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PROJECT RAND  
RESEARCH MEMORANDUM

SUMMARY OF REAC EXPERIENCE

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Assigned to \_\_\_\_\_

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## SUMMARY OF REAC EXPERIENCE

W. F. Gunning - A. S. Mengel

INTRODUCTION

This memorandum summarizes some aspects of our experience with the Reeves Electronic Analogue Computer (REAC) installation at RAND. A rather thorough knowledge of both the engineering details and mathematical principles of a standard REAC is assumed.<sup>(1)</sup> Part I consists of an outline of the modifications made both by Reeves and by RAND. The emphasis is on engineering details. Part II contains examples of generalized operational techniques of an unusual nature, combined with a scrutiny of several problems solved at RAND. Part III is a discussion of personnel requirements, and calibration and maintenance procedures.

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(1) REAC Brochure and Instruction Manual, Reeves Inst. Co.

## PART I

### THE MODIFICATION PROGRAM OF THE RAND REAC

The many changes that have been made in the basic REAC will be discussed under subheadings of the component modified. We begin with:

#### (1a) INTEGRATOR SCALE FACTOR:

The standard REAC integrators have scale factor choices of 1, 4, and 10 volts per second output per volt input. A preliminary consideration of problem types to be solved at RAND indicated that a range of scale factors allowing extended solution times would be useful for three reasons: first, because manual tracking of arbitrary functions from input tables was to be used; second, the accuracy of high speed solutions is curtailed by the frequency component limitations of the multiplying servos and demodulator filters; third, it was felt that improved performance with respect to integrator drift could be achieved. Consequently, Reeves was given an order to construct a special REAC in which the integrator input scale factors were made 1, 0.25, and 0.1 by an increase in the value of the input resistors. Additional reduction of integrator scale factor is obtained by the use of a bank of about 190 microfarads of auxiliary integrating condensers available on the patch bay. The condensers were individually calibrated by a comparison method described in Section (10b) and connected in groups accurately totaling 9 microfarads and 10 microfarads. When two groups of condensers totaling, say 29 microfarads, are patched across an integrator, the scale factors available become  $1/30$ ,  $1/120$ ,  $1/300$  volt per second output per volt input. This arrangement allows the scale factors on integrator inputs to be made large on problems requiring long solution times.

The addition of extra feedback condensers, rather than further extension of the value of input resistance, is preferred when the integrator scale factor is to be reduced, since this allows the signal current fed into the summing junction at the amplifier grid to be maintained at a reasonable value with very small integrator scale

factors. The deleterious effects of leakage and grid currents need not be increased when long solution times must be used. The self time constant of a large bank of condensers is a limitation to this method of extending time scale. If a condenser is considered as a three terminal device with the grounded case as the third terminal, it is only the "direct" resistance from terminal to terminal that is significant in reducing the self time constant. Leakages to ground are relatively unimportant since (a) on the grid side, the potential difference and, consequently, the error current are extremely small (b) on the output side, leakage resistance is effectively in parallel with the low output impedance of the amplifier. Leakage between the jacks on the patch bay is, by far, the most significant component of direct leakage resistance - measuring, in some cases, to as low as  $2 \times 10^{10}$  ohms. It is hoped that error from this source can be substantially reduced by the use of polystyrene insulating materials in the plugboard system of problem patching (Section 1d).

The importance of the function of extra integrating condensers as a means for effectively reducing integrator drift has been reduced by the development of the chopper type drift reduction amplifier.

#### (1b) CONTACT MODULATED AMPLIFIER FOR REDUCTION OF DRIFT

The use of contact modulators (also called vibrators, converters, choppers) as a means to stabilize d-c amplifiers for such applications as thermocouple potential recorders, etc. is well known.<sup>(2)</sup> The ingenious application of this technique to stabilize a wide band feedback d-c amplifier is believed to have been first used by A. W. Vance in connection with Project Typhoon.<sup>(3)</sup> The particular configuration used at RAND (Fig. 1) is only slightly different from the circuit published by RCA.<sup>(3)</sup> The chief deviations are (a) the use of a Leeds and Northrup vibrator (catalogue No. 3338-special) which has five available contacts and very good electrostatic and magnetic shielding of the driving coil and (b) the addition of an error indicator circuit, originally suggested to us by Raleigh McCoy of Reeves.

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(2) Liston et al Rev. Sci. Inst. p 194, May 1946.

(3) RCA Report "Determination of Minimal Component Accuracy for Simulator Systems" Robert Serrell - Feb. 1948, Contract No. N6-Ori-196.

The chopper amplifier proper,  $V_1$  and  $V_{2a}$ , Fig. 1, produces an overall d-c gain of about 2000. As used with a computing amplifier, the equivalent input d-c noise voltage is about 50 to 100 microvolts. For short circuit input to the chopper amplifier, the noise drops to between 10 and 30 microvolts. Very careful shielding is required of components and leads connected with the input circuit. Dielectric absorption in condensers  $C_1$  and  $C_2$  proved to be troublesome until "Glassmike"<sup>(4)</sup> condensers were installed. Low plate voltage on  $V_{2a}$  limits the signal fed to the vibrator on severe overloads. The vibrator contact arrangement allows a phase sensitive peak detection scheme to be used which reduces the detector ripple filtering problem.

The error indication system causes an appropriate indicator lamp to glow whenever the absolute magnitude of the error voltage between the signal current summing junction and ground exceeds about 1 millivolt. The original REAC balancing system indicator light positions are used. Audible indication (of restrained volume) that one or more amplifiers are in error is provided. The circuitry is associated with tubes  $V_{2b}$ ,  $V_3$ ,  $V_4$ ,  $V_5$ , of Fig. 1. It is important to note that the system just described is superior operationally since it will indicate the presence of an error per se and not merely that the output of an amplifier has exceeded some absolute value. For example, linearity errors due to absence of a boost resistor in an amplifier will not cause the original error system to indicate. Further, the amplifiers are capable of providing output signals considerably in excess of the arbitrarily chosen  $\pm 100$  volt limit without nonlinearity or error.

The REAC, as received, was wired in such a manner that circulating ground currents produced errors of several millivolts. It was necessary to remove all d-c power grounds from the individual computer chassis. This change has been incorporated by Reeves in later production runs.

When the Vance drift correction system is used, the recovery characteristics of the amplifier after a severe overload are very poor in that the chopper amplifier output filter,  $R_{10}$ ,  $R_{11}$ ,  $C_7$ , Fig. 1, and the main amplifier input coupling condenser circuits,  $C_1R_1$ , Fig. 2, have extremely long time constants. Slow recovery of an amplifier following

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(4) Condenser Products Co., Chicago, Ill.

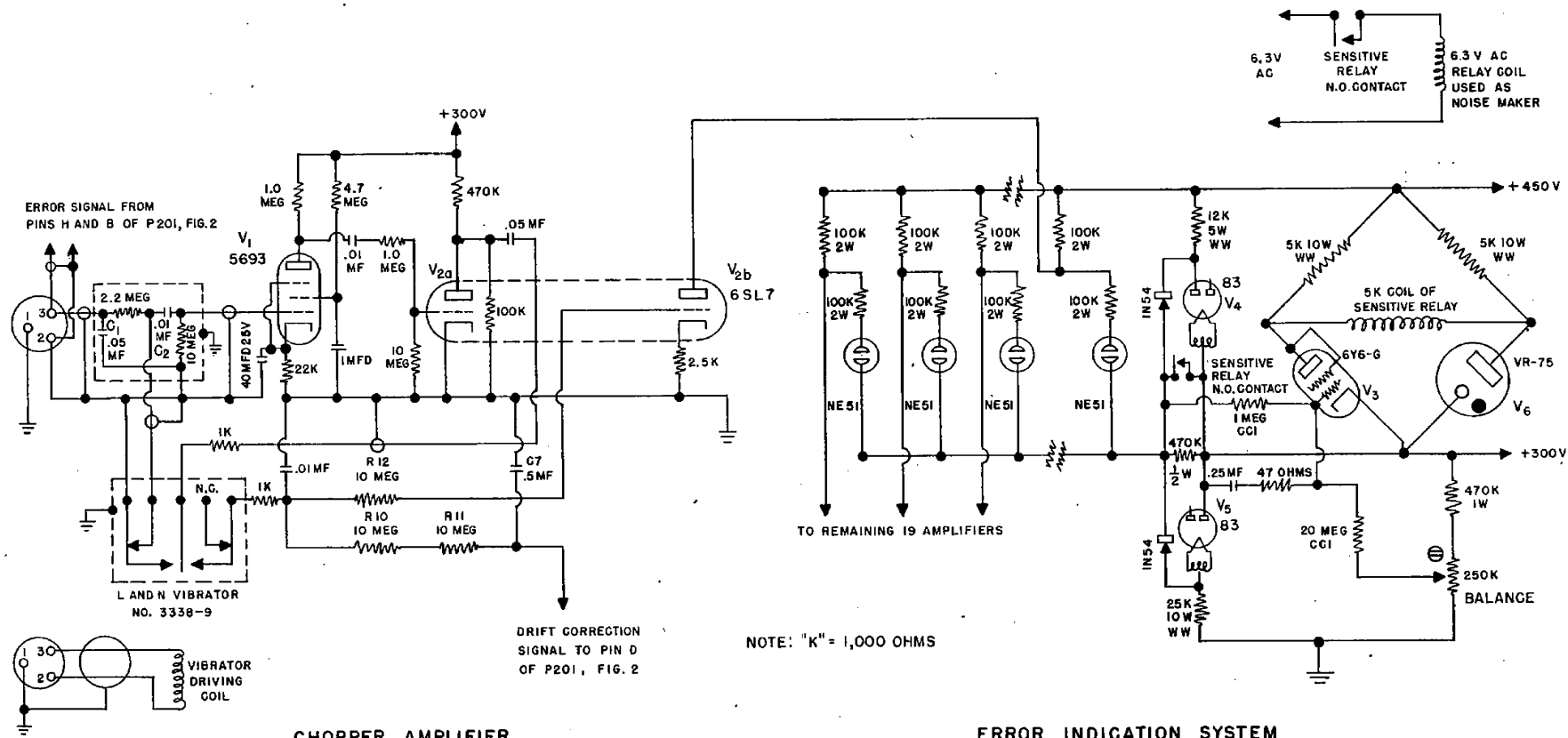


FIG. 1

an overload is never pleasant, but this disagreeable behavior is reluctantly accepted in this case as a reasonable price to pay for the improved computing performance that the system provides. There are useful operating techniques and certain problems (Section 4c) that force a computing amplifier to be overloaded in the sense that the "error voltage" at the input grid is large. A "Manual-Automatic" function switch is provided on each amplifier ( $SW_1$ , Fig. 2) to allow the circuit to be restored to a conventional d-c amplifier when quick recovery following overload is essential.

Figure 2 shows the circuit values used in the modified computer amplifiers. The high frequency cut off shaping has been revised to insure stability when the complete feedback loop is closed. The overall open-circuit gain of an amplifier with a normal load of 20,000 ohms is approximately  $6 \times 10^7$ . The input "coupling" condenser  $C_1$  in Fig. 2 is critical as to leakage and dielectric absorption for a period following overload. Such a coupling network was not used in the original RCA report.<sup>(3)</sup> It was independently developed at RAND and other places and is now in general use. The function of the condenser is to reduce errors caused by grid current of tube  $V_{1a}$ , Figure 2. This current, typically about  $10^{-9}$  amps. for the Reeves amplifiers, flows to ground through  $R_1$  rather than into the computing current summing junction as before.

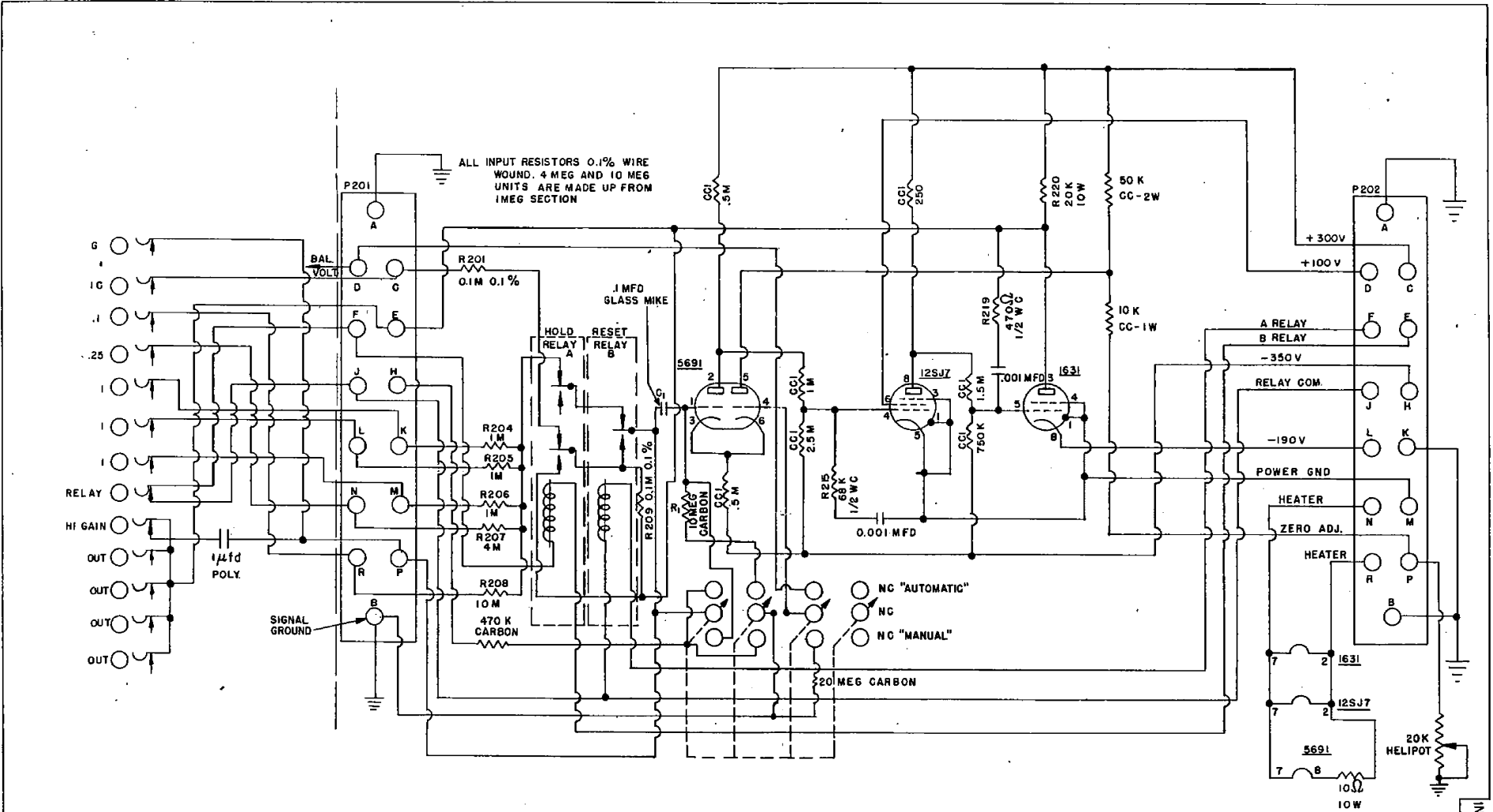
The d-c signal path is through the chopper amplifier and into the grid of  $V_{1b}$ , Figure 2. Frequency components of the signal too high to be passed through the chopper amplifier go directly to the grid of  $V_{1a}$  via  $R_1C_1$ .

The RAND version of the chopper amplifier (Fig. 3) is packaged two to a chassis unit. Ten of the units plug into a special rack which occupies the space in the lower part of the computer cabinet vacated by the original step-by-step balancing mechanism.

### (1c) FUNCTIONAL MODIFICATION OF COMPUTING AMPLIFIERS

Figure 2 shows the revised wiring of the jacks in a REAC patch bay to allow the local feedback path of an integrator to be broken. Use of the operational flexibility provided by this change will be discussed in detail in Section (4a). Briefly, (a) implicit functional relations can be directly solved and (b) certain other functions such as automatic hold and accurate limiting can be performed.





NOTE: RELAY COILS 1300Ω, 20 Ma @ 26V  
 CGI CONTINENTAL CARBON  
 NOBLELOY X RESISTOR  
 R201 THROUGH R209 ARE 0.1% WW  
 ALL OTHERS NOT NOTED ARE 1% TEMP  
 COMPENSATED

REVISED INTEGRATOR  
 SCHEMATIC DIAGRAM

FIG. 2

COMPUTER CAB  
 INTEGRATING AMPLIFIER

Since this arrangement allows integrating amplifiers to be converted into summing amplifiers, it is also necessary to provide means for modifying the control relay functions from the patch bay. For example, when an integrating amplifier is made to sum by substitution of one of its input resistors for the feedback condenser, it is also desirable, in most cases, to prevent the hold relay from opening the amplifier input when the main control switch is in hold position. This can be accomplished by inserting an unwired plug in the jack marked "relay" in Figure 2. Conversely, when an inverting amplifier is used as an integrator, the hold relay function must be added. This is also accomplished by proper patching on the front panel. An additional jack has been wired to allow remote operation of all hold relays.

In connection with the operation of control relays, it was found necessary to equalize the drop out times of the integrator "reset" relays by adjusting the spring tension. The magnitude of the error produced by starting the integrations of different variables at different times is insignificant if the solution time is greater than, say, 20 seconds.

The two sets of contacts and the operating coils of each of two extra relays have been wired to extra jacks on the patch bay. These relays are used in conjunction with high gain amplifiers (i.e. an amplifier without feedback) to provide accurately controlled discontinuous operations. Operational details are in Section (4c).

The use of differentiating circuits in an analogue computer is usually impractical. This function is obtained by patching the desired value of capacity to the grid of an amplifier which has resistance feedback. In the REAC, stray 60 cycle hum voltages and noise generated by servo and functional potentiometer granularity are spurious signals which lie in a part of the frequency spectrum which is usually of no interest. The gain of a differentiator amplifier, which must increase linearly with frequency, is high in the region occupied by these noises. In spite of this serious shortcoming, there are rare occasions when the differentiator can be used - e.g. in problems requiring integration with respect to a variable other than time when the time derivative of that variable was not already present at some point in the machine (see Section 5b).

(1d) PLUGBOARD

Our experience with the operation of the REAC indicates that utilization of the machine could be very significantly increased by a drastic revision of the patch bay. We propose to install a separable plugboard which is a slight modification of the device developed by International Business Machines Corporation. All of the circuits on the present patch bay are to be wired to a single receptacle or sub-panel. Several separable plugboards are to be available for the interconnection of circuits representing a particular problem. Such a board can be prepared completely independently of the computing machine. Further, after completion of the wiring, the board may be turned upside down to allow a buzzer or ohmmeter to be used to verify the accuracy of the wiring and the electrical continuity of the patch cords before entering the machine room.

After a problem plugboard is wired and checked in this manner, it can be connected to the machine in a matter of seconds. If improper behavior of the computer is suspected, the plugboard containing the problem can be removed and a special permanently wired checking board inserted in its place. If mathematical difficulties arise or if subsequent parameter choices depend on analysis of the results to date, the problem can be removed immediately and another one started. As presently operated, problems have been left on the machine for hours or days while the machine stood idle since tearing out, replacing, and checking the wiring is so time consuming. It is felt that multiple shift operation of a REAC installation is practical only with such a change in problem wiring facilities.

A layout of REAC functions on a standard I.B.M. board <sup>(5)</sup> has been made and typical problems wired. Figure 4 is a direct comparison of patching the first problem listed in Section 8a into the REAC and into a plugboard.

The materials used by I.B.M. in fabrication of their boards and patch wires are not satisfactory for use with a REAC. A sample plugboard of polystyrene has been constructed and tested and is con-

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(5) The plugboard for the Type 405 Tabulator was used for trials. A paper mask covered the original labels and allowed the REAC components to be identified.

sidered to be satisfactory for REAC application. Both the stationary sub-panel and the plugboards are to be fabricated from this material. The lead wires and plugs will be insulated with polyethylene or some similar material.

#### (1e) MISCELLANEOUS MODIFICATIONS

The operational procedure of using a servo as an accurate voltmeter to verify the setting of initial conditions and scale factor potentiometers and for reading results is simplified by the following change. The arm of the amplifier selector switch for the test meter has been wired to a jack. A servo patched into this jack can be switched to the output of any of the computing amplifiers without further patching.

It has been necessary to increase the number of multiples by a factor of 4; the number of patch cords by a factor of 3; and the number of boost circuits by a factor of 2.

The ambient air temperature in the REAC computer room has been lowered approximately 10°F. by a computer cabinet exhaust system. Individual ducts from the top of the REAC cabinets and sides, adjacent to the servo amplifiers, were connected to the main exhaust air duct of the room's ventilating system.

#### (2a) SERVO AMPLIFIER INPUT

The servo amplifiers have been modified to the circuit of Figure 5. The input transformer has been replaced by a resistor - condenser network. The servos are normally used as summing amplifiers. The indication on the servo dial is the negative of the sum of the input signals except for the effect of the load of a 1 megohm feedback resistor on the follow-up potentiometer. In order to produce a voltage at the follow-up potentiometer arm which is equal in magnitude but opposite in sign to the sum of the input signals, the servo must turn "too far". That is, the dial will indicate a value slightly larger in magnitude than the correct sum. The correction is a maximum of about 0.13 per cent when a center-tapped follow-up potentiometer is used. This mode of operation will produce a correct product provided the multiplying potentiometers are also loaded with the customary 1 megohm since all potentiometers will be loaded equally.

The switch SW<sub>1</sub> allows the circuit to be restored to a positive gain servo which functions in a manner similar to that of the original design. In this position, the equivalent loading impedance seen by the source driving the servo at In-1 is very high. Similarly, the follow-up potentiometer is effectively unloaded.

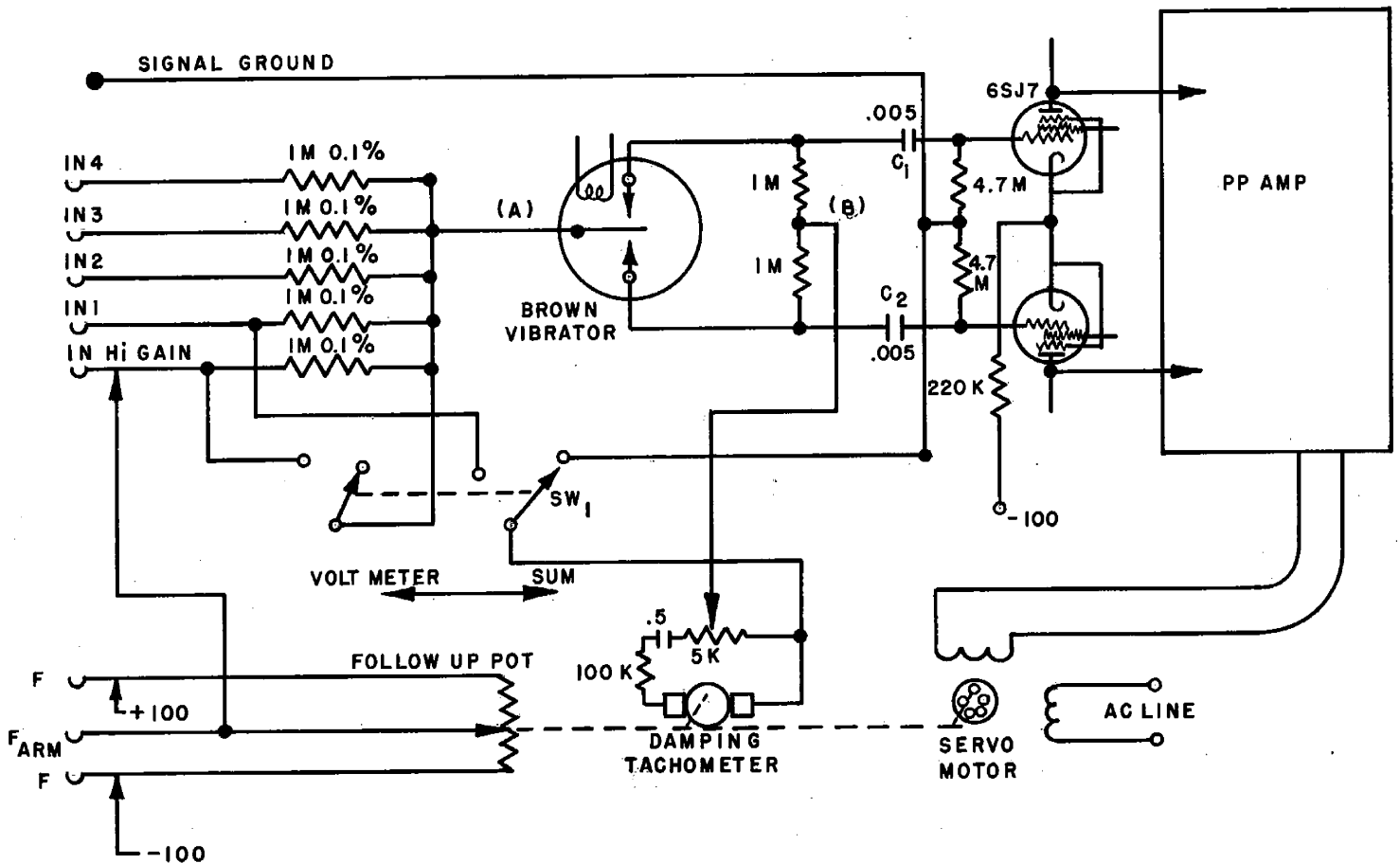


Figure 5 - Simplified Schematic Diagram of Summing Servo Amplifier

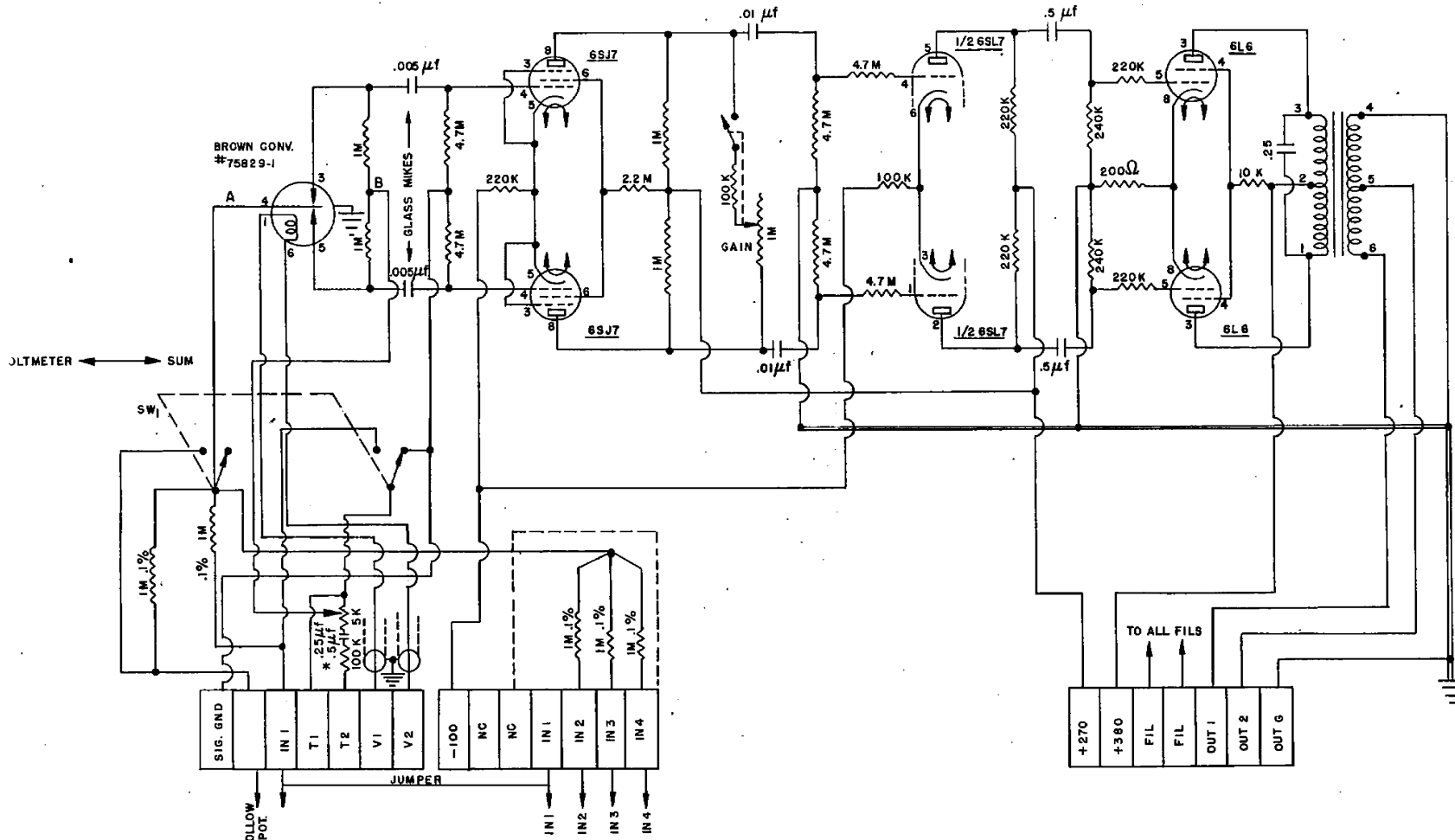
Consequently, as in the original design, the dial indication is as accurate as the follow-up potentiometer since the gear trains appear to be very well made. The off-balance input impedance of the revised servo is of the order of 1 megohm in either "Voltmeter" or "Summing" position. This allows the servo to be switched from point to point during a solution without appreciably affecting the signals to be observed.

The complete schematic diagram of the revised servo amplifier as presently used is shown in Figure 6. On "Voltmeter" position, the point B is at input signal potential rather than near ground and consequently the condensers  $C_1$ ,  $C_2$ , Figure 6, must be of high quality to prevent errors due to leakage. The large common cathode resistors in the first two stages allow the amplifier to operate without blocking in the presence of large rates of change of signal at the point B. Stated differently, the in-phase gain is low but the push-pull gain is high.

As supplied, all three of the Brown vibrator contacts bridge for an appreciable part of the cycle. Adjustment screws are provided inside the shield can which allow the timing to be changed. The performance of either the original or revised circuit is improved by backing off the stationary contacts until the bridging is just eliminated. Care should be taken to retain a symmetric wave form. In our experience, the phase stability of the vibrators is better when the driving voltage is 6 to 7 volts. The center tap of the source of driving voltage should be grounded. When this is done, the residual noise at the amplifier input is equivalent to a signal of about 1/2 millivolt. This could be reduced by feeding the vibrator driving coil voltage in at the top of the shield can in a manner similar to that used by Leeds and Northrup.

#### (2b) SERVO POTENTIOMETERS

In the manufacture of ten turn precision potentiometers (Helipot, Micropot, Multipot) it appears to be difficult to hold the electrical length of the resistance winding to exactly 10 turns. Often, there exists a small length of slide wire at an end of the winding which is beyond the limit of travel of the slider. This is called the end coil resistance. In a REAC servo, the dial indicates the actual angular rotation of the shaft. If the follow-up potentiometer winding is linear but has end coil resistance, the servo will indicate a value which is too high. The fractional error is approximately the ratio: end coil resistance / 1/2 total resistance. Alternately, consider the case where the servo dial is set to, say, 0.970, and an A potentiometer has end coil resistance. Then, if +x and -x volts are patched to High A and Low A, the voltage at the arm will be less than 0.970 x by the above ratio. In our case, the follow-up potentiometers were all acceptable



\* .5 μf FOR SERVO AMPS.  
 \* .25 μf FOR MODULATOR AMPS.

FIG. 6

SERVO CABINET  
 SERVO AMPLIFIERS

with respect to end coil resistance but the A and B potentiometers were not by 0.8 per cent in the worst case.<sup>(6)</sup> This situation presents the irritating dilemma, (a) leave the multiplying potentiometers alone and obtain as a reward a correct servo dial indication, but as a penalty, an incorrect product; or (b) add padding resistors to make the effective end coil resistance of all potentiometers equal to the worst one (which will have no padding resistors). The latter procedure will allow correct products to be formed but will make all servo dial indications incorrect. The computer was operated in condition (b) for about 10 months. The Helipot Corporation was given an order for a set of special potentiometers with electrical length equal to  $3600^\circ$  (plus  $0^\circ$ , minus  $20^\circ$ ). It was then possible to pad the short windings to an equivalent length of  $3600^\circ$  such that both correct indication and correct computing could be obtained. The mechanical stops on the servos prevent the slider from leaving the winding at the extreme ends of travel.

The center taps of the servo potentiometers allow the accuracy to be improved since the end coil padding resistors may be chosen independently. There are other operational advantages mentioned in Section (4b).

#### (2c) TRIGONOMETRIC EQUIPMENT

The accuracy of one of the Arma resolvers has been checked and found to be within the maker's guarantee of  $\pm 0.1^\circ$ . This is a hollow victory since the associated modulator and demodulator equipment will not match this figure. The overall performance of the trigonometric equipment is marginal with respect to accuracy (errors of one per cent of full scale are common) and speed of response (delays of the order of one second have been observed).

The servo amplifiers used to drive the modulator servo motors have been converted to the circuit of Figure 6. Further, a gear train and extra potentiometers have been added to allow the modulators to be used as multiplying servos. The space limitations require that one inch diameter ten-turn center-tapped potentiometers be used.<sup>(7)</sup> Five

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<sup>(6)</sup> The dual unit Helipots were particularly bad.

<sup>(7)</sup> Ford Engineering Co., 2738 1/2 Colorado Blvd., Pasadena, Calif.



extra potentiometers can be added if required. Figure 7 is a photograph of the modified modulator assembly.

The output of the filtered a-c power supply used to energize the modulator potentiometers is affected by variations in temperature, load impedance, and line voltage. These variations are believed to be responsible for a large portion of the system errors. Methods for stabilizing the output have been considered but not put into practice due to the possibility of a major system revision to remove the demodulator filter lags. Any such regulating system should stabilize the a-c voltage with respect to the  $\pm 100$  volt supply used in the rest of the computer.

The demodulator units evidenced a reduction of gain (a-c input to d-c output) of about one per cent when the d-c output exceeded approximately 60 volts. This error appeared to be produced by (a) parasitic oscillations in the 6SN7 phase detector and (b) overload of the second amplifier stage. The oscillations can be removed by adding a small condenser across the grounded winding of the switching transformer. Point (b) introduces the demodulator filter problem since the amplifier overload can be removed by an increase in filter input impedance. The present filter produces excessive time lags. A revision of the filter design and perhaps, an increase in the operating frequency of the entire trigonometric system may be required. Operationally, the use of the resolver equipment has been avoided by various methods to be discussed in Part II. It is hoped that a satisfactory modification of the design can be worked out since the substitute methods use more equipment.

#### (2d) MISCELLANEOUS MODIFICATIONS

In order to accommodate the extra input connections to the servo cabinets, it has been necessary to replace the original 24 position jack strips with 48 position strips. An additional 48 jacks have been added for additional multiples.

It is standard practice to use a servo to verify the setting of scale factor and initial condition potentiometers. This technique will compensate for any fixed differences between the  $\pm 100$  volt supplies in the computer and in the servo. However, variations of any such

difference will produce errors. Further, such differences in computing voltage make the direct use of the calibrated dials on the scale factor potentiometers awkward. As received, the servo and computer  $\pm 100$  volt power supplies were regulated against separate battery standards and were located in separate power supply cabinets. The relative stability was improved by a factor of five by (a) regulating the computer  $\pm 100$  volt supply with respect to the servo  $\pm 100$  volt supply and (b) by relocating the resistors of the voltage comparison network of the regulators such that all resistors were at approximately the same temperature. A separate comparison meter is used to allow fine adjustment of the  $\pm 100$  volt regulators. Precise setting has been facilitated by the installation of Helipot in the regulator input circuits. The present drift is about  $\pm .05$  volts for a day's run.

#### (2e) PROPOSED MODIFICATIONS

A method for obtaining continuous variation of the rate of problem solution, discussed in Section (5b), requires a servo with a potentiometer for each integrator in the problem. The servo used in the original step-by-step automatic zero balancing unit is to be made available for this purpose.

Several problems have been solved on the REAC in which only end points were of interest such that the answers were taken directly from the servo dials. Usually these problems require a very large number of runs. It is proposed to add a commutator system to several of the servo trays which would allow the position of the servo to be recorded on standard I.B.M. cards. The type 517 Summary Punch seems to be well suited to this job. When coordinates along a trajectory are to be recorded, the hold switch would be energized at regular intervals long enough to allow the I.B.M. machine to operate. This process can be made completely automatic by use of the techniques discussed in Section (5c).

#### (3a) AUTOMATIC TRACKING OF ARBITRARY FUNCTIONS

The input tables supplied with our REAC are equipped with a simple direct displacement hand-wheel tracking system. These units were used successfully for many months as designed. During the last six

weeks we have been using a scheme of automatic tracking similar in principle to the arbitrary function potentiometers supplied by Reeves. The function to be tracked is plotted point by point on a standard 11 x 19 inch graph paper. A piece of about 0.015 inch diameter nichrome wire is first straightened by stretching it slightly beyond the yield point. It is then formed to pass through the plotted points of the function, and is held in position by small double needle point clamps (Fig. 8). This operation can replace the usual French curve fitting that is necessary when the function is to be drawn for manual tracking. Next, the wire is cemented to the graph paper using a solution of about equal parts of General Cement Co. No. 30-8 Radio Service Cement and No. 31-8 Radio Service Solvent. After about five minutes the clamps may be removed to allow the rest of the wire to be cemented down. The paper may be left flat during the drying time which is about one half hour. The top of the wire is then cleaned with crocus cloth to insure good contact with the linear slide wire which is mounted parallel to the axis of the input table drum, Figures 9a and 9b. The linear slide wires are 20,000 ohms  $\pm 1$  per cent total resistance, 10 inches  $\pm .02$  inches in length with a  $\pm 0.1$  per cent linearity tolerance. The replacement cost is about \$5.00.<sup>(7)</sup>

The slide wire mortality rate was initially high but the present technique of wired curve preparation and the reduction of slide wire pressure by the addition of counter-weights has extended the life span to perhaps 5000 operations.

Note that it is possible to have more than one wired curve on a given input table if intersection of the functions can be avoided. Obviously, the functions must be of the same independent variable. This procedure permits automatic tracking of a function of two independent variables by means of linear interpolation between parametric plots. More operational details are given in Section (4d). When several wires are present on the same piece of graph paper, it is necessary to use a thin rubber cushion over the input table drum to insure reliable contact of all function wires with the linear slide wire, Figures 9a and 9b. Satisfactory contact is obtained with a net downward force at the slide wire of 3 oz. per wired curve. This value depends somewhat on the thickness of rubber cushion used.

The overall accuracy of automatic tracking was checked by preparing a linear wired curve, and plotting the derived voltage on the output table. The error of the result was less than  $\pm 0.15$  per cent of full scale at all points.<sup>(8)</sup> Similar accuracy has been obtained when an original function was compared with a replica produced on the output table.

### (3b) OUTPUT TABLE REVISION

The lead screw mechanism used in the first input-output tables built by Reeves was not sufficiently accurate for the REAC applications at RAND.<sup>(9)</sup> Consequently, a redesign was undertaken<sup>(10)</sup> which included the following modifications, Figure 10:

(A) A guide rod parallel to the lead screw was installed to carry a ball bushing. The ball bushing sleeve supports the pen carriage and an arm extending down to the lead screw. The guide rod takes up all twisting moments such that the lead screw need only supply translational forces.

(B) A ball bearing anti-friction lead screw was installed in conjunction with a revised low inertia gear train. The full-scale travel time of the pen is now about three seconds.

(C) A mechanism was installed which lifts the pen from the paper whenever the REAC control switch is on any position except "operate" or when the machine is turned off. It is proposed to provide a pulsing circuit to lift the pen at regular intervals to identify equal increments of time on trajectory problems.

On our REAC, Helipots used as follow-up potentiometers for the drum and pen positioning servos of the input and output tables are tapped  $90^\circ$  from each end. The  $\pm 100$  volt potentials are connected to

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(8) The combined errors of the drum-positioning servos of the input and output tables plus the error of the output table pen positioning servo are present in the figure given. It is probable that the accuracy of the automatic tracking itself is somewhat better.

(9) Subsequently, an improved lead screw has been designed by Reeves. Replacement units were supplied to us without charge. The current production input-output tables use the improved lead screw design.

(10) Mr. Gardner Johnson of RAND and Mr. E. C. Burkhart of the Genisco Co., Los Angeles, were responsible for the design of the modifications of this device.

these taps. The gearing is arranged to make 9-1/2 turns of the Helipot shaft correspond to full scale travel. The mechanical stops were originally set to correspond to this full-scale travel. Wear and tear of the stops when the input tables must be reset to the edge of the graph can be reduced by resetting the mechanical stop position to permit some over travel. However, the length of follow-up potentiometer winding beyond the +100 volt tap is all at the same potential. Thus there is no unique null position for the servo in this region. High value resistors were connected from the -190 and +300 volt power supplies to the ends of the windings. The resultant voltage gradient beyond the +100 volt tap forces the servo to be stationary only at the appropriate 90° tap when a +100 volt (or zero volt if one end of the follow-up potentiometer is grounded) signal is applied to the servo input.

All input and output table servo amplifiers have been converted to the circuit of Figure 6 with the modification that two-megohm input and feedback resistors are used to reduce the nonlinearity due to loading of the follow-up potentiometer.

### (3c) ESTERLINE ANGUS RECORDERS

Three one-milliampere Esterline Angus recorders are used to provide auxiliary recording facilities (Fig. 11). No amplifier is required and the accuracy is one per cent of full scale. The three charts are driven from a common synchronous motor which is equipped with a solenoid clutch. The clutch is normally connected to be energized only when the REAC control switch is in "operate" position. The scale factor of the chart record for each meter is adjusted by means of a Helipot used as a series resistance. The frequency response is adequate for most problems solved on the REAC.

PART II

OPERATING TECHNIQUES

The modifications outlined in the previous pages have had considerable influence on the planning of problems for REAC solution. This Part will contain a summary of those techniques that have been found particularly useful in the computing work at RAND.<sup>(11)</sup> Wherever it is possible, actual problems solved on the REAC will be given as examples.

(4a) HIGH-GAIN AMPLIFIER

The high-gain amplifiers are employed most frequently in implicit function techniques.<sup>(12)</sup> For example, assume we have an explicit function  $z = Z(x_1, x_2, \dots, x_n)$  in which  $Z$  is either difficult or impossible to compute, while the equivalent implicit function  $F(x_1, x_2, \dots, x_n, z) = 0$  offers no such complication. If  $F(x_1, x_2, \dots, x_n, z)$  is fed into an amplifier of gain minus  $\mu$  whose output is used as  $z$  in the computation of  $F(x_1, x_2, \dots, x_n, z)$ , it can be seen from Figure 12 that the high-gain amplifier now has a feedback loop closed through the circuits generating  $F(x_1, x_2, \dots, x_n, z)$ . It is also apparent that the system is actually setting  $F$  equal to minus  $Z/\mu$  instead of zero, but since the magnitude of  $\mu$  at low frequencies is greater than  $10^7$ , the resulting error is negligible.

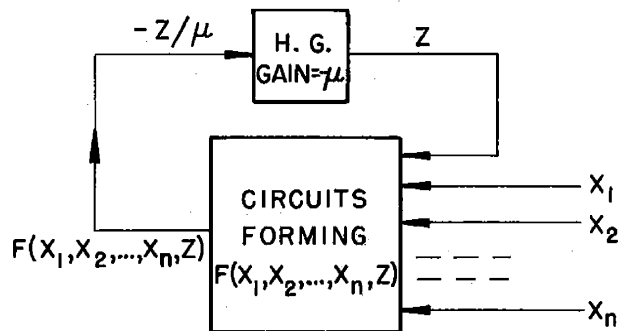


Figure 12 - Use of a High-Gain Amplifier to Solve an Implicit Equation.

(11) Many of the techniques were suggested by Dr. George Brown. Wesley Melahn, who has been responsible for about half of the REAC computations, also has made several contributions.

(12) A good discussion of such techniques appears in "Electronic Instruments", Greenwood, Holdam, and MacRae, Rad. Lab. Series, No. 21, McGraw-Hill Corp., 1948, pp 15-21 and 131-136.

There is always the possibility that the feedback signal may be of such a nature as to cause instability. It can be shown<sup>(12)</sup> that if the output of the high-gain amplifier is designated as  $z$ , the circuit will be stable with plus  $F$  fed back if  $\frac{\partial F}{\partial z} > 0$  in the region of interest. If  $\frac{\partial F}{\partial z} < 0$ , minus  $F$  must be fed back to the amplifier to assure stable operation. Quite often problems are encountered in which  $\frac{\partial F}{\partial z}$  may be of either sign. Stable operation is possible in this case by minimizing  $F^2$  instead of setting  $F$  equal to zero. This can be accomplished by solving for the root of the implicit equation  $\frac{\partial F}{\partial z} * F = 0$ . Stable operation also results for all cases mentioned above if  $\dot{z}$  or  $\int z dt$  replaces  $z$  in the output of the amplifier. In this manner, integration may be performed by differentiation, and differentiation performed by integration, since  $z - \int \dot{z} dt = 0$  and  $\dot{z} - \frac{d}{dt} \left( \int z dt \right) = 0$ .<sup>(13)</sup>

Where  $n$  feedback loops are used to solve a set of  $n$  simultaneous equations by feeding  $+F_i$  into  $n$  high-gain amplifiers, the characteristic roots of the Jacobian of the functions must be positive for stable operation.

If an analytic analysis is not feasible, less elegant methods may be employed to determine the stability of a system. An odd number of amplifiers in the feedback loop usually assures the stability of that individual system. Another check is to trace an assumed  $+\epsilon$  error in  $z$  through the feedback loop. If the result is a decrease in the  $z$  output, the system is stable, but a resultant increase in  $z$  indicates instability.

It has been assumed in the above remarks that  $-\mu$  is a constant. Since in practice the gain of an amplifier must always decrease at high frequencies due to inevitable capacities, a rigorous determination of stability must take the variation of  $-\mu$  into account.<sup>(14)</sup> It is sometimes found that the "parasitic" phase shift of the high gain amplifier

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<sup>(13)</sup> A simple integrating amplifier is an example since the current fed back is proportional to the derivative of the output voltage and the signal current is proportional to the input voltage.

<sup>(14)</sup> Unwanted change in  $-\mu$  produces a phase shift of  $45^\circ$  at about 0.006 c.p.s. when the chopper amplifier is in the loop. The zero frequency gain for this condition is about  $|\mu| \approx 6 \times 10^7$ . When the basic amplifier only is present  $[|\mu| \approx 3 \times 10^4$  at zero frequency]. The phase shift becomes  $45^\circ$  at about 10 c.p.s.

will cause a system to be unstable when analysis assuming  $\mu =$  constant predicts stability. In such cases, often it is possible to "sneak up" on a solution by starting with a small value of effective gain and gradually increasing the gain to the verge of system instability. The error can be determined by measuring the input to the "high gain" amplifier.

Division may be accomplished by multiplication through the use of implicit function techniques. The explicit function  $z = Z(x_1, x_2) = \frac{x_1}{x_2}$  is put into the implicit form  $F(x_1, x_2, z) = x_2 z - x_1 = 0$ . Figure 13 shows the schematic diagram for the REAC solution of this equation.<sup>(15)</sup> In this case  $\frac{\partial F}{\partial z} = \frac{\partial (x_2 z - x_1)}{\partial z} = x_2$  and the system is stable for positive  $x_2$  only. Since  $x_2$  will never be negative, the upper tap of each potentiometer is grounded.

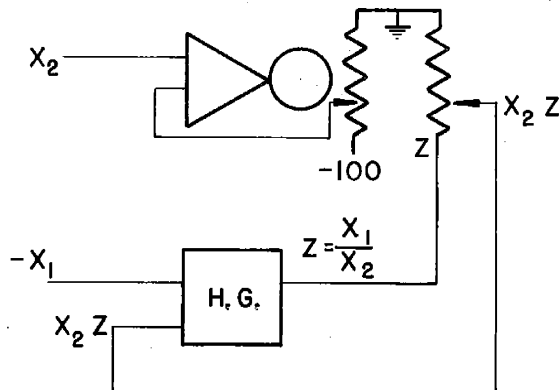


Figure 13 -- Division Using a High-Gain Amplifier

Implicit function techniques permit computing a square root by squaring. For example, instead of solving  $z = \sqrt{x_1^2 + x_2^2}$ , we make use of the equivalent relationship  $F = z^2 - x_1^2 - x_2^2 = 0$ . The schematic diagram for the solution of this equation is shown in Figure 14. The system will be stable for  $z$  greater than zero since

<sup>(15)</sup> Notice that in summing servos the feedback voltage on the follow-up potentiometer arm is the negative of the input voltage.



$\frac{\partial F}{\partial z} = \frac{\partial}{\partial z} (z^2 - x_1^2 - x_2^2) = 2z$ . Notice that there is only one amplifier in the feedback loop, although at first glance the servo amplifier may appear to be included in the loop.

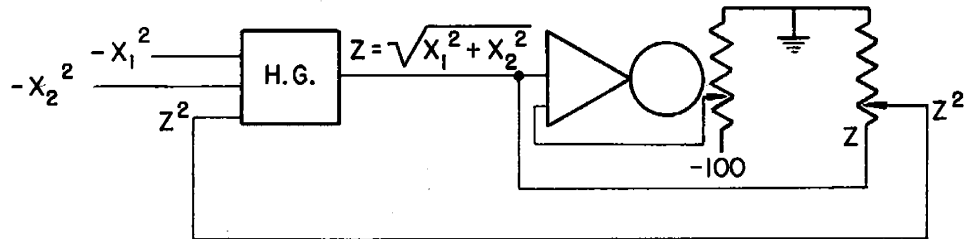


Figure 14 - Computing a Square-root by Implicit Function Techniques.

It has been mentioned that the derivative of  $z$  may be used in place of  $z$  as the output of the high-gain amplifier. As an example of such a technique, consider the computation of  $z = \tan^{-1} \frac{x_1}{x_2}$ . The derived implicit equation,  $x_2 \sin z - x_1 \cos z = 0$  is fed into a high-gain amplifier. Since  $z$  is considered to be some  $z(t)$ , the output of the amplifier is called  $\frac{dz}{dt}$ . The derivative is necessary for the continuous generation of  $\sin z$  and  $\cos z$  which are required in the implicit equation (see Section 5b). The values of  $x_1$  and  $x_2$  are assumed known at  $t = 0$ . Notice that

$$\frac{\partial}{\partial z} (x_2 \sin z - x_1 \cos z) = x_2 \cos z + x_1 \sin z = \sqrt{x_2^2 + x_1^2} > 0$$

for all values of  $z$ .

It is often desirable to have the arm of a servo potentiometer go to an amplifier at a gain greater than unity. However, as mentioned in Section 2a, unless the input resistance of the amplifier is one megohm, potentiometer loading errors result. A high-gain amplifier whose output is fed back to the input through a one megohm resistor from a potentiometer with a setting  $k$  has a gain of  $\frac{1}{k}$ , and hence is useful for the above application. This is a convenient method for producing an adjustable "high-gain" amplifier for implicit function applications where the stability is marginal.

(4b) MODIFIED SERVO-MULTIPLIER APPLICATIONS

The most obvious advantage of having the high and low taps of the follow-up potentiometers brought to the front panel as mentioned in Section 2a, is that grounding of one end is possible when the multiplier is always of the same sign. This technique, shown in Figures 13 and 14, eliminates the requirement for a sign changing amplifier for each multiplying potentiometer. Division is easily accomplished when it is desired to have the quotient on the servo shaft. Figure 15 illustrates this application.

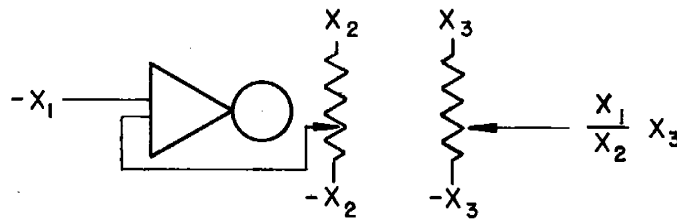


Figure 15 - Simultaneous Multiplication and Division

Actually, this circuit can be considered as a high-gain servo-amplifier having  $z = \frac{x_1}{x_2}$  as an output and  $zx_2 - x_1$  as an input. This system is not stable unless  $x_2$  is greater than zero, and it has the obvious scale factor restriction that  $x_2$  must be larger than the absolute magnitude of  $x_1$ .

All servo potentiometers have center taps that come to the front panels to jacks normally closed to ground. This permits supplying a voltage at only one tap of the multiplicand potentiometers for a multiplier that is always of one sign but can become zero.<sup>(16)</sup> This change has greatly reduced the number of amplifiers required for several problems. Center tap potentiometers permit easy determination of the absolute magnitude of a variable as illustrated in Figure 16. Figure 16 shows how limiting can be accomplished using a servo with center tapped potentiometers.

<sup>(16)</sup> Grounding of one end of all potentiometers is not possible if the multiplier becomes less than about one volt because of the mechanical stops.

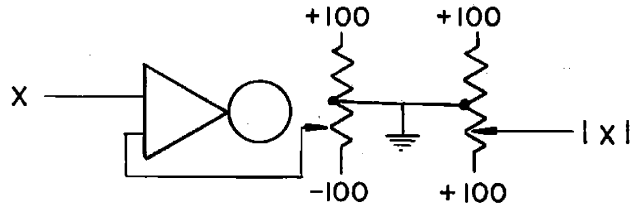


Figure 16 - Determination of the Absolute Magnitude of a Variable

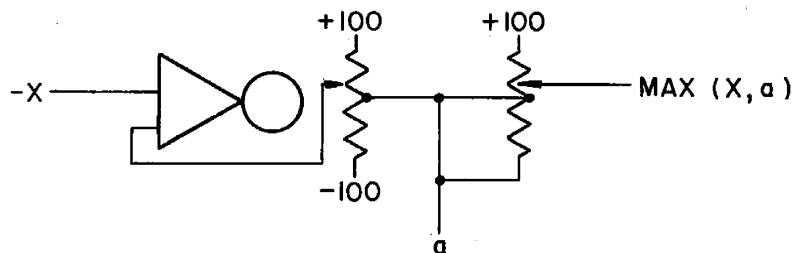


Figure 17 - Limiting With a Servo

A high-gain servo results if the follow-up arm is opened; such a servo may be used in the same manner as a high-gain amplifier, i.e. for implicit function applications, and is especially useful when the derived variable is required as a multiplier. The following thermodynamics problem illustrates such an application.

$$k_1 \left[ \left( \frac{T_B}{100} \right)^4 - \left( \frac{T_S}{100} \right)^4 \right] + \left\{ a_1 + b_1 \left( \frac{T_B + T_C}{2} \right) \right\} (T_B - T_C) - a_3 + b_3 T_B = 0$$

$$k_1 \left[ \left( \frac{T_B}{100} \right)^4 - \left( \frac{T_S}{100} \right)^4 \right] - \left\{ a_1 + b_1 \left( \frac{T_S + T_C}{2} \right) \right\} (T_S - T_C) - \left\{ a_4 + b_4 \left( \frac{T_S + T_N}{2} \right) \right\} (T_S - T_N) = 0$$

$$- \left\{ a_4 + b_4 \left( \frac{T_S + T_N}{2} \right) \right\} (T_S - T_N) + k_2 \left[ \left( \frac{T_N}{100} \right)^4 - \left( \frac{T_T}{100} \right)^4 \right] + \left\{ a_2 + b_2 \left( \frac{T_N + T_A}{2} \right) \right\} (T_N - T_A) = 0$$

$$k_2 \left[ \left( \frac{T_N}{100} \right)^4 - \left( \frac{T_T}{100} \right)^4 \right] - \left\{ a_2 + b_2 \left( \frac{T_T + T_A}{2} \right) \right\} (T_T - T_A) = 0$$

$$\frac{dT_C}{dL} = k_3(a_3 - b_3 T_B)$$

$$\frac{dT_A}{dL} = k_4 \left\{ a_4 + b_4 \left( \frac{T_S + T_N}{2} \right) \right\} (T_S - T_N)$$

Solutions were requested for 50 sets of parameters.

Ordinarily, the analysis of the stability of a circuit for the solution of such a set of implicit equations would be quite difficult. However, the physics of the problem and an inspection of the equations showed that there was a dominant variable in each equation and all powers of the dominant variable were of the same sign. Hence, stability could be assured quite readily. The first four equations were used to determine  $T_B$ ,  $T_S$ ,  $T_N$ , and  $T_T$  respectively. In each case the sum on the left side of the equation was fed into a high-gain servo multiplier since multiplications by the variables were required. The integrations indicated by the last two equations were not initiated until the high-gain servos had settled to a steady state condition.

The summing property of the servo amplifiers has also proved helpful not only in reducing the need for summing amplifiers, but also in augmenting the number of summing amplifiers for large problems. Care must be taken not to exceed current limitations in the follow-up potentiometers, however, and usually no more than two potentiometer loads should be used. A servo so loaded cannot be used for multiplication unless the loading is compensated as described in Section (5a).

#### (4c) RELAY APPLICATIONS

In Section (1c) it was mentioned that leads permitting the simultaneous operation of all hold relays have been brought to a jack on the front panel. One convenient application of this is the

automatic stopping of a problem whenever a variable exceeds a certain limit. Figure 18 shows one way in which this may be accomplished. Current will flow through the relay, holding it open until  $x$  becomes greater than  $a$ , at which time the output of the high-gain amplifier goes from its maximum positive value to its maximum negative value and current flow is prevented by the diode of a limiter.

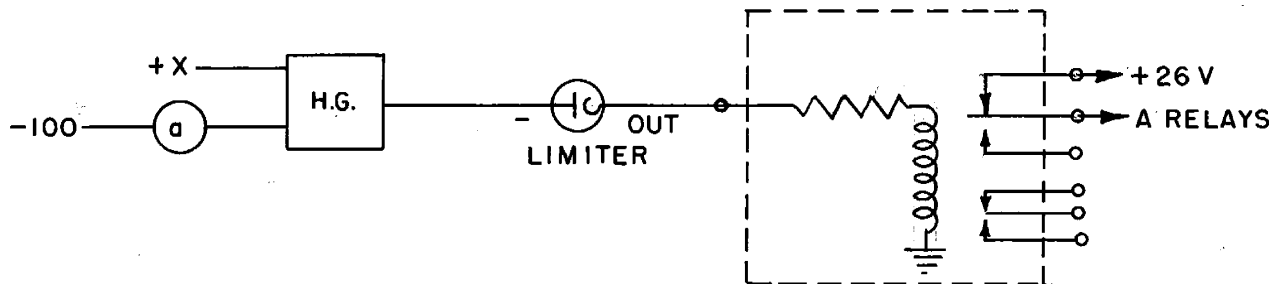


Figure 18 - Automatic Hold Circuit

It was often necessary to stop the computation at a specified limit. To obtain a reasonable accuracy prior to the installation of this circuit, the solution time had to be relatively long to allow for operator reaction time. The precision of the automatic circuit at normal computing speeds has been found to be better than 0.1 per cent over several days of computing.

The leads to individual hold relays of five amplifiers have also been brought to jacks on the front panel, permitting opening of individual amplifier inputs in a manner similar to that described above. The following problem required this technique as a limiting device in the steady state solution of the equations:

$$H_i = \max \left[ K \sum_{j=1}^3 A_{ij} x_j, 0 \right]$$

$$\frac{dx_i}{dt} = H_i - \lambda x_i$$

$$\lambda = \sum_{i=1}^3 H_i$$

$$A_{ij} = \begin{bmatrix} 0 & \alpha & -\beta \\ -\alpha & 0 & \gamma \\ \beta & -\gamma & 0 \end{bmatrix}$$

for various sets of  $K, \alpha, \beta, \gamma$  and initial  $x_i$ 's. The equations for the  $H_i$ 's were first set up using the limiters provided in the REAC, but the "softness" of their limiting action caused the variables to oscillate quite badly about their roots. The use of high-gain amplifiers driving the hold relays of amplifiers with  $H_i$ 's as inputs gave much more satisfactory results, but the finite opening and closing time of the relays apparently caused small oscillations with quite a long damping time.<sup>(17)</sup> The overloading and high frequency components present at the start of the solution caused the final  $x_i$ 's to be low although of the proper ratios. Making

$$\lambda = \sum_{i=1}^3 H_i - \left( 1 - \sum_{i=1}^3 x_i \right)$$

corrected this fault. With  $K = 4$  the oscillations about the steady state solutions dropped to about one per cent in 15 seconds and converged with a 0.2 per cent error after one minute.  $K$ 's greater than 4 produced instability due to the resulting great overload and high frequency components. Solutions with the  $\sum H_i$  term removed from the new  $\lambda$  were as satisfactory as those with the term present.

#### (4d) INPUT TABLE APPLICATIONS

The most obvious advantage of the automatic input tables is the elimination of the cost of hand tracking. The first problem to use the automatic input tables was a trajectory problem requiring five input functions. The elimination of the expense of hand tracking for this problem saved enough to pay for the modifications required to make automatic tracking possible. Moreover, (a) it was possible to use a computing speed four times faster and (b) the reproducibility was better than that obtained in a similar problem run previously using hand tracking.

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<sup>(17)</sup> At the time of this computation, center tap pots were not installed; hence, limiting as in Figure 17 was not available.

Since most limiting operations are but a special form of function generations, an input table may be utilized in various limiting applications. Backlash, hysteresis, and other similar phenomena are readily handled by the input tables, although hand tracking is required for double valued functions.

The installation of automatic input tables has greatly modified the planning of schematics for problems. Previously, input functions were replaced wherever possible by approximations or generation of the functions by the REAC itself. However, since a wired curve for the input table takes little more time to prepare than that required for drawing a curve through the points with French curves, the use of input tables even for simple functions is becoming a standard practice at RAND. The resulting simplification of circuitry reduces scale factor problems and the probability of machine errors, and simplifies any debugging that may be necessary. The following problem illustrates such a use of input tables.

The equations governing the radial error,  $\rho$ , and the angular error,  $\epsilon$ , of a specified guidance system are:

$$\ddot{\rho} - (R + \rho)(\dot{\phi}_0 + \dot{\epsilon})^2 + \frac{\mu}{(R + \rho)^2} = B_r \cos \epsilon$$

$$(R + \rho)\ddot{\epsilon} + 2(\dot{\phi}_0 + \dot{\epsilon})\dot{\rho} = -B_r \sin \epsilon.$$

Letting  $x = \rho/R$ ,  $y = \frac{\epsilon}{\phi_0}$  and modifying to a more satisfactory form for computation we obtain:

$$\ddot{x} = \dot{\phi}_0^2 \left\{ x + (1 + x)(2 + \dot{y})\dot{y} + \frac{\mu}{R^3 \dot{\phi}_0^2} \cdot \frac{(2 + x)x}{(1 + x)^2} - \frac{B_r}{R \dot{\phi}_0^2} (1 - \cos \epsilon) \right\}$$

$$(1 + x)\ddot{y} = -2(1 + \dot{y})\dot{x} - \frac{B_r}{\dot{\phi}_0 R} \sin \epsilon$$

The approximations  $\cos \epsilon = 1 - \frac{\epsilon^2}{2} + \frac{\epsilon^4}{24}$  and  $\sin \epsilon = \epsilon - \frac{\epsilon^3}{6}$  further simplify the computation and improve the accuracy since the use of modulators and demodulators is avoided.

Since the above system is unstable, caution was necessary in planning the computing procedure and in operating the REAC to meet the request for plots of asymptotic behavior for small initial errors in  $\dot{\rho}$ ,  $\rho$ ,  $\dot{\epsilon}$ ,  $\epsilon$ , and  $\epsilon$ .

At first, a solution was attempted computing  $\frac{x(2+x)}{(1+x)^2}$  and  $\frac{1}{1+x}$ , but the drift voltages made it impossible to use as small starting values as desired and still obtain good accuracy. A second set of runs was made generating  $\frac{1}{1+x}$  and  $\frac{x(2+x)}{(x+1)^2}$  on input tables.<sup>(18)</sup> The resultant reduction in the circuitry permitted use of the small initial values desired.

Test runs with the automatic generation of a function of two variables have given encouraging results. Figure 19 illustrates one circuit that has been used.<sup>(19)</sup> The second variable was represented by a three curve parametric plot with the servo linearly interpolating between the two curves bounding the second variable.

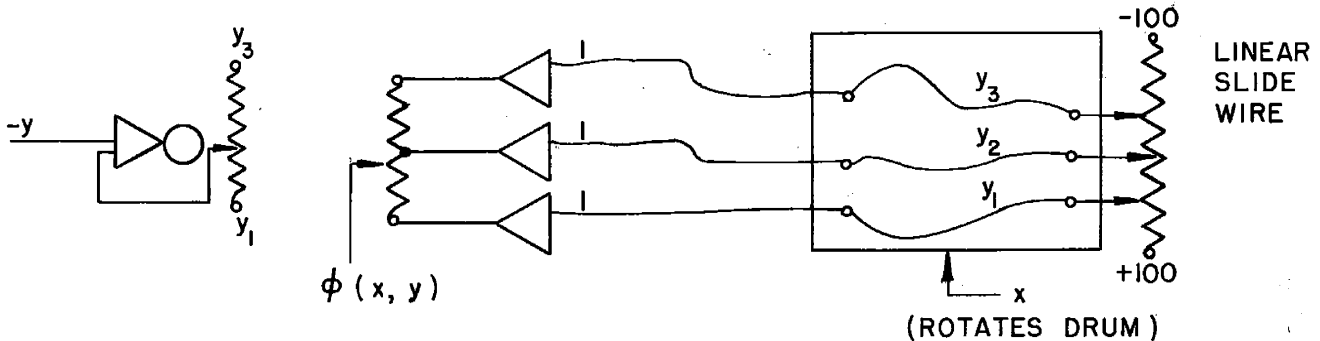


Figure 19 - Automatic Generation of a Function of Two Variables.

The test runs were restricted to three family curves since only potentiometers with three taps were available on the servos. A five family set of curves has been successfully tracked.<sup>(20)</sup> A potentiometer with several taps is to be installed on one servo to accommodate functions of two variables requiring more complete definition in the second variable. In such a situation, a switch geared to the servo

(18) The function  $\frac{2+x}{(x+1)^2}$  was plotted and  $x$  put across the input table linear slide wire.

(19) The idea of using a tapped potentiometer on the servo was suggested by Raleigh McCoy of Reeves Inst. Co.

(20) The slide wire counterweights must be properly adjusted as mentioned in Section (3a).



shaft may be used to allow only three amplifiers to drive the appropriate points on the tapped servo potentiometer. For example, referring to Figure 20, suppose  $y_2 \leq y \leq y_3$  and  $y \rightarrow y_3$ . It is

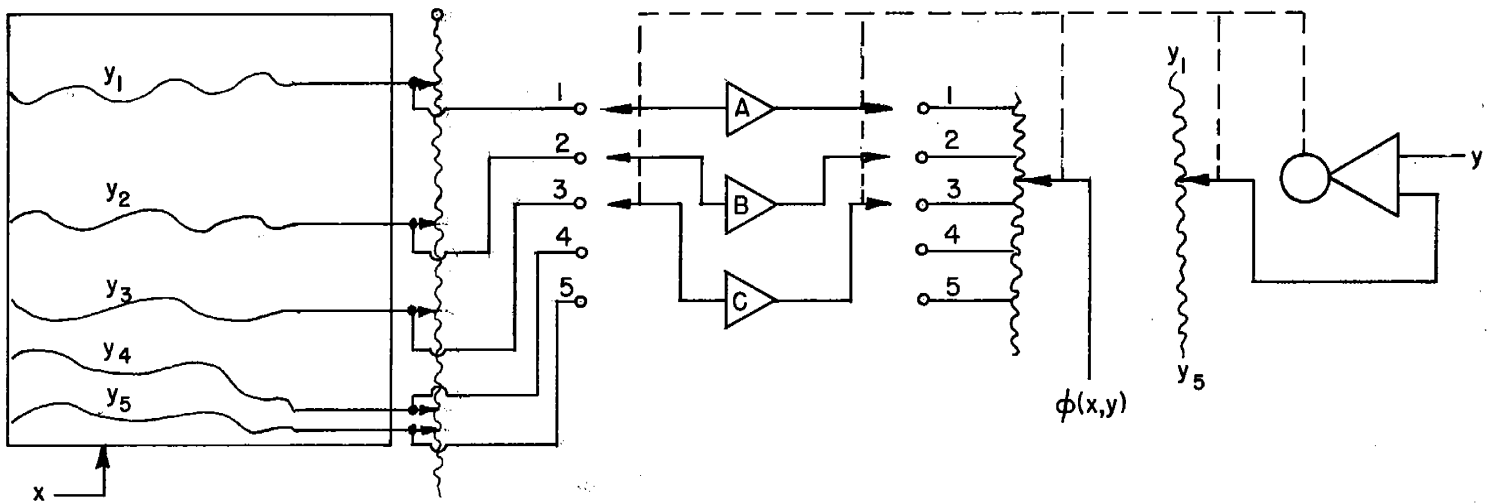


Figure 20 - Generation of Function of Two Variables From Five Parameter Family.

necessary to arrange the switch controlling amplifier A to transfer from position 1 to position 4 for some  $y$  in  $(y_2, y_3)$  the exact point (or interval) of transition being unimportant since the value of  $\phi(x,y)$  is determined by the outputs of the two amplifiers connected to the taps bridging the particular value of  $y$ . Theoretically, it is possible to use only two amplifiers and a "suitable" switch but the extreme precision required of switch operation timing would probably make this an impractical bit of elegance. Alternately, the potentiometer loading compensation techniques of Section (5a) may be used. If the parametric curves cross, one or two additional tables driven by the same variable must be used.

(5a) POTENTIOMETER LOADING COMPENSATION

Since it is usually impossible to connect the arm of one potentiometer to another potentiometer, a large number of amplifiers are often required purely for isolation. However, if the loading is properly taken into account, potentiometer to potentiometer connections are quite feasible.

For example, it is possible to load multiplying potentiometers on a servo multiplier if the follow-up potentiometer is identically loaded. This scheme depends on the total resistance of multiplying and follow-up potentiometers being equal.<sup>(21)</sup> However, as the following well-known analysis shows, variations in total potentiometer resistance have a second order effect.

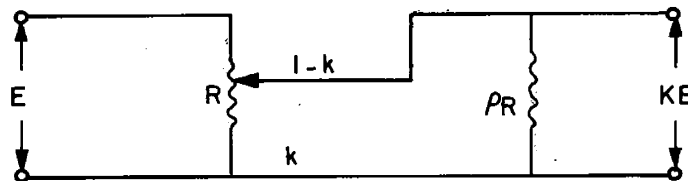


Figure 21 - Potentiometer Loading.

The apparent setting of a potentiometer loaded by another potentiometer is given by:

$$K = \frac{k\rho}{k - k^2 + \rho}, \text{ where}$$

K = apparent setting,  
 k = setting of first potentiometer,  
 ρ = ratio of resistances.

Since  $\frac{dK}{K} = \left(1 - \frac{K}{k}\right) \frac{d\rho}{\rho}$ ,

and  $\frac{d}{dK} \left(\frac{K}{k}\right) = \frac{(2k - 1)\rho}{(\rho + k^2 - k)^2}$ , then, for  $\rho \approx 1$ ,

$$\left(\frac{K}{k}\right)_{\min} \approx 0.8 \text{ at } k = \frac{1}{2}.$$

<sup>(21)</sup> The Helipot Corp. allows +5 per cent variation in total resistance of their standard potentiometers. Those in our REAC were all within +2 per cent of the nominal 20,000 ohms.

It follows that

$$\left(\frac{dK}{k}\right)_{\max} = \frac{1}{5} \left(\frac{d\rho}{\rho}\right).$$

Hence a resistance difference of one per cent would result in a maximum loading compensation error of 0.2 per cent. If care is taken in matching potentiometers, a reasonable accuracy should result.

The solution of the following problem on the RAND REAC required such loading of eight servo potentiometers.

The equations of motion of a missile may be given by:

$$\dot{y} = \left(\frac{kI}{W} - \frac{C\sigma V^2}{W}\right) \sin\beta + \frac{B\sigma V^2}{W} \alpha \cos\beta - g + \frac{V_x^2}{R}$$

$$\dot{x} = \left(\frac{kI}{W} - \frac{C\sigma V^2}{W}\right) \cos\beta - \frac{B\sigma V^2}{W} \alpha \sin\beta - \frac{V_x V_y}{R}$$

$$\dot{y} = V_y$$

$$\dot{x} = V_x$$

$$W = 1 - kt$$

$$V = \sqrt{V_x^2 + V_y^2}$$

where C and B are input functions of V, and  $\sigma$  and I are input functions of y. If terms in  $\frac{1}{R}$  and the partials of C, B, and I are neglected and if  $\frac{d\sigma}{dy}$  is approximated by  $-\alpha\sigma$ , the following additional calculus of variations equations will optimize the flight path,<sup>(22)</sup>

$$\dot{\lambda}_1 = r \left\{ 2 \left( \frac{C\sigma}{W} - \frac{B\sigma\alpha\rho}{W} \right) V_y + \frac{B\sigma\rho}{W} V_x \right\} - \lambda_3$$

$$\dot{\lambda}_2 = r \left\{ 2 \left( \frac{C\sigma}{W} - \frac{B\sigma\alpha\rho}{W} \right) V_x - \frac{B\sigma\rho}{W} V_y \right\} + 1$$

$$\dot{\lambda}_3 = -ar \left\{ \frac{C\sigma}{W} - \frac{B\sigma\alpha\rho}{W} \right\} V^2$$

$$r = \sqrt{\lambda_1^2 + \lambda_2^2}$$

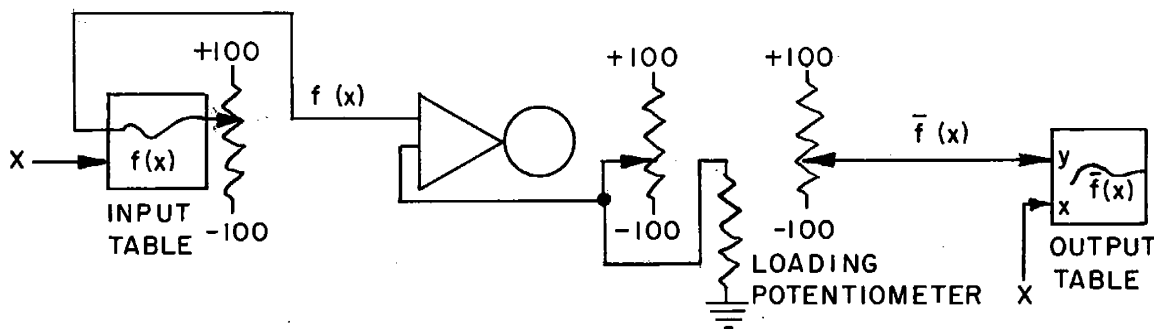
$$\rho = \frac{B\sigma V^2\alpha}{kI + B\sigma V^2 - C\sigma V^2}$$

$$\beta + \rho = \tan^{-1} \frac{\lambda_1}{\lambda_2}$$

$$\beta - \alpha = \tan^{-1} \frac{V_y}{V_x}$$

The solution of this problem would require an additional thirteen amplifiers if it were not for center tap potentiometers and loading compensation.

It is possible to go from an input table slide wire to a potentiometer if the curve is redrawn prior to computation to take loading into account.<sup>(23)</sup> Figure 22 shows how the REAC can be utilized to perform the transformation for a function of one variable. The function  $\bar{f}(x)$  is taken from the output table and used in place of  $f(x)$  on an input table in the solution of the main problem.



$f(x)$  = true value of function

$\bar{f}(x)$  = compensated value

Figure 22 - REAC Correction of Input Data to Permit Potentiometer Loading

(23) This technique should be used only where problem complication has made computing amplifiers unavailable as some loss of accuracy is to be expected in most cases.

A more complicated circuit is required for a function of two variables but the saving in amplifiers is greater.

(5b) CHANGE OF VARIABLE

If integrations are considered to be of the form

$$\frac{d^n x}{dt^n} = \int \frac{d^{n+1} x}{dt^{n+1}} \cdot \frac{dt}{d\tau} \cdot d\tau$$

and  $t = \int \frac{dt}{d\tau} \cdot d\tau$

certain flexibility of operations results by various choices of  $\frac{dt}{d\tau}$ . Notice that  $\tau$  is the machine time scale variable while  $t$  is the independent variable. For example, all derivatives can be put across servo multiplying potentiometers before going to the integrators and the servo positioned by a hand set potentiometer permitting operator selection of the computing speed. This technique is useful for slowing the computing rate at crucial periods when hand tracking or stopping the solution by hand. If both signs of the derivative are put across the potentiometers the solution may be stopped, or even reversed, by proper setting of the input potentiometer. Figure 23 illustrates a typical schematic.

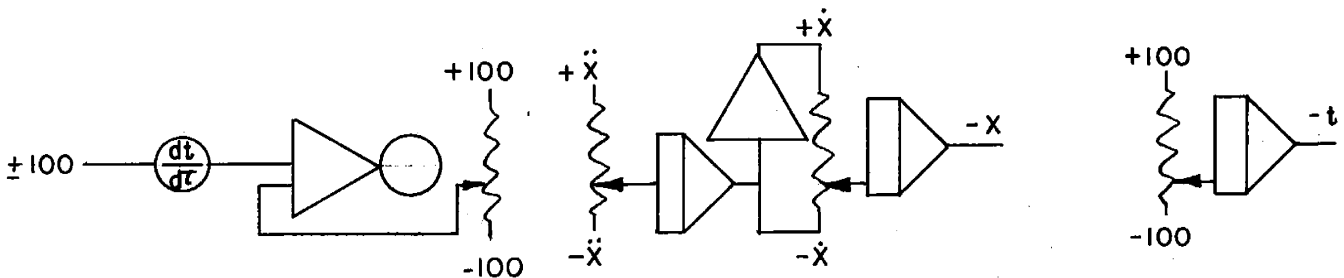


Figure 23 - Manual Control of Computing Speed

Integration with respect to a variable other than the independent variable is similarly achieved, i.e.,

$$\frac{d^n y}{dx^n} = \int \frac{d^{n+1} y}{dx^{n+1}} \cdot \frac{dx}{d\tau} d\tau = \int \frac{d^{n+1} y}{dx^{n+1}} \cdot \frac{dx}{dt} \frac{dt}{d\tau} d\tau.$$

The following problem illustrates automatic time scale adjustment and integration with respect to a variable other than the independent variable.

It was desired to find P as a function of x, such that  $.20 \leq P \leq .99$  where

$$P(x) = 1 - \frac{1}{\pi} \int_0^\pi \operatorname{erf}\left(\frac{3.412}{\sqrt{1+x+x\cos\theta}}\right) d\theta$$

in which  $\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt.$

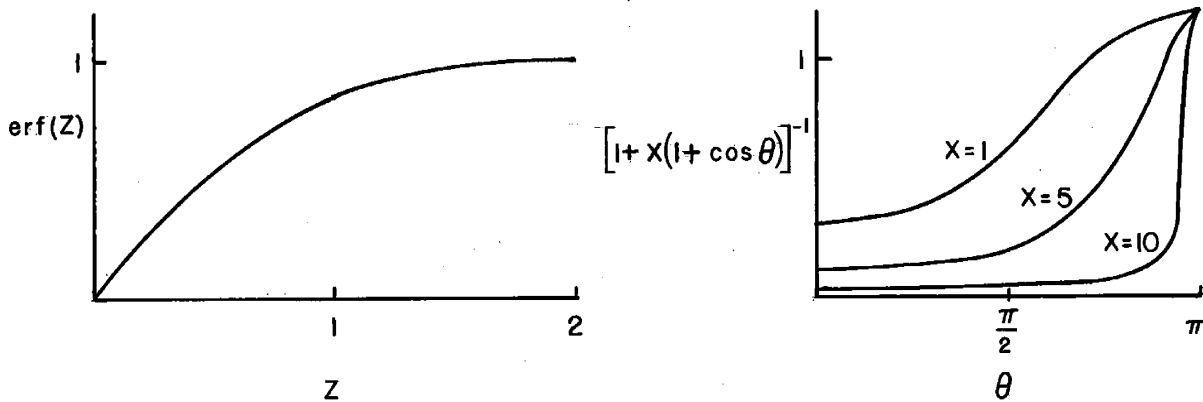


Figure 24 - Function Values for Example.

It turns out that large values of x are necessary to give the values of P(x) of interest. The argument of the error function and thus the integral are such that the major contribution to P(x) occurs for a small range of  $\theta$  near  $\pi$  (Fig 24). Therefore, let  $\theta = \pi(1 - e^{-\tau})$  in order to spread out the integrand. Thus,

$$P(x) = 1 - \int_0^\infty e^{-\tau} \operatorname{erf}\left[\frac{3.412}{\sqrt{1+2x\left(\frac{1+\cos\theta}{2}\right)}}\right] d\tau = 1 - \int_0^\infty e^{-\tau} \operatorname{erf}\left(\frac{3.412}{\sqrt{1+y^2}}\right) d\tau$$

where  $y = \sqrt{2x} \cos \frac{\theta}{2}$ . Here  $w = e^{-\tau}$  is obtained as a solution of

$$\frac{dw}{d\tau} = -w \text{ and } y = \sqrt{2x} \cos \frac{\theta}{2} \text{ is a solution of } \frac{d^2y}{d\theta^2} = -\frac{1}{4}y.$$

However, since the REAC will integrate only with respect to time,  $y$  must be obtained as a solution of  $\frac{dy}{d\tau} = \frac{dy}{d\theta} \cdot \frac{d\theta}{d\tau} = \frac{dy}{d\theta} (-\pi w)$ . The term  $\frac{dy}{d\theta}$  may be obtained by solving  $\frac{d}{d\tau} \frac{dy}{d\theta} = \frac{d^2y}{d\theta^2} \frac{d\theta}{d\tau} = \frac{\pi}{4}yw$ . The function  $\text{erf} \frac{3.412}{\sqrt{1+y^2}}$  was plotted against  $y$  and tracked from an input table.  $w$ , which was obtained from the REAC, was applied to the input table slide wire. The product was then integrated with respect to time; this quantity subtracted from 1 then gave the value of  $P$  corresponding to the value of  $x$  set as an initial condition in the solution for  $y$ .

Changes of variables that simplify computations or improve the accuracy are often possible. The following problems illustrate such techniques. Certain scale factor difficulties in the van der Pol equation  $\frac{d^2x}{dr^2} + \mu(x^2 - 1)\frac{dx}{dr} + x = 0$  could be avoided by considering the equivalent equation  $\frac{d^2x}{dt^2} + (x^2 - 1)\frac{dx}{dt} + \epsilon^2x = 0$  where  $t = \mu r$  and  $\epsilon = \frac{1}{\mu}$ . As  $\mu$  is increased to make the van der Pol equation highly non-linear, the voltage representing the derivative increases rapidly as the sides of the wave form become more nearly vertical. In the transformed equation this increase in voltage is partially off-set by expanding the time scale as  $\mu$  increases. Solutions to this equation were plotted in the  $(x,t)$  plane and in the  $(x, \frac{dx}{dt})$  plane for  $\epsilon = .001$  to 100. Rayleigh's equation,  $\frac{d^2z}{dr^2} + \mu(\frac{1}{3}x^3 - x) + z = 0$ , where  $x = \frac{dz}{dr}$ , is equivalent to the system  $\frac{dx}{dt} = -(\frac{1}{3}x^3 - x) - y$ ,  $\frac{dy}{dt} = \epsilon^2x$  where  $y = \epsilon z$ ,  $t = \mu r$ , and  $\epsilon = \frac{1}{\mu}$ . A plot was made of the phase portrait in the  $(x,y)$  plane for the same values of  $\epsilon$  as above.

The following problem resulted from a study of the laminar boundary layer in gases. The simultaneous differential equations to be solved were:

$$g \cdot \frac{d^2 g}{dx^2} + xA = 0$$

$$\frac{dg}{dx} \cdot \frac{dj}{dx} = \frac{d}{dx} \left( \frac{1}{P} \cdot \frac{dg}{dx} \cdot g \right) + g$$

where A and P were both plotted functions of j which were entered by means of input tables. It was desired to obtain end values of j in the interval  $1 \leq j \leq 4$ , the end value being that obtained when  $g = 0$ . In this form, g and j approach final values with steep slopes since  $\frac{d^2 g}{dx^2} \rightarrow \infty$  as  $g \rightarrow 0$ . With the transformation,  $g = \frac{dx}{dt}$ , the equations were reduced to:

$$\frac{dg}{dt} = g \cdot \frac{dg}{dx} ,$$

$$\frac{d}{dt} \left( \frac{dg}{dx} \right) = -xA ,$$

$$\frac{dj}{dt} \cdot \frac{dg}{dx} = \frac{d}{dt} \left( \frac{1}{P} \frac{dj}{dt} \right) + g^2 .$$

In the transformed equations g and j approach the final values asymptotically. About 680 runs were made.

### (5c) INDETERMINATE FUNCTIONS

Indeterminate functions have reared their nasty heads several times in our work and until recently no satisfactory method of handling them was available. The following methods were suggested by Dr. Brown; the first one seems most satisfactory for any problems we have encountered and requires the minimum of additional equipment. The only disadvantage of the method is the requirement of analysis prior to computation.

Assume we wish to compute  $F = \frac{f(t)}{t}$  where F is finite at  $t = 0$ . It is usually possible to form a good approximation of F by another function G in the neighborhood of  $t = 0$ . We then can approximate F continuously by the linear combination

$$F \approx \frac{\lambda(t)G + t \cdot F}{\lambda(t) + t} = \frac{\lambda(t)G + f(t)}{\lambda(t) + t} . \quad \lambda(t) \text{ is chosen such that it dies}$$



out after  $t$  becomes large enough to permit accurate division and before  $G$  becomes a poor approximation. An exponential seems satisfactory for  $\lambda(t)$  since it is simple to generate and satisfies the above requirements. The approximation then becomes

$$F = \frac{be^{-at}G + f(t)}{be^{-at} + t} .$$

Adjustment of  $a$  and  $b$  permits varying the system to satisfy various  $G$ 's and accuracy demands. A few trial runs of a given problem will usually indicate if the particular  $G(t)$  chosen is a suitable approximation.

A second method that has been used demands more machine components. An analysis of  $F$  will usually permit determination of upper and lower bounds of the function for small values of  $t$ . For example, an alternating series approximation gives upper or lower bounds as terms are added. If all terms of the series are positive, the value given after any term has been added is always a lower bound, and an arbitrary additive function can be chosen giving the upper bound. Division can then be done as usual except that the quotient will be limited by the upper and lower bounds determined as above. The course of  $F(t)$  in the first few runs will indicate how well the bounds were chosen.

If the indeterminate function is of the form of a continuous average,

$$F(x) = \frac{1}{x} \int_0^x f(t)dt, \text{ and } f(t) \text{ is monotonic, the mean value theorem}$$

provides a method of solution. We then need to find  $\gamma$  where  $F(x) = f(\gamma x)$ , and  $0 < \gamma < 1$ . A high-gain amplifier can be used to compute  $\gamma$  and has as an input  $x \cdot f(\gamma x) - \int_0^x f(t)dt$ . It is true that  $\gamma$  will not be well determined initially, but at that time  $x$ , and hence the product  $\gamma x$ , will be zero or very small, and we obtain  $F(0) = f(0)$  as desired.

The solution of Bessel's Equation of higher order is a good example illustrating the need for the above techniques. Here we have

$t^2 \frac{d^2y}{dt^2} + t \frac{dy}{dt} + (t^2 - n^2)y = 0$ . Not only is a division by  $t^2$  necessary to compute  $d^2y/dt^2$ , but for  $n$ 's greater than one, there is no driving voltage since the initial conditions for  $y$  and  $\dot{y}$  are zero. It is true, that the drift and noise voltages in the system will eventually force a "solution" to be obtained, however there is no way of determining the scale factor of the output or the location of  $t = 0$ . The first method of approximation was particularly applicable to this problem. The first term of the series expansion proved to be a suitable approximation for  $\ddot{y}$  for small  $t$ .

#### (6a) ACCURACY

The need for an estimate of the maximum errors or confidence limits on every problem done on the REAC cannot be over emphasized. The simplest machine check is the duplication of hand computed runs, but this cannot give complete assurance of obtaining the same accuracy for runs which may be more sensitive to REAC errors than the trial run. Mathematical definitions sometimes permit a continuous check on certain aspects of a problem. ~~For example,~~ A set of simultaneous equations is usually best checked by studying one equation at a time with the integrators for the other equations opened. Reversing all plus and minus input and initial condition voltages and rerunning will determine whether or not drift or unbalanced power supplies are having a deleterious effect.

A numerical check or improvement of REAC solutions either by hand or IBM equipment often seems advisable. The most straightforward approach involves application of the method used in the proof of the existence of solutions. Most equations may be reduced to the form  $\dot{x}_i = f_i(x_1, x_2, \dots, x_n^t)$ ,  $i = 1, 2, \dots, n$ . Plots are made of all  $x_i$ 's as a function of time, permitting numerical computation of the  $f_i$  as functions of time. Numerical integration then yields a better estimate of the  $x_i$ . This method may be repeated until the desired accuracy is achieved. This technique is also useful in improving solutions of problems so complicated as to require approximations to fit on the REAC.

Subsequent numerical checks have proved REAC solutions to be surprisingly good. The following missile trajectory optimization problem gave particularly encouraging results. The basic equations were:

$$\dot{v} = -g \sin \theta - \frac{K_1 \sigma v^2}{1 - K_2 t} (C_x + B_N \sin^2 \alpha) + \frac{g I K_2}{1 - K_2 t} \cos \alpha$$

$$\dot{\theta} = 0 \quad 0 < t < T$$

$$\dot{\theta} = a + \frac{b}{2} t \quad T < t < 40 \text{ sec.}, \quad T = 0, 20 \text{ sec.}$$

$$\dot{y} = v \sin \theta$$

$$\dot{x} = v \cos \theta$$

$$\sin \alpha = \frac{\left(g - \frac{v^2}{R}\right) \cos \theta - v \dot{\theta}}{\frac{K_1 \sigma v^2}{1 - K_2 t} (B_N - C_x) + \frac{g K_2 I}{1 - K_2 t}}$$

where  $C_x$  and  $B_N$  were input functions of  $v$ , and  $\sigma$  and  $I$  were input functions of  $y$ . The customer requested a consistency of results better than 0.1 volt and this request was met during the week required for computation. In fact, on one day, reruns of the first solution, made about every hour, were consistent within  $\pm 0.03$  volts. This precision demanded extreme care in operating.<sup>(23)</sup> Hand checks on the accuracy through the first twenty seconds of the runs, with  $\dot{\theta} = 0$  during that period, checked twenty-three out of twenty-four end conditions to within  $\pm 0.1$  volt. In order to attain this accuracy and precision, the resolver equipment was not used.  $\sin \theta$  and  $\cos \theta$  were generated from the equations:

$$\frac{d}{dt} (\sin \theta) = \frac{d\theta}{dt} \cos \theta, \quad \text{and}$$

$$\frac{d}{dt} (\cos \theta) = -\frac{d\theta}{dt} \sin \theta.$$

---

(23) All problems discussed were run prior to the installation of the chopper type correction circuits for  $\pm 100$  volt power supplies and computing amplifiers.

When  $\theta$  became large, the errors increased but never exceeded 0.5 per cent of full scale.

The problem used to illustrate relay applications gave a good check on the machine accuracy. Since the purpose of the problem was to test a theory and a means for solution, the answers were known. The inaccuracy for this problem was 0.2 per cent. Similar remarks apply to the Bessel Equation solution.

The navigation error analysis problem discussed in Section (4d) is undoubtedly the worst problem we have done from an accuracy viewpoint. The customer wanted an indication of a trend and said he would be satisfied with results containing errors up to 10 per cent. The basic instability of the system and a requested 1000:1 ratio between initial and final values of the variables (without changing scale factor) combined to put the REAC in its worst light from an accuracy viewpoint. A hand check of a few cases gave a maximum inaccuracy of 2 per cent.

#### (7a) COMPARISONS OF COMPUTING TIMES

Comparisons of the REAC computing times with those required by hand or IBM solutions put the REAC in an extremely favorable light. The problems checked were well suited to analogue solution.

The 680 runs of the laminar boundary layer equations discussed in Section (5b) required 64.5 hours of REAC time. The time to do this problem with IBM punched card equipment available at RAND was estimated at 1750 hours.

A pilot computation of the trajectory optimization problem given above under the discussion on accuracy checks required one day (three shifts) per run on the RAND IBM machinery. Individual runs on the REAC required only 40 seconds, but changing graph paper and parameters, and reading final values made the average computing time about four minutes per run. Moreover, the sequential nature of the REAC solutions reduced the number of runs required since previous runs could be analyzed before new parameters were specified. The parallel computing technique required in IBM solutions demands that all parameters be specified prior to the computation.

The navigation error analysis computation mentioned above required one week to solve by hand. The REAC solution time was 50 seconds, but since curves for the two variables and their derivatives were desired on each graph, four runs were required per solution, or a solution time of about five minutes. However, set-up, checking, changing scale factor, preparing input tables, changing graph paper and other such time consuming activities brought the total machine time to five hours to turn out nine sets of graphs.

It is also important to consider that a typical problem requiring weeks for preparation when IBM computing is used, may often be prepared for REAC solution in a matter of days.

(8a) FURTHER EXAMPLES

Some forty odd problems have been computed on the RAND REAC, with computing times ranging from two hours to three weeks. The following problems are included to illustrate the types of problems in which the REAC has been found a useful research tool.

The following equations result from a trajectory study:

$$\ddot{x} = Ae^{-a(h-Z)} \sqrt{(\dot{x}-bZ)^2 + \dot{y}^2 + \dot{Z}^2} (\dot{x} - bZ)$$

$$\ddot{y} = Ae^{-a(h-Z)} \sqrt{(\dot{x}-bZ)^2 + \dot{y}^2 + \dot{Z}^2} \dot{y}$$

$$\ddot{Z} = Ae^{-a(h-Z)} \sqrt{(\dot{x}-bZ)^2 + \dot{y}^2 + \dot{Z}^2} \dot{Z}$$

$$a = 3.26 \times 10^{-5}$$

$$x_0 = y_0 = Z_0 = 0$$

$$b = \frac{1}{.02h}$$

$$\dot{x}_0 = \dot{Z}_0 = 0 \quad \dot{y}_0 = v_0$$

$$A = -1.530 \times 10^{-5}$$

$$\text{and } -7.650 \times 10^{-5}$$

$$h = 20,000; 35,000; \text{ and } 50,000$$

$$v_0 = 500; 700; \text{ and } 900$$

Find  $x(t)$ ,  $y(t)$  and  $Z(t)$  from  $t = 0$  to  $t = t_f$  which corresponds to  $Z = h$ .

The function  $e^{aZ}$  was generated by solving the differential equation:

$$\frac{d}{dt} (e^{aZ}) = aZ e^{aZ}$$

Final values  $e^{ah}$  were used to check the generation of the function during the computation. The square root was obtained by solving the equation  $u - \sqrt{u} \cdot \sqrt{u} = 0$  with a servo as the high-gain element. The rotation of the shaft was proportional to  $\sqrt{u}$  hence one of the multiplications could be performed by a second potentiometer on the shaft of the computing servo.

For given ranges of values of  $\Psi_0, \Psi_1,$  and  $\Psi_2,$  it was desired to find  $x = f(\Psi_0, \Psi_1, \Psi_2)$  and  $y = g(\Psi_0, \Psi_1, \Psi_2)$  as solutions of the following equations:

$$\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

$$A = \frac{1/2 \Psi_0 + E(y)}{E(x) + E(y)}$$

$$E(x) = \int_0^x \phi(t) dt$$

$$B = \frac{E(x) - 1/2 \Psi_0}{E(x) + E(y)}$$

$$\left(\frac{\sigma}{a}\right)^2 = \frac{Ax^2}{B} \left[ B \left(1 + \frac{1}{xz}\right) + A \left(1 + \frac{1}{yz}\right) \right]$$

$$\Psi_1 = \frac{Ax}{\sigma/a} \left[ E(x) + E(y) + \frac{\phi(x)}{x} + \frac{\phi(y)}{y} \right]$$

$$\Psi_2 = \frac{2Ax^2}{B \left(\frac{\sigma}{a}\right)^2} \left[ B \left\{ E(x) \left(1 + \frac{1}{xz}\right) + \frac{\phi(x)}{x} \right\} - A \left\{ E(y) \left(1 + \frac{1}{yz}\right) + \frac{\phi(y)}{y} \right\} \right].$$

To obtain  $x$  and  $y$  as functions of the three variables,  $\Psi_0,$  and  $\Psi_1,$  are considered to be parameters and  $\Psi_2$  is allowed to vary with time. Then  $x$  and  $y$  are obtained as functions of  $\Psi_2,$  for each pair of values of  $\Psi_0$  and  $\Psi_1.$

The quantity  $\left(\frac{\sigma}{a}\right)^2$  was eliminated by substituting into the expressions for  $\Psi_1$  and  $\Psi_2,$  while  $u_x = \frac{\phi(x)}{x}$  and  $z_x = E(x)$  are, respectively, solutions of the differential equations,

$$\frac{du_x}{dx} = -xu_x w_x \quad \left[ w_x = \left( 1 + \frac{1}{xz} \right) \right]$$

$$\frac{dz_x}{dx} = xu_x.$$

However, the quantities,

$$\frac{du_x}{dt} = -xu_x w_x \frac{dx}{dt}, \quad \frac{dz_x}{dt} = xu_x \frac{dx}{dt}$$

must be obtained and integrated (with respect to time) to give  $u_x$  and  $z_x$ . Analogous equations give  $u_y = \frac{\phi(y)}{y}$  and  $z_y = E(y)$ . The simplified equations used to obtain  $x$  and  $y$  are the following:

$$1/2 \Psi_0 + B(z_x + z_y) - z_x = 0$$

$$A = 1 - B$$

$$\Psi_1 [Bw_x + Aw_y] - AB [z_x + z_y + u_x + u_y] = \frac{dx}{dt}$$

$$2 [Bw_x + Aw_y] - 2 [B(z_x w_x + u_x) - A(z_y w_y + u_y)] = \frac{dy}{dt}$$

where  $u$ ,  $z$ ,  $w$  are defined and obtained as above.

For each pair of values of  $\Psi_0$  and  $\Psi_1$ , the value of  $\Psi_2$  is first held constant at its initial condition and the system allowed to operate until  $\frac{dx}{dt}$  and  $\frac{dy}{dt}$  settle to zero. This gives initial for  $x$  and  $y$ . Then  $\Psi_2$  is allowed to vary, and  $x$  and  $y$  are obtained as functions of  $\Psi_2$ .

Another problem was the solution of the three simultaneous differential equations:

$$\ddot{X} = -S(2.315\alpha^2 + C_f)$$

$$\ddot{Z} = 2S\alpha - 32.2 + 4.78(10^{-8})X^2$$

$$\ddot{\alpha} = -s \left[ \frac{4.3(10^{-5})}{2.315} \right] \left[ 2.315\alpha^2 - C_f + .003954 \right] \quad \text{in which}$$

$$s = 156 \left[ \dot{x}^2(10^{-8}) \right] e^{-4.3\left(\frac{Z}{10^5} - 1\right)}$$

$$C_f = - .00585 + .017 Z(10^{-5}) \quad \text{for } Z \geq 105,000$$

$$= .012 \quad \text{for } Z \leq 105,000$$

The solution was straightforward, with all quantities being generated by the REAC (a limiter being used to give  $C_f \geq .012$ ).



### PART III

#### (9a) PERSONNEL

The Numerical Analysis Section at RAND is directed by Dr. George W. Brown and is divided into three groups: I.B.M., REAC, and Hand Computation. A mathematical analysis staff is responsible for the preparation of problems for all groups. At present, the person who prepares a given problem for the REAC is also responsible for the entire process of patching and operating the machine. A machine operator is to be available to relieve the analysis staff on particularly tedious solutions. At present the RAND REAC can be kept busy by about two full-time problem planner-operators.

The engineering and maintenance staff consists of two full-time engineers and two full-time technicians.

It is probable that the ratio of mathematicians to engineers will increase during the next six months if the modification program progresses as presently outlined.

#### (9b) PROBLEM PREPARATION

The first step in preparing a REAC problem is a conference between the "customer" and the mathematician - operator in which the physics of the problem is thoroughly discussed. It has been found that it is often profitable to cast the equations in a form that is quite remote from that originally suggested. All available information about the anticipated range and type of variation to be expected of the variables is also obtained. A tentative machine diagram is made to see if enough equipment is available. The first trial often exceeds the machine capacity and the tricks mentioned in Part II are used to make the problem fit. The scale factors are chosen with an eye to obtaining the maximum possible utilization of the  $\pm 100$  volt range in order to reduce the importance of the effects of amplifier drift, 60 cycle pickup, and servo potentiometer granularity and non-linearity.

It has been found helpful to prepare a wiring list in addition to the schematic block diagram. Forms have been prepared, upon which each computer component is listed followed by spaces for each input and output jack. These forms provide a sort of double entry check on the wiring of a problem. In theory, scale factor changes and wiring modifications are recorded on these forms as they are made. Very often, however, the operator is the only mortal person who can make sense out of a chart by the time a problem is really underway. This is the chief barrier to multiple shift REAC operation without a plugboard.

It has been possible on over 50 per cent of the recent REAC problems to prepare some sort of hand of I.B.M. calculated check to help verify the operation of the REAC.

As mentioned before, this is considered to be extremely important, and it is hoped that all problems will be prepared in this manner soon. In several cases, a given configuration of computer components failed to give a satisfactory solution either due to instability (Section 4a) or cumulative errors. In such cases the problem has been withdrawn for a complete revision.

#### (10a) CALIBRATION OF POTENTIOMETERS

The calibration of 10 turn potentiometers has occupied a considerable fraction of the scheduled outage time of the REAC. A device has been built to allow checking to be performed with a minimum of interferences with problem solution. (Fig. 24). The test potentiometer and the standard potentiometer are coupled to opposite ends of a common shaft. This shaft is driven from the chart drive of the Esterline Angus Recorders through a pair of bevel gears and a clutch. A regulated, isolated, 200 volt power supply is used to energize the two potentiometers according to Fig. 25.

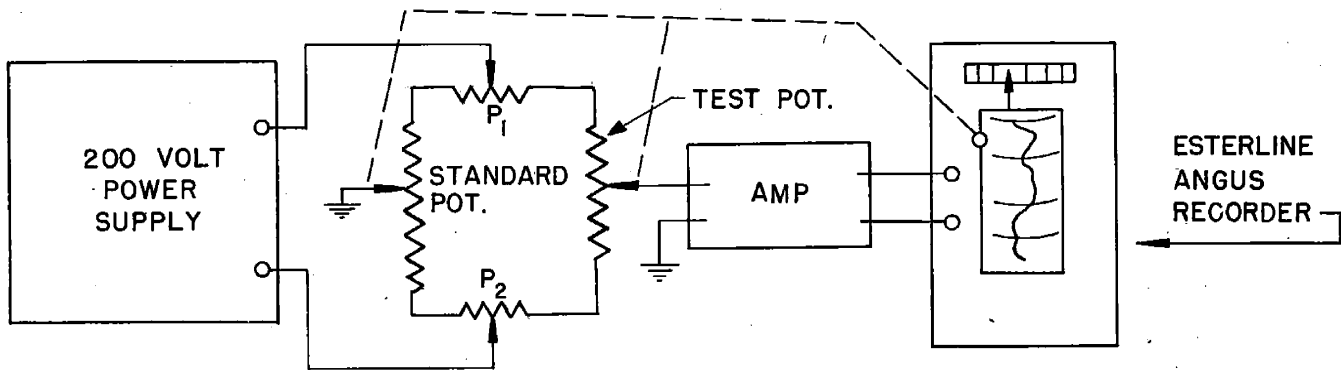


Figure 25. Potentiometer Calibration Circuit.

Potentiometers  $P_1$ , and  $P_2$  allow the system to be balanced at the ends of the 10 turn winding. They are calibrated to give the external end coil resistance in ohms required to trim the test potentiometer (Section 2b).

We were fortunate to find a 10 turn potentiometer with a maximum departure from linearity of  $\pm 0.013$  per cent. This unit was calibrated on a continuous potentiometer checker designed and constructed by Reeves which is good to  $\pm 0.003$  per cent.

It is possible to make a quick check of servo potentiometer mechanical adjustment and linearity by the method of Fig. 26. An indication of the errors to be expected in multiplication is obtained but the servo dial indications are not verified.

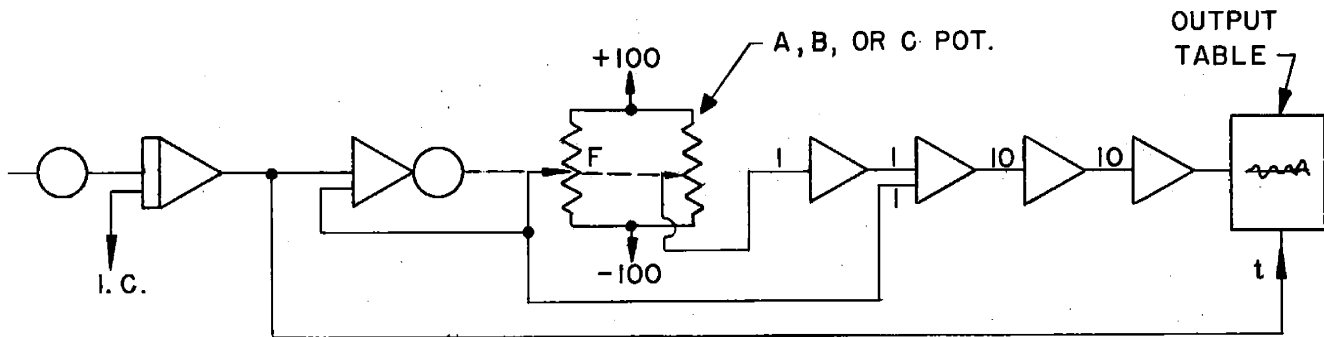


Figure 26. Servo Potentiometer Calibration.

(10b) CONDENSER CALIBRATION

Several elaborate schemes for measuring the capacity of integrating condensers were considered, all of which depended on a measurement of resistance coupled with a measurement of time. However, since the only other link between the computer and the real time of the outside world is through the chart drive mechanism of the Esterline Angus output recorders, it was decided to use the comparison method of Figure 27 to determine the ratio of the various condensers to an arbitrarily chosen standard.

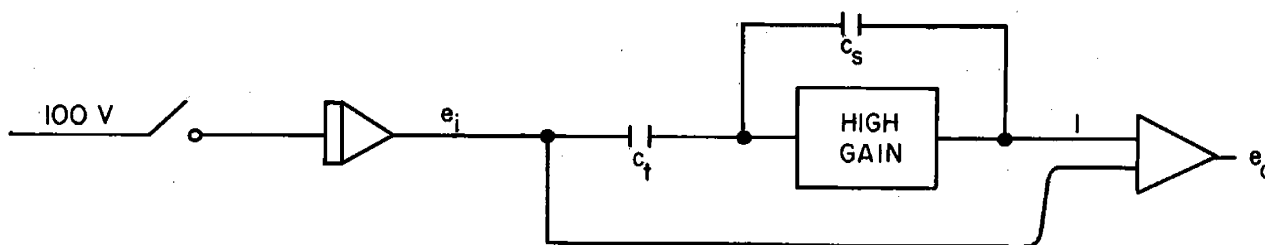


Figure 27. Comparison of Integrating Condensers

The two condensers are initially discharged. The high gain amplifier with input and feedback condensers then behaves like a summing amplifier. When  $e_i = 100$  volts,  $e_o$  in volts gives the approximate percent difference between the two condensers. The test meter switched to one volt full scale becomes a satisfactory indicator. All of the condensers in the REAC (about 200) were individually compared with one condenser. The deviations were tabulated and then another condenser which fell in the middle of the range of deviations was chosen as the standard "REAC microfarad". Thus the individual deviations could be balanced to give groups of 9 and 10 "REAC microfarads" all within  $\pm 0.1$  per cent, without any extra trimming being required. Enough individual "REAC microfarads" were found to allow the scale factors of all integrators to be adjusted to agree within  $\pm 0.1$  per cent and also to make available seven additional "REAC microfarads" ( $\pm 0.1$  per cent) on the patch bay.

All condensers tested were the Western Electric Type D 161270. Although several different manufacturing lots were represented, all capacities fell within a band 0.4 per cent wide.

(10c) MISCELLANEOUS CALIBRATION AND MAINTENANCE AIDS

A 40 turn Helipot with a special large vernier dial has been installed in the REAC panel and wired to jacks. The guaranteed linearity is  $\pm 0.03$  per cent. The absolute value of a computing resistance in the REAC is of secondary importance; ratios of resistance determine the scale factors in all cases. Thus, the 40 turn standard potentiometer may be used to check scale factor potentiometers, servo dial indications, and summing and inverting amplifier gains. The actual ratio of the standard is known to  $\pm 0.01$  per cent at 80 points. These are used when maximum accuracy is required.

Trouble shooting on REAC amplifiers has been facilitated by providing an external test position with all necessary operating voltages wired from the REAC power supplies to an appropriate plug. A pair of extension harnesses allowing a computing amplifier to be examined while it is connected with its regular circuits have been particularly useful in locating intermittent failures.

Although trouble from grid current is significantly reduced when automatic drift correction is used (Section 1b), it is still important to have low grid current tubes for "manual" operation. A simple method has been used for measuring grid current.

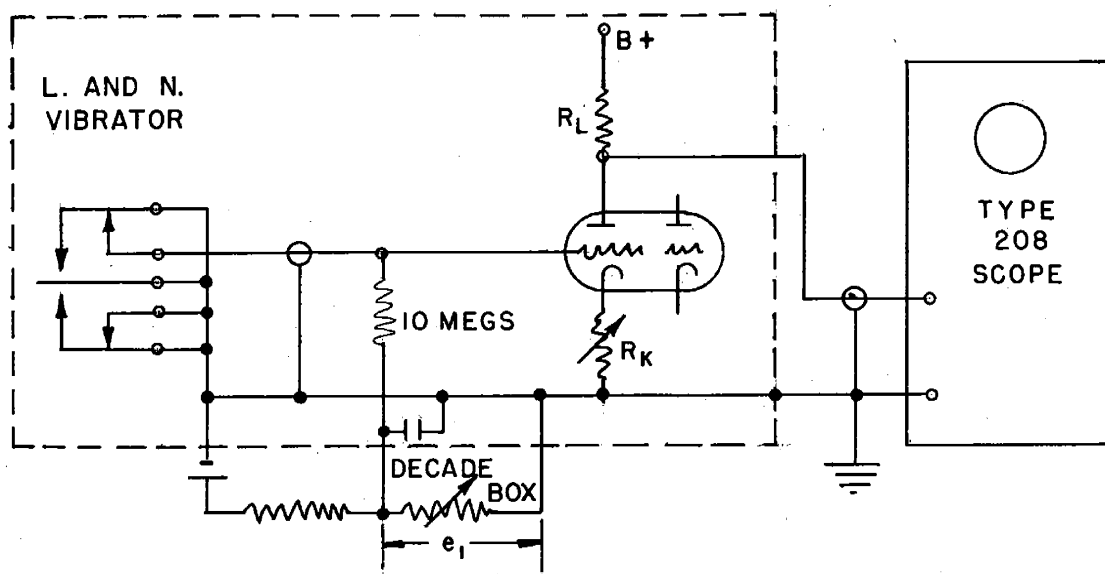


Figure 28. Grid Current Tester

The values of  $B+$  and  $R_L$  (Fig. 28) are first picked to match the equivalent values in the computer amplifier and then  $R_K$  is adjusted to give the corresponding operating plate current. If the decade box is adjusted to null the 60 cycle square wave on the scope, the grid current is given by  $i_g = e_1 \times 10^{-7}$  where  $e_1$  is in volts and  $i_g$  is in amperes. Grid current of  $10^{-11}$  amperes can be detected. The operating point of the input tube of the REAC amplifiers is close to the floating grid potential where electron and gas ion grid currents are balanced.

#### (11a) OUTAGE TIME AND OPERATING COSTS

A daily record of REAC operation has been kept since 20 June 1949. Over this period the total outage time for modification, calibration and maintenance has been about 20 per cent. Of this 20 per cent, less than five per cent was unscheduled outage time resulting from machine breakdown.

Tube failures in the first estimated 2500 hours of operation amounted to about 20 out of the total 353. A test of all tubes at 2500 hours uncovered 68 tubes that were questionable or weak. A tube serial number system is being set up to allow accurate records to be kept.

Component failures have been extremely few. Explosion of an electrolytic filter condenser in the Sorensen d-c filament supply occurred on two occasions in the first month of operation. A new set of condensers (with double rating) were mounted in an external chassis and have operated satisfactorily. One servo amplifier output transformer and one or two computing amplifier by pass condensers have been the only other component failures. A few of the servo potentiometers have become somewhat noisy due to a groove worn in the slider. Repair of such potentiometer defects seems to be feasible.