

## Observations of the fortnightly nutation terms and the "dynamical variation of latitude" with photographic zenith tubes

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Observations made with three photographic zenith tubes show fortnightly nutation terms in good agreement with values determined from Earth tide observations. The radius of the circular "dynamical variation of latitude" occurring with the argument  $(\alpha - 2L)$  is found to be less than the rigid Earth values.

### INTRODUCTION

ONE of the most significant nutation terms is the fortnightly term occurring with argument  $2L$ , where  $L$  is the lunar longitude (Woolard 1953). Observations made with a photographic zenith tube (PZT) will show an error in this term as a lunar diurnal variation in the derived latitude and time.

The gravitational force exerted by the Sun and the Moon on the Earth also produces a motion of the rotational pole of the Earth with respect to the pole of figure. This phenomenon which has been referred to as the "Oppolzer terms" or the "dynamical variation of latitude" has been discussed previously by Oppolzer (1880), Woolard (1953), and recently by Atkinson (1973) and McClure (1973). Observational evidence of its existence has been provided by Morgan (1952), Fedorov (1963), Guinot (1970), and O'Hora (1973).

This variation is the result of the fact that the effects of nutation on a star's position are computed for the instantaneous pole of rotation and not for a coordinate system fixed with respect to the crust of the Earth. The most significant terms are listed by Woolard (1953) where he gives expressions for the variation of the position of the rotational pole with respect to the pole of figure. He includes the Eulerian motion and the lunisolar motion of the pole in his discussion of the theory of the rotation of a rigid Earth. The Eulerian motion, however, is a torque-free motion of the rotational pole with respect to the pole of figure, and will not be discussed here. It has been suggested by Jeffreys (1963) and Atkinson (1973) that the theory of nutation should be changed so that it will be computed for the pole of figure rather than the instantaneous pole of rotation using a rigid Earth theory. No present theory of rotation, however, can be used to predict the motion of the pole of figure in space because of the poorly understood nature of the Eulerian and the forced annual and semiannual motion of the pole of figure with respect to a direction fixed in space (Munk and MacDonald 1960; Rochester 1973). In addition, the deformability of the Earth cannot presently be accounted for with certainty in the rigid Earth theory of nutation.

The U. S. Naval Observatory determines the variation of latitude and time using photographic zenith tubes located in Washington, DC, and in Richmond,

FL. The instrumental design and method of observation of these telescopes is outlined by Markowitz (1960). The purpose of this paper is to present the observations of the fortnightly nutation terms and the "dynamical variation of latitude" derived from the observations of the Washington and Richmond instruments and to combine this information with the recent results of O'Hora (1973). The specific variation in this discussion is that occurring with the astronomical argument  $(\alpha - 2L)$ , where  $\alpha$  is the sidereal time of the star's transit of the instrumental meridian. This is just one of the series of terms given by Woolard (1953) for the "dynamical variation of latitude." The term with the largest theoretical amplitude has an argument of  $\alpha$ , but this effect cannot be separated from the periodic catalog errors existing in the PZT star catalogs (McCarthy 1973) and the effect of seasonal refraction variations (Hughes 1974). Terms other than those with arguments  $\alpha$  and  $(\alpha - 2L)$  are expected to have much smaller amplitudes.

### I. OBSERVATIONS

The PZT is used to photograph stars in a small band of declination as they pass through the local zenith. The measured zenith distance and hour angle of the star at the mean time of the multiple exposures of the star are used to determine the astronomical latitude  $\phi$  and the difference between universal time (UTO) and atomic time at the U. S. Naval Observatory (A.1).

The data used in this analysis is the same used in a recent analysis of the effects of tides in PZT observations (Falzone and McCarthy 1976). It consists of individual star observations of the astronomical latitude (McCarthy 1974) and (UTO-A.1) (McCarthy 1976) which were used to form the residuals analyzed for the lunisolar diurnal motion. Latitude observations made in Washington from October 1915 through December 1973 were used. Only those observations of time made after 1 January 1956 have been used since the A.1 time scale is not available prior to mid 1955. This amounts to 143 671 individual observations of latitude and 57 920 for time. Similarly, Richmond observations made from November 1949 through December 1973 were used, providing 114 606 individual observations of latitude and 87 404 for time. The results

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TABLE I. Fortnighly nutation terms and the amplitude of the "dynamical variation of latitude" found from PZT observations. Units are in seconds of arc.

Instrument	$\Delta\epsilon$	$\Delta\psi \sin \epsilon$	$\gamma$
Washington PZT	$0.0907 \pm 0.0016$	$-0.0832 \pm 0.0015$	$0.0060 \pm 0.0002$
Richmond PZT	$0.0977 \pm 0.0008$	$-0.0896 \pm 0.0007$	$0.0053 \pm 0.0001$
Herstmonceux PZT	$0.0889 \pm 0.0017$	$-0.0816 \pm 0.0016$	$0.0061 \pm 0.0002$
Combination of above	$0.0922 \pm 0.0016$	$-0.0845 \pm 0.0015$	$0.0058 \pm 0.0002$

found by O'Hora (1973) from the Herstmonceux PZT observations were also used.

## II. ANALYSIS

An error existing in the coefficients of the fortnightly nutation terms given by Woolard (1953) would necessitate corrections of  $n$  and  $m$  in the obliquity and longitude terms, respectively. The errors in the time  $t$  and latitude  $\phi$ , derived from a PZT observation of a star with right ascension  $\alpha$  and declination  $\delta$  reduced with Woolard's nutation constants are given by

$$\begin{aligned} dt &= (1/30)(m \sin \epsilon - n) \tan \delta \cos(\alpha - 2L), \\ d\phi &= \frac{1}{2}(m \sin \epsilon - n) \sin(\alpha - 2L). \end{aligned} \quad (1)$$

In these expressions,  $\epsilon$  represents the obliquity for epoch 1900, and only the variation occurring with argument  $(\alpha - 2L)$  is shown.

The "dynamical variation of latitude" will also have an effect on both time and latitude given by

$$\begin{aligned} \Delta t &= \gamma (\tan \phi / 15) \cos(\alpha - 2L), \\ \Delta \phi &= -\gamma \sin(\alpha - 2L), \end{aligned} \quad (2)$$

where  $\gamma$  is the radius of the circular motion expressed in seconds of arc.

The fortnightly nutation terms as well as the term  $\gamma$  are found from theory by the integration of the expressions given in Table 17 of Woolard (1953). Thus, an error in the rigid Earth value of  $\gamma$  is related to  $n$  and  $m$ . Using the constants given by Woolard, this relationship is given by

$$d\gamma = -0.014527n + 0.036501m. \quad (3)$$

Theoretically for a rigid Earth,  $\gamma = 0''.0062$  (Woolard 1953), and for a deformable Earth including the effects of solid Earth tides,  $\gamma = 0''.0065$  (McClure 1973).

The effect which is observed at this frequency is complicated by the existence of the  $O_1$  tide with the same astronomical argument. This produces not only a tilt of the local vertical but also a change in the inertia tensor. The variation in the direction of the vertical will directly affect the observations since the observed time and latitude are referred to the local vertical, while the change in the inertia tensor produces an additional variation in the position of the rotational pole with respect to the pole of figure.

Analyses of the Washington and Richmond data for the periodic effect occurring at this frequency were carried out by correlating the individual star residuals in latitude and time with  $(\alpha - 2L)$ . It appears that neither Washington nor Richmond experiences anomalous tidal effects for the semidiurnal or long-period tide (Falzone and McCarthy 1976). Therefore, it was inferred that the theoretical tides are a good representation of the tidal reaction at the diurnal frequencies also. The correction to be applied, then, to the observed results can be given by the standard expression for the effect of solid Earth tides,

$$\begin{aligned} \Delta\phi &= \Lambda a \cos 2\phi \sin(\alpha - 2L), \\ \Delta(\text{UTO} - \text{A.1}) &= \frac{\Lambda a}{15} \tan \phi \cos(\alpha - 2L), \end{aligned} \quad (4)$$

where  $a$  is the amplitude for a rigid Earth given by Doodson (1922), and  $\Lambda$  is the combination of Love and Shida numbers,  $1+k-l$  to account for the elasticity of the Earth. Numerically,  $\Lambda = 1.13$  (Melchior 1960),  $a = 0''.00652$ ,  $\phi = 38^\circ 55' 17''$  for Washington and  $\phi = 25^\circ 36' 47''$  for Richmond.

The observed variations of time and latitude at Washington, Richmond, and Herstmonceux at this frequency were adjusted by the subtraction of the effect of the theoretical dynamical variation in latitude and time assuming  $\gamma = 0''.0062$  and by the application of Eq. (4). The rigid Earth value of the ratio of  $m/n$  from Woolard was also assumed so that a value of  $n$  could be determined from the observations. From this value  $m$  and  $d\gamma$  were found.

The values of the nutation term along with the mean error determined in this manner are, in obliquity  $(0''.0922 \pm 0''.0016) \cos 2L$ , and in longitude  $(-0''.2124 \pm 0''.0037) \sin 2L$ . These can be expressed as  $\Delta\epsilon = 0''.0922 \pm 0''.0016$  and  $\Delta\psi \sin \epsilon = -0''.0845 \pm 0''.0015$ . The value of  $\gamma$  is found to be  $0''.0058 \pm 0''.0002$ . The results of the individual instruments along with those from the combination are summarized in Table I.

## III. DISCUSSION

The numerical results for the nutation terms are in good agreement with those found by Melchior from the observations of solid Earth tides (Melchior 1973). He finds  $\Delta\epsilon = 0''.0906$  and  $\Delta\psi \sin \epsilon = 0''.0828$ .

These combined observed values are considerably different from theoretical values determined from Earth models as listed by Kakuta (1970), who indicates a range in  $\Delta\epsilon$  from  $0''.0966$  to  $0''.1002$  and in  $\Delta\psi \sin \epsilon$  from  $-0''.0894$  to  $-0''.0929$ .

The value found for  $\gamma$  indicates that the multiplication of the rigid Earth value by the constant 0.76 to account for the deformability of the Earth (Fedorov 1963; *Annual Report of the Bureau International de l'Heure for 1972*) is not valid. This leads to a value of  $0''.0047$  for the radius of the "dynamical" polar motion which does not agree with the observed results.

The effects of  $O_1$  ocean tide have been neglected in this treatment. This should affect not only the tidal response of the individual instruments but also the inertia tensor. However, the amplitude of this tide is small and its effect on the results is expected to be minimal.

#### IV. CONCLUSION

Analysis of Washington, Richmond, and Herstonceux PZT observations shows fortnightly nutation terms  $\Delta\epsilon = 0''.0913$  and  $\Delta\psi \sin \epsilon = -0''.0837$  in good agreement with those found from observations of solid Earth tides and in poor agreement with those from Earth models. The amplitude of the "dynamical variation of latitude" is found to be  $0''.0059$ . These results assume no effect of the  $O_1$  ocean tide, the solid Earth ratio of the axes of the nutational ellipse, and a solid

Earth relationship between the "dynamical variation of latitude" and nutation terms.

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