energy of the robots. Neglecting air resistance, the apex specific extrinsic body energy minus the starting specific extrinsic body energy gives a very close approximation to the specific agility (1) of the leap. The method used to calculate endurance is given in Appendix 1.

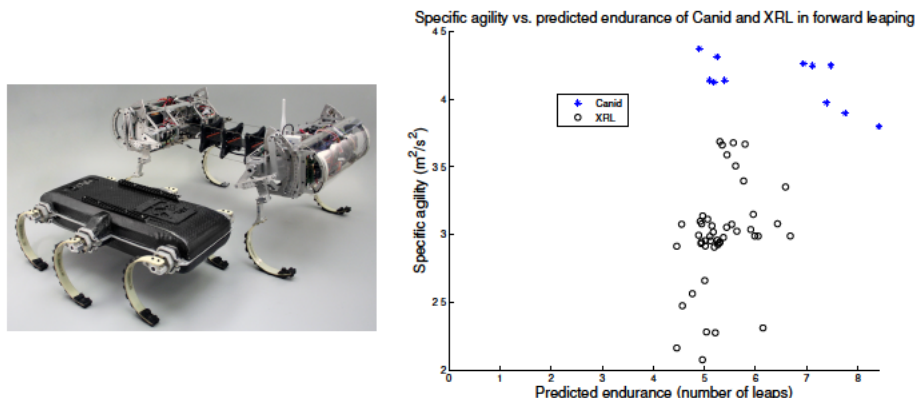


Fig. 1. Canid (top) and XRL (bottom) specific agility vs. predicted endurance during forward leaping. The methods used for calculating these quantities are explained in Footnote 3.

The resulting specific agility and endurance for each Canid and XRL leap is shown in Figure 1. Canid has a better maximum observed specific agility than XRL at a comparable endurance. Although it is likely that we could tune both machines to perform incrementally better, such adjustments would likely further advantage Canid, since these are the very first leaping experiments with Canid whereas XRL leaping has already benefitted from extensive past study and tuning [39]. These results indicate that at least one of the salient morphological differences between Canid and XRL confers upon Canid a significant agility advantage, particularly in light of its reduced (1 fewer) number of actuators used during liftoff.

Additional experiments summarized in Figure 2 were conducted on Canid to quantify the relative agility benefit conferred by Canid’s parallel elastic actuated spine (rather than its four-bar leg transmission) while leaping. Forward leaping data was collected on Canid using 5 different spine stiffnesses varying from rigid to negligible stiffness. Zero agility is recorded in the case where the robot was unable to achieve an aerial phase due to insufficient spine power. For each spine stiffness, Canid was run multiple times while systematically varying its spine motor current limit from 15A to 0A in increments of 5A — always from the same initial condition for every run (except for the rigid case) characterized by a spine preloaded to the same angular displacement prior to its release with lowered current limit reset at the onset of leaping. While successively more compliant spines afford the possibility of successively greater spine pre-loading by a given actuator, we chose to fix the preloading angle because: (1) this avoids the confounding effects of varied initial posture (and attendant variations in control strategy); and (2) the spine motors are capable of breaking the elastic fiberglass plate and it is not yet clear at which point plastic deformation begins. The spine stiffness is approximated by empirical data fit to a torsional Hooke’s law spring as discussed in [16]. Current limits above 15 A are tenable in principle for our actuators, but do not result in substantially different results in any of these cases

because the rapidly extending spine quickly brings them into the no-load regime, as discussed in Section 4.

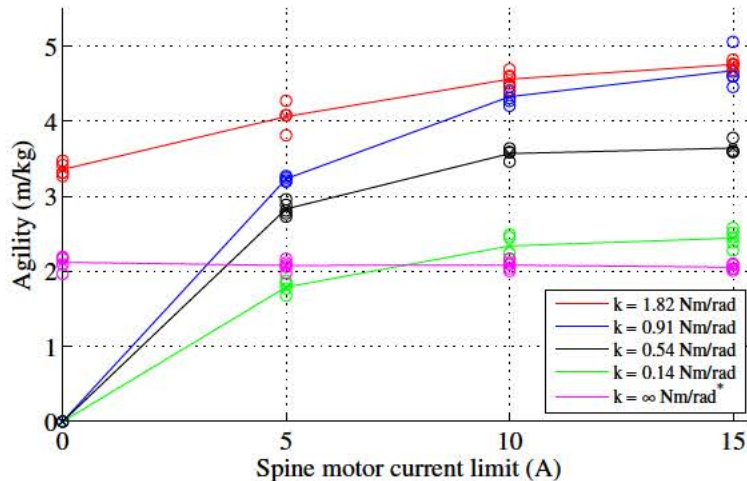


Fig. 2. Canid leaping agility with a variety of spine stiffnesses and spine motor current limits. A total of 80 runs are shown. Canid was allowed to preload its spine to the same angular displacement in every run (except for the rigid case) before setting the lower spine current limit and leaping. Zero agility is recorded in the case where the robot was unable to achieve an aerial phase due to insufficient spine power.

*The $k = \infty$ case is approximated and was not empirically measured for fear of damaging the spine. Rigidity was achieved by locking the spine mechanism with minimal added mass.

4 Experimental Insights: Spine Agility Hypotheses

We now discuss in detail the manner in which these data support the overarching hypotheses regarding the agility benefits of the spine listed in Table 1.

Hypothesis 1: Replacing a rigid back with a spine can increase leaping agility

The results in Figure 2 indicate that — all else being equal — replacing a rigid back with a sufficiently powered spine mechanism (either through releasing initially stored elastic energy or through actuation) can provide a significant morphological advantage for forward leaping agility. The average rigid spine specific agility over the runs was $2.1 \text{ m}^2/\text{s}^2$ which was bested by all actuated spines except for the severely underpowered negligible stiffness $k = 0.14 \text{ Nm/rad}$ spine with a 5A current limit. In the case of unactuated spines, the $k = 1.82 \text{ Nm/rad}$ spine achieves an average specific agility of $3.4 \text{ m}^2/\text{s}^2$ and is thus endowed with

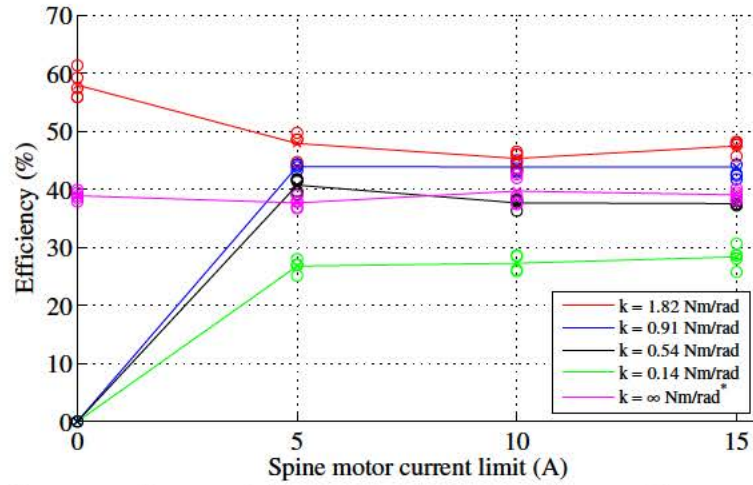


Fig. 3. Energetic efficiency of the experiments shown in Figure 2. Energetic efficiency was calculated by dividing the total change in extrinsic body energy during the leap by the combined mechanical energy output of the motors (comprising of Canid’s two rear leg motors and the spine motor actuating the top spine cables). This motor energy output is calculated at the output shaft before the gearbox and thus doesn’t directly take into account gearbox or transmission inefficiencies.

enough initially stored elastic energy to outperform the rigid back by 62%. The increase of available mechanical power through the spine’s elastic energy release and/or actuation (discussed further in Hypothesis 2) is likely a primary source of this specific agility benefit.

We note that sufficiently-powered core actuation substantially increases rear leg loading in forward leaping. Rear leg stance duration during forward leaping on Canid was observed to be approximately constant over all runs (varying only by a few milliseconds), much to the surprise of the authors given the wide performance range of leaps shown in Figure 2. Thus the larger agility achieved by core actuation must have generated higher rear leg forces during stance when compared to the rigid case. Specifically, the more than 2-fold increase in average specific agility between the rigid back ($2.1 \text{ m}^2/\text{s}^2$) and the best spined runs ($4.8 \text{ m}^2/\text{s}^2$) must have been accompanied by a more than 2-fold increase in average rear leg forces. Canid avoids torque-saturating the rear leg motors with spine forces by operating the rear legs near their kinematic singularity when the spine is doing work. Similar consideration of rear-leg kinematics may be required in general if a spine is added to a legged machine.

	Hypotheses	Supporting Evidence
H1	Replacing a rigid back with a sufficiently powered spine mechanism can provide a significant morphological advantage for forward leaping agility.	Sufficiently powered (either through actuation or releasing initially stored elastic energy) non-rigid spine leaping performance is significantly better than with a rigid spine.
H2	Spine elastic energy release and spine actuation benefit leaping agility both individually as well as in combination.	Motor energy output alone cannot account for the change in extrinsic body energy upon leaping with a non-rigid elastic spine, and increasing spine motor current limits monotonically increases specific agility. Additionally leaping with a full-powered spine and a “tuned” stiffness significantly outperforms leaping with a purely actuated or purely passive spine.
H3	A four-bar transmission increases agility in forward leaping at the expense of reducing the number of other behaviors the robot can perform well.	H3 would be supported if the four-bar diminishes agility of XRL leaping along particular directions relative to others.

Table 1. Hypotheses regarding the performance of Canid and XRL and the proposed series of experiments to support or refute them. Hypotheses 1-2 were shown to be consistent with experimental data from this paper and Hypothesis 3 is the subject of further experiments presently underway.

Hypothesis 2: Both spine elastic energy release and spine actuation benefit leaping agility

Active and Passive Spine Elements in Isolation Figure 2 shows that the $k = 1.82$ Nm/rad spine with no actuation outperforms the fully actuated negligible stiffness $k = 0.14$ Nm/rad spine as well as the rigid spine (both of which initially store a negligible amount of initial elastic energy). The rear legs in the $k = 1.82$ Nm/rad spine case with no spine actuation output on average 65 J of work per leap of which less than 49 J get transferred into the extrinsic body energy due to the rear leg maximum gearbox efficiency of 75% — an overestimate of the true transmission efficiency because we are not accounting the actual gearbox efficiency nor other sources of transmission friction as they are difficult to measure. However the change in extrinsic body energy of these runs averaged 54 J, leaving at least 5 J unaccounted for by the rear legs. Since the only other source of energy in the $k = 1.82$ Nm/rad spine case is the initially stored spine elastic energy, this indicates that initial elastic energy stored in spine bending contributes to forward leaping agility.

There is a monotonic average increase in agility with increased spine actuation power for the runs shown in Figure 2. The decreasing efficacy of motor torque attested by the saturating contours of Figure 2 reflects the no-load speed regime into which the actuators are quickly driven by the rapidly extending spine. Clearly the spine motors can be geared lower to achieve higher agility

at the expense of decreasing endurance for this behavior — as will be taken into account in future Canid design iterations. Notice, as well, at the low end of spine stiffness, that Canid is unable to leap at all without spine actuation. Both observations indicate that the spine motors are directly contributing to forward leaping agility irrespective of spine elastic stiffness — except for of course in the rigid case.

The above results show an individual leaping benefit of spine elastic energy and spine actuation. This should come as no surprise since spine elastic energy release and actuation both augment the available mechanical power output of the machine.

Active and Passive Spine Elements in Parallel Combination

The greatest observed forward leaping performance was achieved with the $k = 0.91$ Nm/rad and $k = 1.82$ Nm/rad spines using the highest spine actuator current limits, averaging a specific agility of 4.7 m²/s². The best purely actuated spine and purely elastic spine experiments on the other hand were only able to achieve specific agilities of 2.6 m²/s² and 3.5 m²/s², respectively. The best “tuned” parallel elastic-actuated spines thus outperform the purely actuated spine by 81% and the purely elastic spine by 34%. This is likely because the nature of parallel elastic-actuators allows the release of the elastic energy stored in the spine to augment the spine motor power during the leap. These results support our hypothesis that using parallel elastic spine actuation outperforms both a purely actuated and purely elastic spine in forward leaping.

However if efficiency is deemed more important than raw agility then the performance using the unactuated $k = 1.82$ Nm/rad spine shown in Figure 3 demonstrates that a sufficiently stiff spine, if properly pre-loaded in stance (either by motors or by a prior maneuver) may offer almost similar agility with considerably greater efficiency. We also note that although at lower actuator limits the $k = 1.82$ Nm/rad spine clearly outperforms the $k = 0.91$ Nm/rad spine, this advantage diminishes as the actuator limits are increased. We are not sure if this reflects the beginning of the “crest” of the “sweet spot” specific agility ridge whose diminishing “other side” is evidenced in these preliminary experiments only by the most extreme $k = \infty$ Nm/rad case. This “sweet spot” is defined by spine stiffness that results in motor torque saturation at the initial spine angular displacement flexion, as this stiffness will store the maximal amount of initial elastic energy. Further experiments are now in progress with still stiffer (but not quite rigid) spines to better fill in the other side of the ridge.

5 Conclusions and Future Work

Specific agility, the mass-specific change in extrinsic body energy accomplished during a stance event, provides a comparative measure for quantifying performance of transitional behaviors such as jumping and accelerating across different platforms using different power budgets. Pairing this measure with endurance, the number of times a transition can be repeated given resource limitations, provides a clearer picture of a robot’s useful operational agility.

Leaping experiments suggest that a significant benefit is conferred by adding a four-bar and a parallel elastic actuated spine [16] to the base XRL robot at no cost to endurance. Further investigation into characterizing the isolated benefit of the spine concluded that — all else being equal — replacing a rigid back with a sufficiently powered spine mechanism (either through releasing initially stored elastic energy or through actuation) can provide a significant morphological advantage for forward leaping agility. There was a measurable individual specific agility contribution from both spine actuation and releasing stored elastic energy in the spine. Furthermore, a parallel-elastic actuated spine confers a larger specific agility benefit to forward leaping than does a purely actuated or purely passive spine. In conclusion, a parallel elastic actuated spine morphology shows a significant agility advantage in forward leaping as compared to a rigid back. Experiments are now in progress to quantify the relative benefit conferred by the addition of a leg four-bar transmission in Canid. Future work will concentrate on comparing the relative benefit of core actuation using a spine to using the same motors instead for additional distal leg actuation.

We are still in the early stages of understanding how to characterize legged agility. Following the tradition of the more mature aircraft [40], aquatic [34], and wheeled [41] vehicle literatures (wherein variously dimensioned agility and maneuverability measures are introduced for different purposes and at different operating points), we explore the utility of a dimensional measure (m^2/s^2) that at the very least proves useful for comparing legged leaps from rest of different machines. Given its (rough) invariance across animal leaping maneuvers, this measure may also have relevance for probing biological energetics. Most immediately, we aim to apply insights provided by the empirical support or refutation of our stated hypotheses toward the design of more agile machines.

Our narrow focus on legged performance presently ignores the fascinating broader question of how to compare agility of such hybrid locomotory platforms against those employing a persistent stance (e.g. cars [42, 43, 41] or boats [34]) or aerial (e.g. jets [40, 34, 44–47]) phase — or even against legged platforms whose limbed manipulation of inertia or momentum in flight significantly enhances their terrestrial locomotory prowess [11]. We trust that further debate and study within the robotics research community along the lines we introduce here will help advance that important goal.

Appendix 1: Endurance Calculations

The endurance of each leap is calculated as follows. The thermal temperature rise ΔT_i incurred by each motor $i \in I$ during the leap is calculated via the thermal model described in Figure 5 of [48]. Let T_F denote the failure temperature of motor i and let T_{i0} denote the motor i 's initial temperature before the leap. The number of times n_i that motor i can perform the leap is approximated by:

$$n_i = \frac{T_F - T_{i0}}{\Delta T_i}.$$

The endurance of the leap is then given by the lowest individual motor endurance, or:

$$\begin{aligned} n &= \inf_{i \in I} (n_i) \\ &= \inf_{i \in I} \left(\frac{T_F - T_{i0}}{\Delta T_i} \right), \end{aligned}$$

so as to extrapolate how many times the leap can be performed sequentially before thermal failure since thermal capacity represents the limiting resource for both Canid and XRL. This method allows us to sidestep the need to run repeated experiments pushing the thermal limits for each machine in order to calculate endurance which would risk motor damage.

Appendix 2: Energy and Power Density for Legged EM Actuators

Assuming that EM motors produce a magnetic field of uniform density, the motor creates force by having this field interact with permanent magnets. This interaction occurs over some area (the air gap) and so is proportional to l^2 . Assuming that the motor does work by rotating through a fixed angle, the transformed displacement through a leg of arbitrary geometry will scale according to the characteristic length, l . The energy produced by the motor (the work done) is therefore proportional to l^3 , so for constant density, specific energy is scale invariant.

Power density scaling is originally presented in [35] pages 176-181, but will be reworked below with more detailed scaling analysis. Assuming energy density is mass-invariant in an actuator, the power density scaling will be considered for a hopping task. Neglecting air resistance the apex height will be constant, and so it follows that the liftoff velocity, v_f , will also be constant. Assuming the system starts crouched at rest, the leg must go through a fixed extension, l , and accelerate the body to v_f . Assuming constant acceleration, a , $v_f = at$ and $l = \frac{1}{2}at^2$ where t is the time the system is in contact with the ground. Substituting for a , $l = \frac{1}{2}v_f t$. Since v_f is constant, t scales according to l . Given constant energy density, power density then scales according to l^{-1} . This means that for specific energy to remain performance limiting, specific power must scale according to l^{-1} . This is in sharp contrast to [49] where specific power scales according to $l^{0.5}$ in support of maintaining dynamic similarity with respect to the pendulous motion of a swinging body characteristic of certain animal climbers [50].

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