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14. ABSTRACT The primary goal of this research has been the optimal control of linear and nonlinear systems driven by fractional Brownian motions and other stochastic processes. For the control of both continuous time and discrete time finite dimensional linear systems with quadratic cost functionals and arbitrary noise processes with finite second moments, explicit optimal controls are determined. Linear-quadratic control problems for stochastic partial differential equations driven by fractional Brownian motions are explicitly solved. For the control of a continuous time linear system with Brownian motion or a discrete time linear system with a white Gaussian noise and cost...					
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Report Title

Final Report: Optimal Control of Stochastic Systems Driven by Fractional Brownian Motions

ABSTRACT

The primary goal of this research has been the optimal control of linear and nonlinear systems driven by fractional Brownian motions and other stochastic processes. For the control of both continuous time and discrete time finite dimensional linear systems with quadratic cost functionals and arbitrary noise processes with finite second moments, explicit optimal controls are determined. Linear-quadratic control problems for stochastic partial differential equations driven by fractional Brownian motions are explicitly solved. For the control of a continuous time linear systems with Brownian motion or a discrete time linear system with a white Gaussian noise and costs that are the exponential of quadratic functionals are solved in a simple, direct way. Linear quadratic stochastic differential games are explicitly solved for noise processes that are arbitrary square integrable processes with continuous sample paths. Some control problems described in rank one symmetric spaces are explicitly solved. Some control problems for infinite dimensional linear systems with fractional Brownian motions and ergodic, quadratic costs are explicitly solved.

Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

<u>Received</u>	<u>Paper</u>
08/10/2011 5.00	T. E. Duncan, Z.Talata. BIC context tree estimation for stationary ergodic processes, IEEE Transactions in Information Theory, (06 2011): 3877. doi:
08/13/2013 18.00	Bozenna Pasik-Duncan, Tyrone Duncan. A direct method for solving stochastic control problems, Communications in Information and Systems, (12 2012): 1. doi:
08/14/2012 15.00	Bozenna Pasik-Duncan, Tyrone E. Duncan. Linear-exponential-quadratic Gaussian control for stochastic equations in a Hilbert space, Dynamic Systems and Applications, (06 2012): 0. doi:
08/14/2012 16.00	Bozenna Pasik-Duncan, Tyrone E. Duncan. Discrete time linear quadratic control with correlated noise, IEEE TRANSACTIONS ON Automatic Control, (12 2012): 0. doi:
08/21/2012 7.00	Tyrone E. Duncan, Bohdan Maslowski, Bozenna Pasik-Duncan. LINEAR-QUADRATIC CONTROL FOR STOCHASTIC EQUATIONS IN A HILBERT SPACE WITH FRACTIONAL BROWNIAN MOTIONS, SIAM Journal of Control and Optimization, (02 2012): 507. doi:
08/21/2013 21.00	Tyrone Duncan. Linear-exponential-quadratic Gaussian control, , (11 2013): 0. doi:
TOTAL:	6

Number of Papers published in peer-reviewed journals:

(b) Papers published in non-peer-reviewed journals (N/A for none)

Received Paper

TOTAL:

Number of Papers published in non peer-reviewed journals:

(c) Presentations

Number of Presentations: 0.00

Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received Paper

TOTAL:

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<u>Received</u>	<u>Paper</u>
08/10/2011 1.00	Bozenna. Pasik-Duncan, Tyrone. E. Duncan. Stochastic Linear-Quadratic Control for Systems with a Fractional Brownian Motion, 49th IEEE Conference on Decision and Control. 15-DEC-10, . . . ,
08/10/2011 2.00	Tyrone Duncan, Bozenna Pasik-Duncan. Control of linear systems with fractional Brownian motions , Mathematical Theory of Networks and Systems. 07-MAY-10, . . . ,
08/11/2012 12.00	T. E. Duncan, B. Maslowski, B. Pasik-Duncan. Control of some stochastic systems in a Hilbert space with fractional Brownian motions, Conference on Methods and Models in Automation and Robotics. 16-AUG-11, . . . ,
08/11/2012 13.00	B. Pasik-Duncan, T. E. Duncan. A stochastic control problem in the two sphere, IEEE Multiconference on Systems and Control. 05-OCT-12, . . . ,
08/13/2013 19.00	Tyrone duncan, Bozenna Pasik-Duncan. Ergodic problems for linear exponential quadratic Gaussian control and linear quadratic stochastic differential games , IEEE Conference on Decision and Control. 11-DEC-13, . . . ,
08/13/2013 20.00	Tyrone Duncan, Bozenna Pasik-Duncan. A solvable stochastic differential game in the two-sphere , IEEE conference on Decision and Control. 11-DEC-13, . . . ,
08/14/2012 11.00	Tyrone E. Duncan, Bozenna Pasik-Duncan. Control of some partially observed linear stochastic systems with fractional Brownian motions., IEEE International on Conference on Decision and Control. 12-DEC-12, . . . ,
08/14/2012 14.00	Tyrone E. Duncan, Bozenna Pasik-Duncan. Linear-exponential-quadratic Gaussian control for stochastic partial differential equations, Mathematical Theory of Signals and Systems. 09-JUL-12, . . . ,
08/21/2012 10.00	Bohdan Maslowski, Bozenna Pasik-Duncan, Tyrone E. Duncan. Control of some linear equations in a Hilbert space with fractional Brownian motions, Proceedings 18th IFAC World Congress. 30-AUG-11, . . . ,
10/06/2014 26.00	Tyrone Duncan, Bozenna Pasik-Duncan. A solvable stochastic differential game in the two-sphere , IEEE Conference on Decision and Control 2013. 15-DEC-14, . . . ,
10/06/2014 27.00	Tryone Duncan, Bozenna Pasik-Duncan. Some stochastic differential games in spheres , Mathematical Theory of Networks and Systems 2014. 07-JUL-14, . . . ,
10/06/2014 28.00	Tyrone Duncan, Bozenna Pasik-Duncan. Some ergodic control problems for linear stochastic equations in a Hilbert space with fractional Brownian motion, Mathematical Theory of Networks and Systems 2014. 07-JUL-14, . . . ,
10/06/2014 29.00	Tyrone Duncan, Bozenna Pasik-Duncan. Linear-quadratic control with general noise processes , IEEE Conference on Decision and Control 2014. 15-DEC-14, . . . ,
10/06/2014 30.00	Tyrone Duncan, Bozenna Pasik-Duncan. Some results on optimal control for a partially observed linear stochastic system with an exponential quadratic cost, IFAC World Congress 2014. 31-AUG-14, . . . ,

10/06/2014 31.00 Tyrone Duncan, bozenna Pasik-Duncan. Some models of control with fractional Brownian motions , IFAC Symp. Advances in Control Education. 10-JUN-12, . : ,

10/08/2014 37.00 Tyrone Duncan, Bozenna Pasik-Duncan. Ergodic problems for linear exponential quadratic Gaussian control and linear quadratic stochastic differential , IEEE Conference on Decision and Control, Firenze. 15-DEC-13, . : ,

TOTAL: 16

Number of Peer-Reviewed Conference Proceeding publications (other than abstracts):

(d) Manuscripts

<u>Received</u>	<u>Paper</u>	
08/10/2011	6.00	Tyrone E. Duncan, Bozenna Pasik-Duncan. Linear-Quadratic Fractional Gaussian Control, Submitted (08 2011)
08/10/2011	8.00	Tyrone E. Duncan. Stochastic Linear-Quadratic Control Revisited, Submitted (08 2011)
10/06/2014	24.00	Tyrone Duncan. 211. Some linear-quadratic stochastic differential games for equations in a Hilbert space with fractional Brownian motions, Discrete Contin. Dyn. Syst. Ser. S (12 2012)
10/08/2014	34.00	Tyrone Duncan. Linear-exponential-quadratic Gaussian control, IEEE TRANSACTIONS ON Automatic Control (10 2013)
10/08/2014	35.00	Tyrone Duncan, Bozenna Pasik-Duncan. Linear-Quadratic Fractional Gaussian Control, SIAM J of Control and Optimization (12 2013)
TOTAL:	5	

Number of Manuscripts:

Books

<u>Received</u>	<u>Book</u>	
08/14/2013	22.00	Tyrone E. Duncan. Models and Methods in Economics and Management Science: Essays in Honor of Charles S. Tapiero, Switzerland: Springer Intern. Publishing, (09 2013)
08/15/2013	23.00	Tyrone E. Duncan. Stochastic linear quadratic control revisited, Hong Kong: World Scientific, (09 2012)
TOTAL:	2	

Received

Book Chapter

10/06/2014 32.00 Tyrone Duncan, bozenna Pasik-Duncan. stochastic adaptive control , New York: Springer Verlag, (12 2014)

10/08/2014 33.00 bohdan Maslowski, Tyrone Duncan, bozenna Pasik-Duncan. Ergodic control of linear stochastic equations in a Hilbert space with fractional Brownian motion, Warsaw, Poland: Banach Center Publications, (12 2014)

TOTAL: 2

Patents Submitted

Patents Awarded

Awards

Bozenna Pasik-Duncan Fellow of the International Federation of Automatic Control Aug. 2014

Graduate Students

<u>NAME</u>	<u>PERCENT SUPPORTED</u>	Discipline
Cody Clifton	0.25	
FTE Equivalent:	0.25	
Total Number:	1	

Names of Post Doctorates

<u>NAME</u>	<u>PERCENT SUPPORTED</u>
FTE Equivalent:	
Total Number:	

Names of Faculty Supported

<u>NAME</u>	<u>PERCENT SUPPORTED</u>	National Academy Member
Tyrone Duncan	0.10	
Bozenna Pasik-Duncan	0.04	
FTE Equivalent:	0.14	
Total Number:	2	

Names of Under Graduate students supported

<u>NAME</u>	<u>PERCENT SUPPORTED</u>	Discipline
Collin Eubanks	0.25	Mathematics
FTE Equivalent:	0.25	
Total Number:	1	

Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period: 1.00

The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:..... 1.00

The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:..... 1.00

Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):..... 1.00

Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering:..... 0.00

The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense 0.00

The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields:..... 1.00

Names of Personnel receiving masters degrees

<u>NAME</u>	
Cody Clifton	
Theodore Lindsey	
Total Number:	2

Names of personnel receiving PHDs

<u>NAME</u>
Total Number:

Names of other research staff

<u>NAME</u>	<u>PERCENT SUPPORTED</u>
FTE Equivalent:	
Total Number:	

Sub Contractors (DD882)

Inventions (DD882)

Scientific Progress

The major focus of this research is the control of stochastic systems with general noise processes. These noise processes include the family of fractional Brownian motions and more generally the family of square integrable continuous stochastic processes. Controlled finite dimension linear stochastic systems driven by arbitrary square integrable continuous processes and having quadratic cost functionals have been solved by providing explicit optimal controls. Infinite dimensional controlled linear systems with fractional Brownian motions that can model stochastic partial differential equations where the control and the noise can be restricted to the boundary of the domain have been explicitly solved. A discrete time controlled linear system with an arbitrary correlated noise and a quadratic cost has been explicitly solved. A controlled continuous time linear system with a Brownian motions and a cost that is an exponential of a quadratic functional is solved in a simple, direct way. Linear quadratic stochastic differential games are solved for systems with arbitrary square integrable continuous noise processes. Control problems with Brownian motion in the two-sphere or real hyperbolic two-space have been explicitly solved. Stochastic differential games in the two-sphere have been explicitly solved. An infinite time horizon linear-exponential quadratic control problem with a long run average quadratic cost is solved directly. It is shown that the direct approach used in the aforementioned problems is much simpler than the well known methods that use either Hamilton-Jacobi-Bellman equations or stochastic maximum principle with backward stochastic differential equations. These methods have been extended to stochastic differential games in spheres of arbitrary dimensions. Ergodic control problems for linear stochastic equations in a Hilbert space with fractional Brownian motions were solved by obtaining explicit optimal ergodic controls.

Technology Transfer