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14. ABSTRACT
Stability analyses of high-speed boundary-layer flow past a 5^{o} half angle sharp cone with the wall-normal injection of air through a porous strip are performed using Navier-Stokes solutions for the mean flow and linear stability theory. The configuration and free-stream parameters are chosen to be similar to the experiments, which were carried out at Caltech's T5 shock tunnel to investigate the effect of CO_2 injection on laminar-turbulent transition. The analysis is focused on pure aerodynamic effects in the framework of perfect gas model. It is shown that the injection leads to destabilization of the Mack second mode in the nearfield relaxation region and its stabilization in the far-field relaxation region. To reduce the destabilization effect it was suggested to decrease the injector surface slope or use suction-blowing of zero net injection. However, the e^n computations showed that these modifications did not improve the injector performance in the near-filed region in general. For special cases of low injection rates in which the N-factors in the near field region are below the critical level, shaping can produce a significant stabilization in the mid- and far-field regions.

15. SUBJECT TERMS

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Stability analysis of high-speed boundary-layer flow with gas injection

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Outline

- Background and motivation
- Baseline configuration and numerical approach
- Stability analysis for the baseline configuration
  - Mean flow
  - Acoustic instabilities
  - N-factors
- How to improve the injector performance
- Shaping of injector
  - Conical shapes
  - Cylindrical shape
- Suction-blowing of zero mass injection
- Conclusions

Background: Delay Transition Using Non-Equilibrium CO_2 to Suppress the Second Mode

PROBLEM: **In hypersonic flight, heating loads are typically a dominant design factor**

Turbulent heat transfer rates can be about an order of magnitude higher than laminar rates at hypersonic Mach numbers

A reduction in heating loads by keeping the boundary layer laminar longer means less thermal protection needed and hence less weight to carry, or conversely more payload deliverable for a given thrust.

OBJECTIVE: **Delay transition from laminar to turbulent flow in the boundary layer of a slender hypersonic body by using nonequilibrium CO_2**

Transition in high Mach numbers occurs through the Mack mode – amplification of acoustic waves traveling in the boundary layer

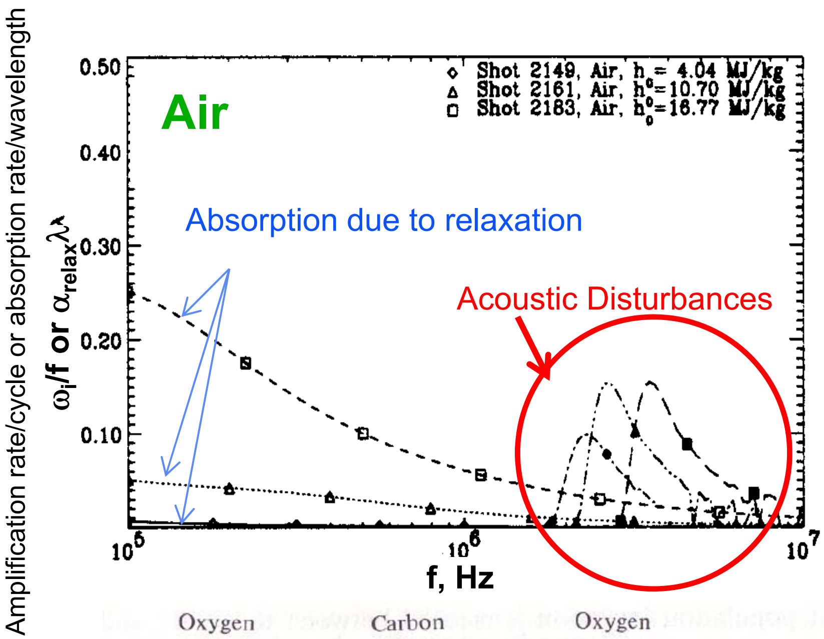
Molecular vibration and dissociation damp acoustic waves

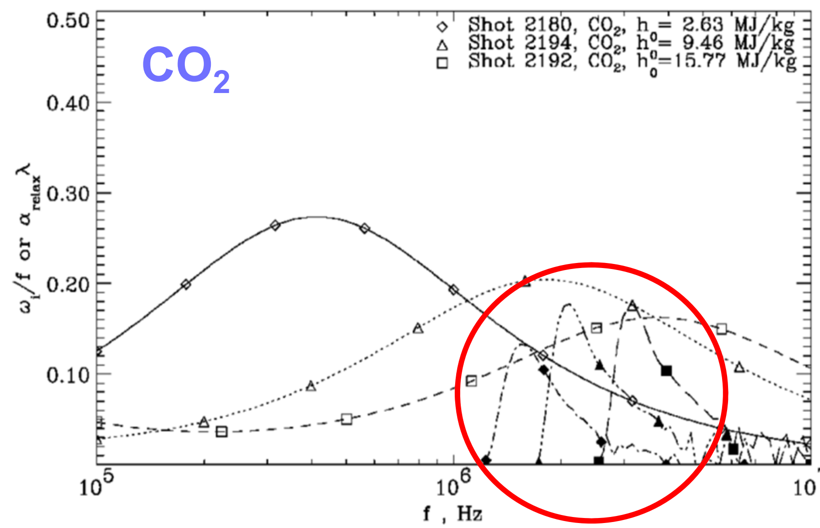
At relevant conditions, CO_2 absorbs most energy at the frequencies most strongly amplified by 2^{nd} (Mack) mode

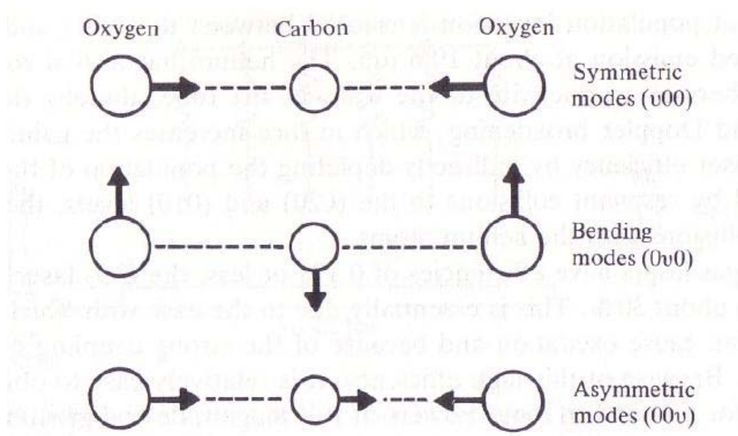
Inject CO_2 to delay transition in air flows of interest

Background

- For CO_2 the broad sound absorption curve peak coincides with the amplification peaks
- This coincidence is most pronounced at enthalpies of ~10 MJ/kg







Θ = 3380 K

Θ = 960 K

Θ = 1993 K

Typical T5
T* ≈ 1700 K (CO_2)

$\tau_{vib} \approx 1\ \mu s$
(Camac 1966)

Plots from: Fujii, K., Hornung, H.G, “Experimental Investigation of High-Enthalpy Effects on Attachment-Line Boundary Layer Transition,” AIAA Journal, Vol. 41, No. 7, July 2003

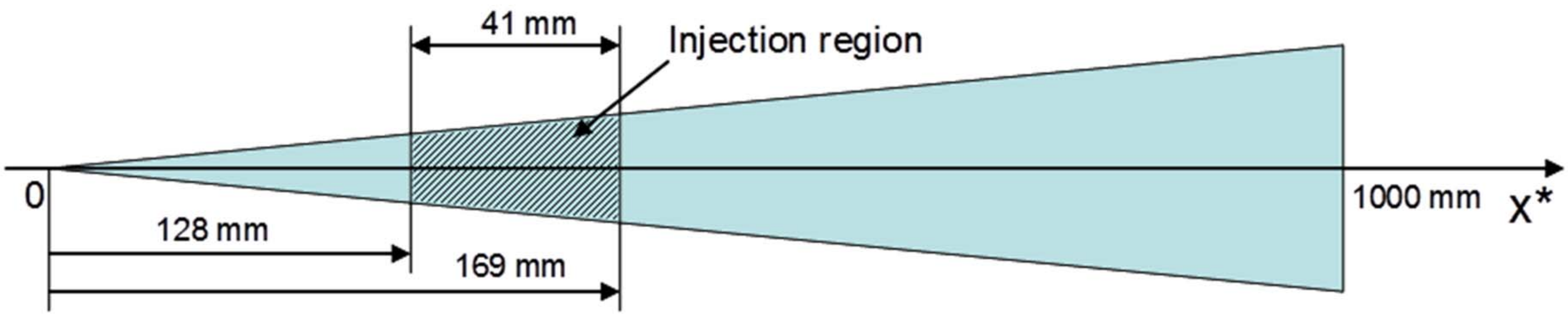
Baseline configuration

Free-stream parameters correspond to Run 2540* in GALCIT T5 shock tunnel

$$M_\infty = 5.3 \qquad \rho^*_\infty = 0.05788\ \mathrm{kg/m^3} \qquad \mu^*_\infty = 4.897\times10^{-5}\ \mathrm{Pa\cdot s}$$

$$T^*_\infty = 1323.77\ \mathrm{K} \qquad U^*_\infty = 3866\ \mathrm{m/s} \qquad L^* = 1\ \mathrm{m}$$

$$p^*_\infty = 21993\ \mathrm{Pa} \qquad T^*_w = 293\ \mathrm{K},\ T^*_w / T^*_\infty = 0.22$$



5-deg half-angle sharp cone with the injector

Gas is injected with the total mass flow rate ranging from 3 g/s to 13.5 g/s

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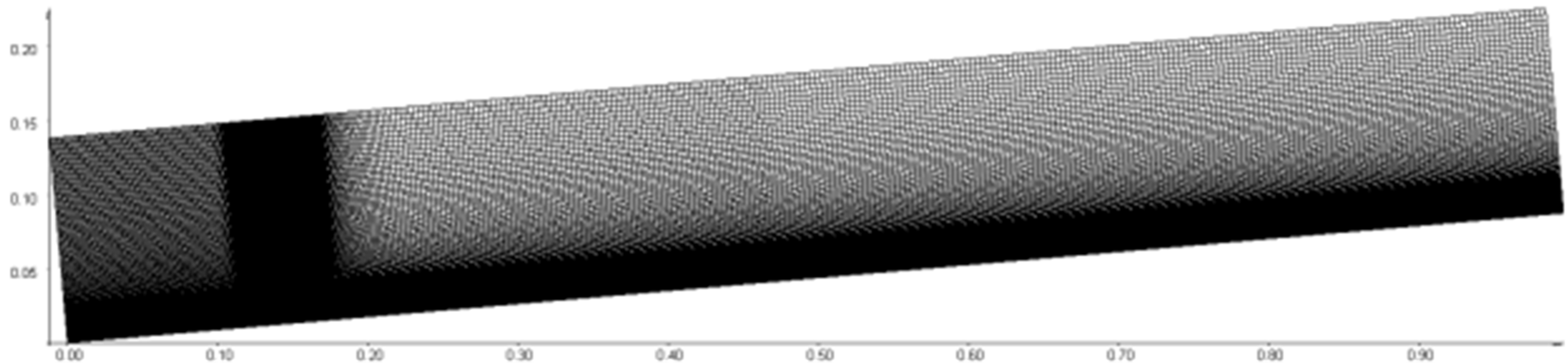
*Parameters are determined using M_∞, T_∞, and ρ_∞ reported by Wagnild, R.M. et al. (AIAA-2010-1244) and perfect-gas model with Pr=0.72 and γ=1.4

Numerical approach for mean flow

In-house Navier-Stokes code HSFlow*

- Perfect gas of Pr=0.72, γ=1.4
- Sutherland viscosity-temperature dependence
- Implicit second-order finite-volume method
- Shock-capturing scheme
- Third-order WENO for advection terms



597×649 grid with

- 50% clustering in the boundary layer
- Clustering near the injector

*Egorov, I.V. et al., Theor. Comput. Fluid Dyn., Vol. 20, No. 1, 2006, pp. 41-54.

Stability analysis

Local-parallel stability computations

- Third-order Rungge-Kutta scheme for integration of stability equation
- Gramm-Schmidt orthogonalization procedure
- Eigenvalues are calculated using a shooting/Newton-Raphson procedure

Disturbance~ $\mathbf{q}(y)\exp(i\alpha x + i\beta z - i\omega t)$

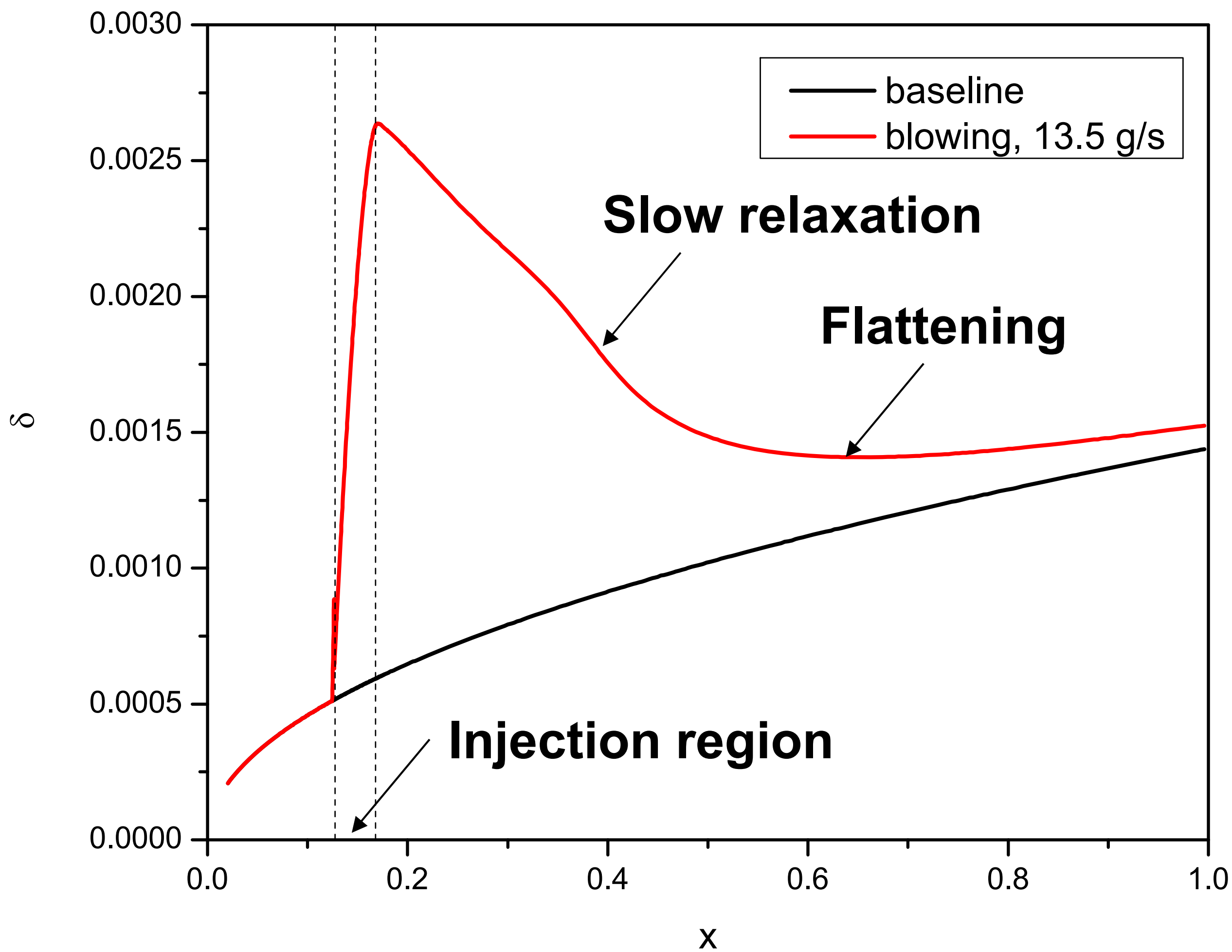
For temporal problem $\omega(\alpha, \beta, x)$ is complex, growth rate= ω_i

For spatial problem $\alpha(\omega, \beta, x)$ is complex, growth rate= $\sigma = -\alpha_i$

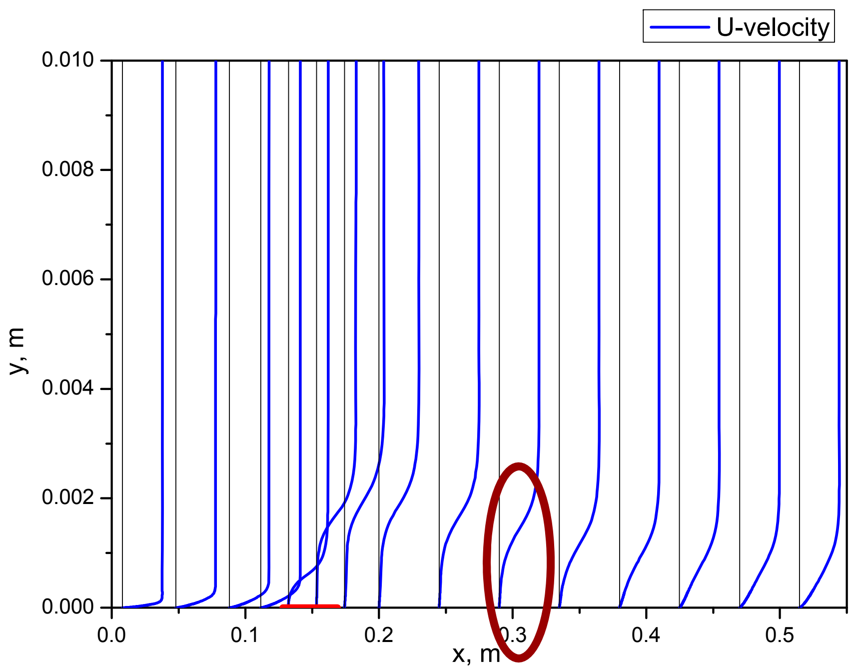
N-factors $N(x,\omega,\beta) = \int_{x_0(\omega,\beta)}^{x} \sigma(\omega,\beta,x)dx$

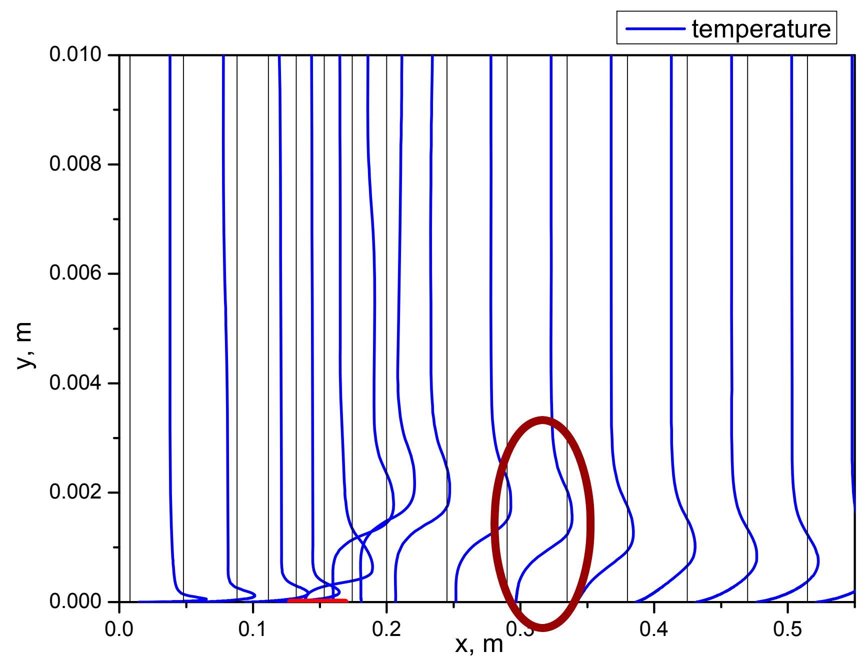
*Egorov, I.V. et al., Theor. Comput. Fluid Dyn., Vol. 20, No. 1, 2006, pp. 41-54.

Boundary layer thickness
baseline configuration with injection rate=13.5 g/s



Mean-flow profiles



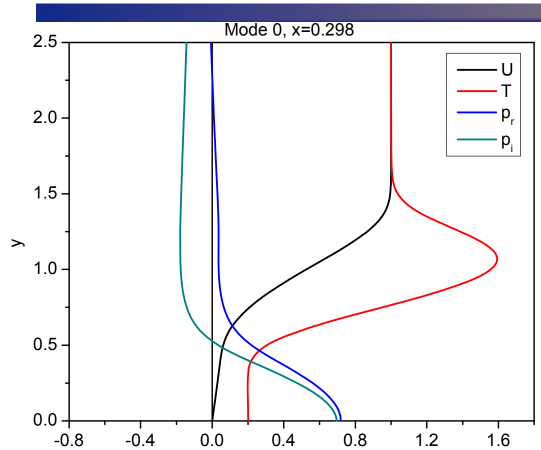


Baseline configuration with injection rate=13.5 g/s

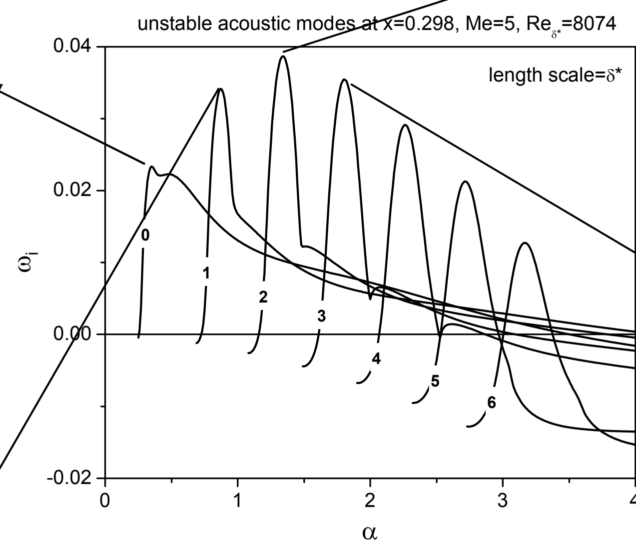
- **Thick region of cold dead flow near wall**
- **Slow relaxation downstream**

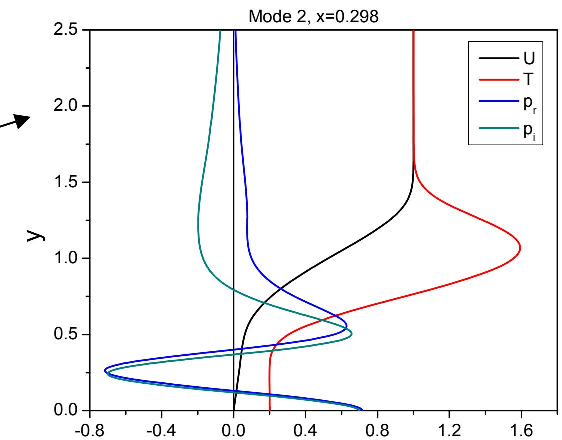
We are dealing with acoustic instability

Temporal stability analysis at x=0.3

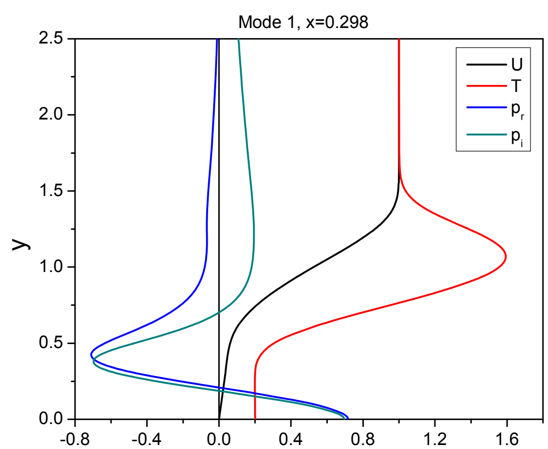


Mode 0=Mack second mode



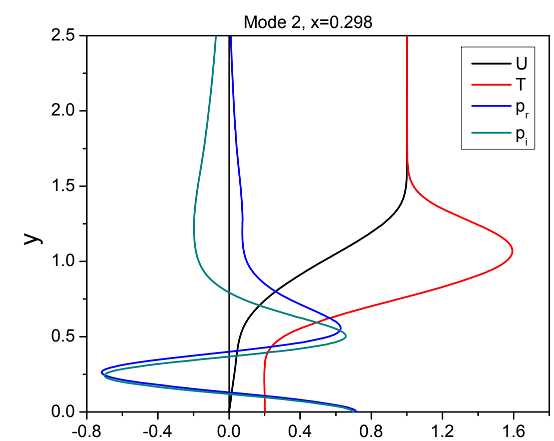


Mode 2=Mack fourth mode



Mode 1=Mack third mode

length scale=δ*

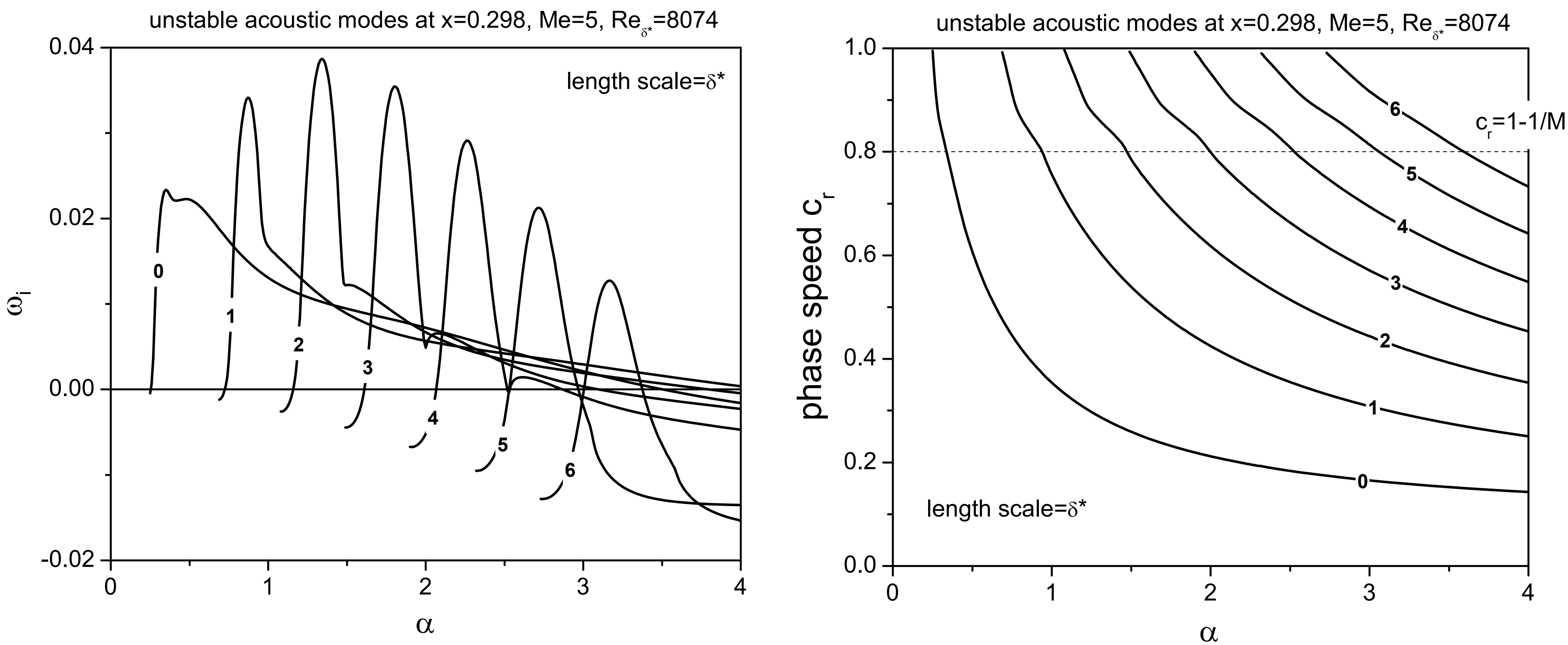


Mode 3=Mack fifth mode

Temporal instability in the relaxation region for baseline configuration

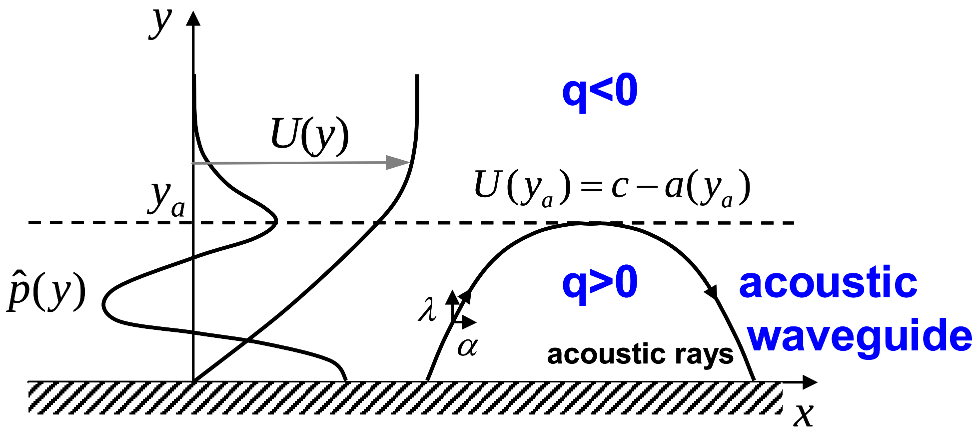
(injection rate 13.5 g/s)

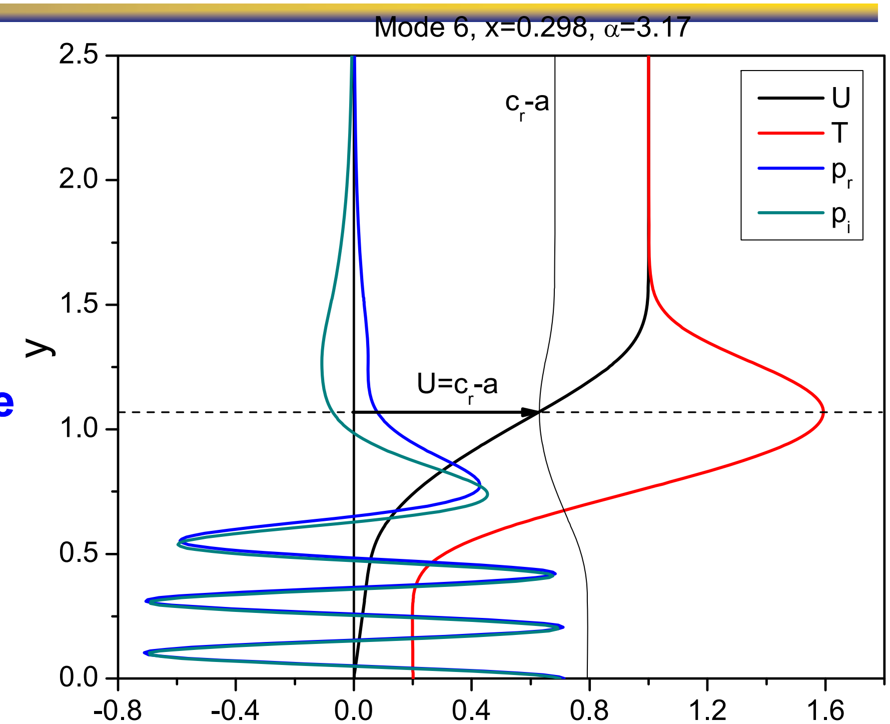
x=0.3



- **There are seven unstable modes!**
- **Mode 2 (Mack fourth mode) has maximal local increment**

We are dealing with acoustic instability (cont'd)





length scale=δ*

Dispersion relation from WKB analysis*,**:

$$\int_0^{y_a} \sqrt{\frac{(\alpha U - \omega)^2 M^2}{T} - 1}\, dy = \frac{\pi}{4} + \pi m$$

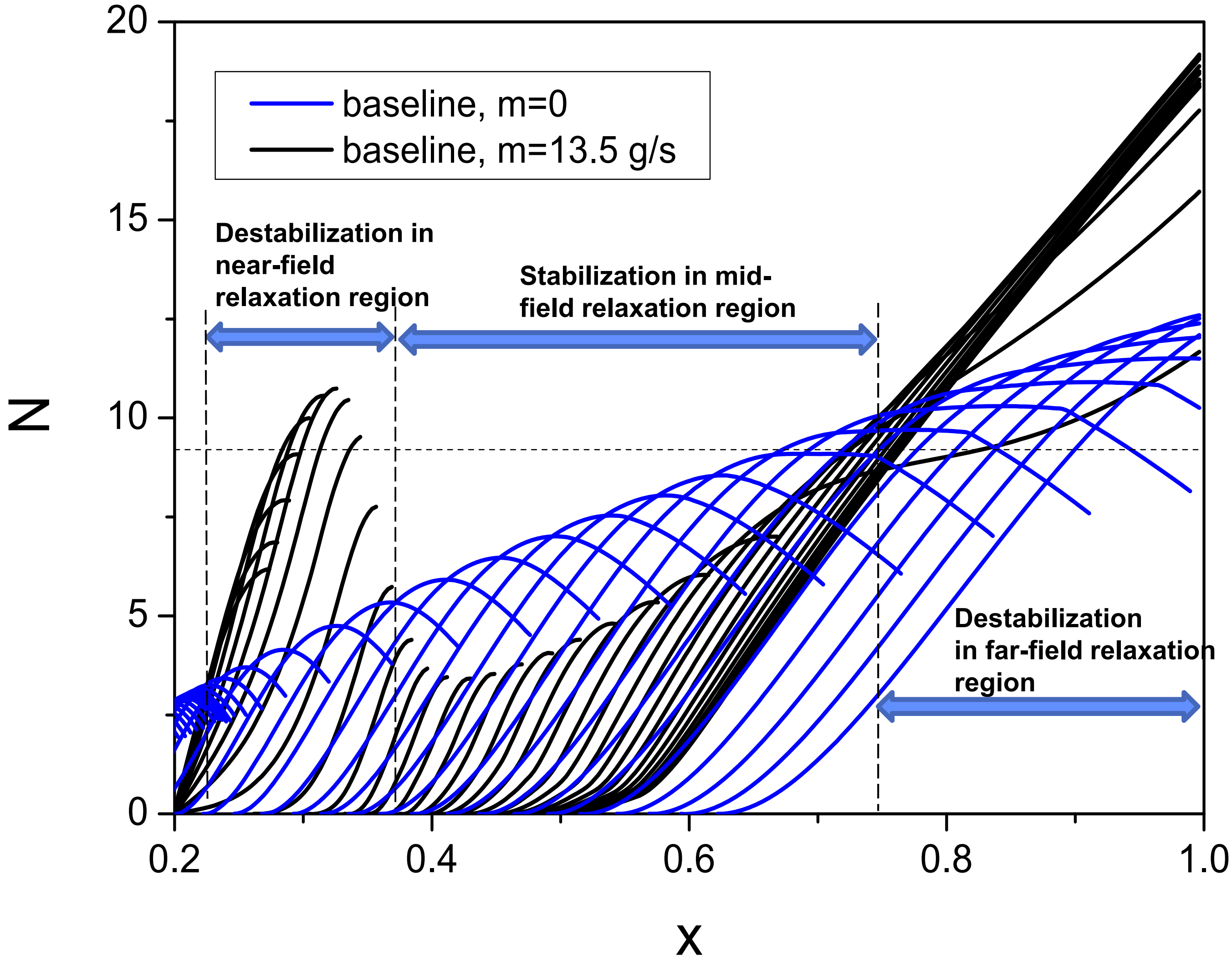
$m = 0,\ 1, \ldots$

Acoustic modes are formed in the waveguide between the wall (y=0) and the relative sonic line $y=y_a$: $U(y_a)=c_r-a(y_a)$

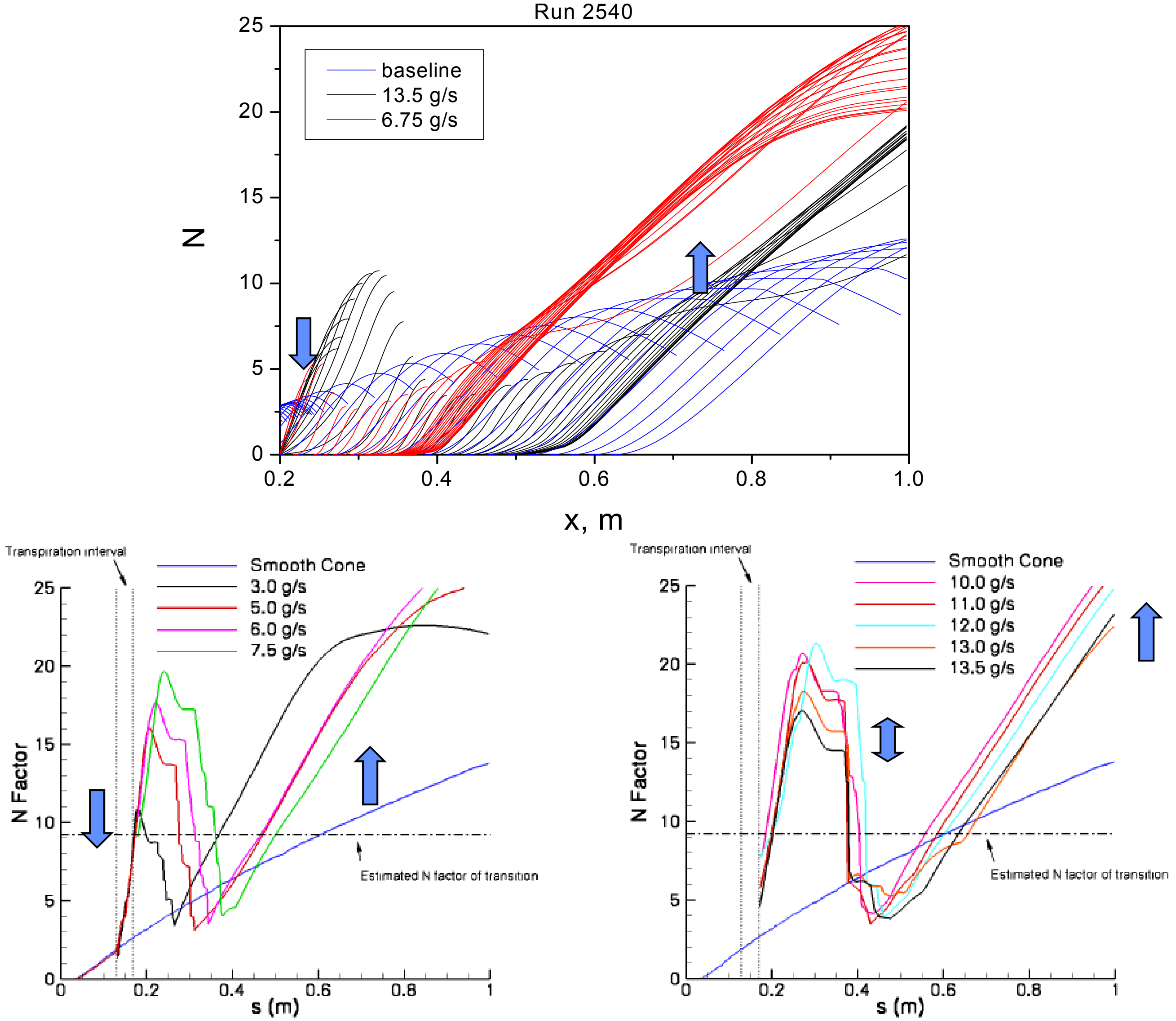
*Guschin, V.R., & Fedorov, A.V., Fluid Dynamics, Vol. 24, No.1, 1989

**Guschin, V.R., & Fedorov, A.V., NASA-TT-20683, April 1990

Injection affects N-factors of Mack second mode



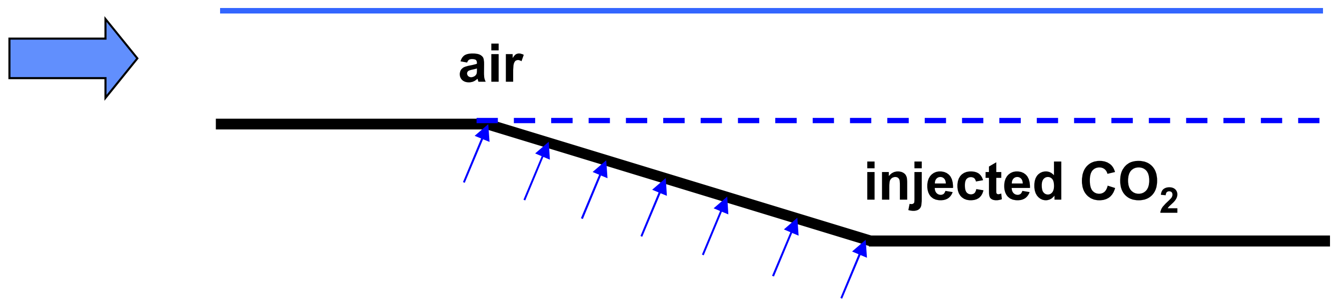
Perfect gas model captures basic trends



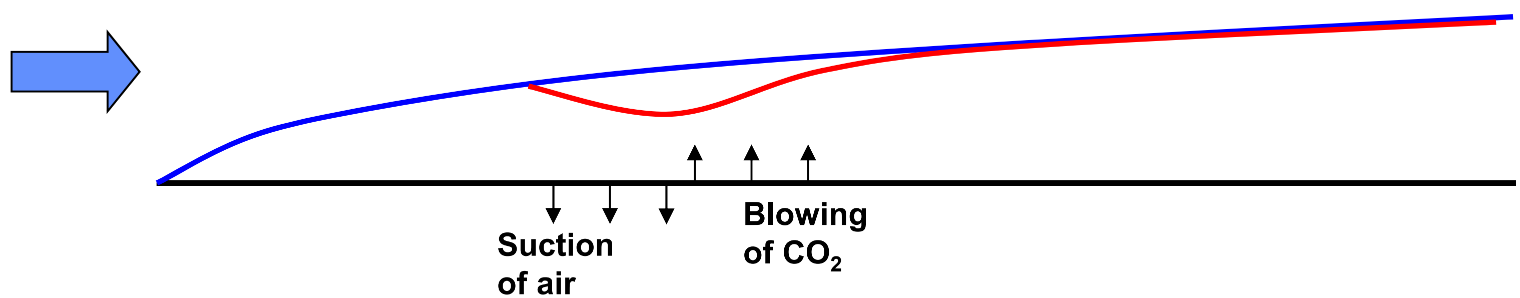
N-factors including real gas effects (Wagnild et al. AIAA-2010-1244)

How to improve the injector performance?

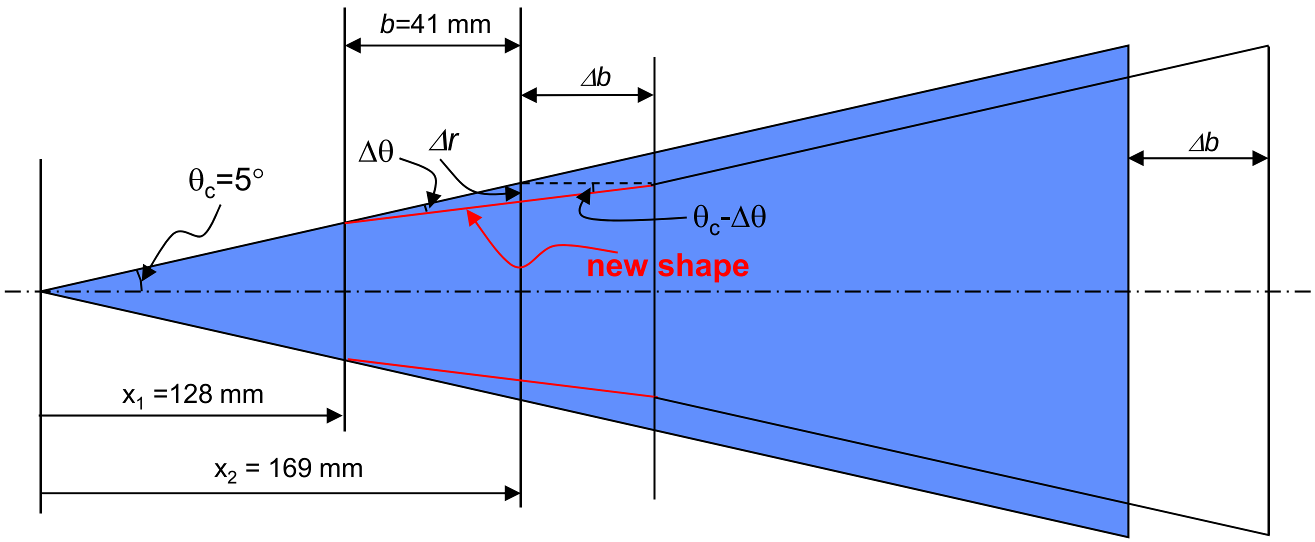
- Negative slope may compensate the blowing effect on the displacement thickness



- Injection of zero total mass addition may help to reduce the relaxation region



Injector of conical shape

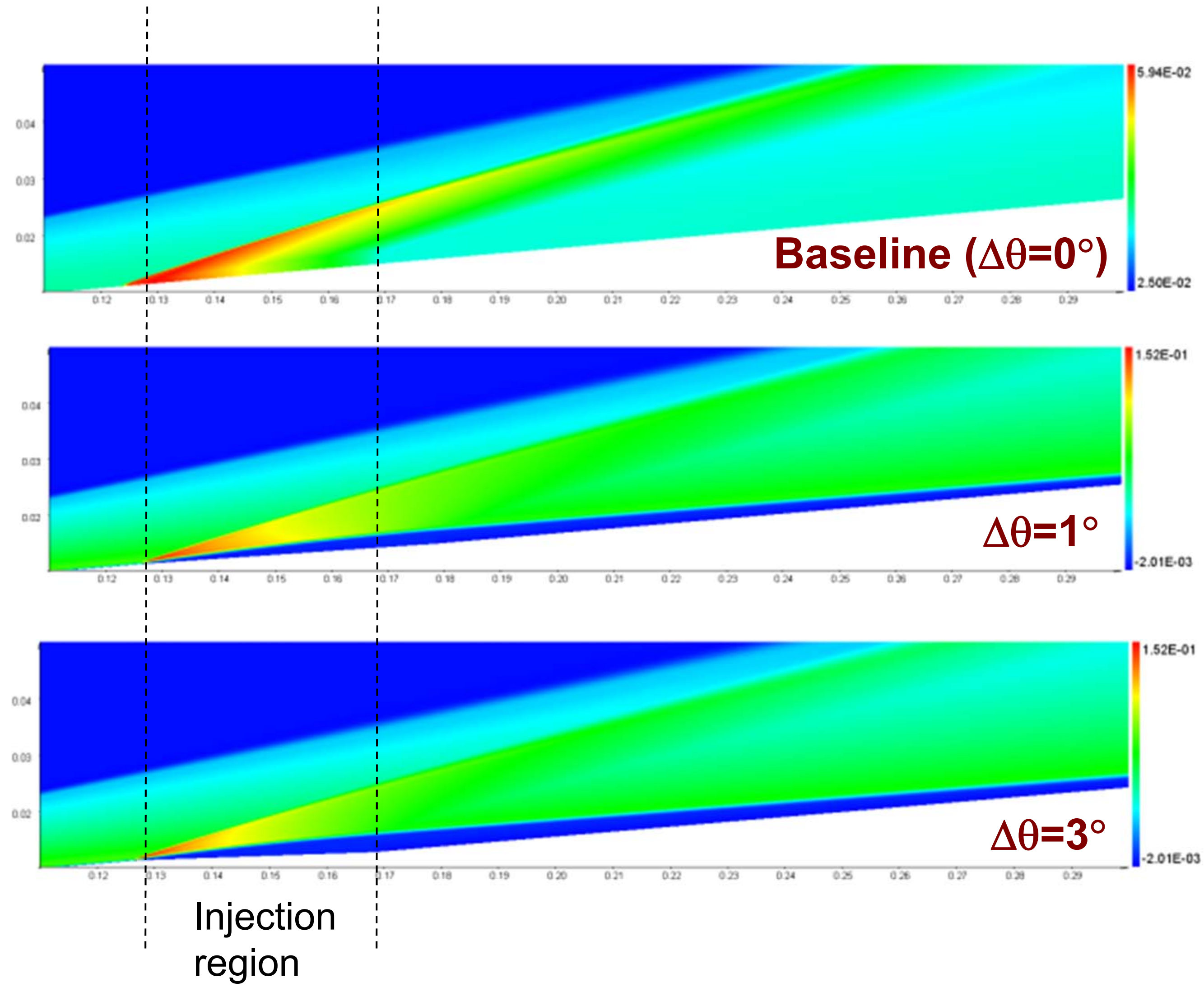


5-deg half-angle sharp cone with the injector having the slope $\theta=\theta_c-\Delta\theta$

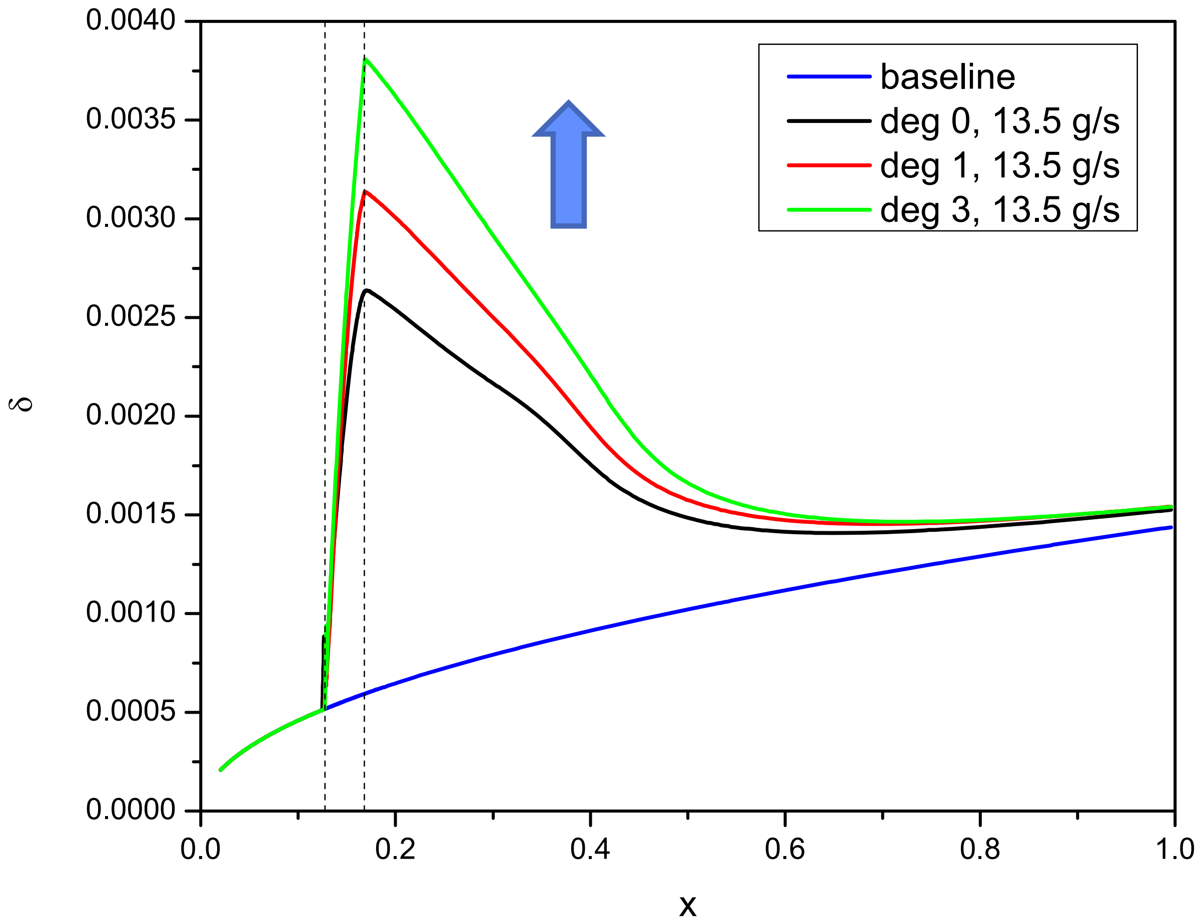
$$\Delta r \approx b[\tan\theta_c - \tan(\theta_c - \Delta\theta)] \approx b\Delta\theta$$

$$\Delta b = \frac{\Delta r}{\tan(\theta_c - \Delta\theta)} \approx \frac{\Delta r}{\tan\theta_c}$$

Mean-flow pressure for conical injectors
(injection rate 13.5 g/s)

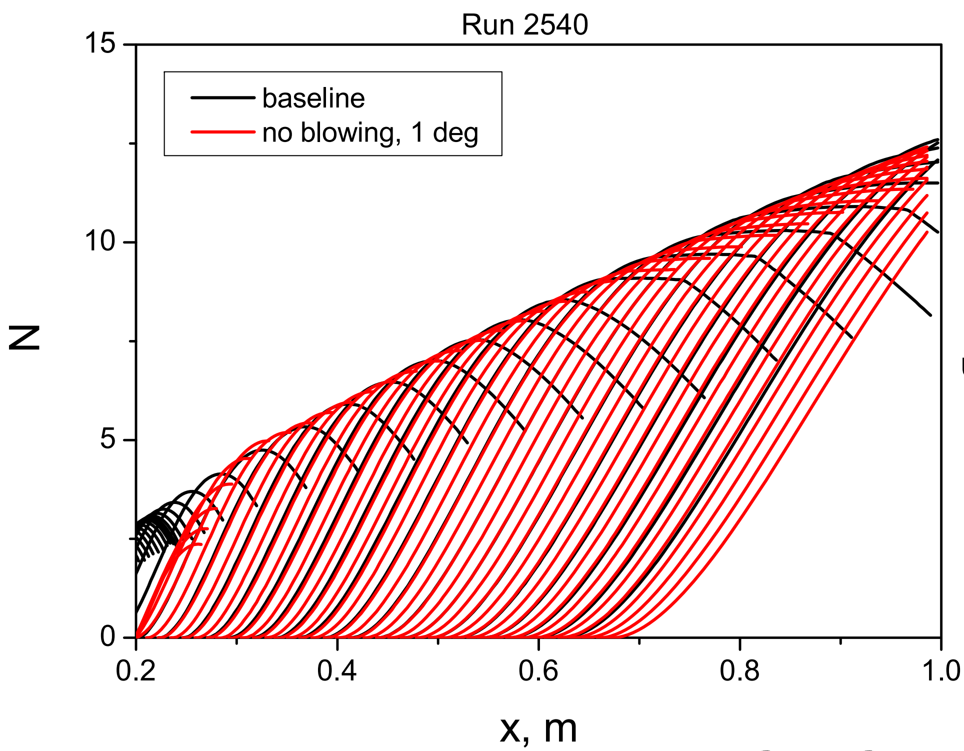


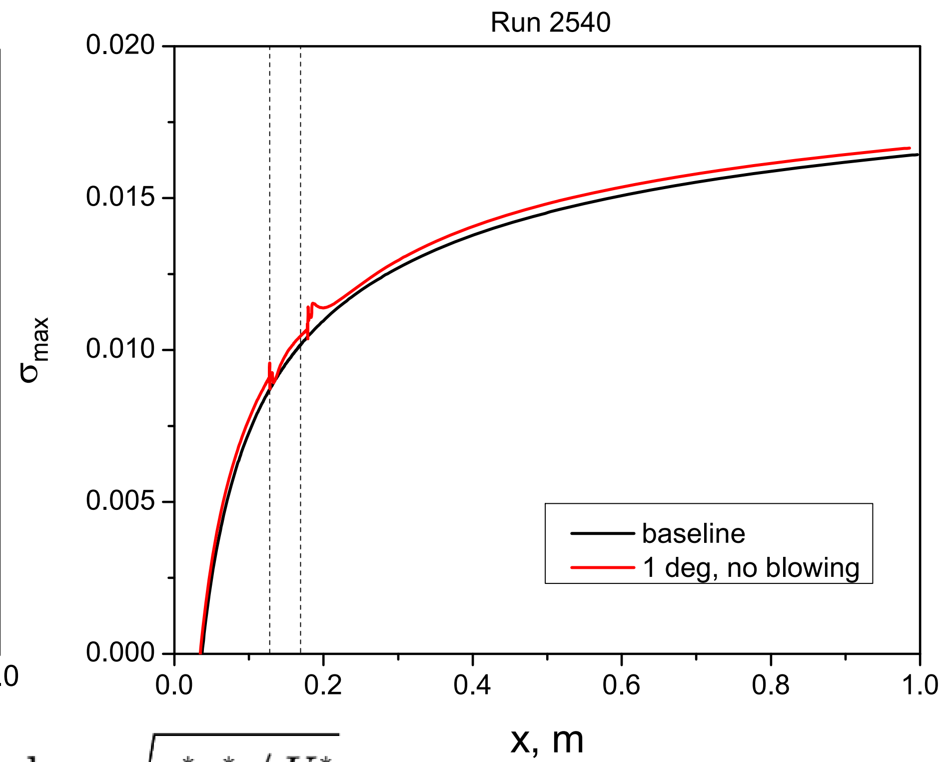
Mean-flow boundary layer thickness for conical injectors
(injection rate 13.5 g/s)



Boundary-layer thickness increases with $\Delta\theta$ in the relaxation region

Shaping Δθ=1° without injection

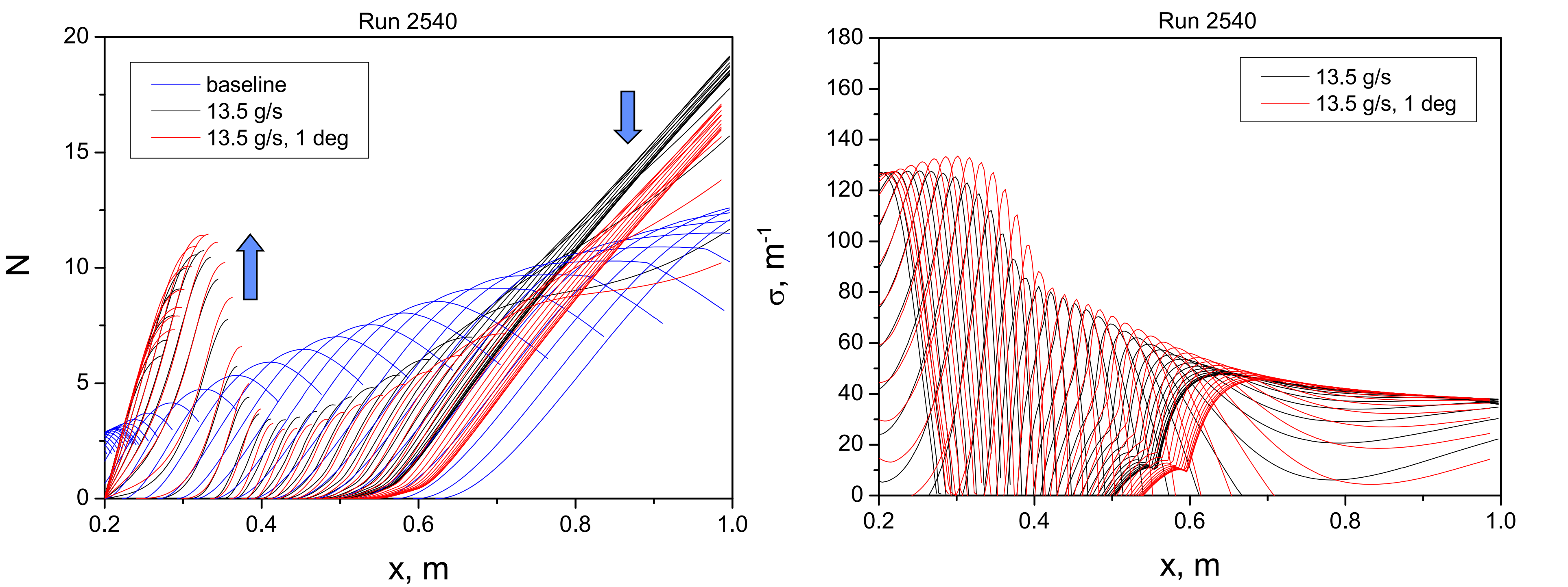




$$\text{length-scale} = \sqrt{\nu_e^* x^* \,/\, U_e^*}$$

Effect of shaping is local and small

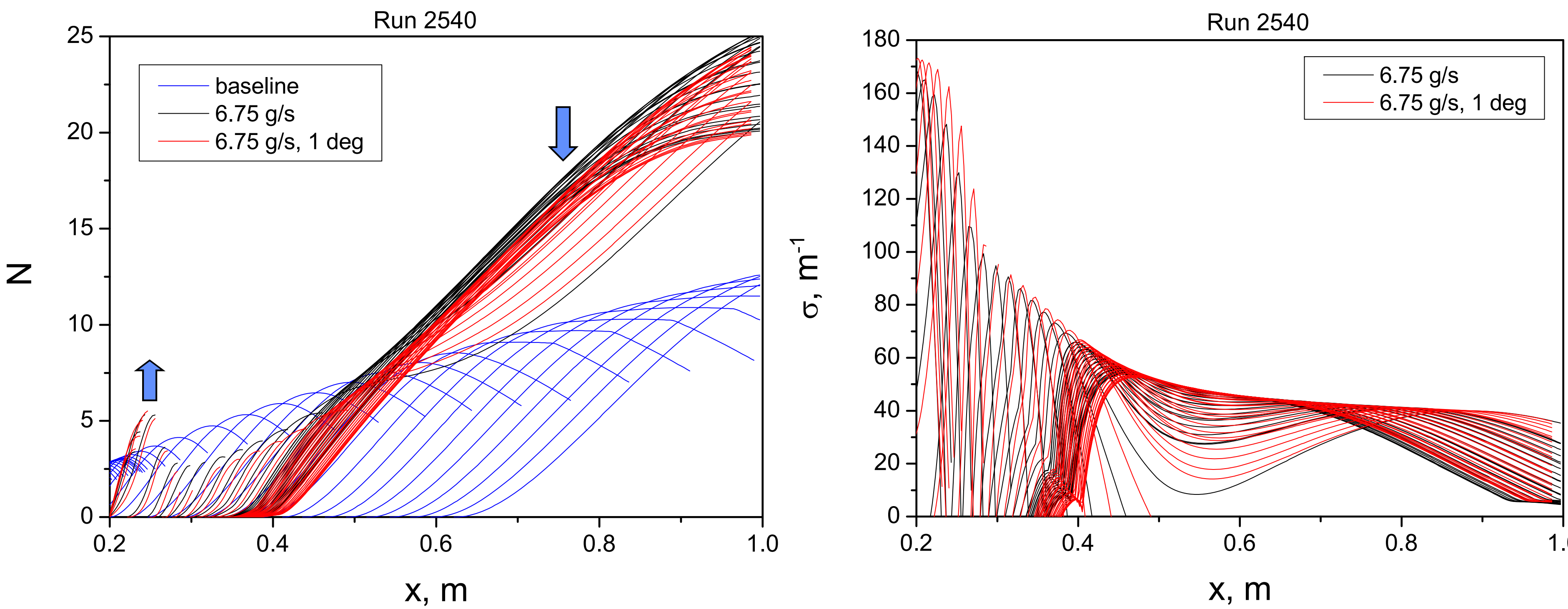
Shaping Δθ=1° with injection of 13.5 g/s



Conical injector with Δθ=1°

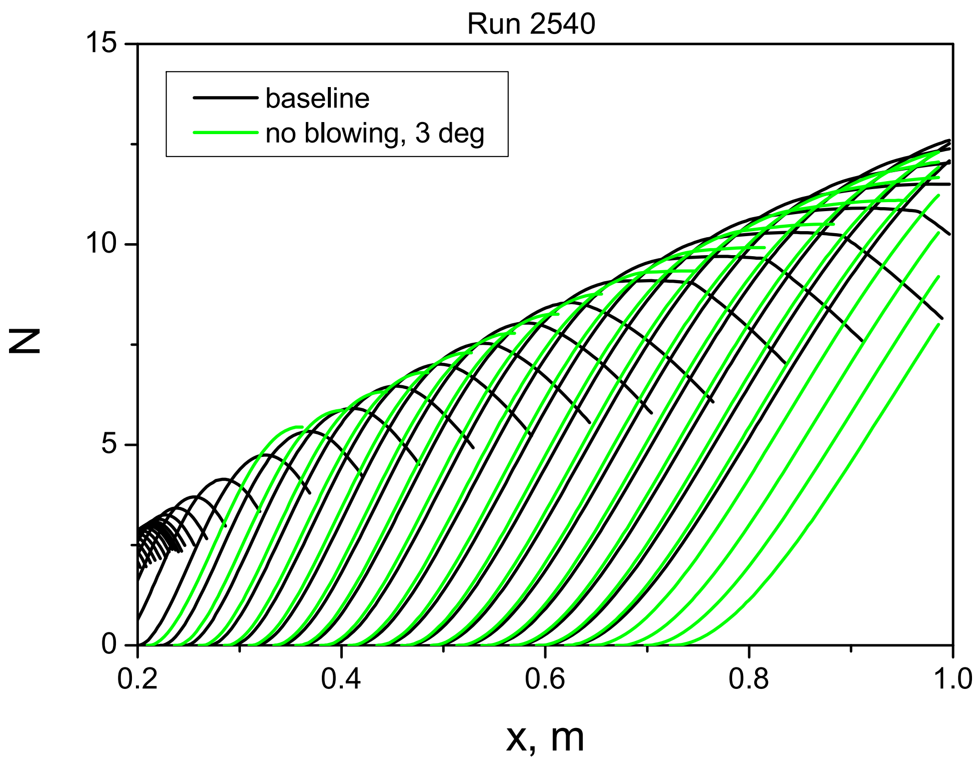
- **Slightly destabilizes flow in the near-field relaxation region**
- **Slightly stabilizes flow in the far-field relaxation region**

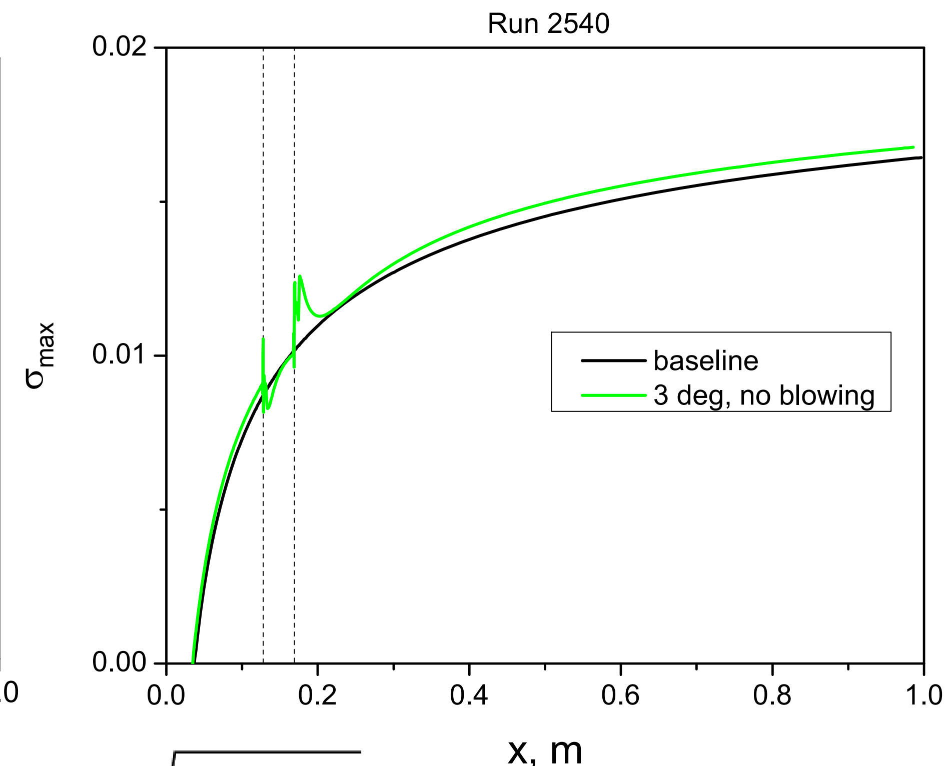
Shaping Δθ=1° and injection 6.75 g/s



Shaping of Δθ=1° produces small effect on stability of flow at the injection rate 6.75 g/s

Shaping of $\Delta\theta=3°$ without injection

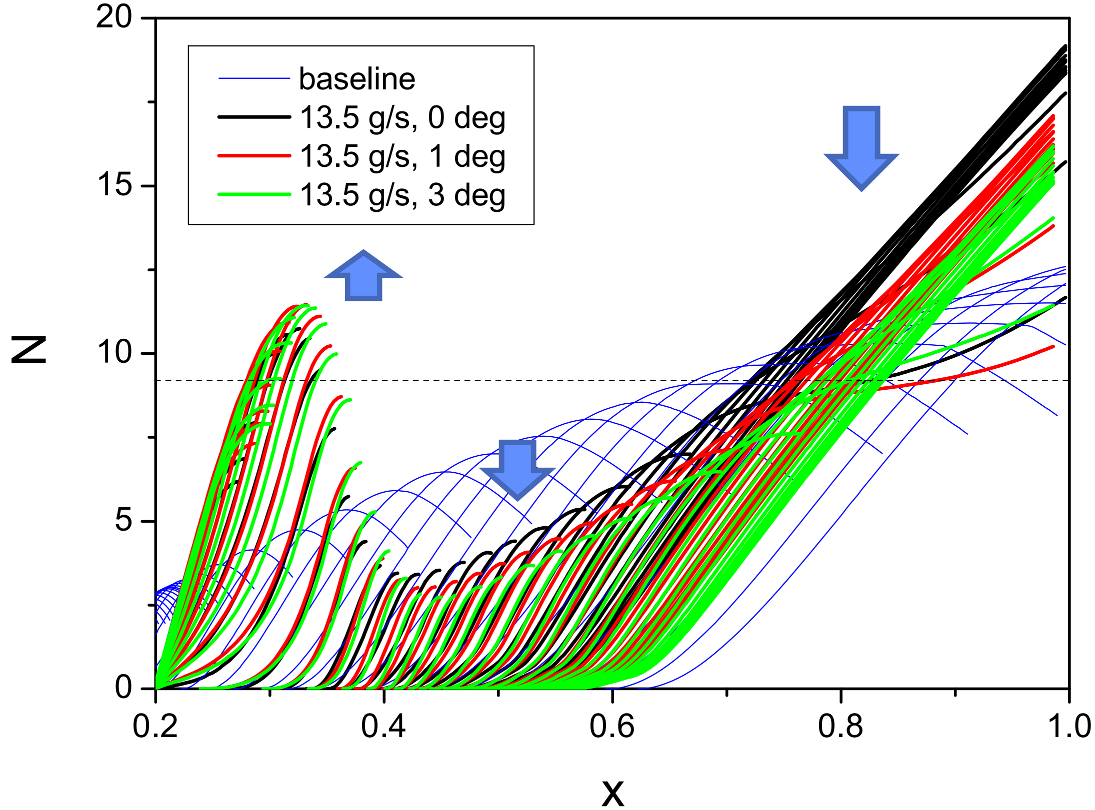




$$\text{length-scale} = \sqrt{\nu_e^* x^* \,/\, U_e^*}$$

Effect of shaping is local and small

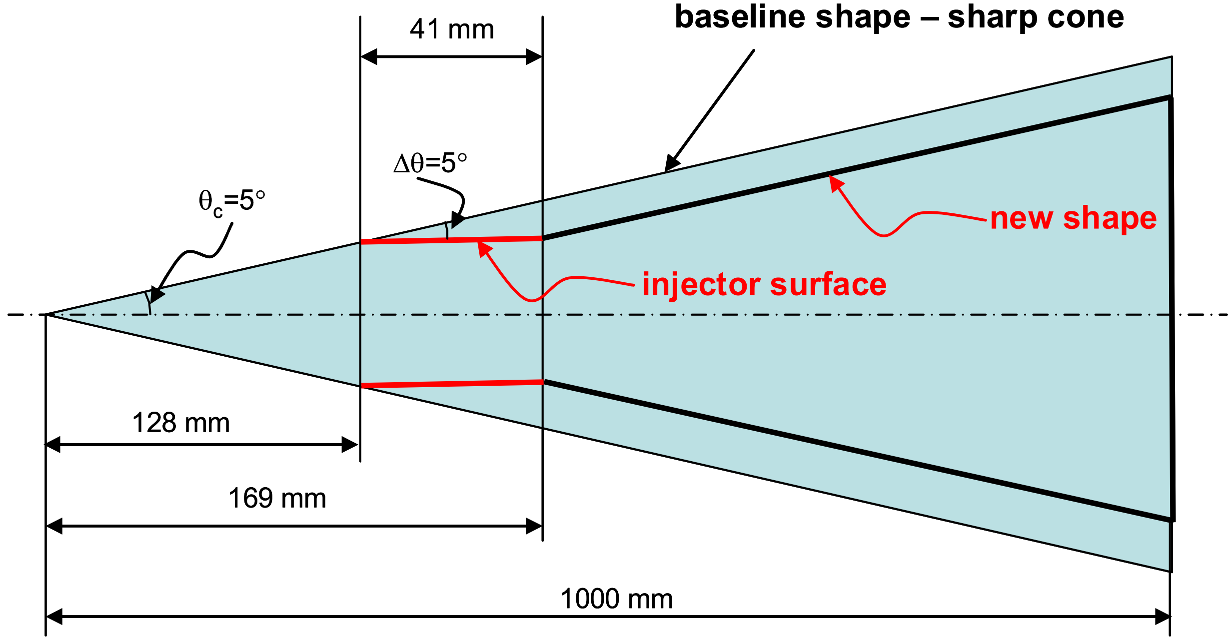
Summary plot of N-factors for conical injectors (injection rate 13.5 g/s)



Shaping with Δθ=1° and 3°

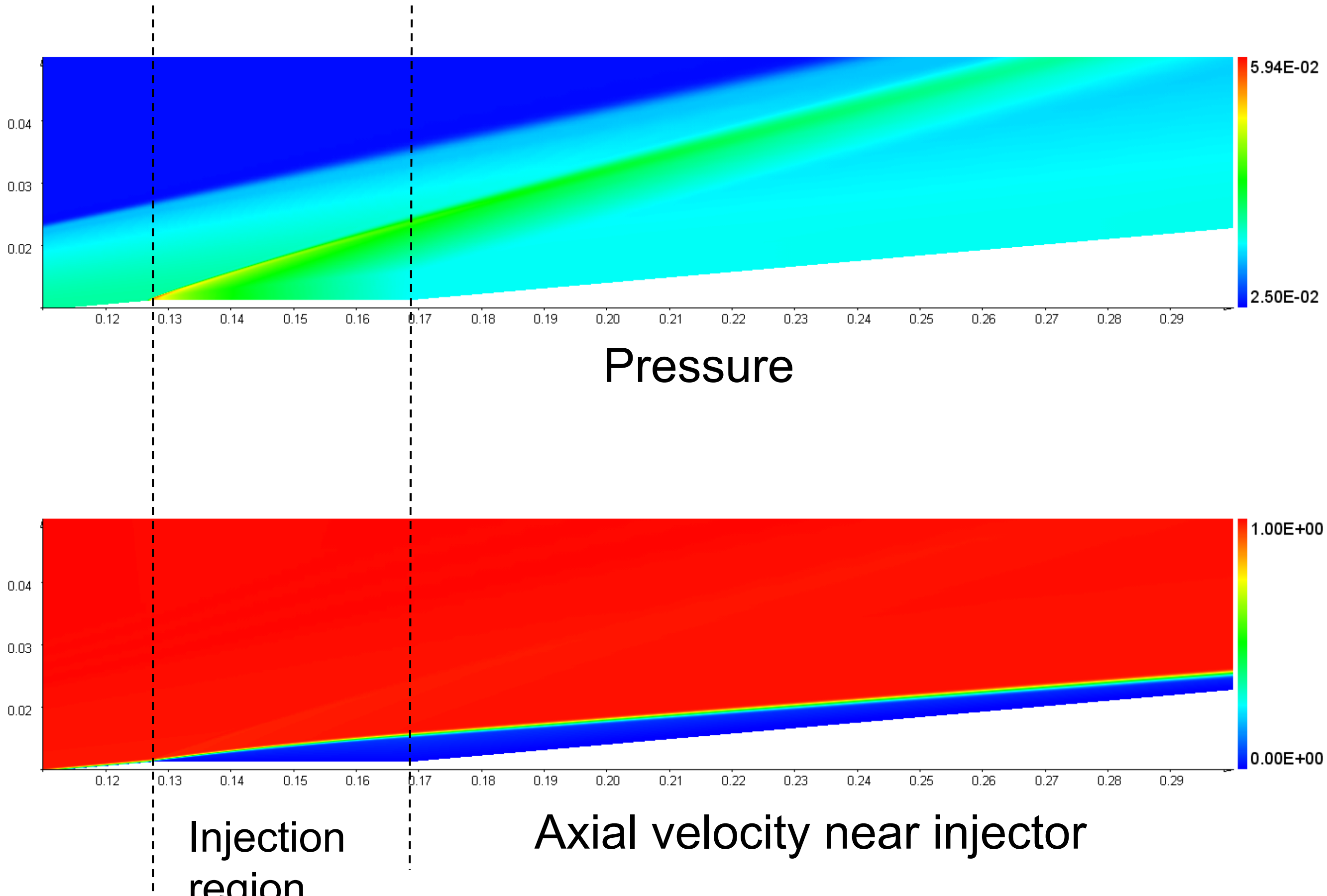
- **Slightly destabilizes flow in the near-field relaxation region**
- **Stabilizes flow in the far-field relaxation region**

Injectors of cylindrical shape

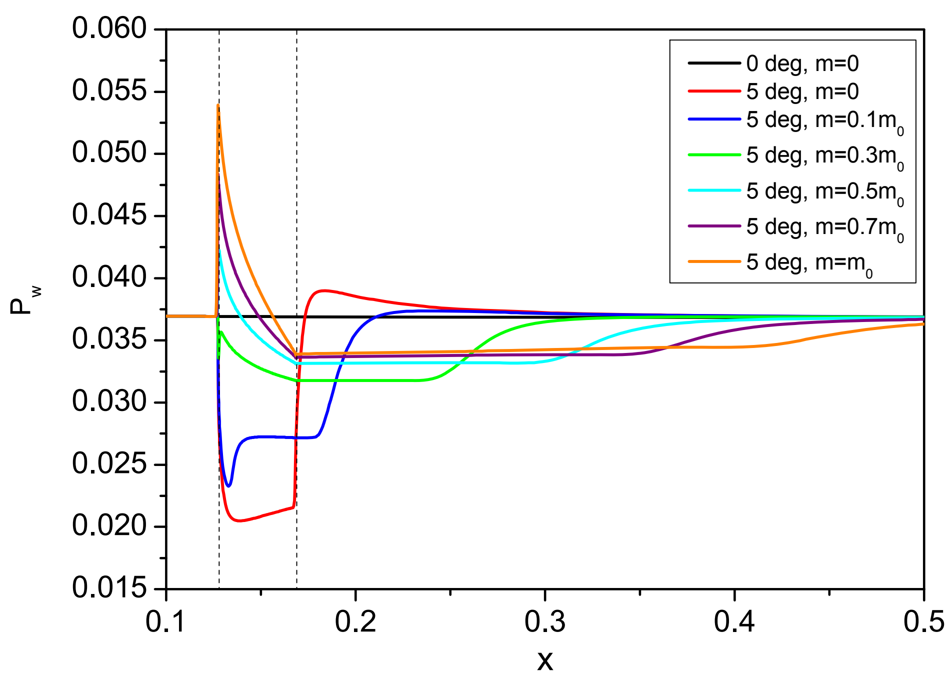


Mean flow for cylindrical injector

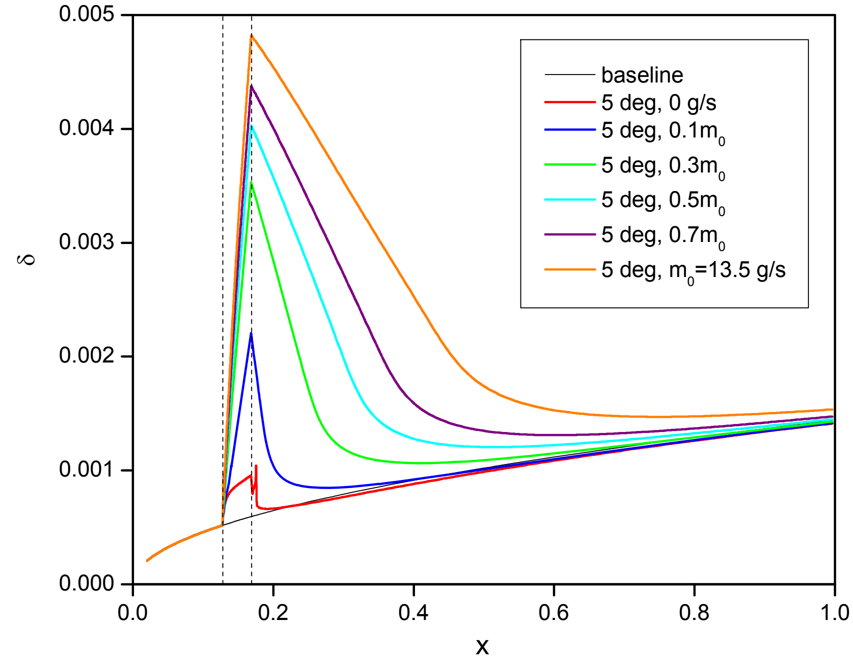
(injection rate 13.5 g/s)



Mean flow for cylindrical injector
(various injection rates)

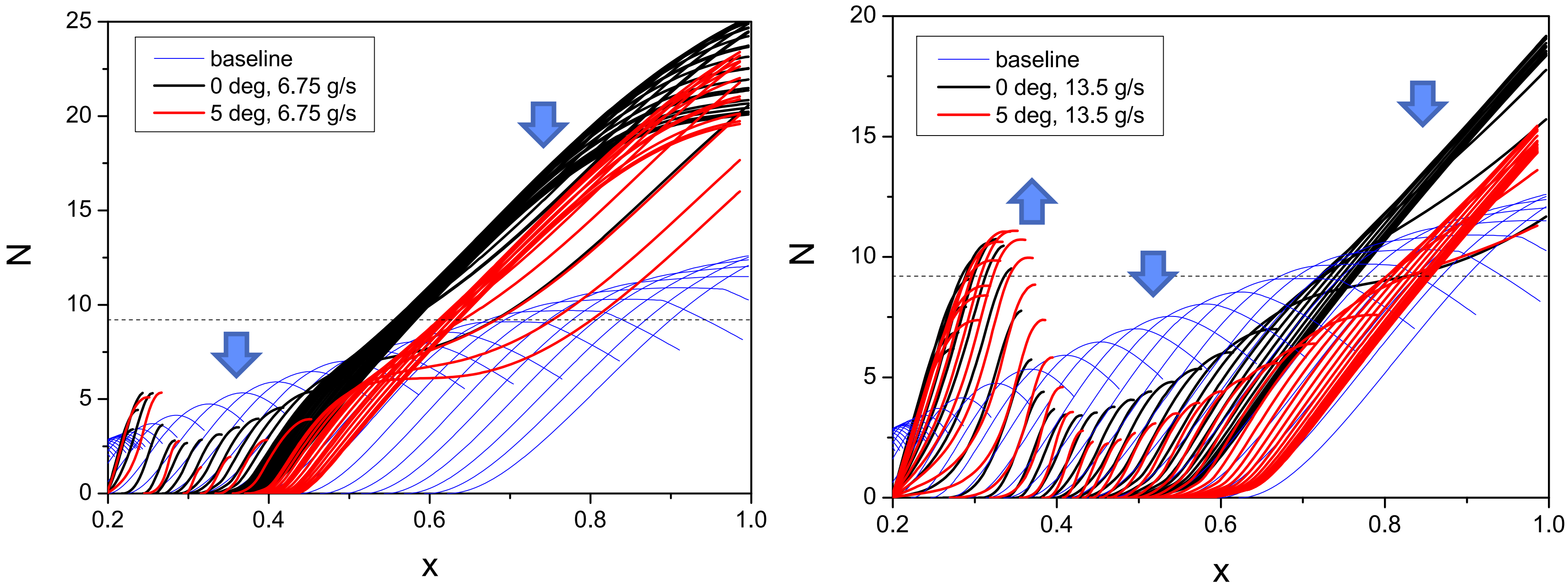


Wall pressure



Boundary layer thickness

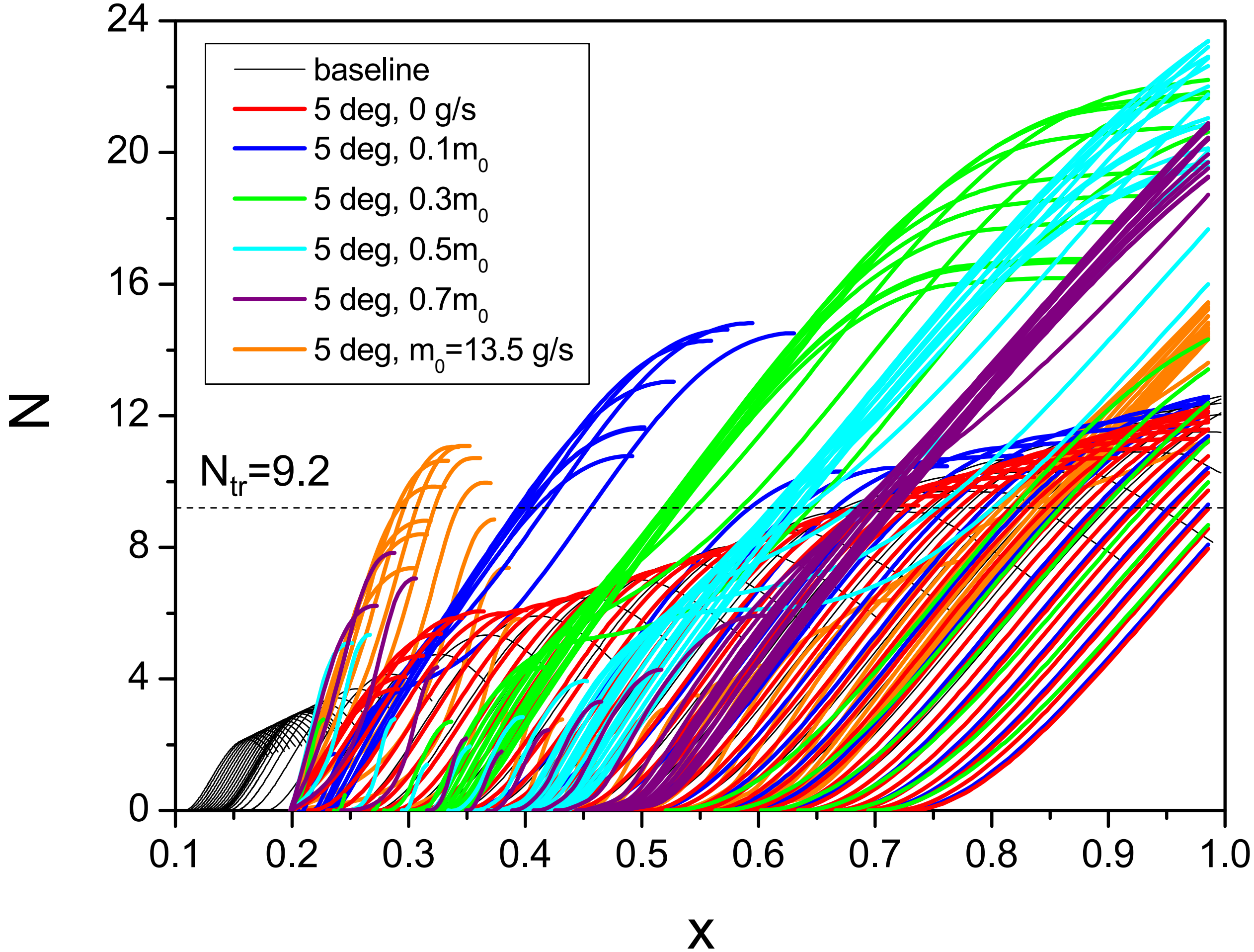
N-factors for cylindrical injector



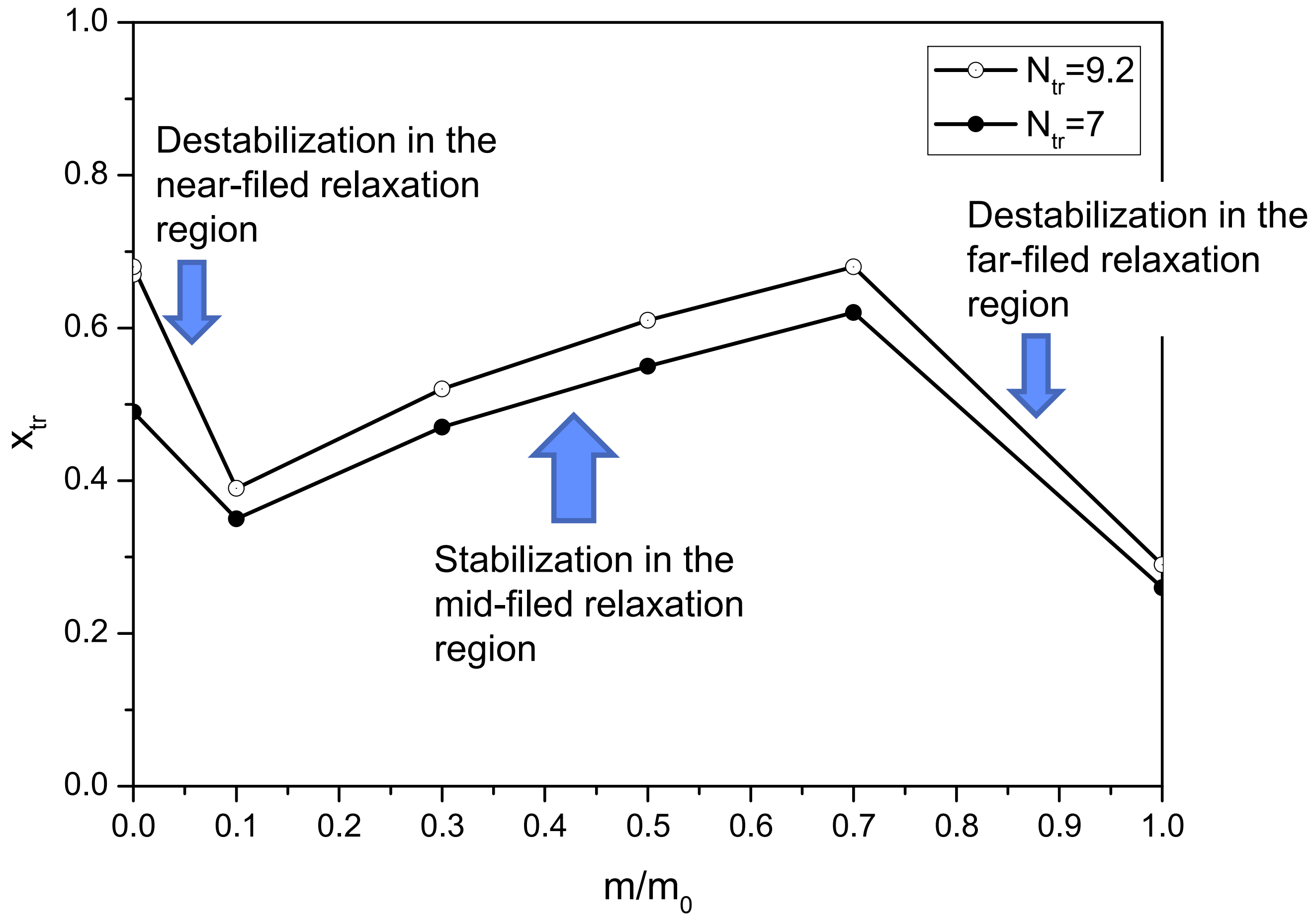
This shaping

- **Slightly destabilizes flow in the near-field relaxation region**
- **Stabilizes flow in the mid- and far-field relaxation region**

Summary plot of N-factors for cylindrical injector
(various injection rates)

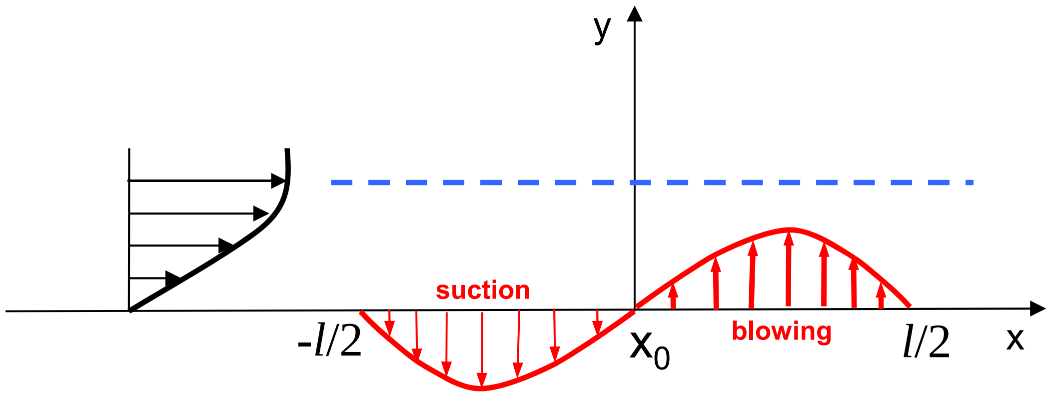


Estimates of the injection effect on the transition onset



Cylindrical injector, m_0=13.5 g/s

Suction-blowing of zero mass addition



$$-l/2 \le x - x_0 \le l/2$$

$$z = (x - x_0)2\pi / l, \quad -\pi \le z \le \pi$$

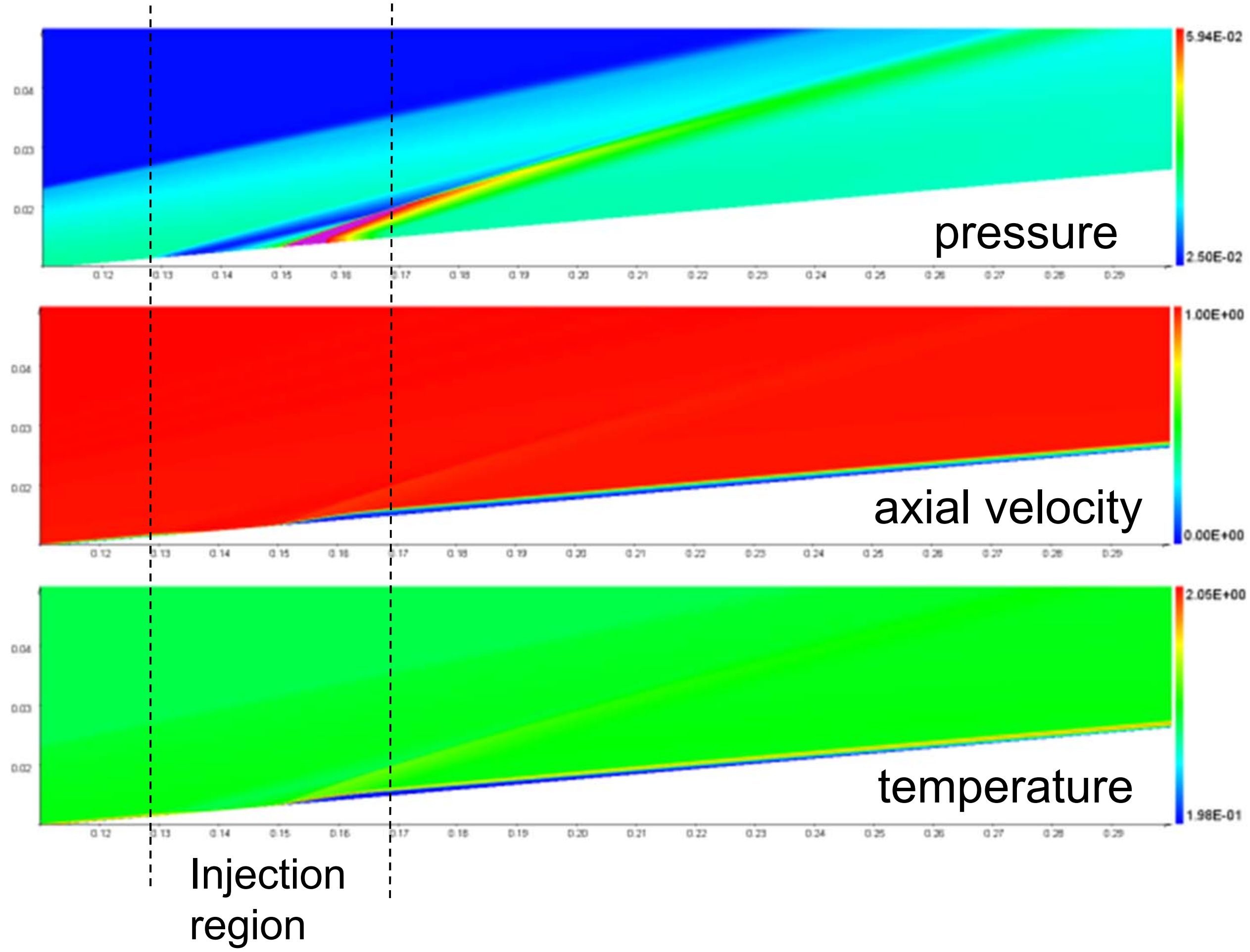
$$q(z) = q_0 \frac{l}{2\pi} \sin z$$

$$\dot{m}_+ = q_0 \frac{l}{2\pi} \int_0^{\pi} \sin z dz = q_0 l / \pi$$

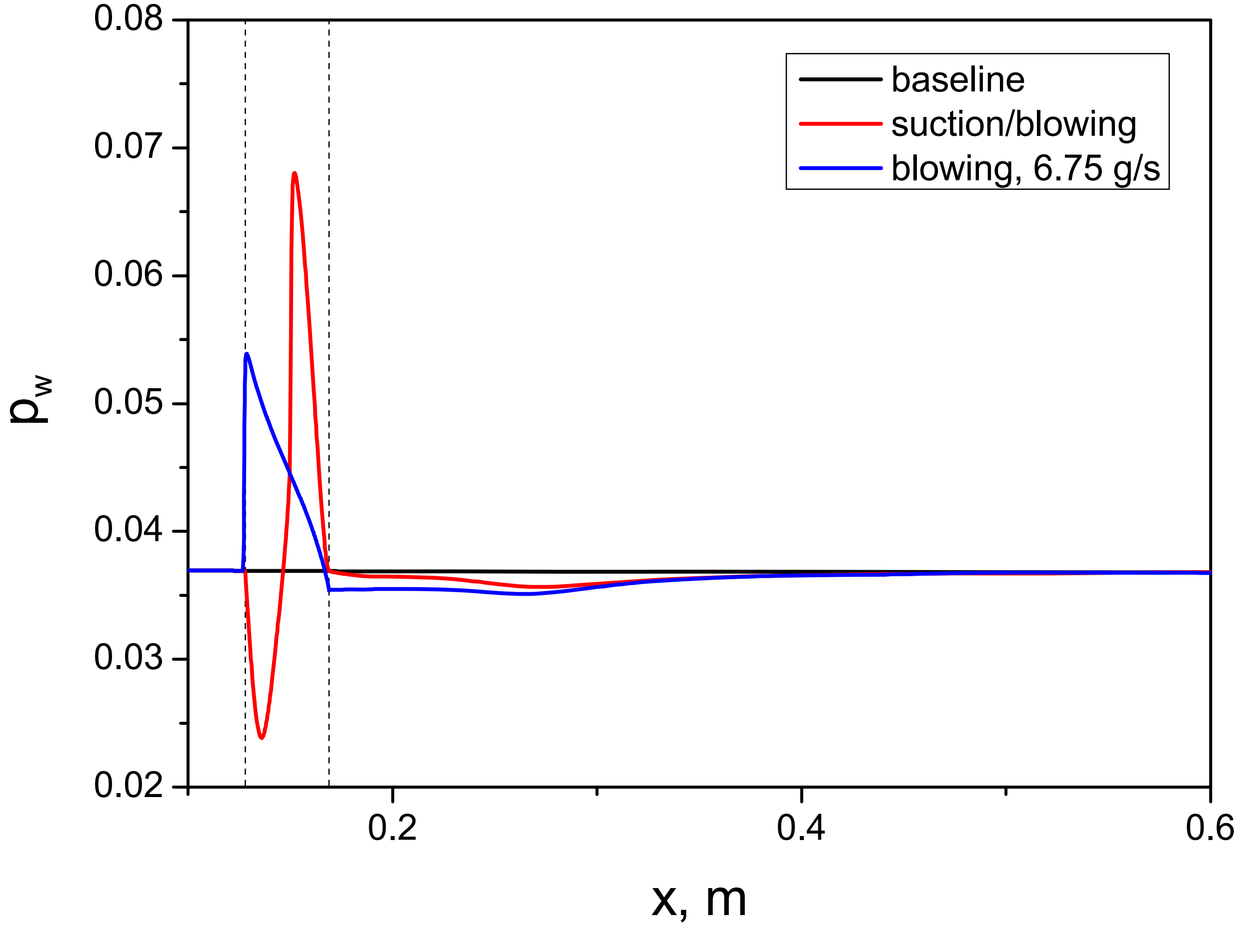
$$\dot{m}_- = q_0 \frac{l}{2\pi} \int_{-\pi}^{0} \sin z dz = -q_0 l / \pi$$

Mean flow for suction-blowing system

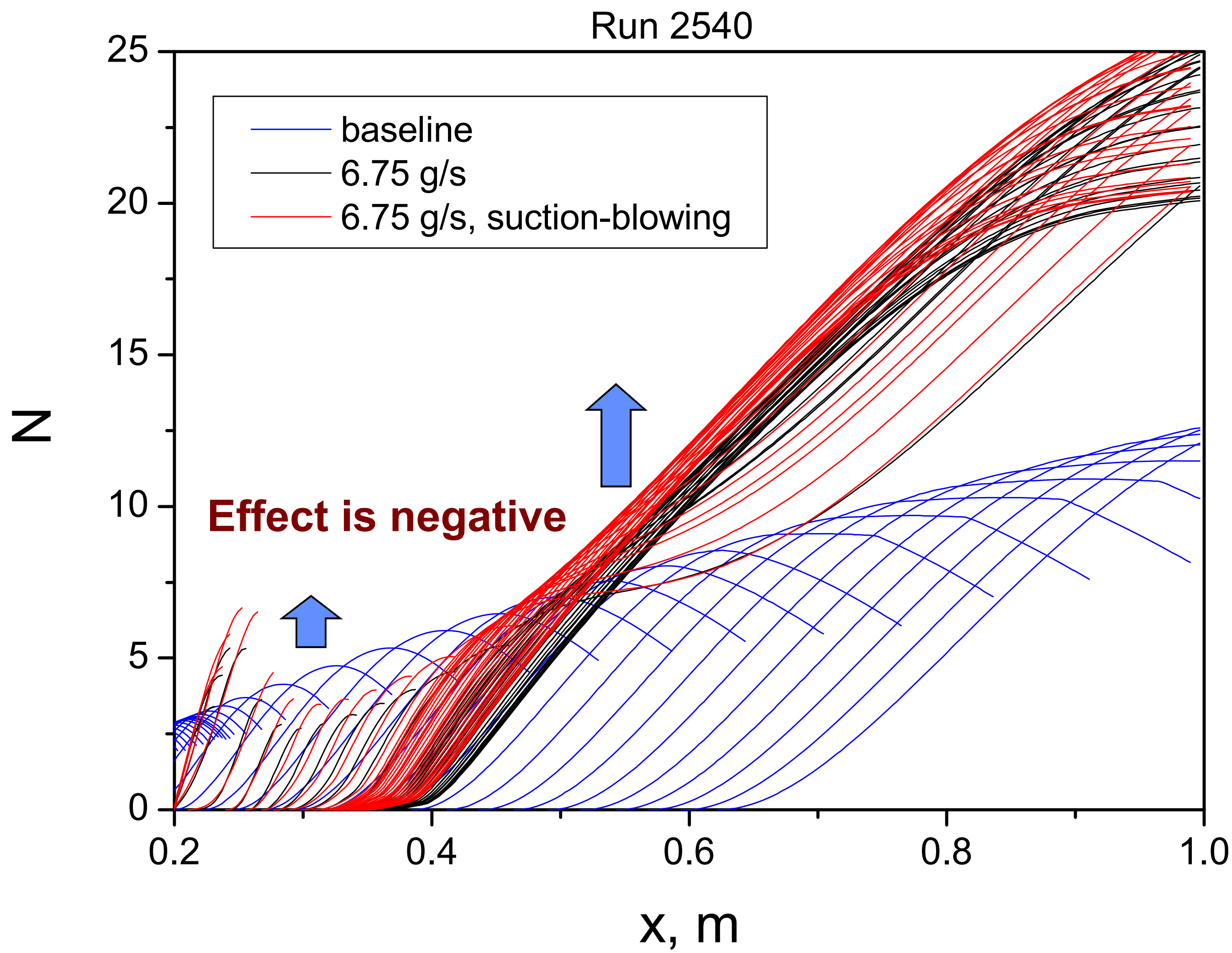
(m_+=6.75 g/s)



Wall pressure distribution for suction-blowing system (m₊=6.75 g/s)



N-factors for suction-blowing system
(m_+=6.75 g/s)



Conclusions

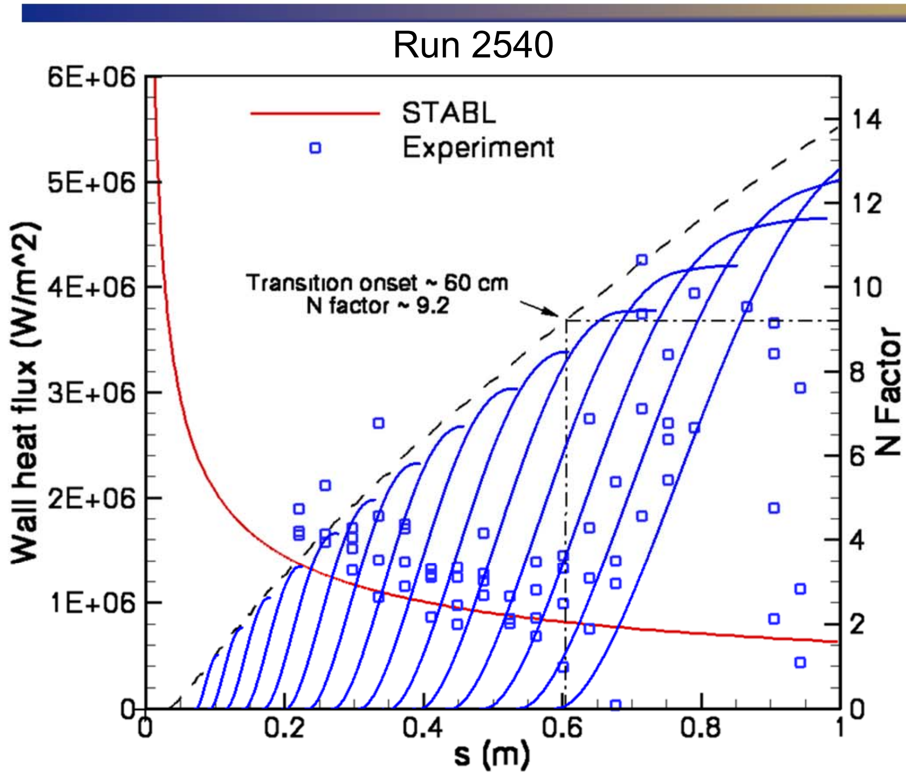
- Injection induces a cold dead-flow layer in the downstream relaxation region
- The near-wall flow behaves as a wave guide which can support several unstable modes of acoustic type
  - The most unstable is the Mack second mode
  - The phase speeds of instability are close to those of slow acoustic waves in the free-stream
  - Instability frequencies are several times smaller than in the no injection case
  - This may lead to dramatic increase of receptivity to free-stream noise

Conclusions (cont'd)

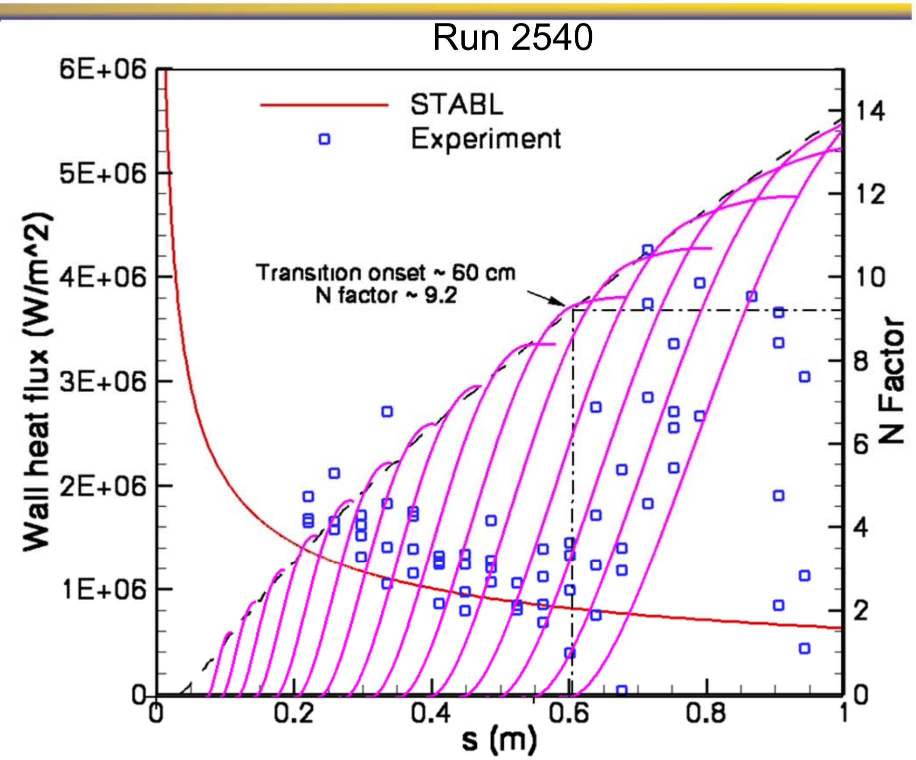
- The e^N computations for baseline configuration showed
  - Injection leads to destabilization of the near-field region, stabilization of the mid-field relaxation region, and destabilization of the far-field relaxation region
  - The level of these effects essentially depend on the injected mass flow rate
- The injector shaping considered
  - Does not stabilize the near-field flow at sufficiently large injection rates
- For relatively small injection rates the shaping produces a significant stabilization effect in the mid- and far-field relaxation regions
- The suction-blowing of zero mass addition destabilizes the flow in the whole relaxation region

Backup

Stability analysis (cont'd)



Blue lines: pres., γ=1.4

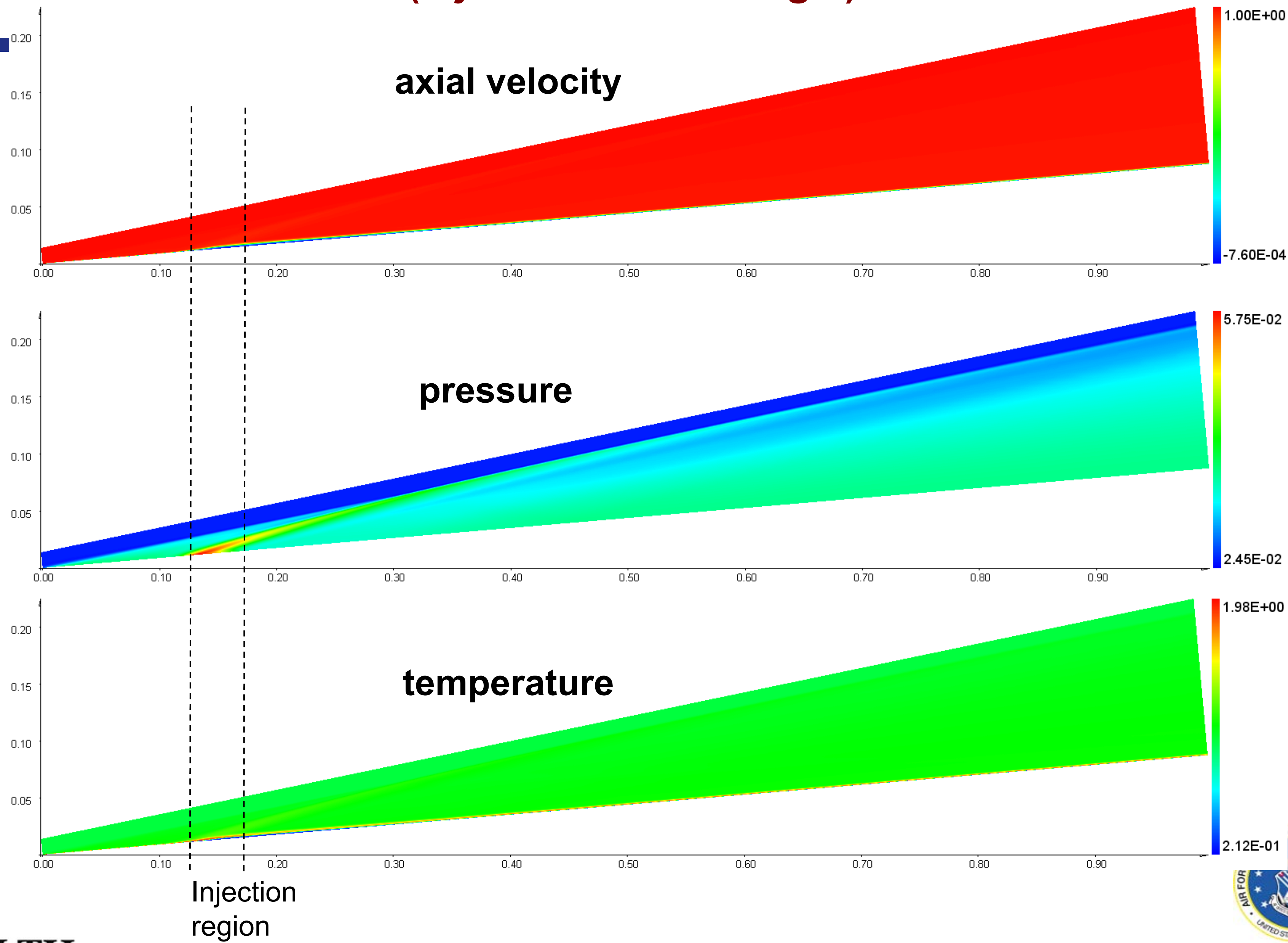


Pink lines: pres., γ=1.38

With small correction of γ, N-factors predicted by the perfect-gas model are close to N-factors predicted by STABL*

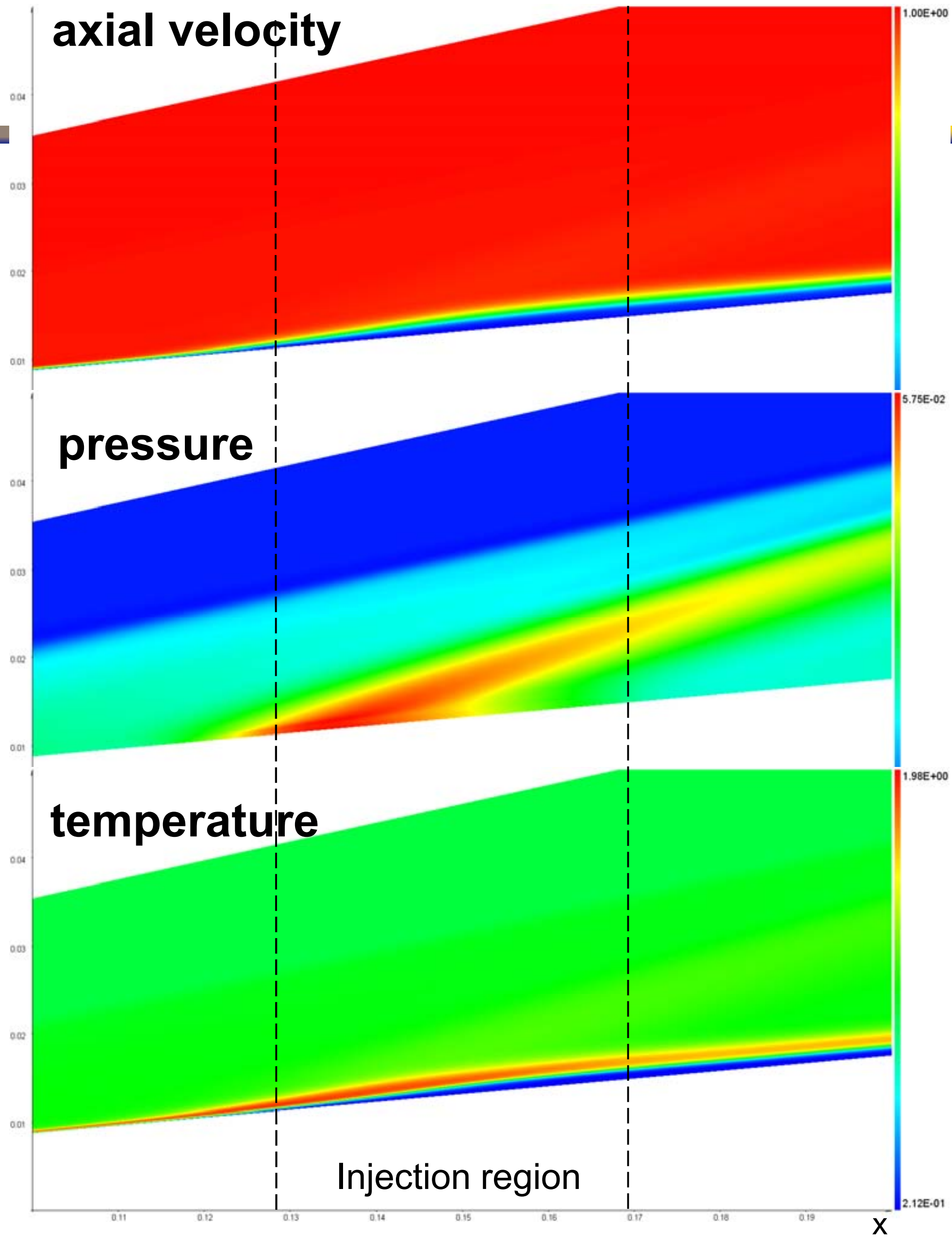
*Wagnild, R.M. et al. AIAA-2010-1244

Mean flow for baseline configuration
(injection rate=13.5 g/s)

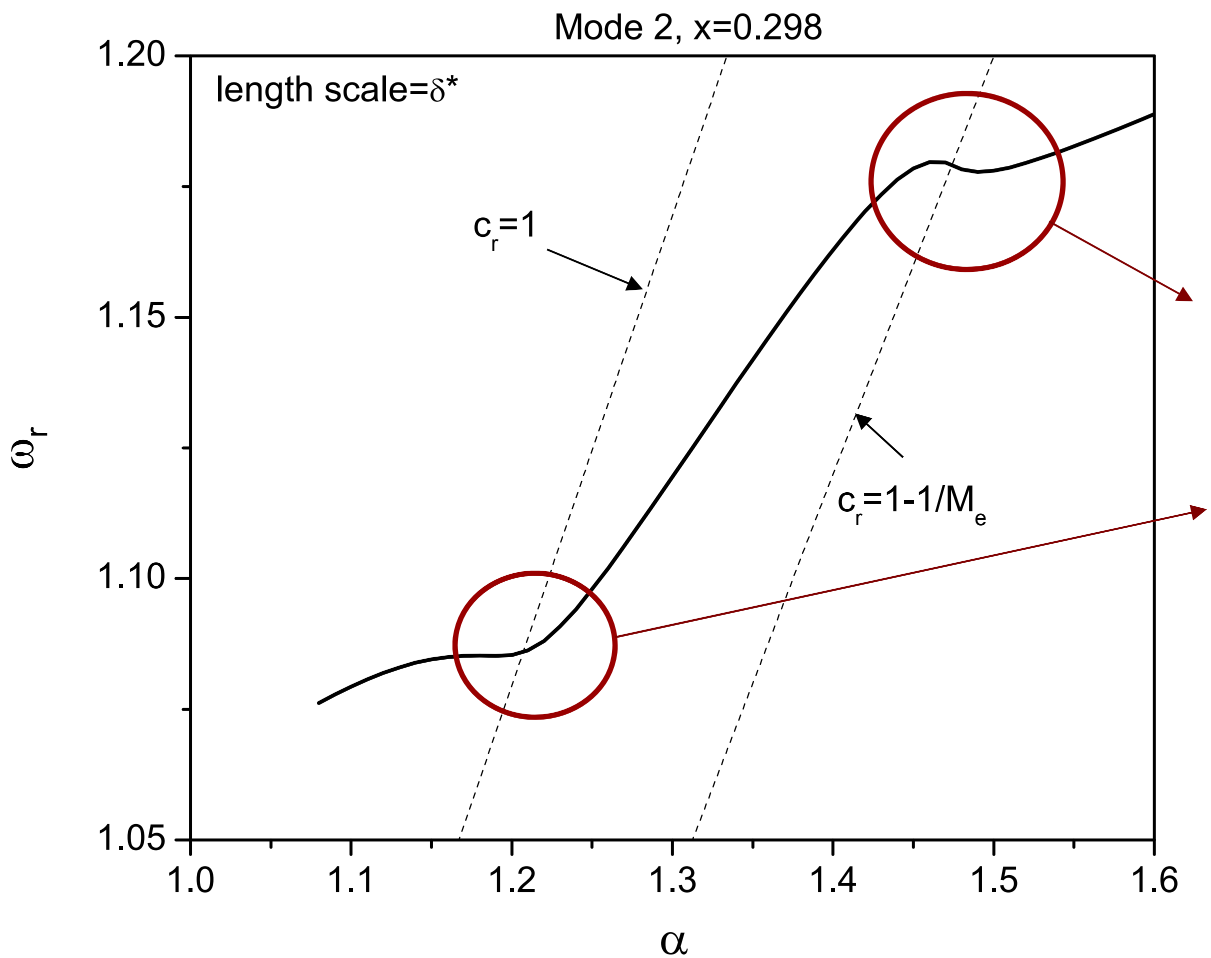


Mean flow near injection

baseline configuration
injection rate=13.5 g/s



It is not easy to convert temporal growth rates to spatial ones



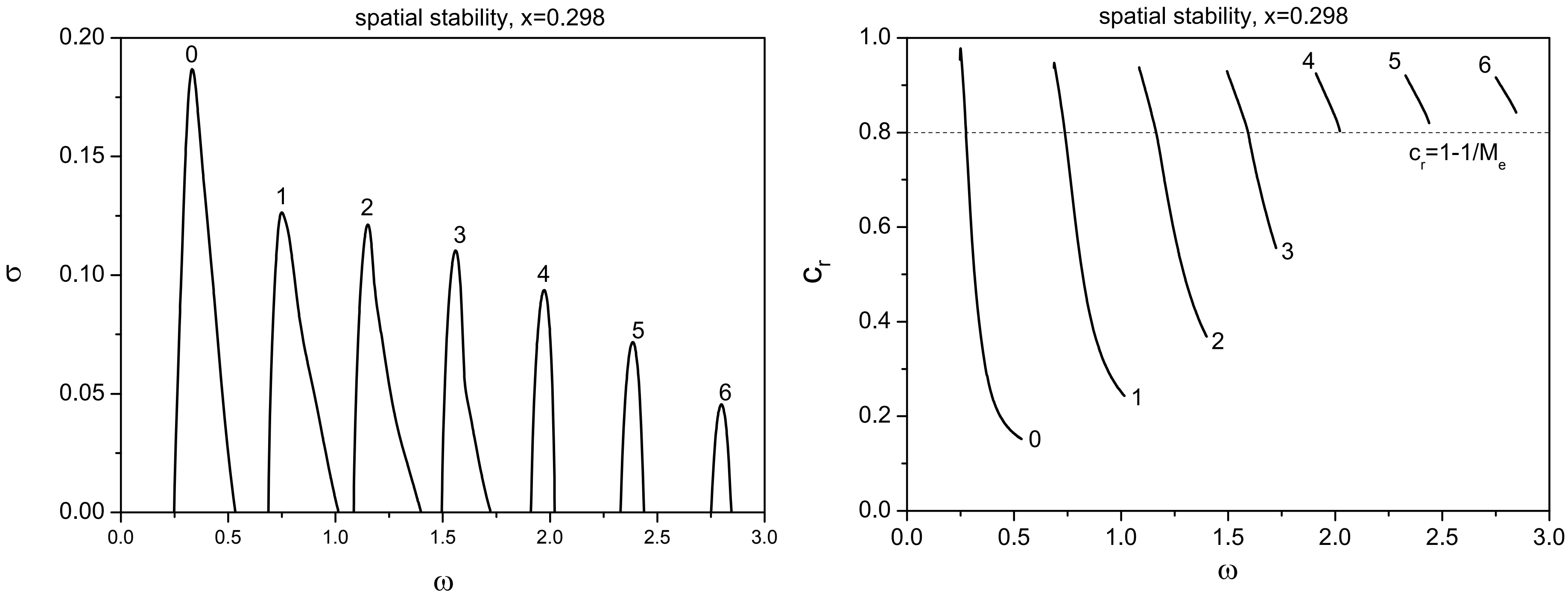
Gaster transformation

$$\alpha_i = -\frac{\omega_i}{V_g}$$

$$V_g = d\omega_r / d\alpha$$

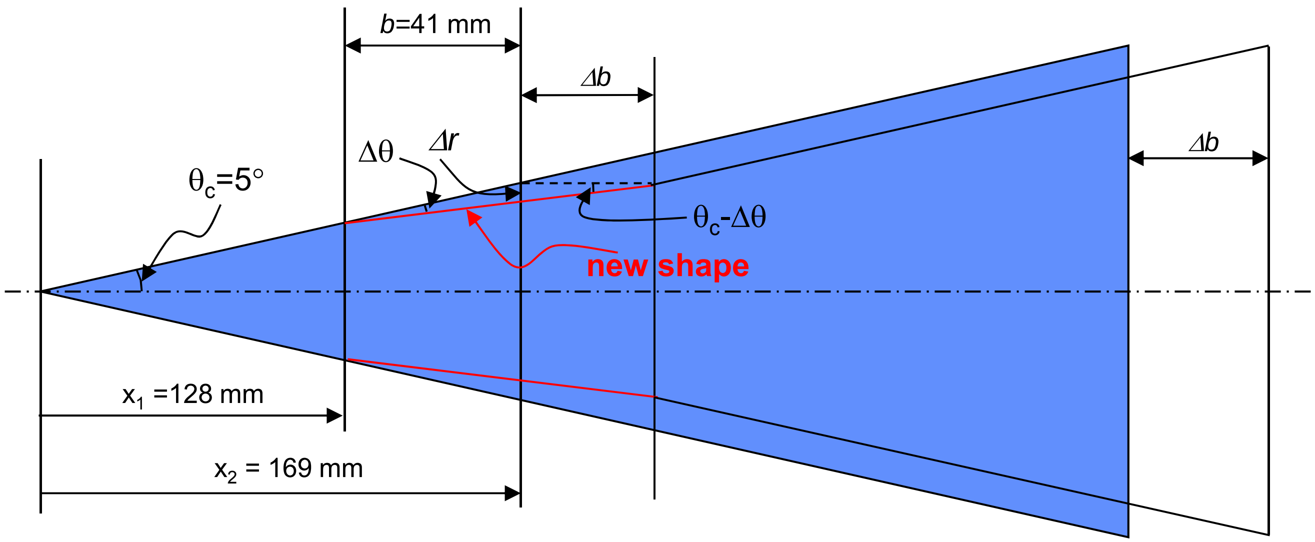
does not work here

Spatial stability analysis in the relaxation region
(injection rate=13.5 g/s, x=0.3)



- Mode 0 (Mack second mode) is most unstable
- Its instability is observed at low phase speeds

Injector of conical shape

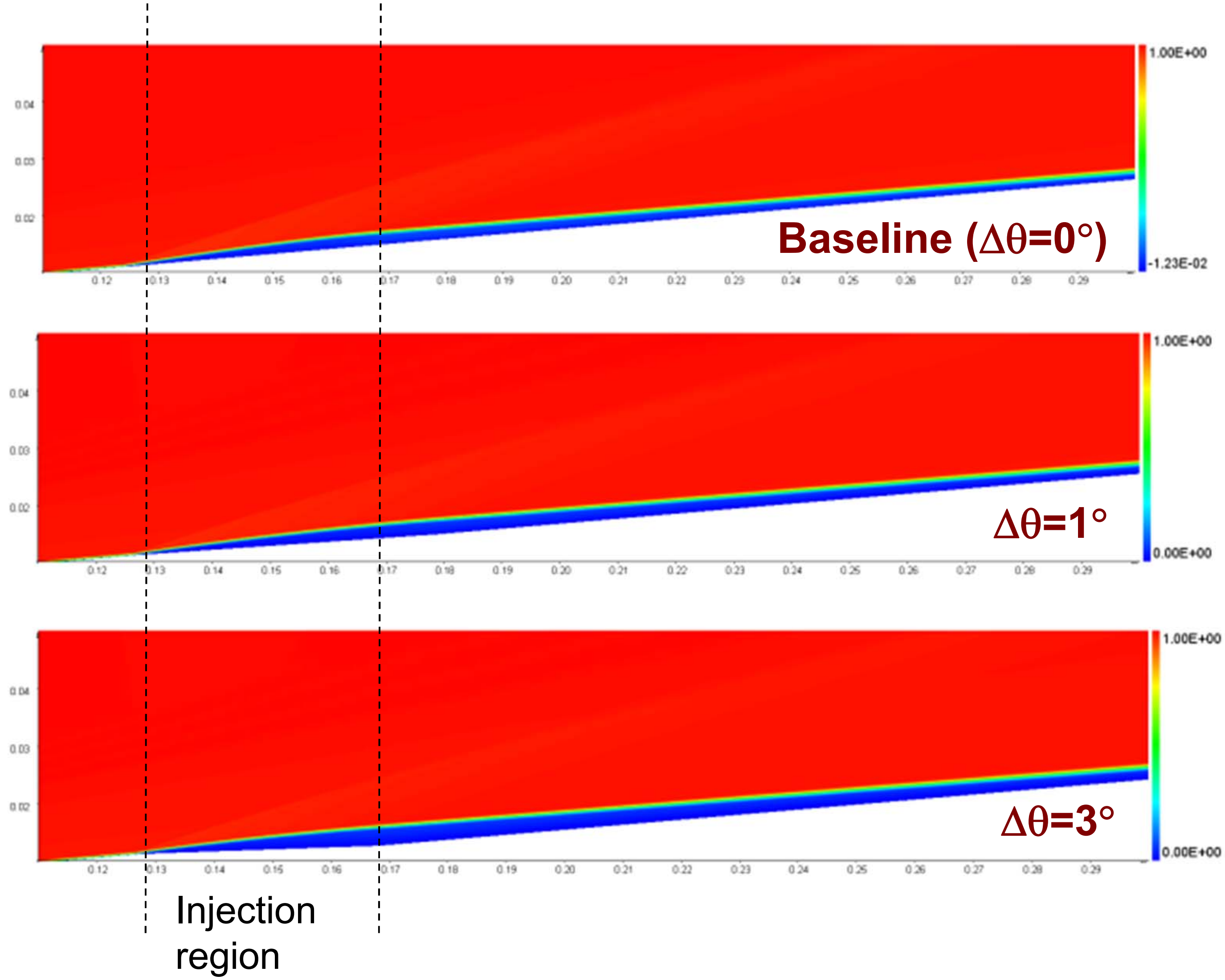


5-deg half-angle sharp cone with the injector having the slope $\theta=\theta_c-\Delta\theta$

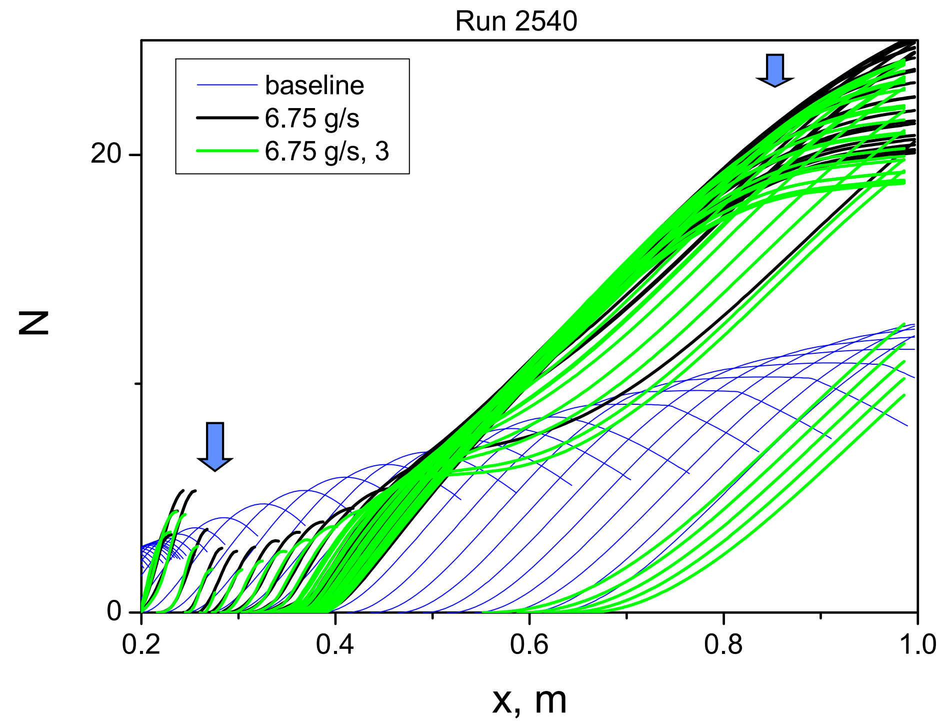
$$\Delta r \approx b[\tan\theta_c - \tan(\theta_c - \Delta\theta)] \approx b\Delta\theta$$

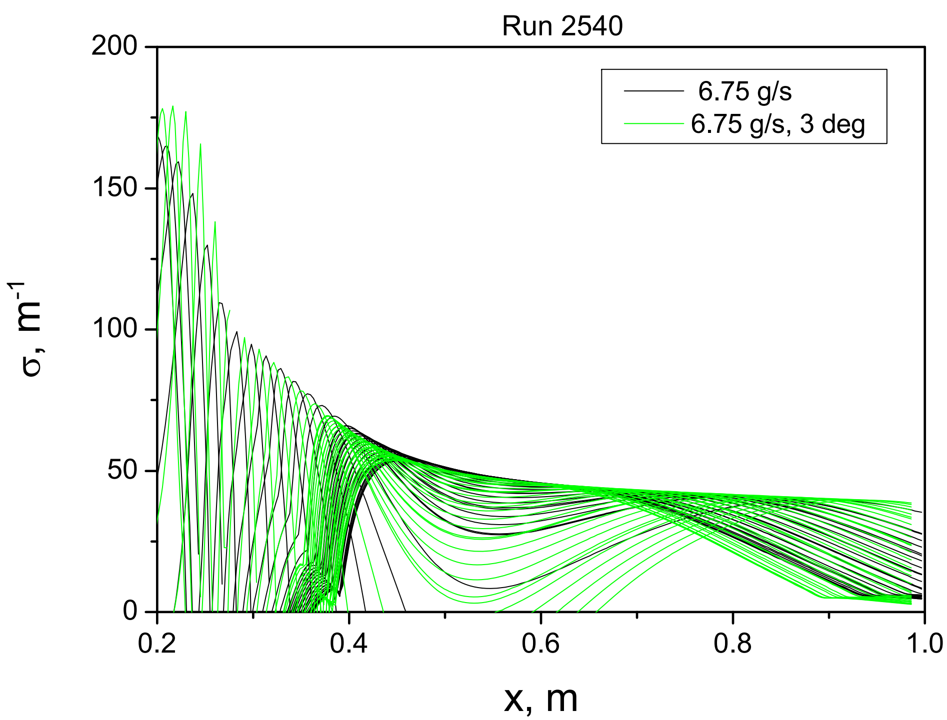
$$\Delta b = \frac{\Delta r}{\tan(\theta_c - \Delta\theta)} \approx \frac{\Delta r}{\tan\theta_c}$$

Mean-flow axial velocity for conical injectors
(injection rate 13.5 g/s)



Shaping of $\Delta\theta$=3° with injection 6.75 g/s

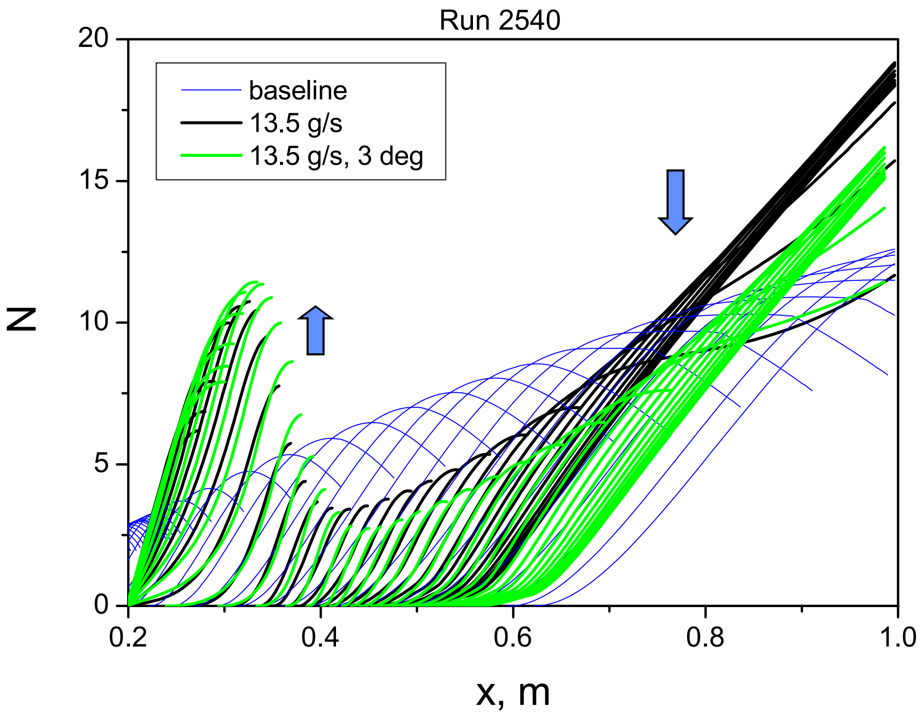


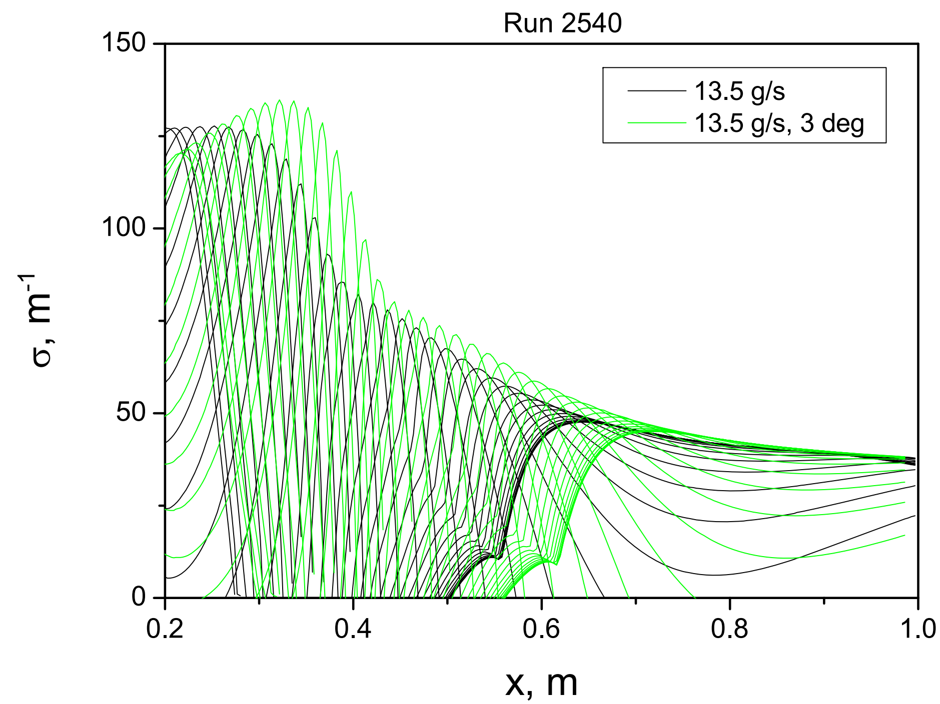


Conical injector with $\Delta\theta$=3° shaping

- **Slightly stabilizes flow in the near-field relaxation region**
- **Almost zero effect in the far-field relaxation region**

Shaping of $\Delta\theta=3°$ with injection 13.5 g/s



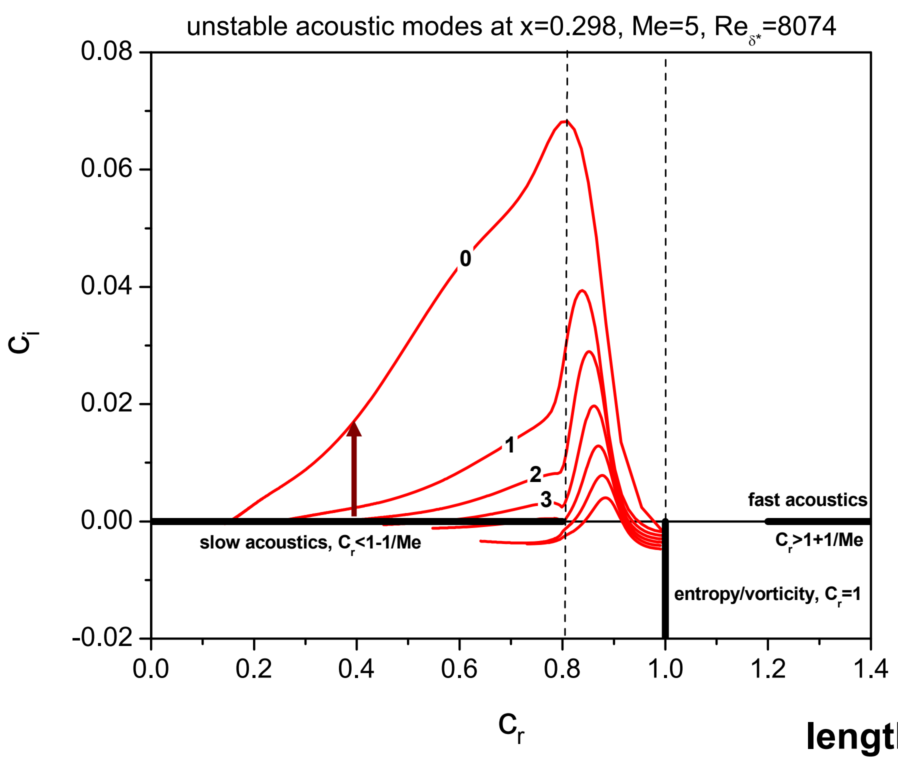


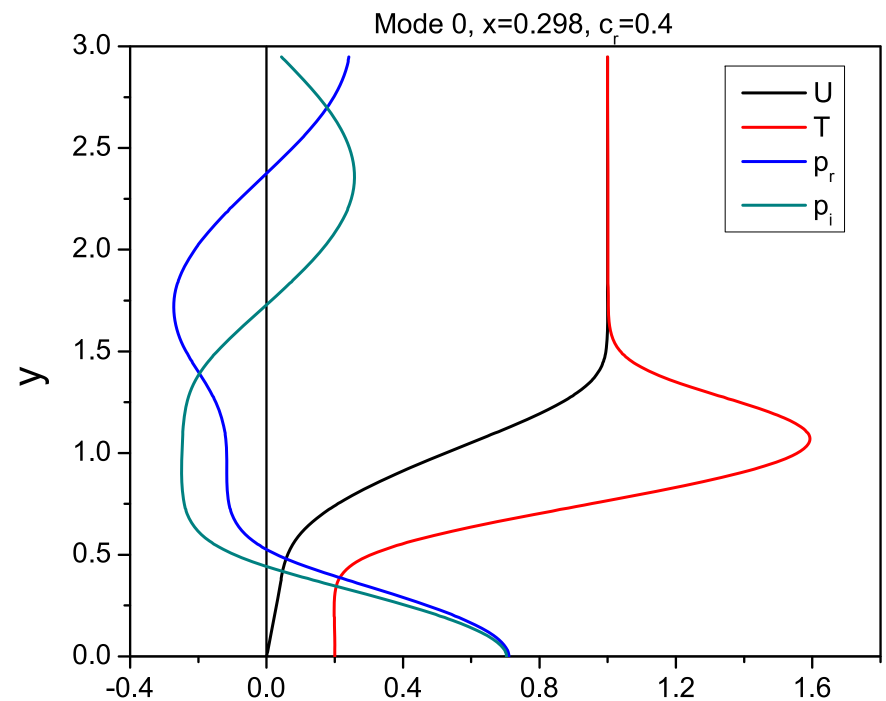
Conical injector with $\Delta\theta=3°$ shaping

- **Slightly destabilizes flow in the near-field relaxation region**
- **Stabilizes flow in the far-field relaxation region**

МФТИ

We are dealing with acoustic instability (cont'd)



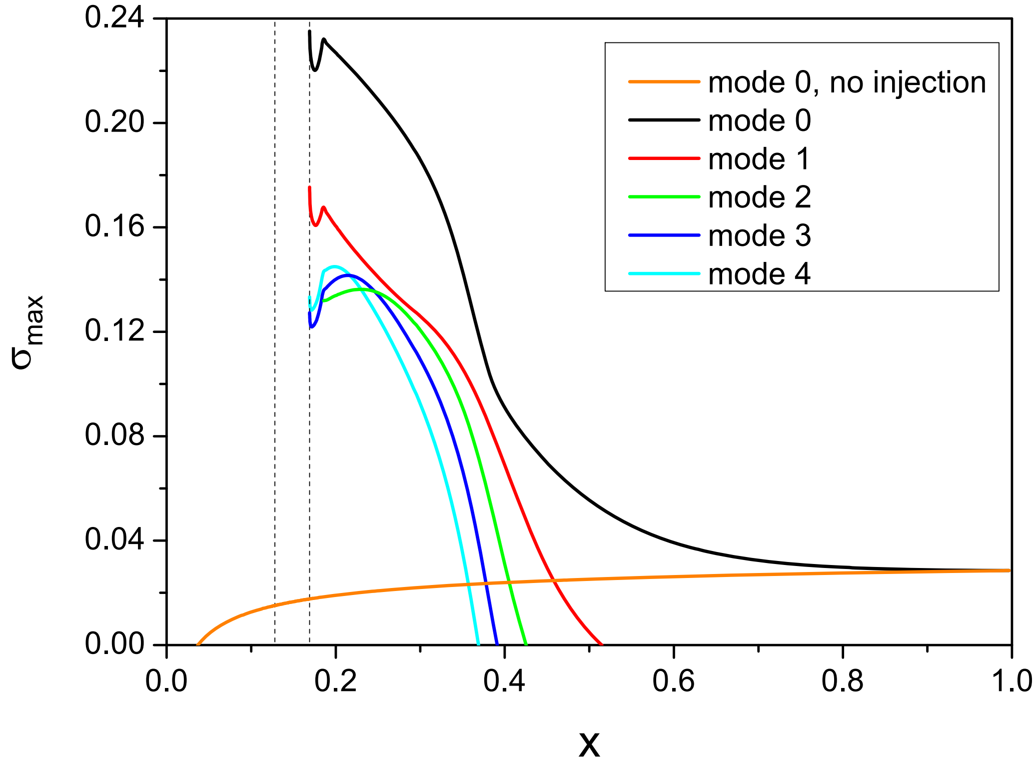


length scale=δ*

eigenfunctions

- **Phase speeds of unstable modes are close to those of slow acoustic waves**
- **Resonant interaction can enhance receptivity to slow free-stream noise**
- **Instability is observed at low frequencies where free-stream noise is higher**

Maximal growth rates in the relaxation region
(injection with 13.5 g/s)



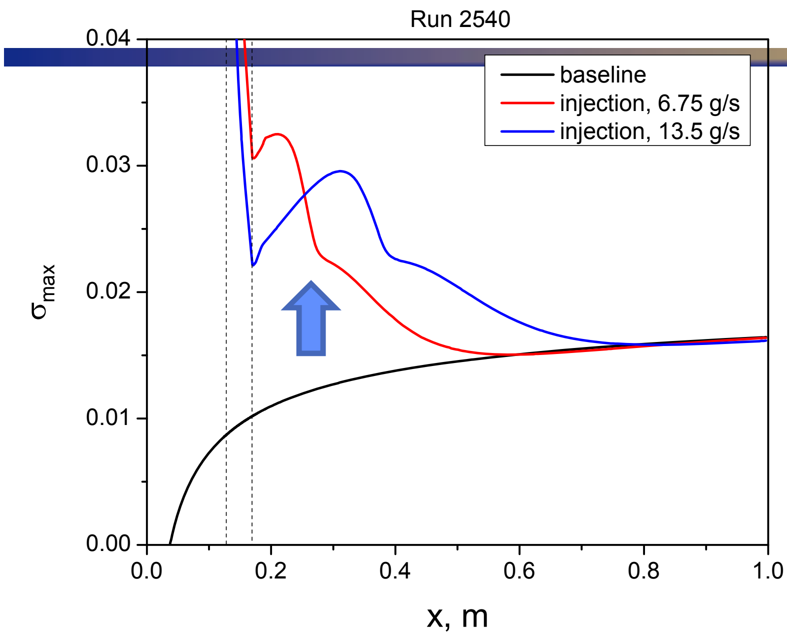
- **The most unstable mode is Mack second mode (mode 0)**
- **Unstable x-region decreases with the mode number**

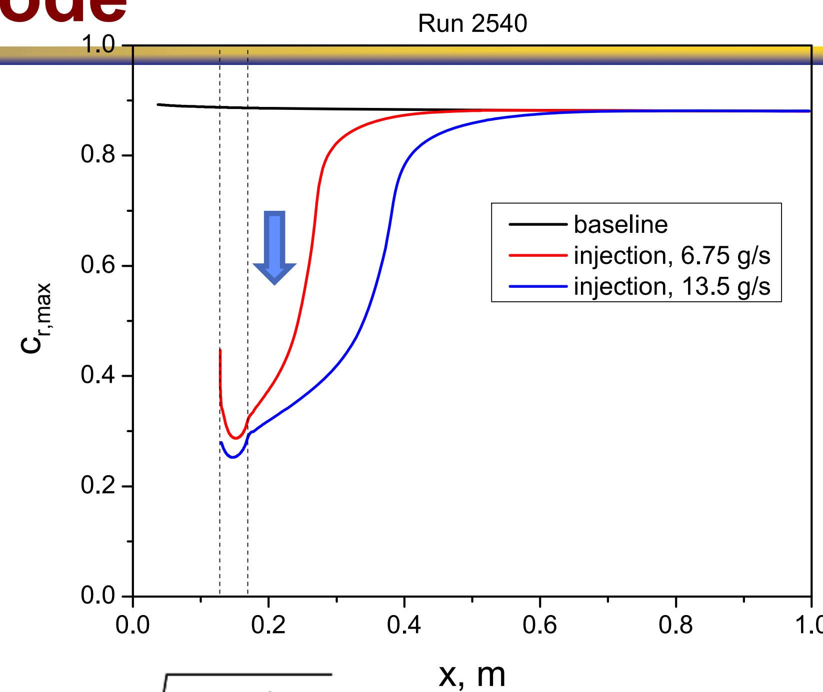
length scale=δ*

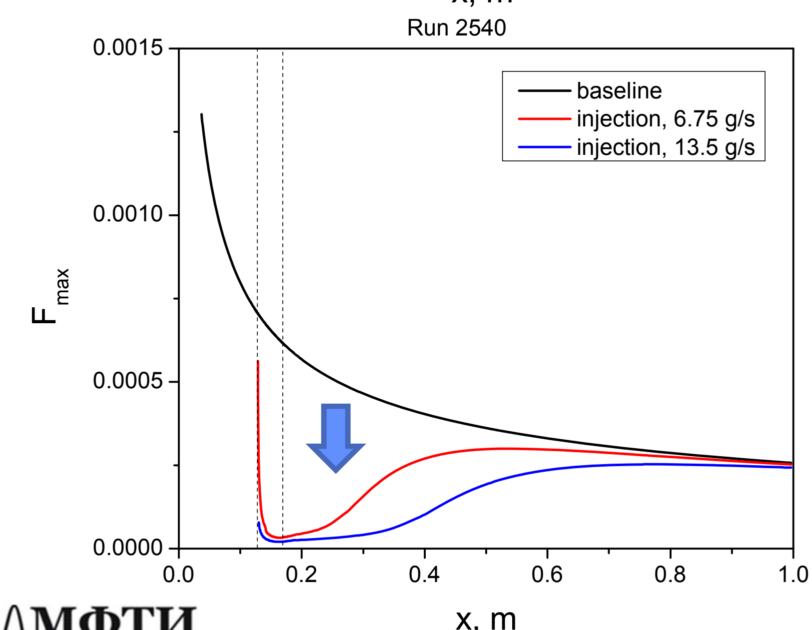
$$\sigma_{\max} = -\alpha_{i,\max} = \max_{\omega}\left[-\alpha_i(\omega)\right]$$

Further analysis is focused on the Mack second mode

Maximal growth rates of Mack second mode





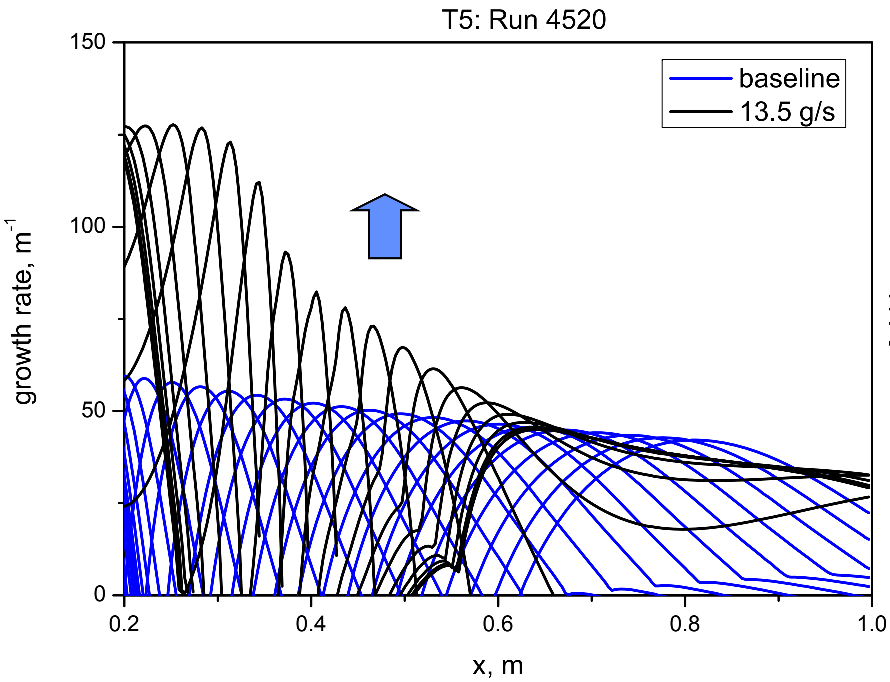


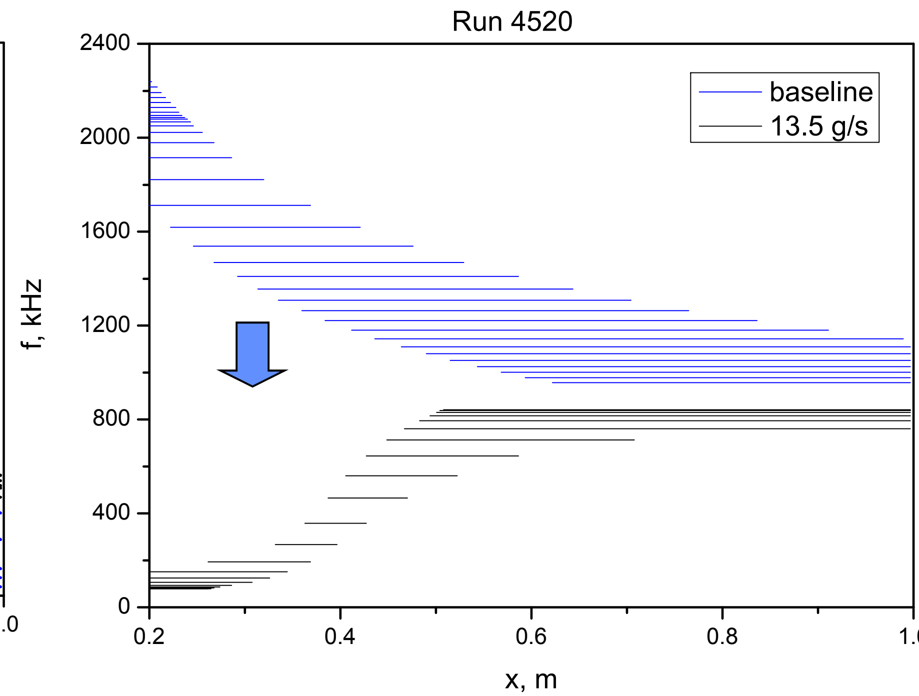
$$\text{length-scale} = \sqrt{\nu_e^* x^* \;/\; U_e^*}$$

In the relaxation region x>0.2 m

- **High maximal growth rates**
- **Low frequencies**
- **Low phase speeds**

Injection affects growth rates and frequencies in the relaxation region

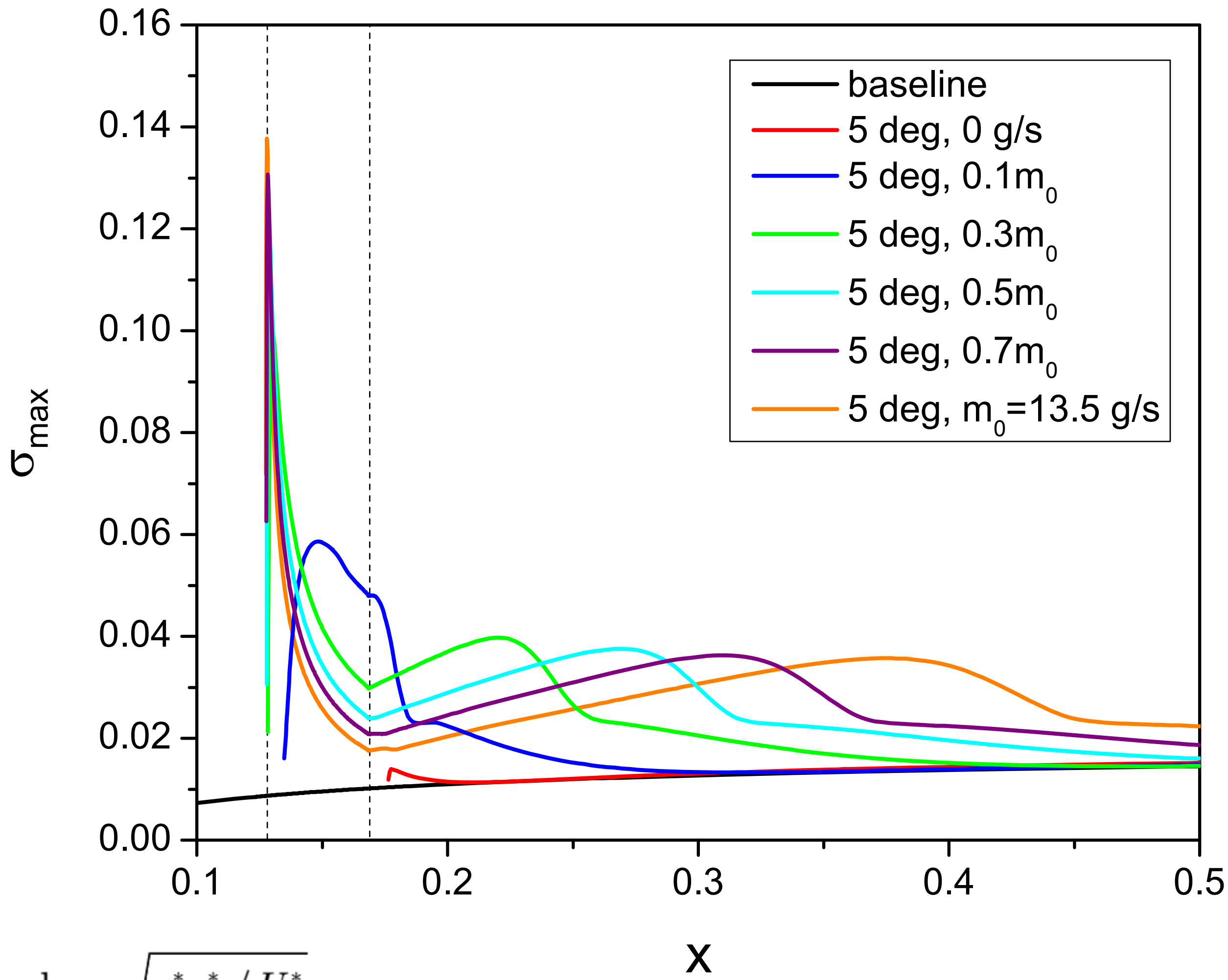




Maximal growth rates are increased
Unstable region is
- **narrowed down for x<0.6**
- **widened for x>0.6**

Frequencies are decreased

Maximal growth rates for cylindrical injector
(various injection rates)



$$\text{length-scale} = \sqrt{\nu_e^* x^* \,/\, U_e^*}$$