Advanced Numerical Methods for Numerical Weather Prediction

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LONG-TERM GOAL

The long term goal of this research is to explore new numerical methods for the next generation global atmospheric model. The reason for considering new methods is due to the paradigm shift in high performance computing from vector computers to distributed memory machines. To take full advantage of the new architectures the global domain must now be partitioned into subdomains/ elements that can then be solved independently on the multiple processors of a distributed memory machine. These new methods, along with the grids used to tile the sphere, must be local in nature to exploit the new architectures.

OBJECTIVES

The objective of this project is to develop a prototype for the next generation global model through the application of advanced numerical techniques. As part of this project, we will develop advanced numerical methods that are local in nature. Because we are developing local methods, we also need to develop local grid generation methods, such as icosahedral and cubic gnomonic grids, which yield uniform spatial representation on the sphere. This combination of grids and local methods simplifies the construction of efficient and high order algorithms for the atmospheric equations. In addition, some of this new technology (such as the semi-Lagrangian method) will be used to improve the current spectral forecast model of the Navy Operational Global Atmospheric Prediction System (NOGAPS).

APPROACH

Our approach is to re-examine existing numerical methods (such as the finite element method) and test and implement advanced numerical methods (such as the spectral element method) to yield improvements to the forecast model in NOGAPS. Finite element methods are local methods – meaning that the equations are solved independently within each individual element (triangle). High order continuous and discontinuous basis functions will be explored. High order finite element methods yield spectral (exponential) convergence, like spectral methods, but unlike spectral methods they are local rather than global methods. Continuous basis functions yields the spectral element method whereas discontinuous basis functions results in the discontinuous Galerkin method, which is conservative, monotone, and high order. Lagrangian formulations (Lagrange-Galerkin, semi-Lagrangian) of the equations will be studied. These methods, like high order methods, have no dispersion errors. This property is important for properly capturing the propagation of atmospheric phenomena (e.g., cyclones and hurricanes). In addition, these methods offer vast improvements in efficiency due to the longer time steps that they permit.

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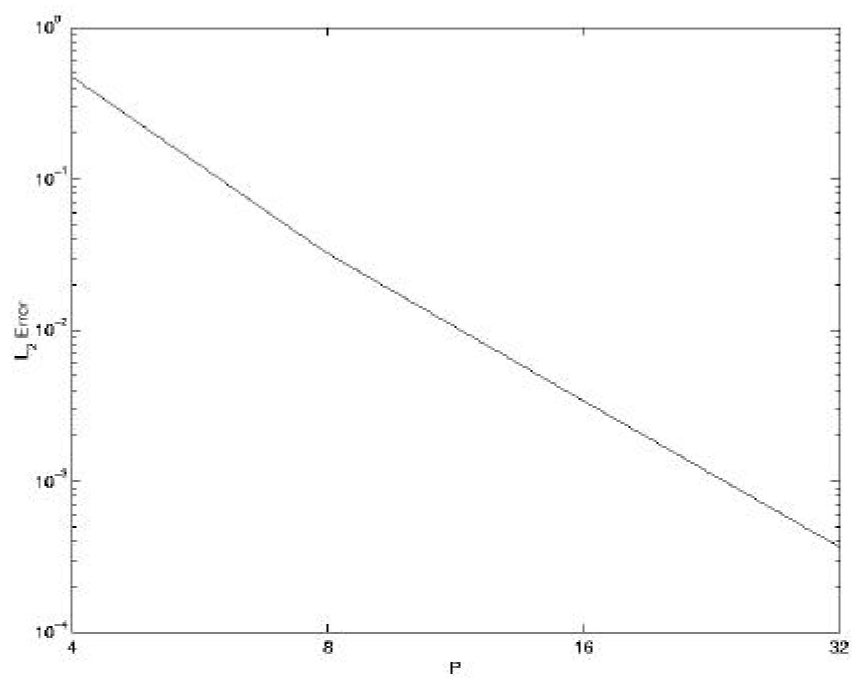
WORK COMPLETED

A new class of icosahedral grids that allow for more control over the number of points and triangular elements has been developed at NRL; two shallow water models on the sphere have also been developed. These new models use high order continuous basis functions (spectral element method) and the Lagrange-Galerkin method on the new class of icosahedral grids. Both of these models have been tested extensively and two papers have been written involving these two methodologies. Finally, a 3D atmospheric model based on the spectral element method has been developed and is currently being tested. This model derives its vertical discretization from NOGAPS – the difference being that the horizontal operators are now discretized using the spectral element method, which is high order but local in nature and thus can be used to exploit distributed memory architectures.

RESULTS

The icosahedral grid NOGAPS has been used to evaluate the relative merits of local finite element and local spectral element methods on quasi-regular grids such as the icosahedral and cubic gnomonic grids. A number of papers and presentations on the results have been published showing that this is a viable method for solving the atmospheric equations on the sphere accurately and efficiently (see the references at the end of this report). The figure below shows the spectral (exponential) convergence of the spectral element method. As the order of the polynomial P is increased by a factor of 2, the L2 error decreases by a factor of 10. This result is for a pure advection problem but the idea is that the order decreases drastically by increasing the order of the polynomial, such as in the standard spectral method. The spectral element method is local in nature and thus is “embarrassingly parallel” meaning that porting this method to distributed memory architectures is quite straightforward.



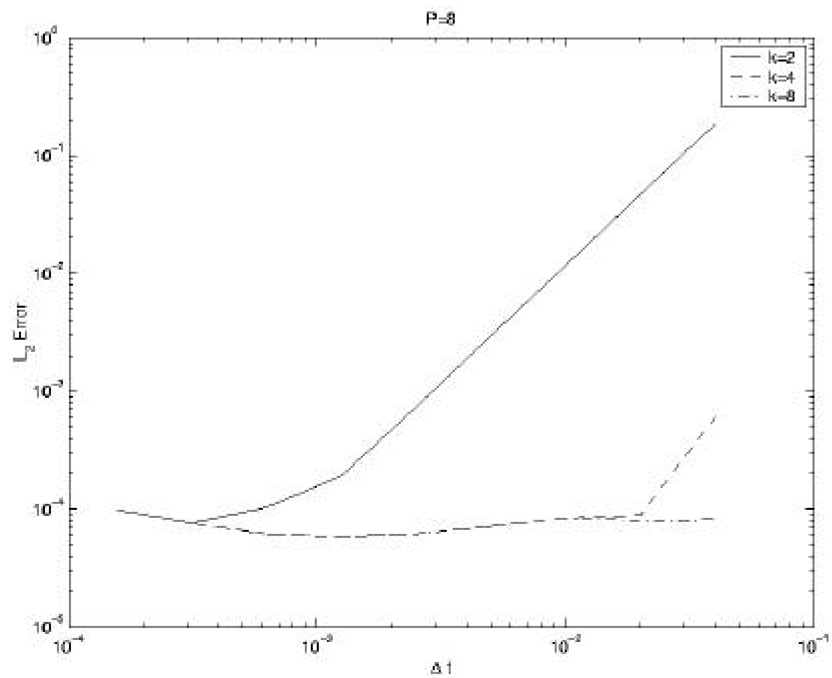
The figure above illustrates the power of the spectral element method; however, the spectral element method combined with the semi-Lagrangian method yields even better results. A theoretical paper written by Giraldo showed how these two methods can be combined to yield incredibly accurate results. Since this paper, other researchers have shown that this method is more powerful than originally thought. The order of accuracy of this hybrid method was shown to be on the order of

$$O\left(\Delta t^K + \frac{\Delta x^P}{\Delta t} \right)$$

where K corresponds to the order of integration of the particle trajectory equation

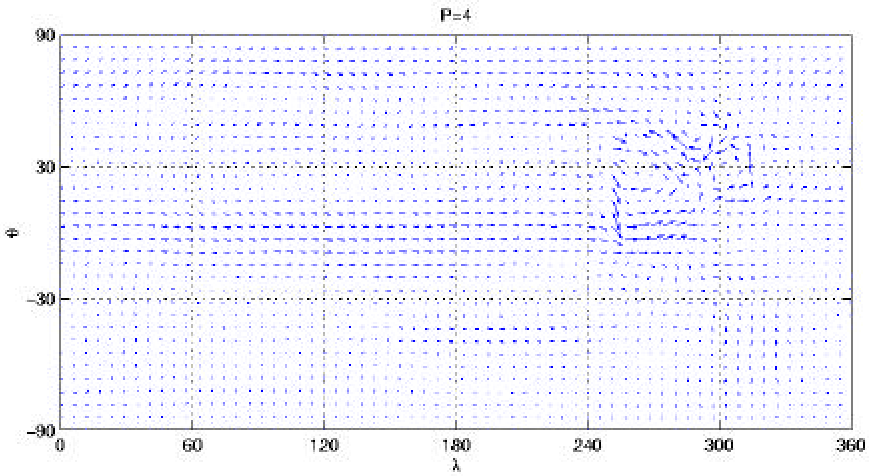
$$\frac{dx}{dt} = u(x,t)$$

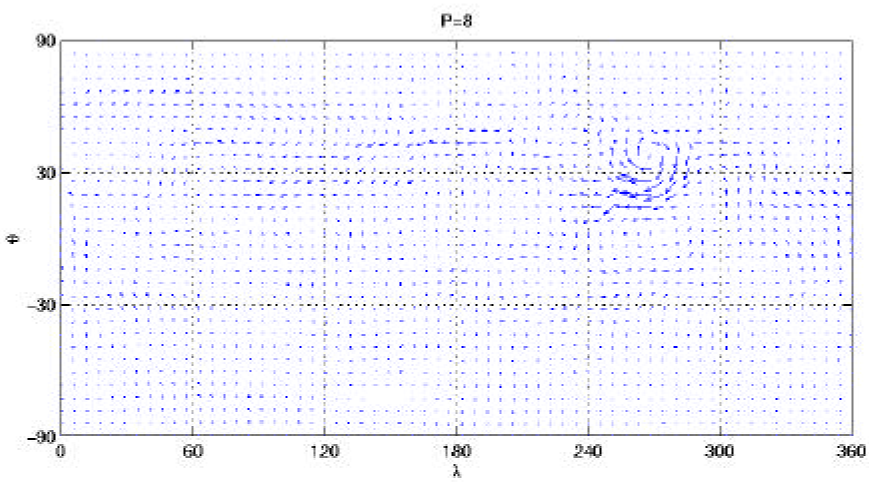
and P corresponds to the order of interpolation. Therefore for any given K and P there exists an optimal time step that balances the time and space errors. The most amazing thing about this order of accuracy result is that as we increase P (as is done in the spectral element method) we can increase the time step without incurring any time discretization errors provided that K is sufficiently high. Below we show the L2 error versus time step for pure advection for P=8 and K=2, 4, and 8.

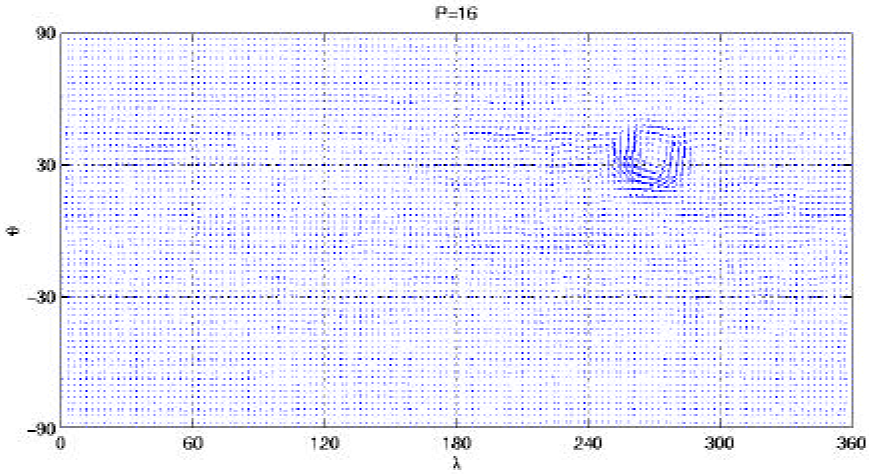


We can see that for K=2 the error increases linearly with time step. However, as we increase K to 4, we see that the error initially stays flat for increasing time step, but then it begins to increase. By increasing K to 8 we see that the error remains constant for all of the time steps used; however, at some point the time discretization error must dominate and the error should increase. The point of this study is to show that the spectral element semi-Lagrangian method (SESL) will not only yield exponentially accurate solutions but it can do so using very large time steps, thereby offering increased accuracy and efficiency over any other numerical method.

The new spectral element 3D global atmospheric model is currently being designed. As a first test to validate the new model, a still isothermal atmosphere was integrated for a hundred days. The result, as expected, was that very little happened. This test was used to check the stability of the algorithms and to ensure that no "numerical sources" arise. As a second test, we took the same initial conditions but then embedded a conical mountain along the Equator. Once again, we executed the model for a hundred days. The model showed completely symmetric flow patterns, thereby showing that no biasing was introduced by either the numerical scheme or the grid. Finally, we ran the same test case with the mountain now located around 30 degrees latitude north. The model showed a vortical flow around the mountain. Below we show results for three different polynomial orders: P=4, P=8, and P=16. Looking at P=4, we can see that the vortical flow is not well developed.







However, when we double the order of the polynomial to P=8 we can now clearly see the ensuing vortical flow. Finally, for P=16 the flow is not only well defined but the resolution is quite impressive. For this test case, we used 10 vertical levels with the following number of grid points per level N=866, 3458, and 13826 corresponding to P=4, 8, and 16, respectively.

IMPACT

NOGAPS is run operationally by Fleet Numerical Meteorology and Oceanography Center (FNMOC) and is the heart of the Navy's operational weather prediction support to nearly all DOD users worldwide. It is also run by many NRL and other Navy researchers to study atmospheric dynamics and atmosphere/ocean interaction. Our work here targets the next generation of this system for the next generation of computer architectures. These architectures are expected to be distributed memory, commodity based systems with enormous theoretical computational power. However, exploiting this capability will require drastically redesigning many important model algorithms.

TRANSITIONS

Improved algorithms for model processes will be transitioned to 6.4 (SPAWAR PMW-185, PE 0603207N) as they are ready, and will ultimately be transitioned to FNMOC with future NOGAPS upgrades.

RELATED PROJECTS

Some of the technology developed for this project will be used immediately to improve the current spectral formulation of NOGAPS. This improvement will complement the work done on constructing an efficient message passing NOGAPS as is being done in the 6.3 CHSSI program for Scaled Software Algorithm Development for Meteorological Models (sponsored by DOD). In addition, the work herein will also complement the improvement of the numerical methods for NOGAPS as is being done in the NRL 6.2 Global Modeling project (NRL BE-35-2-18).

PUBLICATIONS

Giraldo, F. X., 2000: Characteristic-based Spectral Element Methods for the Shallow Water Equations on the Sphere. *European Congress on Computational Methods in Applied Science and Engineering,* (September 2000).

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