EXPECTED COVERAGE WHEN ALL BOMBS ARE AIMED
AT THE CENTER OF THE TARGET

H. H. Germond

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EXPECTED COVERAGE WHEN ALL BOMBS ARE AIMED

AT THE CENTER OF THE TARGET

Summary

If N bombs of individual lethal area a are independently aimed with mean radial error MRE at the center of a circular target of area A_t, the expected proportional coverage is

$$P = 1 - \frac{1}{x} \left\{ \left[-\mathrm{Ei}(-\rho e^{-x}) \right] - \left[-\mathrm{Ei}(-\rho) \right] \right\}$$

where $x = A_t / 4(MRE)^2$, $\rho = Na/4(MRE)^2$, and Ei denotes the exponential integral.

Situation

This deals with a circular target, with a great number (N) of bombs independently aimed at the center of the target, and with these bombs subject to a circular Gaussian distribution law. In that case the bomb density at a distance r from the center of the target is given by

$$\frac{N}{2\pi\sigma^2} e^{-r^2/2\sigma^2}$$

where σ is the standard deviation of aim. If the lethal area of a single bomb is a, then the probability of a point at a distance r from the center not being within the lethal area of any of the bombs is given by

$$\left[1 - \frac{a}{2\pi\sigma^2} e^{-r^2/2\sigma^2} \right]^N \longrightarrow \exp\left[-\frac{Na}{2\pi\sigma^2} e^{-r^2/2\sigma^2} \right] \quad .$$

The expected proportional coverage for a target of radius R is given by the integral

$$P = \frac{E \text{ (Coverage)}}{\pi R^2}$$

$$= \frac{2}{R^2} \int_0^R \left[1 - \exp \left(- \frac{Na}{2\pi\sigma^2} e^{-r^2/2\sigma^2} \right) \right] r\,dr \quad .$$

The substitution

$$t = \frac{Na}{2\pi\sigma^2} e^{-r^2/2\sigma^2}$$

reduces this to

$$P = 1 - \frac{1}{x} \int_{\rho e^{-x}}^{\rho} t^{-1} e^{-t}\,dt$$

where

$$\rho = \frac{Na}{2\pi\sigma^2}$$

and

$$x = R^2/2\sigma^2 \quad .$$

The integral may be recognized as an incomplete gamma function, or as the exponential integral. Using the notation of the latter,

$$- Ei(-y) = \int_y^\infty t^{-1} e^{-t} \, dt,$$

and

$$P = 1 - \frac{1}{x} \left\{ \left[- Ei(-pe^{-x}) \right] - \left[-Ei(-\rho) \right] \right\} \quad .$$

When the product ρe^{-x} is very small this takes on the approximate form

$$P = \frac{1}{x} \left\{ .577216 + \log_e \rho - \rho e^{-x} + \cdots + \left[-Ei(-\rho) \right] \right\} \quad .$$

For large values of ρ, $-Ei(-\rho)$ approaches 0.

If instead of N, the number of bombs and a, the lethal area per bomb, one writes the bomb tonnage W and the lethal area, g, per ton, then

$$\rho = Wg/2\pi\sigma^2 \quad .$$

If the expression be written in terms of the mean radial error (MRE),

$$\rho = Na/4(MRE)^2$$

$$= Wg/4(MRE)^2$$

and

$$x = \pi R^2/4(MRE)^2$$

$$= A_t/4(MRE)^2$$

where A_t is the area of the target.

APPENDIX

Illustration

If the area of the target is 40 square miles and the mean radial error is 2 square miles, then

$$\rho = Wg/4(2)^2 = Wg/16$$

$$x = 40/16 = 2.5,$$

$$e^{-x} = 0.082085,$$

and

$$P = 1 - 0.4 \left\{ \left[-Ei(-0.082055\ Wg/16) \right] - \left[-Ei(Wg/16) \right] \right\}$$

Figure 1 shows the relationship between the expected coverage and the potential coverage (Na or Wg) for a mean radial error of 2 miles and for target areas of 40, 10, and 5 square miles each. From this graph one may estimate the potential bomb coverage needed to achieve a given percentage coverage for each of these targets. These have been so estimated for coverages of 30, 50, 70, and 80 percent. These figures are given in Table I. This table also gives the total bomb coverages in these cases on the assumption that there are 4 of the largest targets, 52 of the intermediate targets, and 43 of the smallest targets. In these figures it is assumed that the bombs used are of such characteristics that the lethal area is 0.001 square miles per ton of bomb.

Comparison of these figures with those presented in the Report RM-170 "Bomb Requirements for Three Target Systems", (B. Brown, M. Halperin, and A. Mood — June 14, 1949 — Confidential) shows these requirements are all greater than for the case there presented.
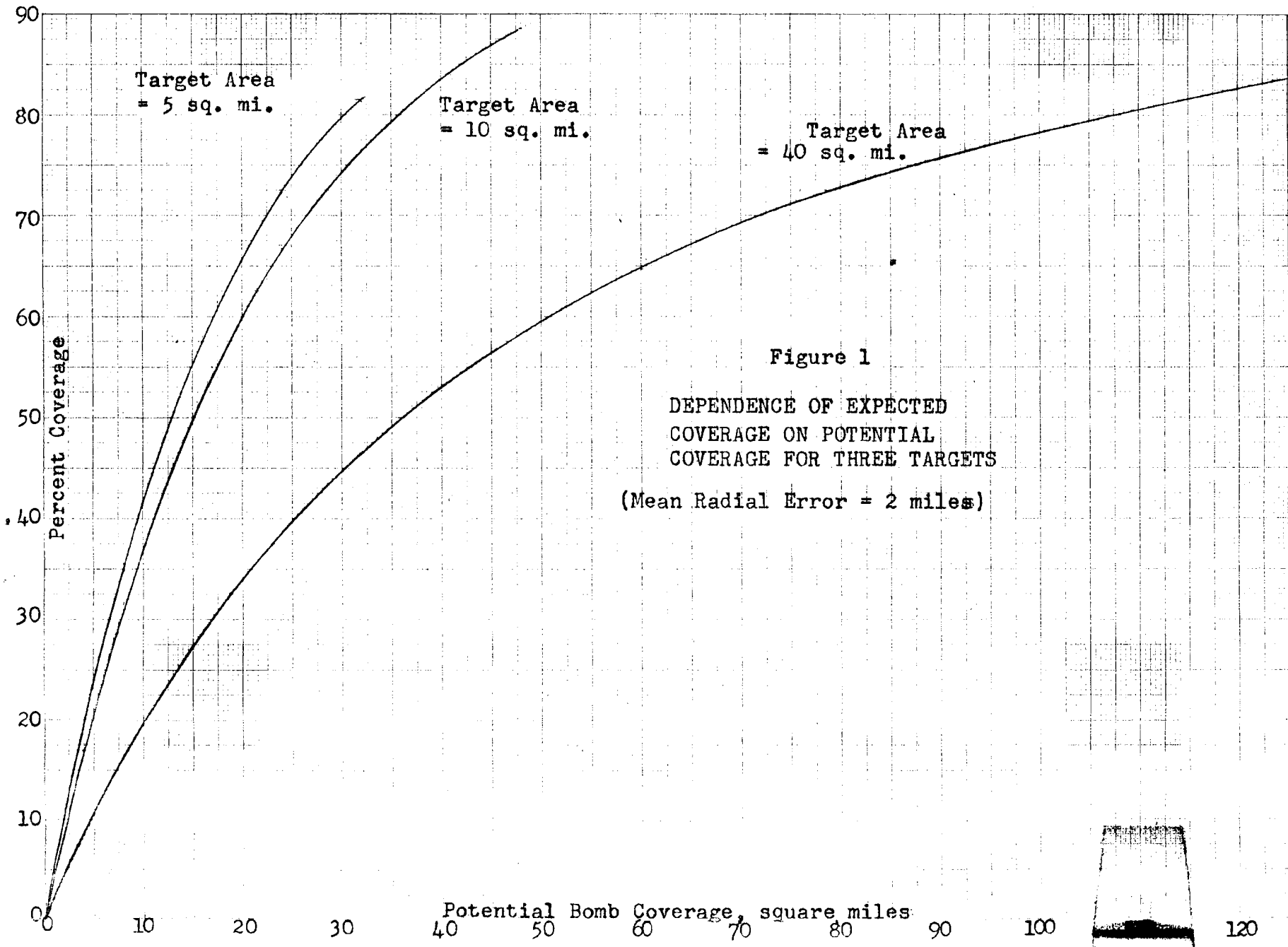
Target Area
= 5 sq. mi.

Target Area
= 10 sq. mi.

Target Area
= 40 sq. mi.

Figure 1

DEPENDENCE OF EXPECTED
COVERAGE ON POTENTIAL
COVERAGE FOR THREE TARGETS

(Mean Radial Error = 2 miles)

Percent Coverage

Potential Bomb Coverage, square miles

Table I

Tons of Bombs Required per Target, and Total Tonnage,
for Various Expected Coverage Percentages.

Potential Coverage of Bombs: 0.001 sq.mi. per ton

Mean Radial Error: 2 miles.

No.	Area, sq. mi.	30 %	50 %	70 %	80 %
4	40	16,800	36,000	71,700	112,300
52	10	7,700	15,200	26,400	35,700
43	5	6,600	12,800	22,500	30,300
Totals		751,000	1,480,000	2,630,000	3,610,000
(From RM-170)		735,000	1,420,000	2,470,000	3,300,000