Near-Axial Interference Effects for Long-Range Sound Transmissions through Ocean Internal Waves

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LONG-TERM GOALS

The long-term goal of this effort is to provide an improved way of interpreting the experimentally observed time-of-arrival patterns in long-range, low-frequency propagation in the deep ocean.

OBJECTIVES

In many long-range propagation studies the source and receiver are placed close to the depth of the waveguide (SOFAR) axis to minimize the interaction of the acoustic field with the ocean's surface and bottom. The most pronounced characteristics of the time-of-arrival patterns for these experiments are early geometric-like arrivals followed by a crescendo of energy that propagates along the axis. In Fig.1 adapted from one published in Ref. (1) these characteristics are clearly shown for a time-of-arrival pattern measured during the Acoustic Engineering Test (AET) of the Acoustic Thermometry of Ocean Climate (ATOC) project conducted in the eastern North Pacific Ocean.

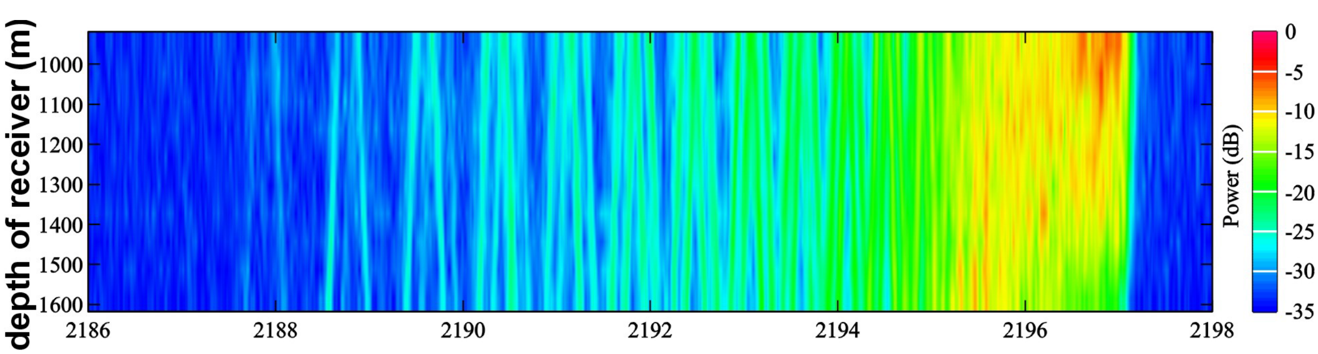

Figure 1. The time-of-arrival pattern measured during the AET experiment

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While the eikonal equation can be solved for the ray paths and travel times can be calculated by integrating the index of refraction along the ray paths, it is impossible to use geometrical acoustics to describe the propagation of energy along the waveguide axis because of the presence of caustics with caustic cusps located repeatedly along the axis. Figure 2 illustrates this pattern of caustics. It is a ray tracing for a source on the axis (and for relatively short ranges). In neighborhoods of cusped caustics a very complicated interference pattern is observed. The neighborhoods of interference grow with range and at long ranges they overlap.

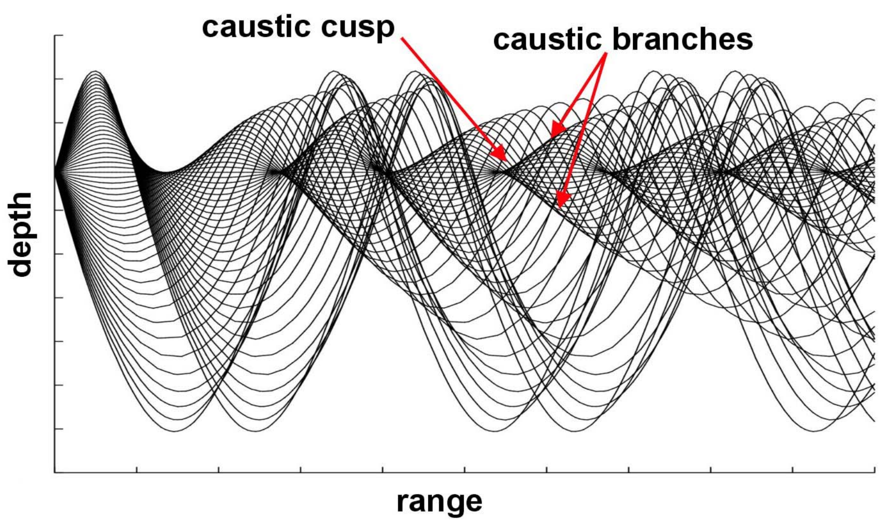

Figure 2. Ray tracing for a source on the axis of a waveguide

The overall goal of the research is to provide a better understanding of the interference effects that are present for sound propagation in a ducted waveguide when the source and receiver are located near or on the axis of the duct. The primary application of this work will be the improved interpretation of time-of-arrival patterns observed in long-range acoustic propagation experiments in the ocean.

APPROACH

It is well known that in the neighborhood of a simple caustic the acoustic field is described by the Airy function. To describe the acoustic field in the neighborhood of a cusp of a caustic it is necessary to use the Pearcey integral. The longitudinal and transverse sizes of the neighborhood of a cusp of a caustic, where the geometrical acoustics formulas are not applicable, increase with range. As a result, at a certain propagation range the neighborhoods of adjacent cusps overlap. In this case the acoustic field cannot be described by the Pearcey integral and the more complicated interference structure appears. For a very idealized model of a symmetric waveguide, Buldyrev in Ref. (2) showed that the interference of the wave fields that correspond to near-axial rays, and is associated with the cusped caustics, leads to formation of a coherent structure that propagates along the waveguide axis like a wave. This structure was called “the axial wave”.

Buldyrev’s work on the axial wave was done before the long-range experiments took place and with the purpose of obtaining a global description of propagation in the presence of cusped caustics. The project entitled “Investigation of Near-Axial Interference Effects in Long-Range Acoustic Propagation

in the Ocean" (Award Number: N00011402M0233; Principal Investigator – N. Grigorieva) was devoted to application of the method proposed by Buldyrev to ocean models typical for long-range propagation experiments. Within the scope of the project the following results were obtained.
The two-dimentional reference point source problem with the parabolic index of refraction squared was studied in detail in Refs. (3) and (4). The integral representation of the exact solution was transformed in such a way to extract ray summands corresponding to rays radiated from the source at angles less than a certain angle, the axial wave, and a term corresponding to the sum of all the rays having launch angles greater than the indicated angle. The obtained results were discussed at the 143rd ASA Meeting in Pittsburg; see Refs. (5) and (6).

For an arbitrary range-independent deep-water waveguide the formula for the axial wave can be obtained as well by transforming the integral representation of the exact solution of the point source problem. But this method is not applicable to the range-dependent medium. That is why it was important to propose another method for obtaining the integral representation of the axial wave admitting the generalization to the range-dependent medium and giving the same result as the transformation of the exact solution. Taking into account the formula for the axial wave in the case of the two-dimensional reference point source problem, it was natural to assume that for an arbitrary deep-water waveguide in a two-dimensional range-independent ocean the axial wave will have the similar integral representation and try to find its integrand in the form of a product of the weight function and asymptotic expansions of solutions to the Helmholtz equation that are concentrated near the sound-channel axis. These solutions have the form of the exponentials multiplied by the parabolic cylinder functions whose arguments are sections of series in –1/2 powers of a cyclic frequency. For an arbitrary range-independent deep-water waveguide the required solutions to the Helmholtz equation were derived in Ref. (7). Using them, the integral representation of the axial wave for an arbitrary waveguide in a two-dimensional range-independent meduim was obtained in Refs. (8) and (9). Numerical computations were carried out for the canonical sound-speed profile. The source frequency was taken equal to 75 Hz, and the propagation range was 3000 km.

The integral representaion of the axial wave obtained in Refs. (8) and (9) was generalized to a three-dimensional range-independent medium in Ref. (10). Through numerical simulations, the dependencies of the axial wave on range, sound-speed profile properties, and geometry of the experiment were studied for two sound-speed profiles: the average profile from the AET experiment and the Munk canonical profile.

The integral representation of the axial wave in the time domain for a range-independent ocean was obtained in Refs. (11) and (12). Numerical computations were done for the Munk canonical sound-speed profile. A signal with the center frequency of 75 Hz and 30-Hz bandwidth was used for modeling. The propagation range was 3250 km. Numerical simulations for the average profile from the AET experiment were included in Ref. (13).

As it was noted above, the main advantage of the proposed method for obtaining the integral representation of the axial wave in a range-independent medium, see Refs. (8) and (9), is that all the steps of this method admit generalization to a range-dependent ocean. It means that to derive the integral representation of the axial wave in a range-dependent ocean it was necessary to start with obtaining the solutions to the Helmholtz equation that are concentrated near the sound-channel axis and have the form described above; see Ref. (14). As opposed to the adiabatic approximation of low-order modes that depend only on medium properties along a vertical line parallel to the Z-axis, the newly obtained solutions to the Helmholtz equation provided in Ref. (14) accumulate the information about medium properties near the range-variable sound-channel axis along the whole propagation range.

In Refs. (15) and (16) the integral representation of the axial wave in a range-dependent ocean was derived in the form of a linear superposition of the solutions to the Helmholtz equation concentrated near the axis of a deep-water waveguide. The weight function was selected in such a way that the localization principle holds. In the limiting case of a range-independent ocean, this principle allows one to use the integral representation of the axial wave obtained in Refs. (9) and (10). Numerical simulations in Refs. (15) and (16) were carried out for a deterministic model of a range-dependent ocean, where the horizontal inhomogeneity results from the change in geographic location. The model is based on the information about sound-speed profiles as a function of range between the source and receiving array derived from the WOA'94 climatology for November along the path of the AET experiment , see Fig. 5 of Ref. (1).

WORK COMPLETED

The effect of a local deterministic inhomogeneity on the axial wave was studied. The perturbation is induced by a cold mesoscale eddy embedded in a sound channel with the sound-speed profile typical for the AET experiment. The frequency of the source was taken equal to 75 Hz. It was shown that after passing over the mesoscale eddy its effect on the diffractive component of the acoustic field as a function of range or depth remains substantial even at a distance equal to 2000 km from the eddy. The axial wave was simulated for the Long-Range Ocean Acoustic Propagation Experiment (LOAPEX) CTD data measured at 7 different ranges from the vertical line array (VLA). At 0-range (at the VLA) the same sound-speed data were used as present at 50 km from the VLA.

To evaluate very roughly what results might be obtained for the transmission loss of the axial wave at the propagation range of 3200 km and at the frequency of 68.2 Hz, a simple range-independent ocean model was used with a background profile corresponding to the sound-speed data measured at 50 km from the VLA. It was shown that the waveguide is characterized by the formation of a very complicated interference pattern. At the frequency of 68.2 Hz the neighborhoods of caustic cusps overlap starting 50 km from the source. At the propagation range of 3200 km and at the frequency of 68.2 Hz, 37 waves interfere with the wave propagating along the sound-channel axis.

For smoothing the initial sound-speed data two polynomials were used; an 11^{th} degree polynomial approximated the sound-speed data in the smaller depth interval 300 – 1100 m and a 14^{th} degree polynomial in the larger depth interval 150 – 1500 m. It was shown that the use of these two polynomials changes the transmission loss of the axial wave about 0.5 dB.

To simulate the axial wave in a range-dependent ocean, the smooth two-dimensional sound-speed field was obtained and the sound-channel axis was calculated as well as the sound-speed at the axis. The depth of the sound-channel axis increases as one proceeds to the array from 862.535 m at 3200 km from the VLA up to 635.678 m.

To evaluate the maximal possible interference effect near the sound-channel axis, the transmission loss of the axial wave in the range interval 3150 – 3200 km was simulated at a frequency of 68.2 Hz with the source and receiver placed on the sound-channel axis. It was shown that the transmission loss changes in the interval (-120 dB, -117 dB). The axial wave falls off with distance as the range to –3/4 power. The transmission loss as a function of depth of the source placed at 3200 km from VLA monotonically decreases as the distance between the source and the sound-channel axis increases. At the frequency of 68.2 Hz when the depth of the source is 500 m, the transmission loss is –192.683 dB. In the experiment the source level was about 194 dB. Thus, the axial wave is not detectable in the case when the propagation range is 3200 km and the depth of the source is 500 m.

At the next two stations T2300 and T 1600 the distance between the source and the sound-channel axis, H, is too large as well. If the propagation range is equal to 2300 km, we get H = 328.1 m and at 1600 km this distance is 421.4 m (for the source deployed to 500 m and 350 m respectively). The axial wave is not detectable as well. The source deployed to 800 m at the stations T1000, T500, T250, and T50 generated axial waves that can be detected.

The results for simulation of the axial wave for the LOAPEX sound-speed data formulated above were included in the talk "Simulation of near-axial interference effects for the LOAPEX sound-speed data" by N. Grigorieva and G. Fridman given at the Ninth North Pacific Acoustic Laboratory (NPAL) Workshop, Borrego Spring, California, April 2006. They were discussed as well at the Applied Physics Laboratory, University of Washington, Seattle, in April 2006.

For the stations T1000 (the distance from the VLA is 990 km) and T500 (the distance from the VLA is 490 km) at the frequencies of 56.25 Hz, 75 Hz, and 93.75 Hz (the bandwidth of signals transmitted in the LOAPEX is equal to 37.5 Hz) the following characteristics of the axial wave and the lowest mode in the adiabatic approximation were calculated: 1) the absolute magnitude of the axial wave as a function of depth at the VLA when the source is at the sound-channel axis of the range-dependent ocean; 2) the absolute magnitude of the axial wave as a function of depth at the VLA in the range-dependent ocean when the depth of the source is 800 m; 3) the absolute magnitude of the axial wave as a function of depth at the VLA when the source is placed at the waveguide axis of the range-independent ocean; 4) the absolute magnitude of a part of the acoustic field corresponding to the lowest mode in the adiabatic approximation as a function of depth at the VLA; the depth of the source in the range-dependent ocean is 800 m; 5) the reference propagation time corresponding to the arrival time of the wave propagating directly along the sound-channel axis (had it existed); 6) the propagation time of the axial wave; 7) the propagation time of the lowest mode in the adiabatic approximation.

To check the stability of the axial wave simulations, the same characteristics of the axial wave were calculated for propagation ranges equal to 1000 km and 500 km.

The orthonormalized modal functions at the VLA form a complete system. Thus, any continuous function, in particular, the axial wave as a function of depth at the VLA can be expanded in the Fourier series with respect to modal functions. The coefficients of this expansion are the scalar product (with integration along the depth) of the axial wave and the modal functions. To evaluate the relationship between the axial wave and first two modal functions, the normalized scalar products of these modal functions and the axial wave were calculated.

RESULTS

We concluded that at the VLA the maximum value of the absolute magnitude of the axial wave as a function of depth is almost independent of frequency (f = 56.25 Hz, 75 Hz, and 93.75 Hz). The received axial wave intensity is about 80 dB for the station T1000 and it is about 95 dB for the station T500 (for the source level equal to 194 dB). The depth interval where the absolute magnitude of the axial wave is equal to a half of its maximal value is about 300 m.

It was learned that the difference between the propagation time of the wave propagating along the waveguide axis (had it existed) and the propagation time of the axial wave decreases with frequency approximately as a cyclic frequency to –2 power. The coefficient of this law depending on waveguide properties near the range-variable sound-channel axis was found. The deviation of calculated values from values given by the derived law increases with frequency. The difference in the propagation time between the wave propagating along the waveguide axis and the axial wave increases linearly with the distance between the source and receiver.

It was learned that at the propagation range of 990 km (station T1000) and at the frequency of 56.25 Hz and 75 Hz the axial wave arrives at the receiver more than 36 ms later than the lowest mode. At the propagation range of 490 km (at the station T500) and at the frequency of 56.25 Hz this delay is about 18 ms. It was shown that computational results obtained for two close distances (990 and 1000 km; 490 and 500 km) are stable.

By calculating the absolute values of the normalized Fourier coefficients of the axial wave at the VLA (as a function of depth) with respect to the normalized modal functions, we concluded that the axial wave is not reduced to the lowest mode or to a sum of the first few modes.

IMPACT/APPLICATIONS

The primary application of this work is the interpretation of time-of-arrival patterns observed in long-range acoustic propagation experiment. The computations show that as the axial wave propagates and accumulates interference effects it arrives later than the lowest mode. The delay is more than 36 ms for the station T1000 at the frequencies of 56.25 Hz and 75 Hz. This effect will be even much more significant for the Kauai source because of the greater range.

RELATED PROJECTS

This work falls within the context of the ONR Ocean Acoustic Program (Code 321OA) S&T Thrust, Long-Range Propagation and complements other Code 321OA theoretical works.

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