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**DEPENDENCE OF CRACK PROPAGATION/  
DEFLECTION MECHANISM ON CHARACTERISTICS  
OF FIBER COATING OR INTERPHASE IN CERAMICS  
MATRIX CONTINUOUS FIBER REINFORCED  
COMPOSITES (POSTPRINT)**

**Craig P. Przybyla  
AFRL/RXCCP**

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CRAIG P. PRZYBYLA  
Composites Branch  
Structural Materials Division

*//Signature//*

---

MICHAEL J. KINSELLA, Chief  
Composites Branch  
Structural Materials Division

*//Signature//*

---

TIMOTHY J. SCHUMACHER, Chief  
Structural Materials Division  
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<b>14. ABSTRACT</b> Toughness in continuous ceramic fiber reinforced ceramic matrix composites (CMCs) with dense matrices are dependent on the properties of the fiber coating or interphase that separate the fibers from the matrix. Multiple criteria have been proposed to describe the mechanism of crack propagation/deflection at the filament scale in brittle matrix continuous fiber reinforced composites; however, most of these criteria fail to account for the presence of an interphase of finite thickness or employ unrealistic boundary conditions (e.g., He MY, Evans AG, Hutchinson JW, Int. J. Solids Struct. 1994; 31:24; 3443-55). Recent simulations employing the extended finite element method (XFEM) have shown that variations in interphase thickness and strength relative to the fibers/matrix can have a significant influence on the mechanism of crack propagation/deflection. It is shown that primary crack deflection most often occurs when conditions favor secondary cracking in the interphase in front of an approaching matrix crack. Although this mechanism is similar to that argued by Cook and Gordon (Cook J, Gordon JE, Proc. Roy. Soc. A 1964; 28; 508-520), variations in the properties of the interphase are simulated to produce large deviations in the local crack growth behavior as a matrix crack grows into interphase.					
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### 6. AUTHOR(S)

C. P. Przybyla - Materials and Manufacturing Directorate, Air Force Research Laboratory, Structural Materials Division  
M. Braginsky - University of Dayton Research Institute

### 7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

AFRL/RXCCP  
Air Force Research Laboratory  
Materials and Manufacturing Directorate  
Wright-Patterson Air Force Base, OH 45433-7750

University of Dayton Research Institute  
300 College Park Drive  
Dayton, OH 45469-0101

# DEPENDENCE OF CRACK PROPAGATION/DEFLECTION MECHANISM ON CHARACTERISTICS OF FIBER COATING OR INTERPHASE IN CERAMICS MATRIX CONTINUOUS FIBER REINFORCED COMPOSITES

M. Braginsky<sup>a,b</sup> and C. P. Przybyla<sup>a\*</sup>

<sup>a</sup>Materials and Manufacturing Directorate, Air Force Research Laboratory, Wright-Patterson AFB, Ohio, USA

<sup>b</sup>University of Dayton Research Institute, Dayton, Ohio, USA

\*craig.przybyla@us.af.mil

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## Abstract

*Toughness in continuous ceramic fiber reinforced ceramic matrix composites (CMCs) with dense matrices are dependent on the properties of the fiber coating or interphase that separate the fibers from the matrix. Multiple criteria have been proposed to describe the mechanism of crack propagation/deflection at the filament scale in brittle matrix continuous fiber reinforced composites; however, most of these criteria fail to account for the presence of an interphase of finite thickness or employ unrealistic boundary conditions (e.g., He MY, Evans AG, Hutchinson JW, Int. J. Solids Struct. 1994; 31:24; 3443-55). Recent simulations employing the extended finite element method (XFEM) have shown that variations in interphase thickness and strength relative to the fibers/matrix can have a significant influence on the mechanism of crack propagation/deflection. It is shown that primary crack deflection most often occurs when conditions favor secondary cracking in the interphase in front of an approaching matrix crack. Although this mechanism is similar to that argued by Cook and Gordon (Cook J, Gordon JE, Proc. Roy. Soc. A 1964; 28; 508-520), variations in the properties of the interphase are simulated to produce large deviations in the local crack growth behavior as a matrix crack grows into interphase.*

## 1. Introduction

One mechanism enabling toughness in continuous ceramic fiber reinforced ceramic matrix composites (CMCs) operates via a weak interface between the fiber and matrix when the matrix is dense or non-porous. As matrix cracks propagate through the matrix they are deflected around the fiber by this weak interface and/or weak fiber coating or interphase surrounding the fiber. Deflection of matrix cracks at the interphase depends on the relative strengths of the matrix, interphase and fiber and the bonding characteristics between the constituent layers. Changes in the interphase strength and the bonding characteristics between the interphase and the fiber were shown by Rebillat et al. [1] to have the potential to double the overall tensile strength of a SiC/SiC CMC with a BN coated fiber. Optimization of composite strength and toughness in CMCs requires a detailed understanding of the fracture process at the fiber/interphase/matrix interfaces.

Several approaches have been proposed to estimate the conditions of crack propagation versus deflection at matrix, interphase, fiber boundary. He and Hutchinson [2] employed traditional fracture mechanics to define a criterion for crack deflection at the fiber-matrix interface in an isotropic material according to

$$\frac{G_d}{G_p} < \frac{\Gamma_i}{\Gamma_f} \quad (1)$$

where  $G_d$  and  $G_p$  are the energy release rates for deflection and propagation and  $\Gamma_f$  and  $\Gamma_i$  are the critical energy release rates or surface energies for the fiber and for a deflecting crack at interface between two semi-infinite planes with plane strain traction free boundary conditions. Others included the effect of anisotropy [3] and demonstrated that the energy for a doubly deflected crack is higher than that for a singly deflected crack in certain cases. Later He et al. [4] and also considered the influence of residual stress. Similar results were shown numerically by Tullock et al. [5]. Ahn et al. [6].

It is important to note that none of the previous studies considered an interphase of finite thickness, which can have a significant influence on the overall response. For example, Martin et al. [7] employed fracture mechanics implemented in a finite element model to consider deflection both with and without a finite interphase. In the absence of an interphase, they predicted that crack deflection is enhanced if the toughness of the matrix is less than that of the fiber. Previous studies neglected any toughness mismatch between the fiber and matrix. In the presence of an interphase, enhanced crack deflection is predicted when the toughness at the fiber-interphase interface is low. Simulations by Parthasarathy et al. [8] employed the FEM to explicitly model the matrix, interphase and fiber to determine the optimal thickness of the interphase for crack deflection. Employing plane strain boundary conditions, they predicted that a thin interlayer with low modulus may significantly enhance crack deflection, which supports what was predicted previously.

The previously cited approaches account for one single dominant crack intersecting the fiber-matrix or matrix-interphase interface, but secondary debonding or cracking at the interphase or at interphase boundaries in many materials is observed at some distance in front of the primary crack [9-12]. This secondary crack arrests or retards growth of the primary crack. Cook and Gordon [13] postulated this type of crack deflection mechanism where interfacial debonding was predicted ahead of the crack tip as a result of the stress component parallel to the crack plane,  $\sigma_{rr}$ , if the interface is sufficiently weak. They predicted that, for an elliptical crack,  $\sigma_{rr}$  is maximal at a distance ahead of the crack tip on the order of the crack tip radius. Pagano and Brown [10] simulated cracking with and without the secondary cracking or interfacial debonding mechanism and predicted much higher energy release rates in the case of debonding of the interface ahead of the crack tip. Leguillon et al. [14] employed an asymptotic analysis to demonstrate that crack deflection is more favorable in the presence of a secondary crack initiated ahead of the primary crack than in the case where no secondary initiation occurs. Later Lacroix et al. [15] extended the previous analysis to establish an energy based criterion based on secondary crack initiation. Martin and Leguillon [16] predicted decohesion always occurs ahead of a propagating crack when the fiber is stiffer than the matrix; whereas, in the case of a stiffer matrix, decohesion is unlikely. In another approach, Pompidou and Lamon [17] employed the Cook and Gordon model [13] to estimate

the propensity for the interface ahead of the crack tip to debond based on the elastic mismatch and relative strengths of the fiber and matrix assuming elliptical crack with finite radius.

The effort described here employs the extended finite element method XFEM [18-20] implemented in Abaqus [21] to study fiber/matrix crack deflection in SiC/SiC CMC with a BN fiber coating. In all the previous studies cited, the cracks and/or crack propagation were defined a priori. Employing the XFEM here allow the simulation of crack initiation and growth without predefinition. This is a continuation of the effort previously described by Braginsky and Przybyla [22] where different mechanisms of crack progression/deflection could be observed depending on the relative strengths of the matrix, interphase, and fiber. In the previous study, 2D plane strain boundary conditions were employed, similar to many of modeling approaches cited previously. In this current work, it will be shown that the boundary conditions can significantly affect the operant mechanism for crack deflection as well as the relative strength of the constituent properties. Thus, both plain strain and axisymmetric boundary conditions are considered. Material properties of a commercially produced SiC/SiC CMC (HiPerComp®) were taken as the basis for the simulations, but different ranges of these properties were considered to understand the impact of relative strengths on the operant mechanisms. Reduced integration was not employed in any of the simulations. The fracture energy of the interface between the matrix/interphase and interphase/fiber was taken to be the same as the interphase (the weakest phase).

## **2. Simulation methodology**

A SiC/SiC composite similar to that of the melt-infiltrated processed HiPerComp® material produced by GE Aviation® was explicitly modeled with the strength of the matrix being less than that of the fiber as shown experimentally. Basic properties of the fiber (SiC), matrix (SiC) and fiber coating (BN) were assumed based on experimental results [23]. The thickness of the coating was fixed at 1  $\mu\text{m}$ . The material properties employed in the simulations are summarized in Table 1. To evaluate the influence of damage initiation and propagation parameters on the behavior of the system, a range of strengths was considered for the matrix and the fiber coatings. The range of properties for the axisymmetric simulations described here is extended slightly compared to the one employed in plain strain simulations previously in [22].

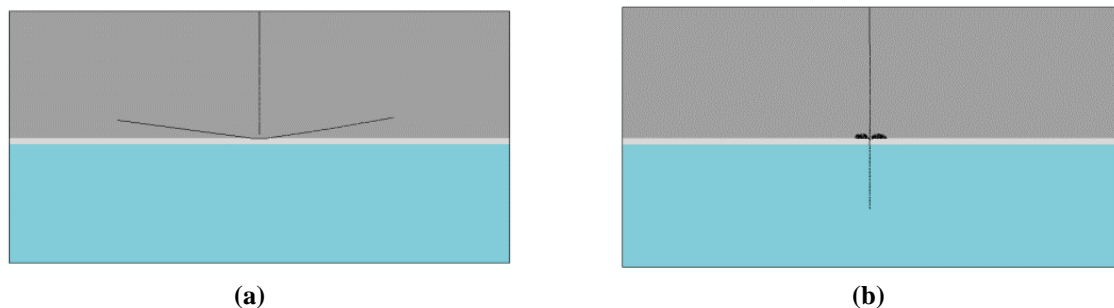
An initial crack was incorporated into the model to study crack propagation characteristics as the cracks approaches the fiber/matrix interphase. Loading in all the simulations was strain-controlled with uniform displacements applied at the boundaries parallel to the initial crack. The remaining boundary conditions were traction free. Damage initiation is predicted based on a maximum principal stress initiation criterion, and values of this initiation stress are referred to as “strengths” of respective materials. Crack evolution is governed by cohesive laws where the fracture energy defines the rate at which cohesive stiffness is degraded once the initiation criterion is met. The modeling is quasi-static, which requires viscous regularization to overcome convergence difficulties in ABAQUS Standard defined by the “damage stabilization” parameter. The additional parameter in ABAQUS “tolerance” controls time incrementation.

Base Properties	Variation of damage parameters	
<b>Matrix</b>	$\sigma_I$	Fracture energy
Elastic properties $E=360$ GPa , $\nu=0.185$		
Initiation stress $\sigma_I=0.8$ GPa (maximum principal)		
Fracture energy $36$ J/m <sup>2</sup>		
Tolerance 0.05; Damage stabilization 0.005		
<b>Coating (BN)</b>		
Elastic properties $E=10$ GPa , $\nu=0.05$		
Initiation stress $\sigma_I=75$ MPa (maximum principal)		
Fracture energy $5$ J/m <sup>2</sup>		
Tolerance 0.05; Damage stabilization 0.01		
<b>Fiber</b>		
Elastic properties $E=380$ GPa , $\nu=0.185$		
Initiation stress $\sigma_I=2.6$ GPa (maximum principal)		
Fracture energy $50$ J/m <sup>2</sup>		
Tolerance 0.05; Damage stabilization 0.005		
	<b>Matrix</b>	
	0.4 GPa	36 J/m <sup>2</sup>
	0.8 GPa	
	1.2 GPa	
	<b>Coating</b>	
	50 MPa	5 J/m <sup>2</sup>
	75 MPa	10 J/m <sup>2</sup>
	100 MPa	15 J/m <sup>2</sup>
	150 MPa	20 J/m <sup>2</sup>
	200 MPa	30 J/m <sup>2</sup>
	300 MPa	
	<b>Fiber</b>	
	2.6 GPa	50 J/m <sup>2</sup>

**Table 1.** Base Properties of HiPerComp® SiC-SiC CMCs [23] and variation of damage parameters

### 3. Results and Discussion

Previous simulations performed by the authors [22] with applied 2D plane strain boundary conditions based on a limited subset of the range of properties given in Table 2 resulted in two possible scenarios of crack propagation at the interface as shown in Figure 1.



**Figure 1.** Two distinct scenarios in plane strain simulations: (a) deflection; (b) penetration

Crack deflection in that study was characterized by the secondary crack initiation near the coating/matrix boundary when the primary crack’s tip was at a finite distance (i.e., several widths of the interphase thickness) from the coating with subsequent growth of this secondary crack into the matrix as shown in Figure 1-(a); we used differing lengths of secondary cracks’ branches in those simulations to quantify the overall tendency to deflect, [24]. Crack penetration of the primary crack through the coating/matrix interface and into the fiber with is shown in Figure 1-(b). In this latter case, secondary cracking was observed in the coating/matrix interface, but these secondary cracks did not prevent penetration of the primary crack.

The main material parameters that determined whether an approaching crack would deflect or penetrate were determined to be the relative strength of the matrix and interphase; the fracture



energy was not predicted to have a significant influence in these simulations. Additionally, the simulations that exhibited crack deflection were further separated into different groups based on the secondary crack *leverage* or the horizontal distance from the secondary crack tip to the primary crack. Larger leverage was thought to indicate more prominent deflection. Leverage was also found to depend mainly on the relative strengths of the coating and the matrix.

When axisymmetric boundary conditions were imposed, an increased variety of scenarios was observed. Shown in Figure 2 are several different observed cases for crack deflection and crack penetration, respectively. Simulations pictured in Figure 2-(a), (d), (e), and (g) appear to most closely resemble the classical view of crack deflection, while (b) and (c) are the closest crack configuration to the deflection patterns of the plain strain simulations as shown in Figure 1-(a).

This variety of cracking scenarios in axisymmetric simulations makes quantification of the tendency to deflect more challenging than in plane strain, in particular because the XFEM implementation in ABAQUS does not allow for crack intersections. The latter leads to artificially stiff regions when neighboring cracks are within two to three finite elements from each other. A natural way to distinguish between different simulations would be to allow the simulations to progress till complete fracture; however, in the absence of crack coalescence and the resulting artificially stiff regions render subsequent parts of simulations to full fracture unreliable. Properties used to generate simulations in Figure 2 are given Table 2.

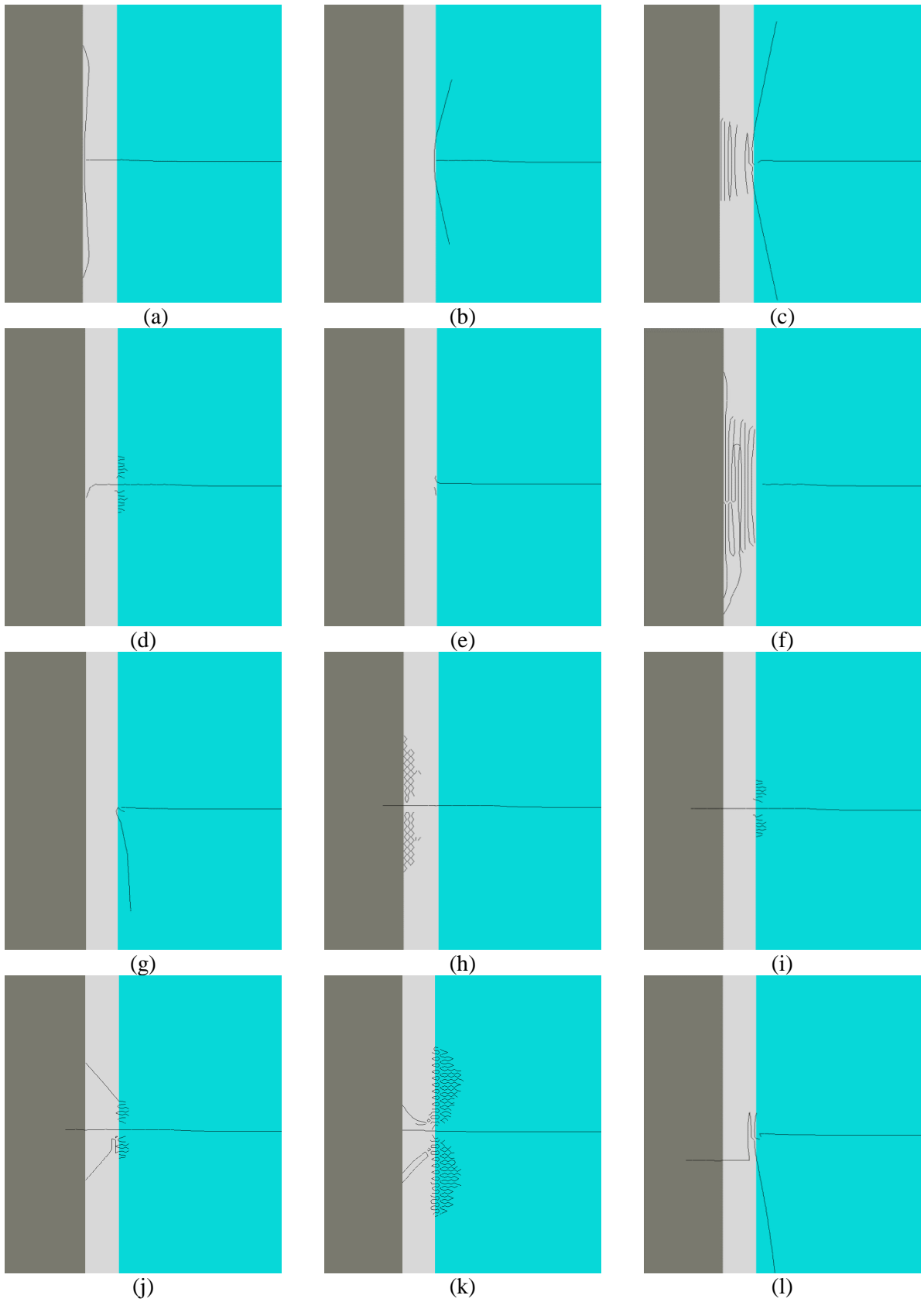
While the main material parameters that determined whether crack would deflect or penetrate were determined to be relative strengths of the matrix and coating, the fracture energy of the coating<sup>1</sup> was very important in several cases. In particular, the only difference between the simulations shown in Figure 2-(d), (h), and (i) is the fracture energy of the coating. With smaller fracture energy ( $5 \text{ J/m}^2$ ), the primary crack grows into the fiber with secondary cracks appearing near the fiber/coating interface. With the fracture energy  $15 \text{ J/m}^2$ , the primary crack turns near the fiber/coating interface without penetrating it. Increasing the fracture energy to  $30 \text{ J/m}^2$ , results in the primary crack penetrating the fiber. In this case, secondary cracks initiate in the interphase near the matrix/coating interface, but are not in the path of the primary crack. In other simulations, the influence of the fracture energy of the coating is less drastic. Such behavior was never observed in the simulations with applied plane strain boundary conditions.

#### 4. Conclusions

A numerical study of the classical problem of crack propagation/deflection at an interface of dissimilar materials presented is quite complex. When axisymmetric boundary conditions are imposed, results are similar to what has been observed experimentally and demonstrated by other modeling approaches. The results from the axisymmetric modes are more realistic and exhibit more varied crack configurations than previous studies demonstrated with applied 2D plane strain boundary conditions. For the chosen parameter set representing the melt-infiltrated SiC/SiC CMC HiPerComp®, the parameters having the greatest influence on the primary crack interaction with the coating are relative strengths of the matrix and the coating. Unlike the previous results from plane strain analysis, the fracture energy of the coating was found to be quite important in many cases of axisymmetric simulations.

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<sup>1</sup> the fracture energy of the matrix was fixed in all presented simulations



**Figure 2.** Distinct scenarios of cracking in axisymmetric simulations; deflection: (a) – (g), penetration: (h) – (l)

		<b>Matrix Strength MPa</b>	<b>Coating Strength MPa</b>	<b>Coating Fracture Energy J/m<sup>2</sup></b>
<b>Deflection</b>	a	800	50	5
	b	1200	100	30
	c	400	75	5
	d	1200	300	15
	e	400	100	20
	f	1200	50	5
	g	800	150	10
<b>Penetration</b>	h	1200	300	5
	i	1200	300	30
	j	1200	200	15
	k	400	150	10
	l	800	100	5

**Table 2.** Simulation parameters employed in the axisymmetric matrix-interphase-fiber crack model.

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