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# Acoustic Nondestructive Testing and Measurement of Tension for Steel Reinforcing Members

Part 1–Theory

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**PURPOSE:** This Coastal and Hydraulics Engineering Technical Note (CHETN) describes theoretical work supporting the development of a nondestructive evaluation (NDE) technology that can quantitatively measure tension in structural steel reinforcing members. In large concrete structures such as locks and dams, the corrosion of tensioned steel components can lead to loss of tension and consequent severe problems such as cracking of the concrete or fracturing of the steel. The theory and application address the problem of determining tension in concrete-embedded pre- and post-tensioned steel reinforcement rods.

**BACKGROUND:** Many concrete structures contain internal pre- and post-tensioned steel structural members that are subject to fracturing and corrosion. Corrosion of steel components can lead to loss of tension and consequent severe problems, such as cracking of the concrete or fracturing of the steel. The major problem with conventional tension-measurement techniques is that they use indirect and non-quantitative methods to determine whether there has been a loss of tension. We have developed an acoustics-based technology and method for making quantitative tension measurements of an embedded, tensioned steel member.

Acoustic waves are nondestructive, are capable of traveling long distances in engineered structures, and can thoroughly interrogate a structure's integrity. Acoustics are dual purpose: they can determine bulk material properties, such as tension; and they can detect small defects, such as fractures. Measurements can be performed very quickly, usually in real time, although post-processing may be required. In this Technical Note, a theoretical basis for bulk tension measurements and an acoustic propagation model is described. The theory and model are verified using simple steel rods as test specimens. A companion Technical Note, ERDC/CHL CHETN-IX-38 (Part 2–Field Testing), describes the field testing of a technology application that addresses the problem of determining tension in the concrete-embedded pre- and post-tensioned reinforcement rods used in large Civil Works structures.

**BASIC ULTRASONIC THEORY:** A material's ultrasonic properties are fundamental to the understanding of wave behavior. There is much literature describing the theory and applications of ultrasonic waves. Some common references are Auld 1990, Ensminger 1973, Filipczynski et al. 1966, and Krautkramer and Krautkramer 1990.

Ultrasonic waves can propagate as both longitudinal and shear waves. The two different propagation modes are displayed in Figure 1. For longitudinal waves, the direction of particle motion is the same as the direction of propagation. For shear waves, the direction of particle motion is perpendicular to the direction of propagation. Longitudinal waves can exist in all media; shear waves exist only in solids. In steel, shear waves move about half the speed of longitudinal waves. The shear wave is 4 – 10 times more attenuated than the longitudinal wave.

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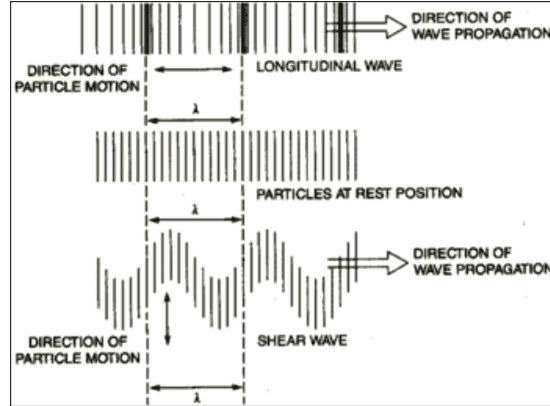


Figure 1. Illustration of acoustic wave propagation (<https://www.nde-ed.org/EducationResources/CommunityCollege/Ultrasonics/Physics/wavepropagation.htm>).

**TENSION-MEASUREMENT THEORY:** Derivation and development of the acoustic tension-measurement technique are described in detail in Carlyle et al. 2004a and Carlyle et al. 2004b.

Given equations describing linear elastic deformation and ultrasonic wave velocity, an equation describing how to determine tension inside a homogeneous material using pure ultrasonic measurements can be derived. The elastic deformation quantities include Hooke's Law, the bulk modulus, Young's modulus, the shear modulus, and Poisson's ratio.

General linear elastic deformation is described using Hooke's Law, which relates stress to strain:

$$\sigma_{ij} = c_{ijkl} \times \varepsilon_{kl} \quad (1)$$

where  $\sigma_{ij}$  is the stress in the  $i, j$  direction,  $\varepsilon_{kl}$  is the strain in the  $k, l$  direction, and  $c_{ijkl}$  is the elasticity tensor (which has 36 independent stiffness constants). For the isotropic case, Equation 1 reduces to a much simpler form of Hooke's Law that involves only two independent constants:

$$\sigma_{ij} = \lambda \delta_{ij} \varepsilon_{aa} + 2\mu \varepsilon_{ij} \quad (2)$$

where  $\delta_{ij}$  is the Kronecker delta, and  $\lambda$  and  $\mu$  are known as Lamé's constants. Lamé's constants can be used to calculate several other important material properties, such as the bulk modulus:

$$K = \lambda + 2\mu/3 = \sigma_{xx}/3\varepsilon_{xx} = \sigma_{yy}/3\varepsilon_{yy} = \sigma_{zz}/3\varepsilon_{zz} \quad (3)$$

Young's modulus:

$$E = \mu(3\lambda + 2\mu)/(\lambda + \mu) = \sigma_{zz}/\varepsilon_{zz} \quad (4)$$

The shear modulus:

$$\sigma_{zx} = 2\mu \varepsilon_{zx} \quad (5)$$

And Poisson's ratio:

$$\nu = \lambda/2(\lambda + \mu) \quad (6)$$

A fundamental property of ultrasonic waves is that their propagation velocity,  $v$ , is a function of the stiffness constants,  $c_{ij}$ , and the density of the material,  $\rho$ , in which they are traveling:

$$v_l = (c_{11} / \rho)^{1/2} \quad (7)$$

$$v_s = (c_{44} / \rho)^{1/2} \quad (8)$$

where  $v_l$  is the longitudinal wave velocity,  $v_s$  is the shear wave velocity,  $c_{11}$  is the stiffness constant governing oscillatory motion along the direction of wave propagation, and  $c_{44}$  is the stiffness constant governing oscillatory motion transverse to the direction of wave propagation.

The definitions for bulk modulus, Young's modulus, shear modulus and Poisson's ratio given by Equations 3 – 6 are substituted into Equation 2 along with the longitudinal and shear wave velocities (given by Equations 7 and 8).

The seven fundamental equations shown in Figure 2 were used to derive Equation 9, which relates tension to the velocities of the longitudinal and shear waves.

<b>Hooke's Law</b>	<b>Young's Modulus</b>
$\sigma_{ij} = \lambda \delta_{ij} \epsilon_{aa} + 2 \mu \epsilon_{ij}$	$E = \mu ( 3 \lambda + 2 \mu ) / ( \lambda + \mu ) = \sigma_{zz} / \epsilon_{zz}$
<b>Bulk Modulus</b>	
$K = \lambda + 2 \mu / 3 = \sigma_{xx} / ( 3 \epsilon_{xx} ) = \sigma_{yy} / ( 3 \epsilon_{yy} ) = \sigma_{zz} / ( 3 \epsilon_{zz} )$	
<b>Shear Modulus</b>	<b>Poisson's Ratio</b>
$\sigma_{zx} = 2 \mu \epsilon_{zx}$	$\nu = \lambda / 2 ( \lambda + \mu )$
<b>Longitudinal Wave Speed</b>	<b>Shear Wave Speed</b>
$V_l = (c_{11} / \rho)^{1/2}$	$V_s = (c_{44} / \rho)^{1/2}$

**Figure 2. Fundamental laws and equations governing material properties.**

$$\sigma = \frac{R(v_l^2 - C v_s^2)}{C(v_l^2 - v_s^2)} \quad (9)$$

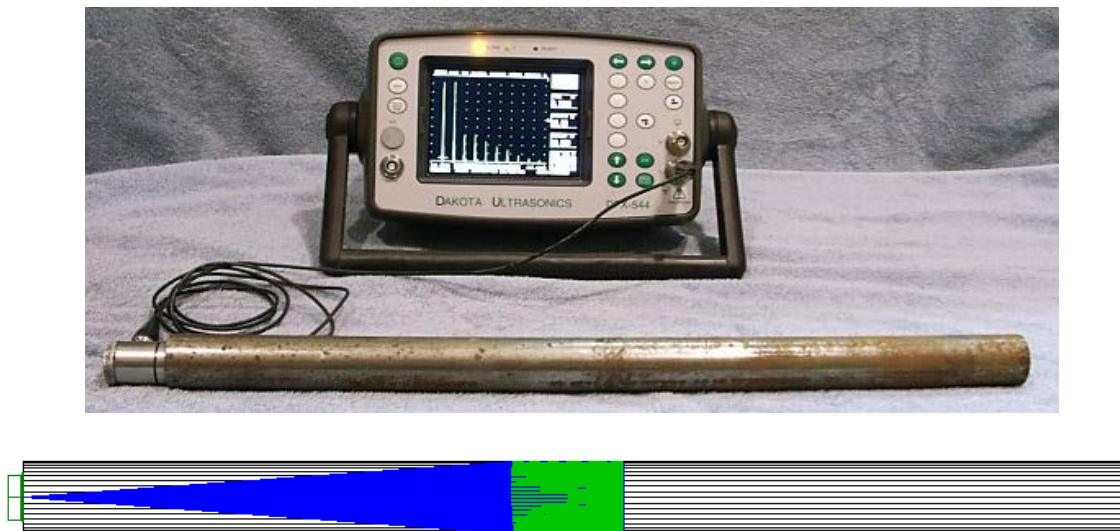
Tension ( $\sigma$ ) is dependent on acoustic longitudinal ( $v_l$ ) and shear ( $v_s$ ) wave velocities (Equation 9), and material properties R and C (Equation 10):

$$C = \frac{c_{11}}{c_{44}} \quad (10)$$

where  $c_{11}$  and  $c_{44}$  are two of the material's thirty-six stiffness constants.

R and C are constants, and do not vary with changes in tension. Thus one needs only to determine, either through measurement or modeling, the velocities of the longitudinal and shear waves.

**ULTRASONIC SIMULATION:** A modeling and ray-tracing software package, Imagine3D<sup>1</sup>, was used to model a 19 in. plain steel rod measuring 1.25 in. diameter. This seemingly simple object actually involves a great deal of complexity due to beam divergence and mode conversion<sup>2</sup>. The experimental setup and the acoustic wave propagation simulation of the rod are shown in Figure 3. The transducer is mounted on the left end of the rod. The acoustic energy travels down the rod, reflects off the right end of the rod, and travels back to the transducer. The ultrasonic beam from the transducer diverges (i.e., spreads) as it travels. The beam divergence is represented in the lower plot of Figure 3, where the blue rays emitting from the transducer have a clear conical shape. As the beam interacts with the longitudinal surface of the rod it reflects and mode-converts in accordance with Snell's Law<sup>3</sup>. The reflected ultrasonic rays are shown in green. Because of the number of rays involved, the reflected beam seems to fill the rod, but in reality each ray is traveling at an angle to the surface of the rod. Mode conversion is also taking place when the surface reflection occurs, and although the software simulation is producing rays that correspond to shear waves, they are not seen in the chosen simulation view.



**Figure 3. Ultrasonic instrument showing received signal from transducer mounted on left end of rod (top); simulation of signal propagation within the rod (bottom)**

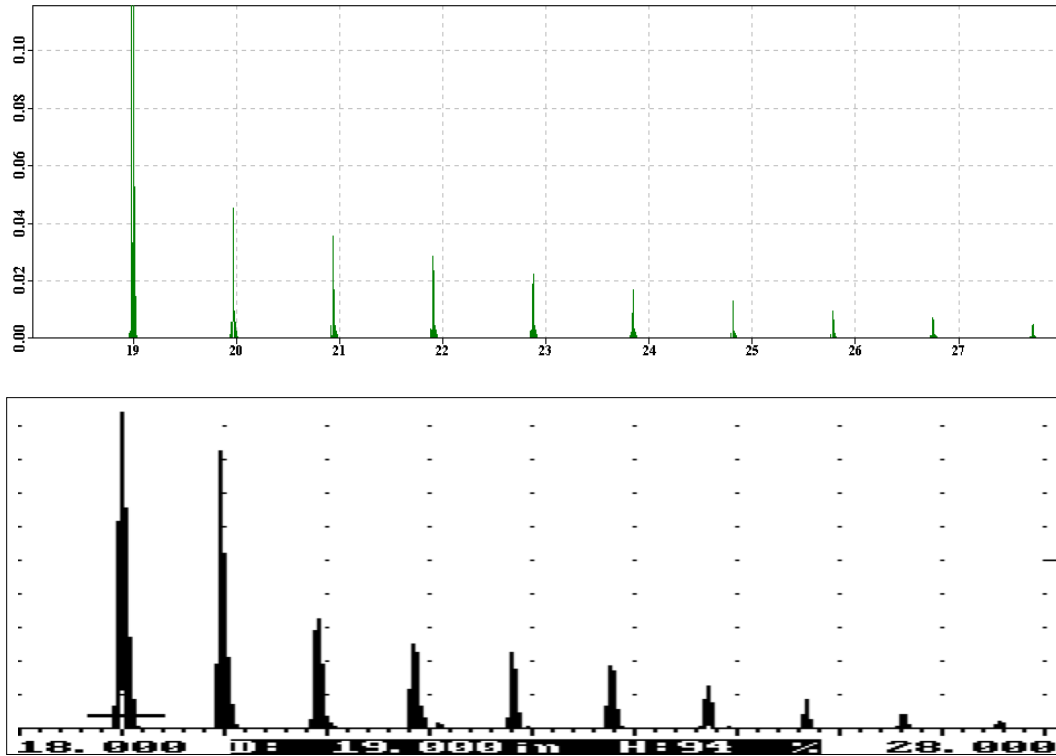
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<sup>1</sup> Microsoft Windows version discontinued by Impulse, Inc., after this modeling project was completed.

<sup>2</sup> Conversion from a longitudinal to a shear wave and vice versa.

<sup>3</sup> According to Snell's Law, the wave also refracts into the adjacent medium. When the adjacent medium is water, for example, 12.17% of the wave energy refracts and 87.83% reflects back into the steel. However, when the adjacent medium is air, only a vanishingly small portion of the energy (0.0036%) refracts, with 99.9964% of it reflecting back into the steel. In our simple model, the adjacent medium is air, so we neglect the minuscule loss of energy due to refraction of the wave at the longitudinal surface.

Figure 4 shows the results of the beam divergence and mode conversion taking place inside the test rod when the simulator sums all of the received echoes. The rod is 19 in. long so, as expected, a strong echo appears at 19 in. from the transducer. This is produced by the longitudinal wave traveling straight down the rod, reflecting off the far end of the rod, and echoing back to the transducer.



**Figure 4. Simulation of echoes in sample rod (top)  
and actual screen shot from ultrasonic instrument display (bottom).**

Note that other echoes appear after 19 in. These are caused by mode conversion of the original longitudinal ultrasonic beam into shear waves, as the diverging beam interacts with the longitudinal surface of the rod. Since shear waves travel in steel about half as fast as longitudinal waves, when they arrive back at the transducer they will lag behind the faster longitudinal. The agreement between the simulation and the result obtained from the measurement is very good.

In addition to placing all the echoes at the proper distance, the modeling software also did a good job of calculating the expected amplitude for the echoes. The only significant disagreement between the result obtained from the measurement and simulation appears in the echo at 20 in. The measurement recorded a stronger echo than predicted by the simulation. All the other echoes, from about 21 – 28 in., agree well between measurement and simulation.

**TENSION MEASUREMENTS USING ULTRASOUND:** The key to computing tension in a component is to obtain the necessary ultrasonic measurements with a high degree of accuracy. There are two ways to do this: (1) modeling software can be used to compute the value of the shear velocity from two echoes and the measured velocity of the longitudinal wave, and (2) the longitudinal and shear wave velocities can be measured directly.

The tension-measurement model consists of an equation together with an accurate ultrasonic echo model. Our equation calculates a material's bulk stress,  $\sigma$ , in terms of ultrasonic quantities. This equation permits us to measure the tension in an embedded steel rod, such as a tainter gate anchor rod, by measuring two ultrasonic properties and dividing the calculated stress value by the circular area (i.e., cross section) of the rod. The physical changes produced by tension in a rod can be determined using the isotropic version of Hooke's Law,

$$E = \sigma_{zz} / \epsilon_{zz} \tag{11}$$

Stress,  $\sigma$ , is force (tension) divided by area, while strain,  $\epsilon$ , is change in length divided by the original length. The following equation shows the relationship for the length change that results from a given tension in a rod:

$$\Delta L = FL / AE \tag{12}$$

where

- $F$  = applied force
- $L$  = original length of the rod
- $A$  = circular area of the rod
- $E$  = Young's modulus.

The change in length for various applied loads to a 19 x 1.25 in. (diameter) steel rod is calculated using Equation 4 and presented in Table 1.

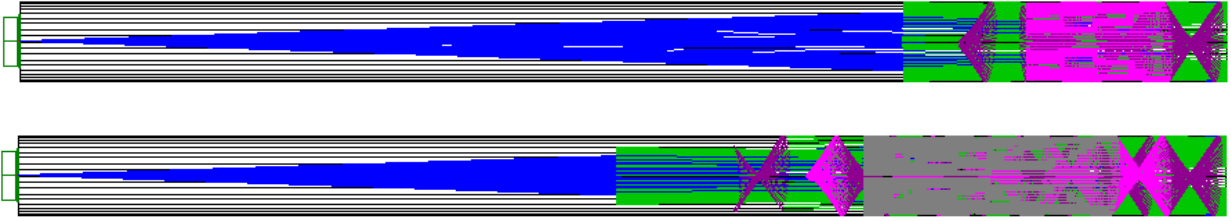
**Table 1. Load-induced length change in a 19 x 1.25 in. (diameter) steel rod.**

Load (lb)	Length Change (in.)
100 pounds	0.000525 inch
500 pounds	0.002627 inch
1000 pounds	0.005253 inch
5000 pounds	0.026267 inch
10000 pounds	0.052534 inch

From the information in Table 1, the modeling software can be used to predict the exact position of the ultrasonic echoes that will result when the length of the example rod is increased by these lengths. Thus, the effect of tension on ultrasound echoes in the steel bar can be observed. This accomplishes two things: (1) it permits us to see the magnitude change of any given tension state and (2) it permits us to see the effect of tension on any particular echo. The first point is important because it allows us to determine the precision with which the ultrasonic velocity measurements will need to be conducted.

The echoes shown in Figure 5 are produced in the modeling software by specifying the wave modes that can exist in the rod. The first echo, at 19 in., is the result of a longitudinal wave propagating down the length of the rod and another longitudinal wave reflecting off the far end and returning to the transducer. This can be seen clearly in the top and bottom plots of Figure 5; the blue rays are the longitudinal wave propagating down the length of the rod and the green rays correspond to this longitudinal wave, reflecting off the far end and returning to the transducer.

There are no shear waves in the first echo because the shear waves are created by mode conversion of the reflecting longitudinal wave, and they travel at about the half the velocity of the longitudinal wave.



**Figure 5. Model of 19 x 1.25 in. (diameter) steel rod showing how first echo (green rays) and second echo (magenta) form (top); model showing how the first echo (green rays) and third echo (gray rays) form (bottom).**

The second, third, and higher echoes consist of a mix of longitudinal waves and shear waves. The top plot of Figure 5 also shows how the second echo is created. A longitudinal wave (blue) propagates down the rod, a reflected longitudinal wave (green) reflects off the far end, and a reflected shear wave (dark purple) is created when the reflected longitudinal wave mode-converts at the longitudinal surface of the rod. Another reflected longitudinal wave (magenta) is created when this shear wave mode-converts on the opposite surface of the rod. This last longitudinal wave returns to the transducer to create echo 2. This event can be expressed with the following equation for the time of echo 2,  $t_{e2}$ .

$$t_{e2} = \frac{x_2}{v_1} + \frac{x_3}{v_1} + \frac{x_4}{v_s} + \frac{x_5}{v_1} \quad (13)$$

where  $v_1$  and  $v_s$  are the longitudinal and the shear wave velocities, respectively, and  $x_2, x_3, x_4$ , and  $x_5$  are the path lengths for their respective rays in the rod.

The bottom plot of Figure 5 shows how the third echo is created. A longitudinal wave (blue) propagates down the rod and a reflected longitudinal wave (green) reflects off the far end. A reflected shear wave (dark purple) is created when the reflected longitudinal wave mode-converts at the longitudinal surface of the rod, and another reflected shear wave (magenta) is created when the first reflected shear wave reflects off the opposite side of the rod. Finally, a reflected longitudinal wave (gray) is created when this second shear wave mode-converts on the opposite side of the rod. This last longitudinal wave returns to the transducer to create echo 3. This can be expressed with the following equation for the time of echo 3,  $t_{e3}$ .

$$t_{e3} = \frac{x_2}{v_1} + \frac{x_3}{v_1} + \frac{x_4}{v_s} + \frac{x_6}{v_s} + \frac{x_7}{v_1} \quad (14)$$

where  $v_1$  and  $v_s$  are the longitudinal and the shear wave velocity, respectively, and  $x_2, x_3, x_6$ , and  $x_7$  are the path lengths for their respective rays in the rod. Note that  $x_2, x_3$ , and  $x_4$  are common to echoes 2 and 3.



An equation for the velocity of the shear wave as a function of the velocity of the longitudinal wave and the echoes in the rod is obtained by subtracting Equation 13 from Equation 14.

$$v_s = \frac{(v_1 \times x_6)}{(v_1 \times (t_{e3} - t_{e2}) + x_5 - x_7)} \quad (15)$$

The modeling software can compute the values of the four path lengths  $x_4$ ,  $x_5$ ,  $x_6$ , and  $x_7$ . The software can also provide the times of the two required echoes,  $t_{e2}$  and  $t_{e3}$ . These computed quantities, in combination with the measured value of the longitudinal wave velocity,  $v_1$ , are all that is needed to obtain the last desired quantity,  $v_s$ . To obtain the tension in the rod one uses Equation 9 and the measured longitudinal velocity,  $v_1$ , and the computed shear velocity,  $v_s$ .

**MEASUREMENT ISSUES:** There are many possible difficulties with ultrasonically measuring the tension of anchor rods in situ. The greatest is the attenuation of the acoustic signals, especially that of the shear wave. Additional wave propagation problems are reflection, refraction, beam spread and coupling of the signal to the rod from the transducer. Unfortunately these difficulties are entirely due to the physics of the problem and are very difficult to overcome. The measurement will also be dependent on the medium surrounding the rod, e.g., air, grease, or grout.

The preferred measurement method is to measure the longitudinal and shear wave velocities directly and compute the tension using the formula in Equation 1. This gives a quantitative result. Although modeling software can be used to compute the value of the shear velocity from the measured velocity of the longitudinal wave and two echoes, this is considered quantitative since the technique is dependent on the model used. For example, the current model did not take into account the media surrounding the rod. The model would need to be redone to incorporate the results of the grouted rod tests.

**SUMMARY:** A theory and testing technique was developed for quantitatively determining tension in concrete-embedded structural steel members. In this work, a theoretical basis was established for bulk tension measurements and an acoustic propagation model was developed and implemented using commercially available software. The technology and method of application were awarded a U.S. patent (McInerney et al. 2009).

When perfected, the benefits of this acoustic NDE tension-measurement technology will include

- rapid, noninvasive tension measurement of embedded steel rods in the field
- ease of measurement where at least one rod end is available, even where access is difficult
- cost reduction by a factor of 10 compared with the present lift-off testing method
- facilitation of more frequent testing and improved structural evaluation.

**FUTURE WORK:** Further laboratory studies of wave propagation will be done using the ERDC Anchor Rod Test Bed, located at the Engineer Research and Development Center, Coastal and Hydraulics Laboratory, Vicksburg, MS. This facility accommodates the testing of rods approximately 60 ft long under tension with several different types of conduit-filler materials.

**POINTS OF CONTACT:** This CHETN is a product of the Acoustic Nondestructive Testing work unit of the Navigation Systems Research Program (NavSys) being conducted at the U.S. Army Engineer Research and Development Center, Construction Engineering Research Laboratory, Champaign, IL. Questions about this technical note can be addressed to Mr. Michael K. McInerney at 217-373-6759; e-mail [Michael.K.Mcinerney@usace.army.mil](mailto:Michael.K.Mcinerney@usace.army.mil). For information about the NavSys Research Program, contact the Program Manager, Charles E. (Eddie) Wiggins at 601-634-2471; e-mail [Charles.E.Wiggins@usace.army.mil](mailto:Charles.E.Wiggins@usace.army.mil). This CHETN should be cited as follows:

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