

## **DECISION TOPOLOGY ASSESSMENT IN ENGINEERING DESIGN UNDER UNCERTAINTY**

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### **ABSTRACT**

The implications of decision analysis (DA) on engineering design are well known. Recently, the authors proposed decision topologies (DT) as a visual method for making design decisions and proved that they are consistent with normative decision analysis. This paper addresses the practical issue of assessing DTs for a decision maker (DM) using their responses, particularly under uncertainty. This is critical to encoding decision maker preferences so that further analysis and mathematical optimization can be performed using the correct set of preferences. We show how multiattribute DTs can be directly assessed from DM responses. Four methods are shown to evolutionarily assess DTs among which one that requires the DM to rank alternatives and another where a utility function is first assessed. It is also shown that preferences under uncertainty can be easily incorporated. In addition, we show that topologies can be constructed using single attribute topologies similarly to multi-linear functions in utility analysis. This incremental construction simplifies the process of topology construction. The reverse problem of inferring single attribute DTs is also presented. The proposed assessment methods are used on a design decision making problem of a welded beam.

### **1. INTRODUCTION**

Engineering design does not happen in vacuum, instead it is a decision making process. This notion is

well documented in the engineering design literature (e.g., Howard (1989), Thurston (1991) and Hazelrigg (1998)). However, despite all possible benefits, widespread adoption of prevalent decision making tools is impeded by the complexity of these tools to the average person. Most practicing engineers implement “best practices” when it comes to making design decisions because to them the required effort seems higher than the gained benefit - much to the disappointment of researchers in design decision making. Even when a DM is convinced of the benefits, errors are introduced when eliciting preferences, affecting the quality of decisions to be made. These errors may result from problems associated with stated and revealed preferences (Train, 2003). It is also possible that the facilitator (expert helping the decision maker making the decision) overwhelms the DM when assessing utility functions. This problem is exacerbated when preferences over multiple attributes are collected, particularly when the DM is not comfortable or experienced in giving responses to lottery questions.

To alleviate some of the roadblocks to successful implementation of DA in design, the authors recently proposed an alternative called Decision Topologies (DTs). DTs offer many advantages (Pandey and Mourelatos, 2013) over classical methods of encoding preferences and making decisions. They are also entirely consistent with decision analysis at the discretization limit (described later). In this paper, we make theoretical advances in DTs by focusing on their assessment directly or from a utility function. Our goal is to bring a theoretically sound method to the level that

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it can be used in actual engineering design decision making. We propose four methods to construct DTs if the decision maker:

1. Provides ranking of alternatives involving multiple attributes.
2. Answers lottery questions.
3. Provides single attribute utility functions or decision topologies.
4. Has provided MADTs and an attribute is to be removed from consideration.

The paper is arranged as follows. Section 2 discusses the decision topologies and Section 3 presents theoretical results in DT assessment. The design of a welded beam is presented in Section 4 demonstrating the topology assessment and how it is used in decision making. Finally, Section 5 concludes and discusses directions for future work.

## 2. DECISION TOPOLOGIES

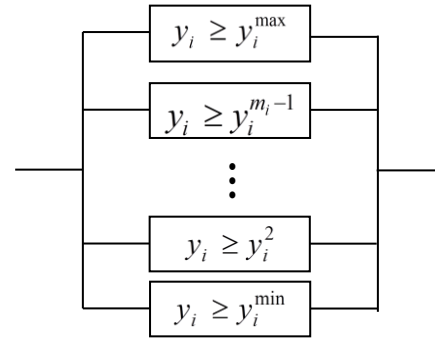
Decision topologies are block diagrams similar to the reliability block diagrams used in reliability engineering. In reliability engineering, a system is operational if its block diagram representation has a continuous path from one side of the diagram to another. The decision topology extends this notion to decision making. If there are no paths from one end to the other, the DT is assigned a score of zero. Otherwise, all paths from one side of the diagram to the other are counted providing an overall score for the DT. This score is a positive linear transformation of the decision maker's utility function, a claim that has been substantiated in Pandey and Mourelatos (2013). We provide a sketch of the proof here for continuity.

### 2.1 Consistency with decision analysis

We prove that DTs are consistent with decision analysis by showing that the behavior of any continuously differentiable utility function which is monotonic in attributes can be modeled by decision topologies. This is *in addition* to binary attributes, which can be trivially incorporated.

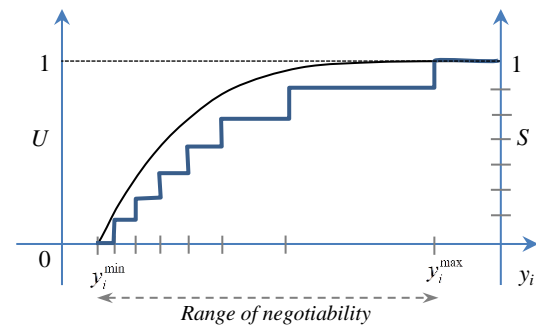
In a single attribute case, the objective of decision making methods is to assign a score corresponding to an attribute level. We can represent the decision topology associated with an attribute visually as shown in Fig. 1. The decision topology consists of blocks where each block tests the binary condition that the attribute level is greater than a partition  $y_i^j$  of the attribute. If this is true, the block is considered active. The raw score for an attribute level  $y_i$  is equal to the number of paths from right to left (or left to right) through the active blocks. For example, if  $y_i \geq y_i^{\max}$

all blocks will be active because all inequalities are satisfied. Similarly, if  $y_i < y_i^{\min}$  none of the blocks will be active. For  $y_i^{\min} < y_i < y_i^{\max}$ , a number of blocks less than  $m_i$  will be active. Fig. 2 shows pictorially the score from a DT as the attribute level is steadily increased and the blocks become progressively active. As shown, this increase is step-wise.



**Figure 1.** A decision topology for a single attribute case

How the score varies with respect to an attribute depends on the chosen partitioning for the attribute. A non-uniform partitioning should be used to draw the topology. If the density of partitions around a particular value of  $y_i$  is proportional to the derivative of the utility function at that value of  $y_i$ , the normalized score will be equal to the utility value (Fig. 2). This is guaranteed if  $m_i \rightarrow \infty$  since the score approaches the Riemann sum under the utility density function (derivative of the utility function). A rigorous proof is given in Pandey and Mourelatos (2013).



**Figure 2.** Comparison of a utility function with a decision topology score when the first partitioning is proportional to the local value of the derivative of the utility function

If we assume that the multiattribute utility function is continuously differentiable and increasing in individual attributes, the proof is straightforward. Let's

start with the point  $\{y_1^{\min}, \dots, y_n^{\min}\}$  where the utility function and the score  $S(y_1^{\min}, \dots, y_n^{\min})$  are both zero. If the attribute vector is perturbed by a small amount  $\{dy_1, \dots, dy_n\}$  in any general direction that keeps the attributes in the range of negotiability, the new utility can be approximated by

$$U(y_1^{\min} + dy_1, \dots, y_n^{\min} + dy_n) \approx \frac{\partial U}{\partial y_1} dy_1 + \dots + \frac{\partial U}{\partial y_n} dy_n \quad (1)$$

As in the single attribute case, if the first partitioning at every point along any attribute is proportional to the partial derivative along that attribute, we have

$$S(y_1^{\min} + dy_1, \dots, y_n^{\min} + dy_n) \approx U(y_1^{\min} + dy_1, \dots, y_n^{\min} + dy_n) \quad (2)$$

The approximation of Eq. (2) improves as the partitioning becomes finer. As we move on a path from  $\{y_1^{\min}, \dots, y_n^{\min}\}$  towards any point  $\{y_1', \dots, y_n'\}$ , the decision topology makes successive linear approximations of the utility function, and

$$S(y_1', \dots, y_n') \approx U(y_1', \dots, y_n') \quad (3)$$

Eq. (3) proves that if the partitioning of the attribute space is sufficiently fine, and the distribution of the partitions at any point along any direction is proportional to the partial derivative of the utility function along that direction, the decision topologies will provide the same score with the value of the utility function. An algorithm we have developed, called Evolutionary System Topology Approximation (ESTA) to evolve decision topologies from limited data can find creative arrangements of blocks which can concisely represent the tradeoff information (Pandey and Mourelatos, 2012a). Furthermore, the partitioning does not even have to be fine in order to approximate a utility function well. These claims are substantiated in the results section.

## 2.2 Clarifications

A decision topology is not simply a decision tree. It is a visual representation of the entire decision-making situation. The main function of decision topologies is to replace the utility function and make decision making visual. Decision trees quickly become intractable as more nodes are added. However, decision topologies provide a good picture of the decision situation without

becoming intractable. In addition, uncertainty can be incorporated in the decision topology by definition. Similarly to calculating the expectation of a utility function in decision analysis, we can calculate the expectation of the score provided by the decision topology. To assess topologies that incorporate preferences under uncertainty, we simply evolve topologies using tests that are influenced by uncertainty.

## 3. ASSESSING DECISION TOPOLOGIES

Our previous work (Pandey and Mourelatos, 2012b) has shown that system topologies and hence decision topologies, can be provably deduced using the Evolutionary System Topology Approximation (ESTA) algorithm. Initially developed for reliability engineering studies, ESTA requires input-output test data. In assessing a system topology, a test provides input and output information. The input includes the component states (working / nonworking) and the output indicates if these states lead to system failure or not. If multiple such “tests” ( $\mathbf{t}$ ) and the corresponding system responses given by vector  $\mathbf{r}$  are available, we can use them to approximate the system topology as follows.

First a set of candidate topologies  $T$  is created from a set of feasible topologies  $T_f$ . Each of these topologies is evaluated using the tests and the response is stored in the vector  $\mathbf{s}$ . A candidate topology may output a failed system state or a working system state, for a given input. If this output matches the output of the original system, the score for the topology is incremented, otherwise it is decremented. The method then evolves better performing topologies to achieve a good concordance between a candidate topology and the true system. It is proven in Pandey and Mourelatos (2012b) that for sufficient number of tests, the ESTA algorithm will return the actual system topology  $T^*$ . For that, the following optimization problem is solved:

$$T^* = \arg \max_T F(\mathbf{s}(T, \mathbf{t}), \mathbf{r}(\mathbf{t})) \quad (4)$$

s.t.  $T \in T_f$

where the function  $F$  is the measure of similarity between the responses to the tests provided by a topology and the actual response of the system. For example, in the case study of Section 4 we use a Spearman's rank correlation coefficient for  $F$ .

### Method 1: Decision maker provides rankings

If the alternatives are deterministic, the utility functions and therefore the MADTs, only need to capture the tradeoff behavior between the attributes and not the risk attitude of the DM. In this case, we propose having the decision maker rank different multiattribute tuples according to their tradeoff preferences. The

ESTA can then be used to approximate the multiattribute decision topologies (MADT) by maximizing the Spearman's rank correlation between the ranking provided by the DM and that provided by a candidate topology. The output of the algorithm is the topology that provides the highest rank correlation with the rankings of the decision maker. While ESTA has provable convergence, we may not run the simulation long enough because of time constraints. The results even in this case are very good as our recent work has shown (Pandey and Mourelatos, 2012b).

Only a minor modification to the original ESTA (Pandey and Mourelatos, 2012b) was made in order to assess decision topologies from decision maker responses. The decision maker is asked to rank multiple alternatives and many different attribute combinations are acquired. These tests must be ideally unique so that the more *different* combinations we use, the more we can learn about the actual utility function. If a particular attribute combination is however, encountered more often than others in the tests, the ESTA will use this information to better represent the utility function in that region of the attribute space. The ESTA aims to find a topology that will give the highest Spearman's rank correlation between it and that provided by the decision maker.

## Method 2: Decision maker answers lottery questions

Lottery questions are used to elicit the decision maker's preferences over uncertain choices. A standard question is: "Would you prefer option A or a lottery which gives option B with probability  $p$  and option C with probability  $(1-p)$ ". In case the decision maker answers lottery questions, we first fit a multiattribute utility function which will provide a ranking of alternatives and then we apply method 1 directly. Another way to implement ESTA is by minimizing the root mean square error between the normalized topology score and the utility value as

$$T^* = \arg \min_T \|s(T, \mathbf{t}) - \mathbf{u}(\mathbf{t})\| \quad (5)$$

s.t.  $T \in T_f$

where  $\mathbf{s}$  is the vector of normalized scores provided by a topology  $T$  corresponding to each alternative in  $\mathbf{t}$ , and  $\mathbf{u}$  is the vector of utility values for the same alternatives in  $\mathbf{t}$ . This approach can be also used to model the risk attitude of the decision maker. Despite using utility functions, the visual benefits of DTs are retained. If there is little confidence in utility function modeling because of non-fulfillment of independence conditions, we can assess the DTs directly by including uncertain choices in the alternatives presented to the DM similarly to the lottery questions in the DA literature.

The DM is offered a combination of deterministic and lottery alternatives and he/she is asked to rank them. Table 1 shows for example, a small subset of ranking questions.

As in the case of system topology generation (Pandey and Mourelatos, 2012b), ESTA creates a set of candidate decision topologies. The rankings created by a candidate topology are then compared with that given by the decision maker based on the Spearman's rank correlation metric. For uncertain alternatives, we calculate the expected score for a candidate topology. This approach is not different from that of ESTA. It also retains its convergence properties. The relative number of uncertain and deterministic options in the alternatives is a function of the amount of effort the assessor and DM are willing to invest, similarly to any utility assessment procedure (Thurston, 2001). Our approach guarantees that the DT assessment will improve with an increasing number of asked questions. The assessor must ensure that the questions are different from each other so all regions within the ranges of negotiability are properly modeled.

**Table 1.** Sample ranking questions to assess decision topologies directly from DM responses

Rank alternatives A-E in the order of your preference:	
A	$\mathbf{y}_A = \{y_1^A, \dots, y_n^A\}$
B	20% chance of $\mathbf{y}_{BP} = \{y_1^{BP}, \dots, y_n^{BP}\}$ and 80% chance of $\mathbf{y}_{BQ} = \{y_1^{BQ}, \dots, y_n^{BQ}\}$
C	$\mathbf{y}_C = \{y_1^C, \dots, y_n^C\}$
D	$\mathbf{y}_D = \{y_1^D, \dots, y_n^D\}$ where $y_1^D$ 's are uniformly distributed between $\{y_1^{D\min}, \dots, y_n^{D\min}\}$ and standard deviation $\{y_1^{D\max}, \dots, y_n^{D\max}\}$
E	$\mathbf{y}_E = \{y_1^E, \dots, y_n^E\}$

## Method 3: Decision maker provides single attribute utility functions or decision topologies (SADTs)

In general, the assessment of single attribute utility functions is much easier compared to multiattribute utility functions. Using Method 2, single attribute utility functions can be used directly to obtain SADTs.

We now address the issue of assessing multiattribute decision topologies (MADTs) from SADTs. It is well known that the multilinear expression of Keeney and Raiffa (1994) requires that the attributes are preferentially and utility independent. These conditions can somewhat be relaxed so that other

functional forms can be used to obtain MAUFs from SAUFs directly, as shown in Abbas (2009).

Abbas showed that utility copulas can be used to model MAUFs as joint distributions. Functionally, many utility functions have some of the same properties as joint CDFs (constrained between 0 and 1, and increasing in all attributes). Therefore copulas can be used to model utility functions just as they are used to model CDFs. The SAUFs can be also combined using a *utility copula* to get a multiattribute utility copula (which is technically a MAUF). Certain functional forms where the mixed derivative of the MAUF is negative - not true for probability distributions - can also be modeled using utility copulas as Abbas (2009) has shown. In addition, the requirement that probability copulas are zero when one of the variables is at its lowest level is not an impediment because there are utility copulas that do not need to satisfy the grounding condition (see Abbas, 2009). SADTs can be thus used to generate MADTs. As a result, when SADTs are available, one has access to the proper discretization of the concerned attributes. Recall that these discretization points define the blocks used in DTs. To assess MADTs, we can simply use the blocks identified in the SADTs without having to obtain the “right” discretization. The case study of Section 4 demonstrates this method.

#### Method 4: Single attribute decision topologies (SADTs) are needed when a MADT or MAUF is available

If preferential and utility independence conditions are satisfied we can obtain MAUFs from SAUFs. The inverse problem is also encountered when one or more attributes must be removed from the attribute set. For the case of maximizing utility functions over attributes, we define a SAUF as

$$U_j(x_j) = \frac{\lim_{x_i \rightarrow \infty, i \neq j} U_M(x_1, \dots, x_n)}{\lim_{x_i \rightarrow \infty} U_M(x_1, \dots, x_n)}. \quad (6)$$

This can be achieved in a topology by simply removing the blocks for other attributes and connecting the ends together. The attribute discretization does not affect this method.

It is of particular interest to accurately assess DTs if the DM provides responses using a combination of the above techniques. It is preferred to use multiple methods in order to avoid biases and modeling errors creeping into an assessment. We can use the utility functions if available (method 2), to get a ranking of alternatives. These rankings can then be combined with raw rankings if available. If there are discrepancies, the DM can be made aware of them. If single attribute utility functions are available, we can use them to get

the correct partitioning of each attribute as shown before. ESTA can then be used to get DTs using the available rankings and partitioning.

## 4. CASE STUDY

In this section, we apply the four DT assessment methods of Section 3 using a modified welded beam design example (Deb, Pratap and Moitra, 2000) where the bound constraints  $g_5(\mathbf{x})$  and  $g_6(\mathbf{x})$  are added as shown in the Appendix.

The problem involves minimization of the cost ( $C$  in dollars) and the deflection ( $D$  in inches) of the weld. Although our focus is not on the optimization part, we use the information from the generated Pareto front that the cost ( $C$ ) and deflection ( $D$ ) attributes can be feasibly realized in the ranges of  $[10, 260]$  and  $[0.001, 0.05]$ , respectively. In this example, we are only concerned with finding a topology consistent with the utility function.

We first define an exponential utility function over each attribute individually as

$$U_C(C) = \left(1 - e^{-\frac{260-C}{100}}\right) / \left(1 - e^{-\frac{250}{100}}\right) \quad (7)$$

$$U_D(D) = \left(1 - e^{-\frac{0.05-D}{0.02}}\right) / \left(1 - e^{-\frac{0.049}{0.02}}\right) \quad (8)$$

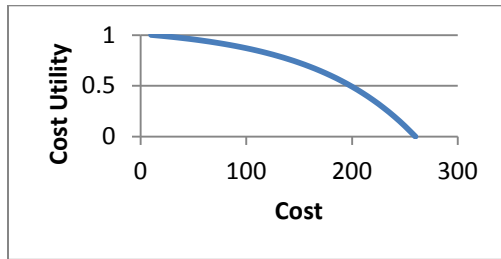
and combine the above utilities into a multiattribute function using the following multilinear form

$$U(U_C, U_D) = 0.7U_C + 0.5U_D - 0.2U_CU_D. \quad (9)$$

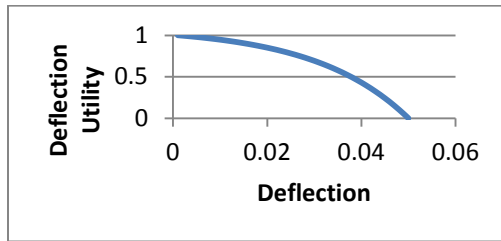
We assume that these utility functions are unknown to the DT assessor even though they model the decision maker's preferences correctly. Notice that the  $U$  of Eq. (9) does not satisfy the grounding condition; i.e., it is not equal to zero when one of the attributes is at its lowest possible level. In addition, it exhibits a negative mixed derivative and a direct comparison with probability copulas is therefore, not correct. Utility copulas must be used instead. Our results show that a topology can be still approximated well using only a limited number of DM responses.

We generate survey questions using the utility functions of Eqs (7) and (8). However, using the proof discussed earlier in Section 2, we obtain the SADTs directly from the derivative characteristics of the two utility functions plotted in Fig. 3. For the SADTs to work, we must only partition the domain proportionally to the derivative of the utility functions. Table 2 shows this partitioning. We observe that more points are placed in regions where the function has a higher slope.

As such, this case study utilizes a combination of the four proposed methods of Section 3; i.e, single attribute utility functions (Methods 3 and 4) and provided rankings from utility functions (Methods 1 and 2).



(a)



(b)

**Figure 3.** Utility functions associated with the two objectives. Note that the utilities are decreasing with attributes. This does not affect the applicability of the method

Table 3 shows the training set. The DM is asked to rank the alternatives in the order of desirability. This step simply utilizes the multiattribute utility function of Eq. (9). For the case of uncertain alternatives, an expected criterion is used. Some of the alternatives are probabilistic as shown in the table. The total number of alternatives is significantly less than that used in our previous work (Pandey and Mourelatos, 2013). This is possible because we have already extracted useful derivative information from the SAUFs (Fig. 3).

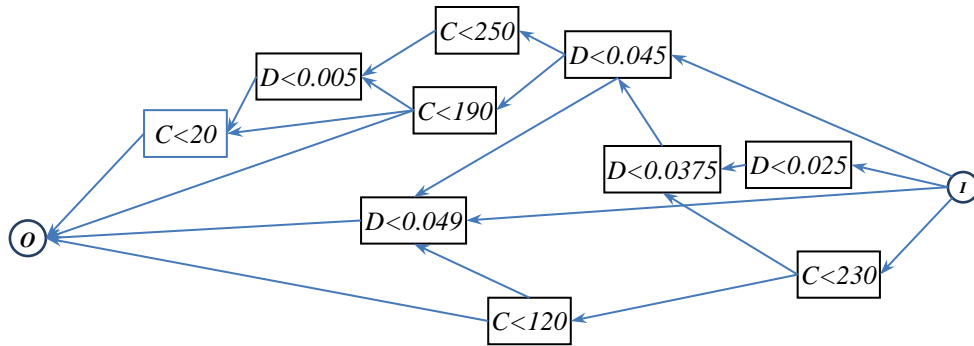
**Table 2.** Partitioning of the ranges of negotiability of the two attributes using the derivative information of Fig. 3

Cost (\$)	Deflection (in)
20	0.005
120	0.025
190	0.038
230	0.045
250	0.049

**Table 3.** Ranking provided by the DM for the 15 alternatives

	No.		Cost (\$)	Deflection (in)		Cost (\$)	Deflection (in)	Ranking provided
<b>Deterministic outcomes</b>	1		234.71	0.0088				9
	2		218.69	0.0446				13
	3		247.67	0.0486				15
	4		226.93	0.0468				14
	5		158.32	0.0149				3
	6		148.45	0.0138				2
	7		204.12	0.0024				7
	8		76.60	0.0208				1
	9		235.91	0.0144				10
	10		124.81	0.0316				5
<b>Probabilistic outcomes</b>		<b>Probability</b>	<b>Outcome 1</b>		<b>Probability</b>	<b>Outcome 2</b>		
	11	0.15	86.25	0.0488	0.85	193.03	0.0066	6
	12	0.65	238.29	0.0218	0.35	109.56	0.0484	11
	13	0.2	157.75	0.0092	0.8	137.72	0.0439	8
	14	0.8	228.69	0.0218	0.2	174.44	0.0473	12
	15	0.3	139.40	0.0407	0.7	99.52	0.0261	4





**Figure 4.** MADT generated using the responses of decision maker

Using the ranking information of Table 3, the ESTA algorithm was run to create a topology that provides the same rankings (or close to it) as in Table 3. ESTA maximizes the Spearman's rank correlation between the rankings of Table 3 and the scores from a candidate topology. The evolutionary part of ESTA was run with a population size of 350 and a probability of mutation of 0.1. The total run-time on an Intel dual-core machine was less than a minute. Fig. 4 shows the topology generated by ESTA. The topology provides a Spearman's rank correlation of 0.97, which is excellent.

Table 4, shows the adjacency matrix,  $\mathbf{R}$ , associated with the topology of Fig. 4. The above topology can also be evaluated visually or using a linear algebra method (Pandey et al. 2012). The size of matrix  $\mathbf{R}$  is  $(n+2) \times (n+2)$ . It has an entry of 1 if there is an arrow from the block corresponding to the column to the block corresponding to the row, and 0 otherwise. If certain blocks are not active, an updated matrix  $\mathbf{R}'$  is obtained by deleting the corresponding columns. The score of the topology considering all active and inactive blocks, is the  $(1, n+2)^{\text{th}}$  entry of the  $(\mathbf{I}-\mathbf{R})^{-1}$  matrix. The proof of this method is provided in (Pandey et al., 2012).

**Table 4.** Adjacency matrix  $\mathbf{R}$  for the topology of Fig. 4

		Cost (\$)						Deflection (in)					I
		O	<20	<120	<190	<230	<250	<0.005	<0.025	<0.0375	<0.045	<0.049	
Cost (\$)	O	0	1	1	1	0	0	0	0	0	0	1	0
	<20	0	0	0	1	0	0	1	0	0	0	0	0
	<120	0	0	0	0	1	0	0	0	0	0	0	0
	<190	0	0	0	0	0	0	0	0	0	1	0	0
	<230	0	0	0	0	0	0	0	0	0	0	0	1
	<250	0	0	0	0	0	0	0	0	0	1	0	0
Deflection (in)	<0.005	0	0	0	1	0	1	0	0	0	0	0	0
	<0.025	0	0	0	0	0	0	0	0	0	0	0	1
	<0.0375	0	0	0	0	1	0	0	1	0	0	0	0
	<0.045	0	0	0	0	0	0	0	0	1	0	0	1
	<0.049	0	0	1	0	0	0	0	0	0	0	0	1
	I	0	0	0	0	0	0	0	0	0	0	0	0

Table 5 shows the score (column 2) and the ranking (column 3) provided by the best MADT found by ESTA (Fig. 4). The rankings have a very high rank correlation coefficient of 0.97 with the rankings provided by the utility function (column 4). This high correlation guarantees that the decisions made using the MADT, even under uncertainty, will be the same with those made by the decision maker.

**Table 5.** Ranking comparison between the MADT score and the DM score

Alternative	MADT score	MADT rank	DM rank
1	3	12	9
2	2	13	13
3	1	15	15
4	1	14	14
5	7	3	3
6	7	2	2
7	4	6	7
8	9	1	1
9	3	11	10
10	5	5	5
11	3.85	7	6
12	3	10	11
13	3.8	8	8
14	3.4	9	12
15	5.8	4	4
Spearman's rank correlation		0.97	

## 5. DISCUSSION

In this paper, we proposed four practical methods to assess multiattribute decision topologies (MADTs) which are then used to make design decisions. Our recent work has shown that MADTs can be visual and theoretically sound replacements of utility functions. This paper addressed a challenge encountered in any decision making method where the preferences of the DM must be assessed. We showed that the MADTs can be directly assessed using rankings provided by the decision maker using the ESTA evolutionary method we have previously proposed.

Our approach works whether we use rankings provided by the decision maker, their multiattribute utility function, or a combination of single attribute

utility functions or topologies. Even lottery responses can be easily incorporated in MADTs thereby allowing us to model preferences under uncertainty. The theoretical basis behind the assessment methods we proposed was also discussed.

The methods were used on the design of a welded beam. The decision topology utilized only 15 alternatives (5 of them uncertain). The Spearman's rank correlation between the ranking provided by the DM and the ranking from the topology was found to be very high. This is evidence of the validity of both the proposed DTs and the DT approximation algorithms. Future work will focus on applications of the algorithms in design decision making problems where a visual method will be of value.

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## APPENDIX

The mathematical formulation of the welded beam example is given below.

$$\text{Minimize } \begin{cases} C(\mathbf{x}) = 1.10471h^2l + 0.04811tb(14+l) \\ D(\mathbf{x}) = \frac{2.1952}{t^3b} \end{cases}$$

subject to:  $g_i(\mathbf{x}) \leq 0, i = 1, \dots, 5$

where:  $\mathbf{x} = (h, l, t, b)^T$ , and

$$g_1(\mathbf{x}) = \tau(\mathbf{x}) - 13600 \quad g_2(\mathbf{x}) = \sigma(\mathbf{x}) - 30000 \quad g_3(\mathbf{x}) = h - b$$

$$g_4(\mathbf{x}) = 6000 - P_c(\mathbf{x}), \quad g_5(\mathbf{x}) = \mathbf{x} - (10, 10, 10, 5)^T \text{ and } g_6(\mathbf{x}) = (0.125, 0.1, 0, 0)^T - \mathbf{x}$$

In the above expressions:

$$\tau(\mathbf{x}) = \sqrt{\tau_1^2 + \tau_2^2 + \frac{l\tau_1\tau_2}{\sqrt{0.25(l^2 + (h+t)^2)}}} \quad \text{and} \quad \tau_1 = \frac{6000}{\sqrt{2hl}}.$$