Report Documentation Page

Form Approved
OMB No. 0704-0188

1. REPORT DATE **01 OCT 2014**	2. REPORT TYPE **N/A**	3. DATES COVERED **-**

4. TITLE AND SUBTITLE **Probabilistic Analysis of Time Sensitive Systems**	5a. CONTRACT NUMBER
	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)	5d. PROJECT NUMBER
	5e. TASK NUMBER
	5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **Software Engineering Institute Carnegie Mellon University Pittsburgh, PA 15213**	8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSOR/MONITOR'S ACRONYM(S)
	11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release, distribution unlimited

13. SUPPLEMENTARY NOTES
The original document contains color images.

14. ABSTRACT

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT **unclassified**	b. ABSTRACT **unclassified**	c. THIS PAGE **unclassified**	**SAR**	**1**	

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std Z39-18

Probabilistic Analysis of Time Sensitive Systems

Problem Statement

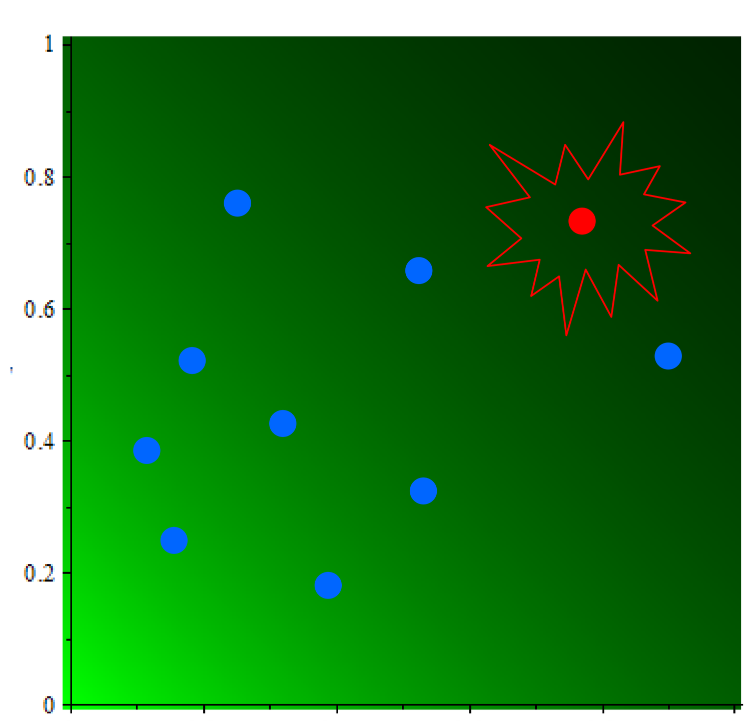
Time-sensitive systems in uncertain environments have complex behaviors. How do we assure correctness of such systems?

- Exact probabilistic verification is infeasible due to model size
- Black box testing does not yield bounded predictions
- Need formal approach for dealing with uncertainty
- Accurate, bounded, probabilistic results
- In reasonable time even for rarely occurring errors

Stochastic Model Checking (SMC)

SMC is a rigorous simulation-based approach for estimating that a property holds in a system.

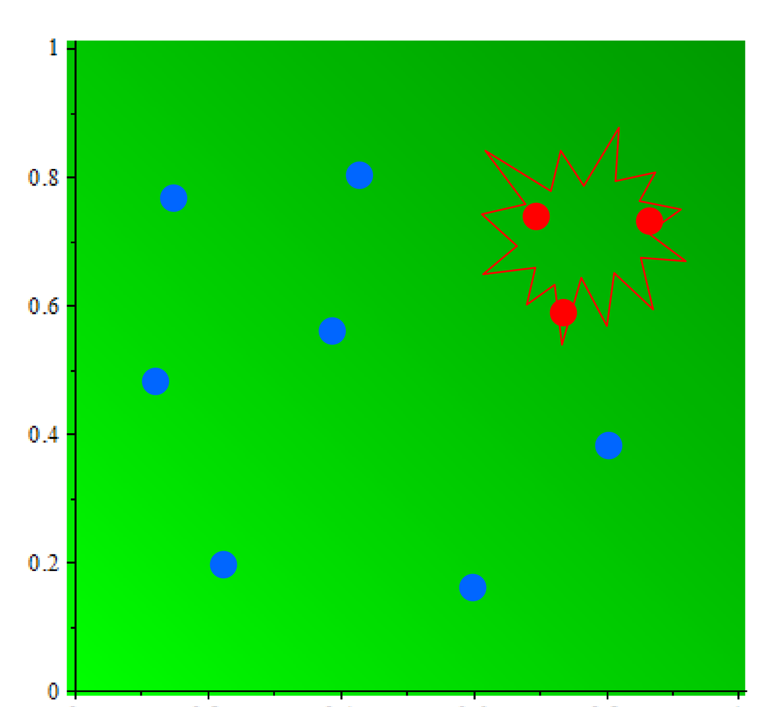
- System properties described in formal language (BLTL, etc.)
- Property is tested on "sample trajectories" (sequence of states)
- Each outcome treated as a Bernoulli trial (i.e., coin flip)



SMC Basics

- Indicator function $I(\vec{x}) = 1$ iff property holds for input $\vec{x}$.
- Relative Error $RE(\hat{p}) = \frac{\sqrt{var(\hat{p})}}{E[\hat{p}]}$ is measure of accuracy.
- Draw random samples from input distribution $f(\vec{x})$ until target Relative Error is met.
- Estimated probability that property holds is:

$$\hat{p} = \frac{1}{N}\sum_{i=1}^{N} I(\vec{x}_i) = \frac{1}{10} = 0.1 \qquad \mathrm{RE}(\hat{p}) = \frac{0.32}{0.1} = 3.2$$



Importance Sampling

- Modify input distribution to make rare properties more visible.
- Weighting function $W(\vec{x})$ maps solution back to original problem.
- Reduced relative error with same number of samples.

$$\hat{p} = \frac{1}{N}\sum_{i=1}^{N} I(\vec{x}_i)W(\vec{x}_i) = \frac{0.2 + 0.5 + 0.3}{10} = 0.1$$

$$\widehat{RE} = \frac{0.18}{0.1} = 1.8$$

Semantic Importance Sampling
A New Approach to Importance Sampling

Input Specification in C

```
#include "osmosis_client.h"

//@dist a=uniform(min=0,max=5)
//@dist b=normal(mean=3,std=1,min=0,max=5)
void simple()
{
  double a = INPUT_D("a");
  double b = INPUT_D("b");
  double c = a + b;
  double d = (a - b)/2.0;

  ASSERT(sin(c)*cos(d/2) < 0.995);
}
```

Translate C model to SMT2 for Analysis.

SMT2 Model

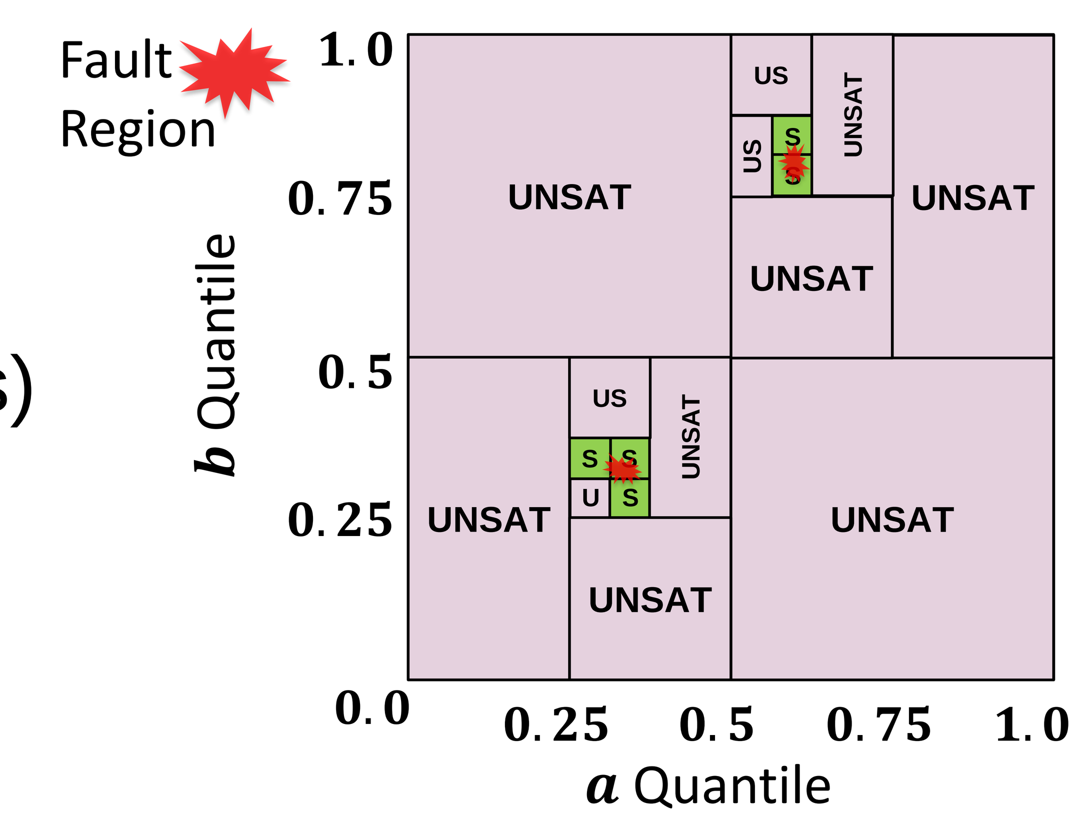
```
(set-logic QF_NRA)
(declare-fun a () Real)
(declare-fun b () Real)
(declare-fun a_1 () Real)
(declare-fun b_1 () Real)
(declare-fun c_1 () Real)
(declare-fun d_1 () Real)
(assert (>= a 0))
(assert (<= a 5))
(assert (>= b 0))
(assert (<= b 5))
(assert (= a_1 a))
(assert (= b_1 b))
(assert (= c_1 (+ a_1 b_1)))
(assert (= d_1 (/ (- a_1 b_1) 2.0)))
(assert (not (< (* (sin c_1) (cos d_1)) 0.9)))
(check-sat)
(exit)
```

Input Cube

ASSERT()

Recursively invoke dReal SMT checker to build abstract model of specification.

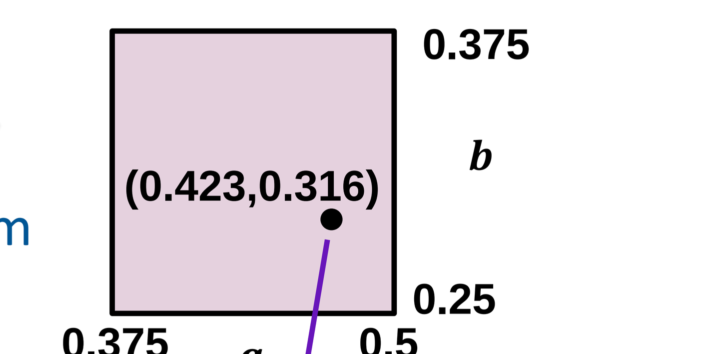
Abstract Indicator Function $I^*(\vec{x})$



Use $I^*(\vec{x})$ to generate random input vectors:

- Randomly pick SAT cube
- Randomly pick point in cube

Input Generation



Apply inverse CDF on each input variable.

$(a, b) = (2.115, 2.503)$

Apply generated inputs to original C model to calculate bounded failure probability estimate.

Raw Prob. Estimate

$$\hat{p}_{raw} = 0.024$$

$$RE(\hat{p}_{raw}) = 0.01$$

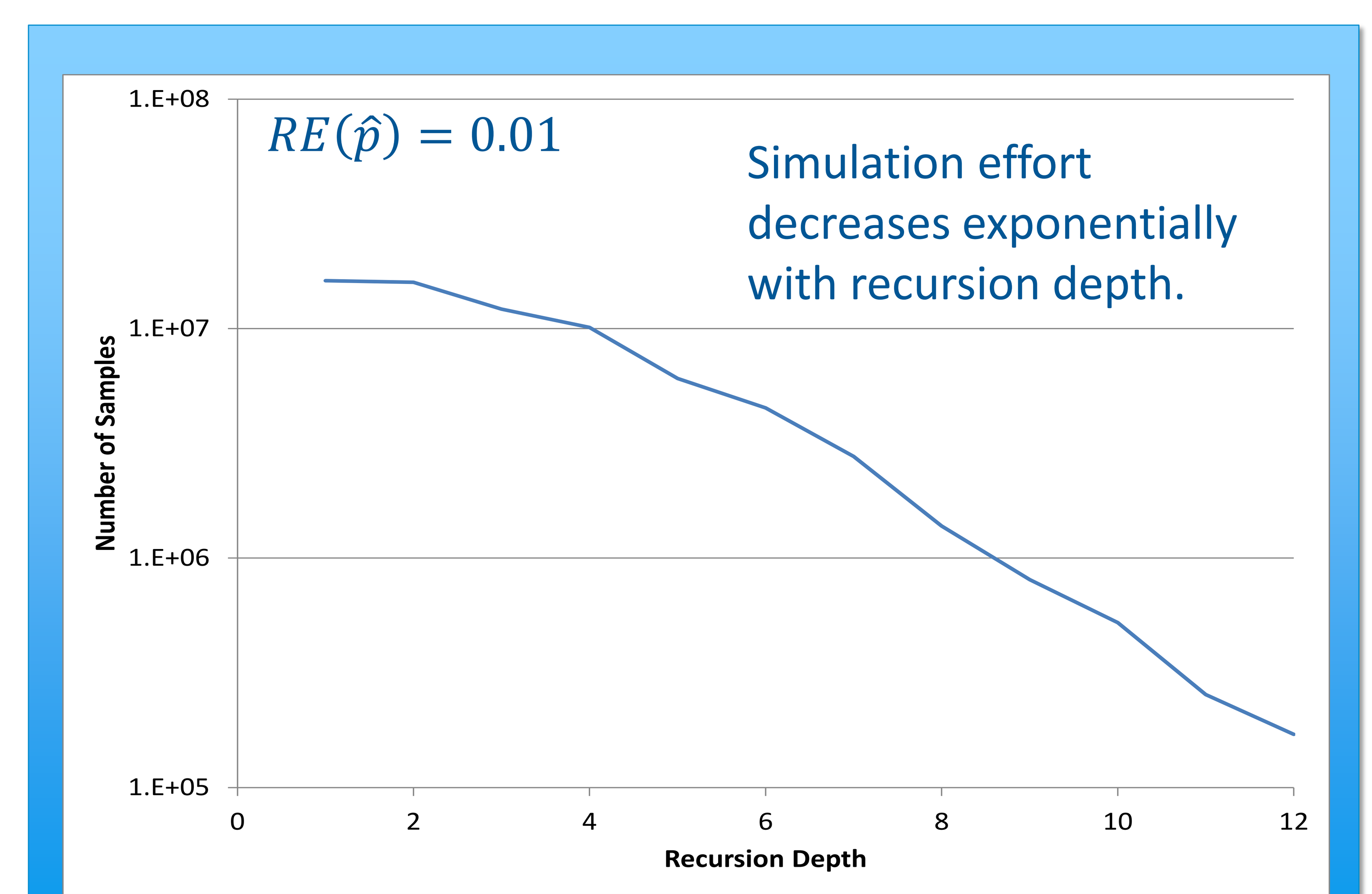
Weight function $W(\vec{x}_i)$ is probability p^* that $\vec{x}$ is in $I^*(\vec{x})$.

Abstract Probability

$$p^* = \frac{5}{2^8}$$

Number of cubes in $I^*(\vec{x})$.

Level of cubes



Final Probability Estimate

$$\hat{p} = p^*\hat{p}_{raw} = 0.00047$$

$$RE(\hat{p}) = 0.01$$

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This material is based upon work funded and supported by the Department of Defense under Contract No. FA8721-05-C-0003 with Carnegie Mellon University for the operation of the Software Engineering Institute, a federally funded research and development center.

DM-0001712