Nonlinear Inversion from Nonlinear Filters for Ocean Acoustics

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LONG TERM GOALS

The long term goals of this research are to develop practical and efficient algorithms for application the nonlinear inversion problems encountered in ocean acoustics. Such algorithms would be used for estimating or accounting for the effects of the environment on acoustic propagation, detection and tracking in shallow water.

OBJECTIVES

The specific objectives of this research are adapt a specific nonlinear filter, known as a Daum filter, for acoustic inversion of shallow water environmental properties, and to assess the performance of this nonlinear filter relative to local linear inversion on the one hand and global methods, e.g. Monte Carl methods on the other hand.

APPROACH

Many inverse problems of interest in ocean acoustics are intrinsically nonlinear, e.g. inverting measured pressure data for bottom and scattering properties. The solution to the nonlinear inversion problem is usually approached in one of two ways. The first way is to assume a starting model, which one hopes is near to the true model, then recursively solve a linearized version of the inverse problem for corrections to the starting model and model covariance. The advantage of this approach is that the numerical implementation of the solution algorithm is relatively straightforward and in a linear problem the statistical properties are well defined and will remain gaussian if they start out gaussian. However linearization of a nonlinear system can produce biased estimates for two reasons: 1. Linearization of the system and/or measurement equations may not be a good approximation, and 2. Nonlinear systems do not maintain gaussian statistics as they evolve even if they are initially gaussian. Another problem with linearizing a nonlinear system is that with a poor starting guess the solution algorithm may never converge to the true answer. If the starting model represents a point near a local minimum of the solution space, the final solution will be trapped in that local minimum, and never converge to the true answer. This can be circumvented by using Monte Carlo techniques to randomly sample the solution space for starting models.

The other class of solution methods attack the nonlinear problem directly by using simulated annealing or genetic algorithms. The disadvantage of these directly nonlinear methods, is that there is no way to conveniently propagate the statistical properties of the solution through to the final result.

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One solution to this problem is to find the global minimum in the solution space, if one exists, then linearize about the solution representing the global minimum and do a statistical analysis about that solution. This was done by Potty et al.(2000), who employed a genetic algorithm followed by linear analysis about the solution determined by the genetic algorithm.

The recursive algorithms commonly employed for the estimation of the model and covariance relative to some initial starting values bear a strong resemblance to Kalman filters, which are commonly employed in target tracking algorithms. The original Kalman filter was derived for strictly linear systems. However, the Extended Kalman Filter can be applied to systems which are weakly nonlinear. In the late 1980s Frederick Daum, a mathematician working at Raytheon Corporation, developed a fully nonlinear formulation to the filtering problem for target tracking (Daum, 1985, 1986, 1987). His theory is elegant, but impractical from an implementation point of view. Sometime later Schmidt (Schmidt, 1993) succeeded in deriving an approximate algorithm based on Daum's original theory, and developed a successful numerical implementation of a nonlinear filter that was a significant improvement to the Kalman and Extended Kalman filters for the type of tracking problem Schmidt was interested in.

This research aims to develop an ocean acoustic inversion algorithm based on Schmidt's (1993) implementation of Daum's nonlinear filtering theory. The purpose is to be able to carry along the statistics of the geoacoustic model parameters through the inversion process. The work is much more than a straightforward "rename the variables and code it up". However, the tracking algorithms do bear resemblance to the iterative inversion algorithms for updating model parameter means and covariances of an iterated inverse problem as in, for example, Menke (1983). Most estimation problems can be cast into an interative form, whereby the state vector, which in our case is the ocean bottom model vector, is updated sequentially as data is added. Estimation filter formulations are also natural for range dependent or time dependent environments. Daum's original theory and Schmidt's practical implementation assumes nonlinear dynamics and a linear relationship between the measurement and state vectors. In our case the measurement vector, complex pressure say, and the state vector, the bottom model, are not linearly related. The filter needs to be re-derived from scratch with the measurement to state vector relationship appropriate for our ocean acoustic application. Once re-derived, it will need to be coded, and checked against results for linear inverse problems. Dosso (e.g. Dosso and Wilmut, 2002) at the University of Victoria has developed a Monte Carlo inversion method for the ocean acoustics problem. This is computationally very intensive, but he does get the full probability density function (pdf) for the model parameters. Because the Schmidt implementation of the Daum theory propagates the additional terms in the mean and covariance of the state vector pdf, it falls in between the standard linear inversion methods and Dosso's Bayesian Monte Carlo methods.

Currently employed algorithms for nonlinear problems such as simulated annealing and genetic algorithms have no mechanism for propagating the statistics. What the nonlinear filter algorithm will do is provide a natural mechanism for updating the statistics as a solution is determined. A comparison of the filter with an algorithm such as simulated annealing would be illuminating, and a valuable check on the filter algorithm itself.

Filter type algorithms are ideally suited to inverse problems with time dependent oceanography or range dependence. We do not anticipate attempting to include time dependent oceanography at this time, but we would like to look at the issue of range dependent inversion. The idea would be to sequentially update parameter estimates as a function of range. Also note that any inversion algorithm can be cast into a filter like algorithm by supplying the data sequentially and updating the model parameter estimates sequentially as data is added to the problem, or a smoother by considering the complete data set, and working both forwards and backwards through the data set. In the end, probably the best formulation to use for a given inverse problem depends on the noise statistics. This is also something we propose to investigate.

Linear inverse problems admit the construction of both data and model resolution matrices. These resolution matrices can be used as metrics with which to estimate model uniqueness and data predictability. We will be able to construct resolution matrices for the nonlinear problem and compare them with their fully linear equivalents.

WORK COMPLETED

As stated above an inverse problem can be recast as a filtering problem. A strictly linear problem becomes a Kalman Filter (KF). A problem that has been locally linearized becomes an Extended Kalman Filter (EKF), and the fully nonlinear problem can be represented as a Daum – Schmidt Filter. We have completed working computer code for a simple EKF, an EKF smoother, an iterated Kalman filter and an interated Kalman smoother with which to compare more complicated inverse models.

Recent papers in the geoacoustics literature (Dosso and Nielson 2002) show estimated probability density functions (PDFs) of ocean geoacoustic parameters via a Monte Carlo method, and the PDFs for some parameters are far from Gaussian – some are bimodal, some are greatly skewed. If linearization methods are used on such problems then the resulting maximum likelihood estimate and/or variance about it may be misleading descriptors of the true solution. While the Monte Carlo methods offer a reliable way to address such non-Gaussian statistics of the inverse solution, they are very slow and may not convey an intuitive understanding of these statistics that an analytic expression might. Our work this year has also been to continue researching analytic means to address these non-Gaussian statistics and their effects on resolution in nonlinear inverse problems such as the ocean geoacoustic problem.

RESULTS

Recent work in the filter community (Daum, 1986) aims to analytically map higher order moments of the state variable PDFs for the solution of a nonlinear problem, given a particular form of PDF. There are close connections between filter estimation theory and geophysical inverse theory, and we have been working to adapt the filter theory moments work to problems in ocean geoacoustic estimation. This year in the ONR nonlinear inversion project, the relation between filter theory and inverse theory for these problems has been explored. Figure 1 shows how the nonlinear smoother's solution steps through the space of candidate solutions, as it homes in on the objective surface minimum at the solution point. The geophysicists' iterated Gauss-Newton method takes this same path. In the simple synthetic example problem shown in Figure 1, noisy wave arrival times "recorded" at 20 receivers (the white triangles) are used to estimate the source location (yellow diamond) in a constant medium; the contours are the objective function corresponding to the data misfit. Figures 2a and 2b show error plots (estimated solution minus "true" synthetic model) for two extended Kalman filter (EKF) and iterated Kalman smoother (IKS) runs. The run in 2a and the run in 2b correspond to two different orderings of the same receiver data, including the same instantiation of the random noise, so the data point order is the only difference between the runs. The horizontal axes in these plots is the number of receivers from which data was brought in, with an implied ordering. Note between 2a and 2b that when the ordering of the receiver data brought in changes, the EKF result changes but not the smoother result. The smoother's result is a flat line for receiver location because unlike in the EKF, the source location is made to be constant regardless of the order that the data were read in.

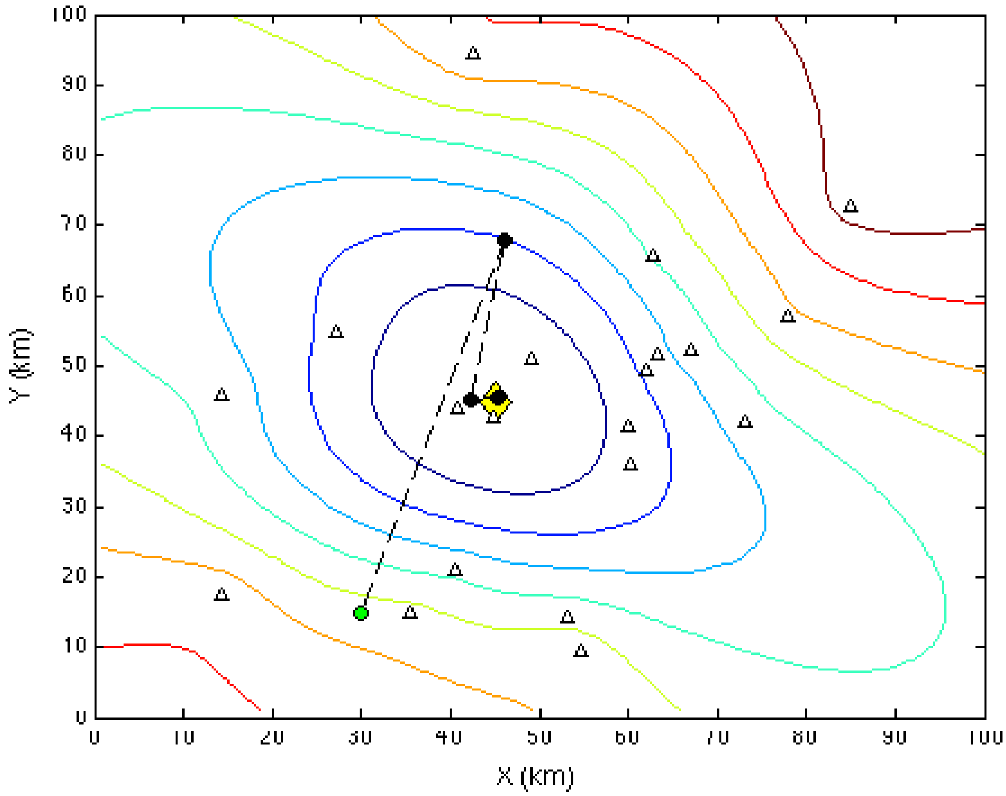
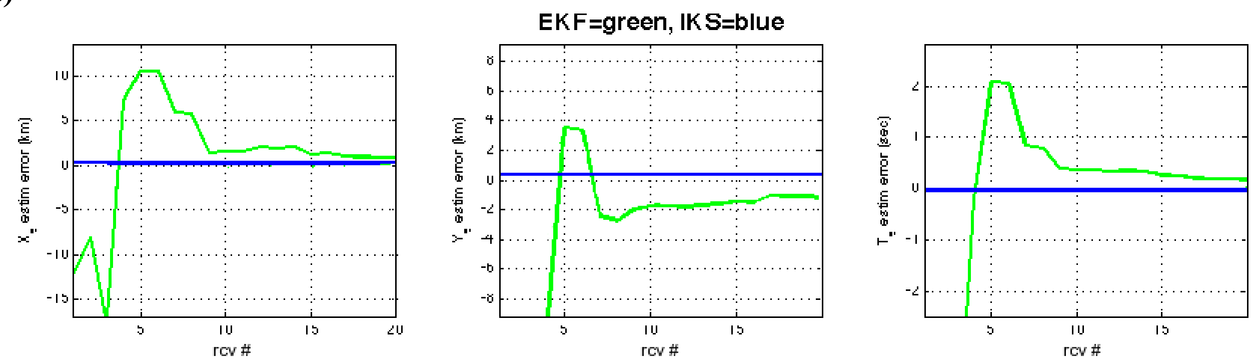


Figure 1: Path of nonlinear inversion iteration steps through the objective space. From the initial solution estimate (and its associated uncertainty) at the green dot the Gauss-Newton process, and the smoother, take steps (the dotted line) toward source location (yellow diamond). The triangles are receiver locations. The contours reflect the problem's objective surface, which is not paraboloidal because this 2D source location problem is nonlinear.

The filter literature used thus far assumes a special form of PDF for the nonlinear problem, but ultimately one would like to generalize this PDF form much further to cover a wide variety of ocean geoacoustic problems. This year a Monte Carlo simulation is being developed to explore the relation between this PDF form and the "true" statistics for an ocean geoacoustic problem based on the DEMUS experiment at the Malta Plateau. Uncertainty and resolution aspects of this DEMUS experiment problem have also been initially explored for multiple receivers in summertime work on

(a.)



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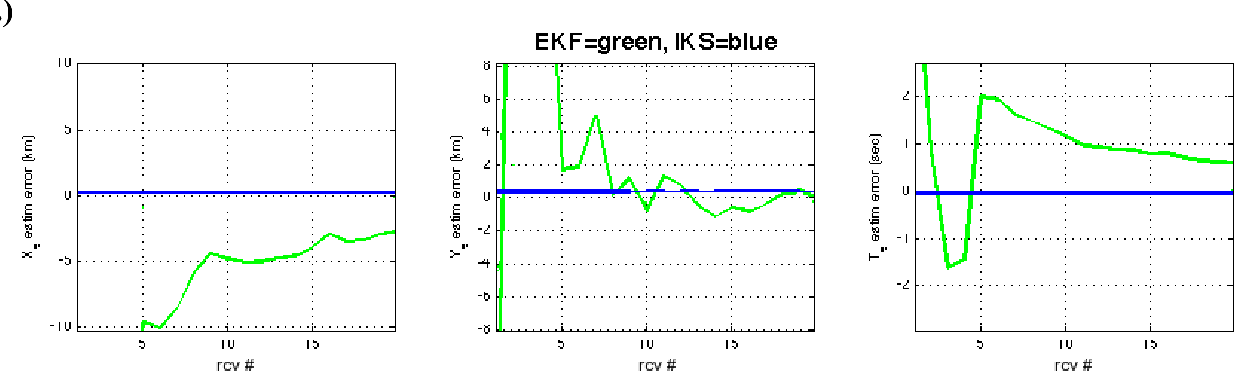

***Figures 2a & 2b**: Error plots (estimated solution minus "true" synthetic model) for two extended Kalman filter (EKF) and iterated Kalman smoother (IKS) runs on the same problem shown in Figure 1. The run in (a.) and the run in (b.) correspond to two different orderings of the same receiver data, including the same instantiation of the random noise, so the data point order is the only difference between the runs. The IKS's result is a flat line for receiver location because unlike in the EKF, the source location is made to be constant regardless of the order that the data were read in. In clarification, the iteration steps in Figure 1 are different than the steps along the horizontal axis here. Both the EKF and IKS curves were iteratively updated, and the IKS results shown here are associated with the final solution point in Figure 1.*

the ONR ARL project managed by John Tague, and presented at the FUSION06 conference in Florence, Italy (Pitton, Ganse, Krout, Anderson, 2006). In this work, a synthetic demonstration inversion of ocean bottom properties based loosely on the geometry of the DEMUS experiment quantified the improvement via decreased uncertainty and increased resolution of bottom loss and scattering functions given the addition of a second receiver. See Figures 3 and 4 for part of the demonstration. The Monte Carlo part of this work is a little bit different formulation than the synthetic demonstration problem – the demonstration problem for the FUSION06 paper estimated the full continuous bottom loss and scattering functions, which are parameterized by many parameters, thus making Monte Carlo simulation computationally intractable in the scope of this work. So for the Monte Carlo simulation the problem is simplified to a small number of parameters representing the properties of the ocean bottom. This is a strong regularization of the problem, based on additional prior information in the form of cores and samples from the area of interest. Finally, in future work, the filter based developments will be applied to this same parameter-estimation problem to compare to the Monte Carlo results.

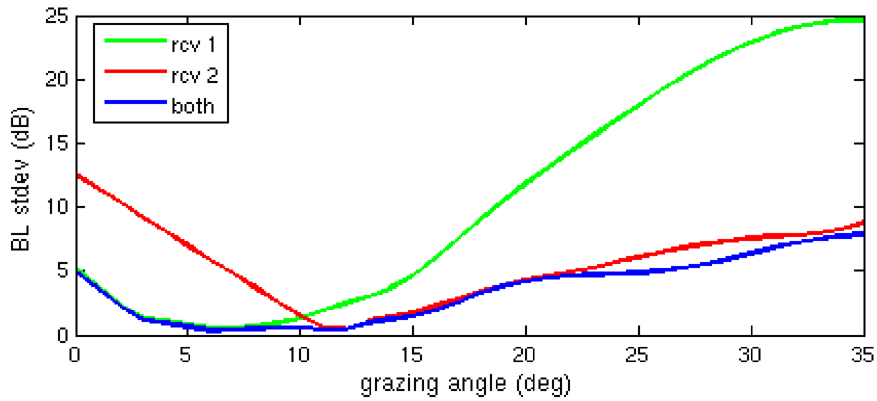

Figure 3: Standard deviations of inverted bottom loss functions of grazing angle in the FUSION06 demonstration problem, showing the improvements via decreased uncertainty when adding another receiver into the inverse problem. Standard deviation curves over angle are compared for each single-receiver inversion and the combined-receiver inversion case. Note that the benefits seen in the individual receiver results are essentially combined when both receivers are used – each receiver provides for a low standard deviation in a particular angular range, which together provides for low standard deviations over a much wider angular range.

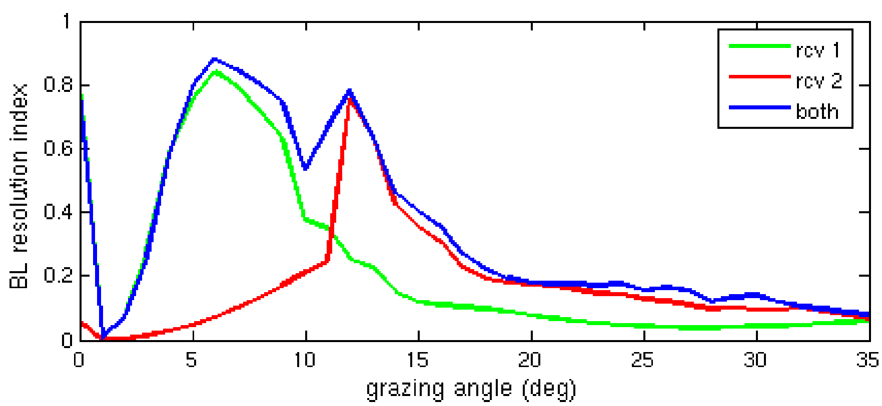

Figure 4: Resolution of inverted bottom loss functions of grazing angle in the FUSION06 demonstration problem, showing the improvements via increased resolution (equivalent to decreased bias) when adding another receiver into the inverse problem. Resolution index curves (proportional to inverse bias) over angle are compared for each single-receiver inversion and the combined-receiver inversion case. Note that the benefits seen in the individual receiver results are essentially combined when both receivers are used – each receiver provides for a high resolution (low bias) in a particular angular range, which together provides for high resolution (low bias) over a much wider angular range.

IMPACT/APPLICATIONS

A nonlinear, well characterized filter-based inversion method and algorithm will have application to environmental estimation and target tracking. A practical method way to compute the resolution for a nonlinear inversion will have an impact on the characterization of uncertainty and uniqueness of environmental estimates required for acoustic propagation.

RELATED PROJECTS

Our research is directly related to other programs studying effects of uncertainty in the environment, measurements, and models on acoustic propagation, and target detection and characterization.

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