TRAVELING CROSSFLOW INSTABILITY FOR HIFiRE-5 IN A QUIET HYPersonic WIND TUNNEL (POSTPRINT)

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JUNE 2013

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# TRAVELING CROSSFLOW INSTABILITY FOR HIFiRE-5 IN A QUIET HYPERSONIC WIND TUNNEL (POSTPRINT)

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A scale model of the 2:1 elliptic cone HIFiRE-5 flight vehicle was used to investigate the traveling crossflow instability at Mach 6 in Purdue University’s Mach-6 quiet wind tunnel. Traveling crossflow waves were measured with surface-mounted pressure sensors. The crossflow instability phase speed and wave angle were calculated from the cross spectra of three surface-mounted pressure sensors. Both quantities show good agreement with computational values from about 30 to 50 kHz. Repeated runs at the same initial condition show excellent repeatability in traveling crossflow wave properties, and give an estimate of the Experimental uncertainty associated with this technique. Additionally, autobispectral analysis showed the onset and development of moderate nonlinear quadratic phase-locking prior to transition, but not for the peak traveling crossflow wave. The bicoherence achieved only moderate values. No traveling crossflow waves were observed when freestream noise levels were intentionally elevated, but transition occurred for a much lower Reynolds number. It appears that the traveling crossflow instability is not the primary transition mechanism in the noisy flow of Purdue's Mach 6 wind tunnel.
Traveling Crossflow Instability for HIFiRE-5 in a Quiet Hypersonic Wind Tunnel

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Abstract

A scale model of the 2:1 elliptic cone HIFiRE-5 flight vehicle was used to investigate the traveling crossflow instability at Mach 6 in Purdue University's Mach-6 quiet wind tunnel. Traveling crossflow waves were measured with surface-mounted pressure sensors. The crossflow instability phase speed and wave angle were calculated from the cross spectra of three surface-mounted pressure sensors. Both quantities show good agreement with computational values from about 30-50 kHz. Repeated runs at the same initial condition show excellent repeatability in traveling crossflow wave properties, and give an estimate of the experimental uncertainty associated with this technique. Additionally, autobispectral analysis showed the onset and development of moderate nonlinear quadratic phase-locking prior to transition, but not for the peak traveling crossflow wave. The bicoherence achieved only moderate values. No traveling crossflow waves were observed when freestream noise levels were intentionally elevated, but transition occurred for a much lower Reynolds number. It appears that the traveling crossflow instability is not the primary transition mechanism in the noisy flow of Purdue’s Mach 6 wind tunnel.

I. Introduction

The Hypersonic International Flight Research Experimentation (HIFiRE) program is a hypersonic flight test program jointly executed by the Air Force Research Laboratory (AFRL) and the Australian Defence Science and Technology Organization (DSTO). The purpose of this research is to develop and validate technologies critical to the development of next generation hypersonic aerospace systems. Candidate technology areas include, but are not limited to, propulsion, propulsion-airframe integration, aerodynamics and aerothermodynamics, high temperature materials and structures, thermal management strategies, guidance, navigation, and control, sensors, and system components. The HIFiRE program consists of extensive ground tests and computation focused on specific hypersonic flight technologies. Each technology program culminates in a flight test. HIFiRE-5 is the second of two flights in the HIFiRE manifest focused on boundary layer transition. The HIFiRE-1 program created an extensive knowledge base regarding transition on axisymmetric bodies that has been summarized in numerous prior publications. The HIFiRE-5 flight is devoted to measuring transition on a three-dimensional (3D) body.

Extended hypersonic flight with lifting configurations requires improved understanding and prediction of 3D transition. Transition on 3D configurations embodies several phenomena not encountered on axisymmetric configurations like HIFiRE-1, including leading-edge or attachment-line transition and crossflow instabilities (including crossflow interactions with other instability mechanisms shared with axisymmetric flow configurations such as first and second mode instabilities). Very limited hypersonic flight data exist for any of these phenomena. The need for a better understanding of 3D transition motivates the HIFiRE-5 experiment.

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Accurately simulating flight conditions in a pre-flight, ground-test environment is very challenging. One often overlooked issue is that of tunnel noise. Conventional supersonic and hypersonic wind tunnels are characterized by high levels of freestream noise. This noise takes the form of eddy Mach wave acoustic energy radiated from the turbulent wall boundary layer. Root-mean-square pitot pressure fluctuations are typically on the order of 0.5-1.0% of the mean for conventional hypersonic tunnels. Reference 16 summarizes noise measurements from many supersonic and hypersonic wind tunnels for Mach numbers ranging from 2 to 24. Flight noise levels, however, can be an order of magnitude lower than those experienced in conventional hypersonic ground facilities.

Elevated noise levels have been shown to decrease the transition Reynolds number by up to an order of magnitude and even change or bypass the normal parametric trends in transition. Reference 17 presents an extensive review of the effect of tunnel noise on transition. References 8, 21 and 22 report an effect of freestream noise on roughness-induced transition. The high levels of noise in conventional tunnels introduce uncertainty in using transition data from conventional facilities to predict transition on flight vehicles.

I.A. Vehicle Description

The HIFiRE-5 configuration consists of a payload mounted atop an S-30 first stage and Improved Orion second stage motor, shown in Figure 1. The term payload refers to all test equipment mounted to the second stage booster, including the instrumented test article and additional control and support sections situated between the test article and the second stage motor. The test article consists of a blunt-nosed elliptic cone of 2:1 ellipticity, 0.86 meters in length.

The elliptic cone configuration was chosen as the test-article geometry based on extensive previous testing and analysis on elliptic cones.26–33 This prior work demonstrated that the 2:1 elliptic cone would generate significant crossflow instability at the expected flight conditions. The 2:1 elliptic cone configuration also possesses ample internal volume for sensors and instrumentation. In order to exploit this prior body of work and expedite configuration development, the 2:1 elliptical geometry was selected as the HIFiRE-5 test article. The half-angle of the elliptic cone test article in the minor axis (x–y) plane is 7.00°, and 13.80° in the major axis, where x is the streamwise direction and y is the spanwise direction. The nose tip cross-section in the minor axis is a 2.5 mm radius circular arc, tangent to the cone ray describing the minor axis, and retains a 2:1 elliptical cross-section to the stagnation point.

II. Traveling Crossflow Instability

For 3D vehicle configurations with significant crossflow, the crossflow instability can become the dominant path to boundary layer transition. Both stationary and traveling modes are possible with stationary vortices nearly aligned with the inviscid streamline direction, and traveling waves inclined at a steep angle to the inviscid streamlines. The wave vector of the most unstable traveling mode has a spanwise component opposite to the direction of the crossflow.34 For incompressible flows the stationary modes are typically dominant in low noise environments of flight and “quiet” wind tunnels, while the traveling modes tend to dominate in conventional tunnels.35 The stationary mode is thought to be seeded by natural surface roughness, while the traveling mode is generated by vortical disturbances in the freestream which are entrained in the boundary

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layer.\textsuperscript{36} Saric\textsuperscript{35,37} suggested that the same behavior should be seen for compressible flows.

Considerable effort has been given to crossflow instabilities for incompressible flow.\textsuperscript{36,38–40} Saric and Reed\textsuperscript{35} detail the pertinence of nonlinear effects when predicting crossflow instability growth and transition for incompressible flow. It has been well established that stationary modes, after a period of linear growth, saturate nonlinearly and give rise to secondary instabilities that then rapidly lead to transition (see, for instance, Reference 41). Most of the extant crossflow work has focused on stationary modes, since this is expected to be the dominant crossflow instability in the low-noise flight environment.

Despite much effort to understand the crossflow instability, little of it has focused on the hypersonic regime. King\textsuperscript{42} made measurements of crossflow-dominated transition at Mach 3.5 in quiet and noisy flow. Poggie and Kimmel\textsuperscript{43} measured traveling crossflow waves at Mach 8 on an elliptic cone with hot films, but were not able to get good agreement with computational results. They suggest that the poor agreement was due to the limitations of the computations. Malik\textsuperscript{15} detailed results from a crossflow-dominated hypersonic flight test including stability computations. Choudhari\textsuperscript{44} details stability computations for HiFiRE-5 for both flight and wind tunnel conditions. Swanson\textsuperscript{45} measured stationary crossflow vortices at Mach 6 in a quiet wind tunnel. Reference 46 describes previous experiments for the HiFiRE-5 geometry that primarily focused on stationary crossflow modes in the BAM6QT. Traveling crossflow waves were also detected, but were not the main focus of the previous studies.

Although the highly nonlinear development of the crossflow instability modes has been well-documented for incompressible flows, the contribution of nonlinear processes at hypersonic Mach numbers is not well understood. The current experiments attempt to verify the presence of the traveling crossflow instability as well as help establish the extent of nonlinear interactions for the traveling crossflow instability in the hypersonic regime. Experimental crossflow instability properties are compared to computations to verify that the measured disturbances are crossflow and not some other instability mechanism.

II.A. Model Description

The model, shown in Figure 2, is a 38.1\% scale model of the flight vehicle. It includes the vehicle features from the nose to the payload/transition-section interface. The model is 328.1 mm long and has a base diameter of 164.6 mm in the major-axis direction and maintains 2:1 ellipticity from the base to the tip. The 76.2 mm long nosetip is fabricated from 15-5 stainless steel, followed by a frustum made of 7075T6 aluminum. For the present experiments, much of the model acreage was spray-painted black. The paint was left on the model after previous fluorescent oil flow visualization. The edges of the paint were sanded with fine-grit sand paper to more smoothly transition the model surface from the bare aluminum finish to the paint layer. A more complete description of the model can be found in Reference 47.

![Figure 2: Wind tunnel model](image)

III. Experimental Overview

Experiments were performed in Purdue University’s Boeing AFOSR /Mach-6 Quiet Tunnel (BAM6QT). Quiet flow was realized for freestream Reynolds numbers up to \(12.2 \times 10^6\)/m. The Purdue tunnel achieves...
quiet noise levels by maintaining a laminar boundary layer on the tunnel walls. A laminar boundary layer is achieved by removing the nozzle boundary layer just upstream of the throat via a bleed suction system. A new, laminar boundary layer then begins near the nozzle throat. The boundary layer is kept laminar by maintaining a highly-polished nozzle wall to reduce roughness effects. The divergence of the nozzle is intentionally very gradual to mitigate the centrifugal Görler instability on the tunnel walls.\textsuperscript{48} The experimental results presented in this paper were obtained with the model at 0 degrees angle of attack and yaw.

Kulite XCQ-062-15A and XCE-062-15A pressure transducers with A screens were mounted flush with the model surface to detect traveling crossflow waves. The Kulite sensors are mechanically stopped at about 1 bar so that they can survive the full stagnation pressure in the BAM6QT and still maintain the sensitivity of a 1 bar full-scale sensor. These sensors typically have flat frequency response up to about 30-40\% of their roughly 270-285 kHz resonant frequency.\textsuperscript{49}

For the current experiments, five instrumentation holes were available in the model. Table 1 shows the locations of the instrumentation holes. Figure 3 shows a sketch of the model and sensor holes. The holes were located in a region of the model that had previously shown strong stationary crossflow vortices.\textsuperscript{36, 47, 50} Although there were five holes, there were only four available pressure sensors for most of the quiet experiments, so some repositioning of the sensors was necessary to gather data from all five sensor hole locations.

<table>
<thead>
<tr>
<th>Hole Number</th>
<th>x (mm)</th>
<th>y (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>310.14</td>
<td>41.21</td>
</tr>
<tr>
<td>2</td>
<td>312.56</td>
<td>45.09</td>
</tr>
<tr>
<td>3</td>
<td>312.56</td>
<td>39.78</td>
</tr>
<tr>
<td>4</td>
<td>315.48</td>
<td>43.04</td>
</tr>
<tr>
<td>5</td>
<td>318.07</td>
<td>39.78</td>
</tr>
</tbody>
</table>

Table 1: Instrumentation hole locations and notation

![Figure 3: Schematic of instrumentation holes](image)

IV. Reynolds Number Sweep

IV.A. Spectra

Sixteen unique runs comprised a Reynolds number sweep from \(Re=5.9-12.2\times10^6/m\) with a fine gradation in Reynolds number (changes of about 35 kPa in initial stagnation pressure). This allowed the development and evolution of the traveling crossflow waves to be studied in great detail. Figure 4 shows power spectral densities (PSD) for some of the Reynolds numbers tested for sensors 1, 3, and 5. A sensor was also installed in hole 4, but at some unknown point during the experiments, it came loose from the hole. Thus, data from

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sensor 4 are regarded as unreliable and are not shown.

Data were sampled at 5 MHz. The time traces at the conditions of interest were divided into 243 windowed segments of 4096 samples, meaning that each PSD is for 199 ms of data. For the BAM6QT, the time it takes for the expansion wave to traverse the length of the driver tube, reflect, and return is approximately 200 ms. Thus, each PSD is computed for quasi-steady conditions with at most one 5-6% reduction in Reynolds number due to the reflecting expansion wave. Freestream Reynolds numbers were estimated using the method developed in Reference 51. Uncertainty in the calculated freestream Reynolds number is largely due to 2 s of massively separated nozzle wall boundary layer just subsequent to tunnel startup.

Figure 4: PSDs for quiet Reynolds number sweep, sensors 1, 3, and 5

For all three sensor locations, the spectra at Re=6.3×10^6/m appear to be laminar without measurable disturbances. For each sensor, as Re is increased, a small peak in the spectra centered near 45 kHz begins to grow above the laminar spectra, which is essentially the noise floor. The magnitude of the peak centered at
45 kHz continues to increase with Re until apparent transition onset is evidenced by peak broadening and elevated power at all frequencies. As Re continues to increase, the main 45 kHz disturbance peak begins to be overwhelmed by power at all other frequencies as the boundary layer begins to transition. For the highest Reynolds numbers, the boundary layer is fully turbulent, and little or no evidence of the traveling crossflow waves remains.

For sensors 1 and 3, the 45 kHz traveling wave is not seen until Re=7.2-7.6×10^6/m. For sensor 5, the disturbance can barely be seen for Re=6.8×10^6/m. For sensor 1, the boundary layer has not completely transitioned at the highest Reynolds number. The spectra for Re=11.1 and 12.2×10^6/m do not quite fall on top of each other. For sensors 3 and 5, however, the spectra for Re=11.1 and 12.2×10^6/m fall nearly on top of each other, indicating that the boundary layer at sensors 3 and 5 is turbulent for Re≥11.1×10^6/m. Additionally, the unit Reynolds numbers for which spectral broadening starts to be significant decreases with increasing downstream sensor location.

Computations for Re=8.3×10^6/m predicted a peak N factor of 11.5 for traveling crossflow waves near the sensor locations at 40 kHz, in good agreement with the experimental peak shown in Figure 4. For reference, an N-factor contour plot, reproduced from Reference 44, for traveling crossflow at Re=9.8×10^6/m is shown in Figure 5. The computations for Re=8.1×10^6/m also show that the pressure eigenfunction for traveling crossflow waves near the sensor locations has its peak amplitude away from the surface. However, the amplitude at the surface is still around 73% of the peak amplitude. Thus, conditions are not unfavorable for detection of traveling crossflow waves with surface-mounted pressure sensors. All of these data are consistent with traveling 45 kHz crossflow instability waves that grow in amplitude for increasing Reynolds number.

![Figure 5: N-factor contour for traveling crossflow for Re=9.8×10^6/m. Reproduced from Reference 44. Sensor locations and notation added.](image)

### IV.B. Calculation of Crossflow Wave Properties

The utilization of four sensors allows the calculation of disturbance phase speed and wave angle as a function of frequency using the cross spectrum. Although the more general case allows four sensors, the minimum number of sensors needed to calculate the quantities of interest is three, since two unique sensor pairs are needed. The method of Reference 53 as used in Reference 43 was followed to calculate the wave angle and phase speed. The autospectrum of a sensor’s signal is given by

\[
G(f) = 2 \lim_{T \to \infty} (1/T) E[|\hat{s}(f, T)|^2]
\]  

Here, the Fourier transform of a measured signal \( s(t) \) is \( \hat{s}(f) \), and the expected value operator is \( E[] \).

The cross spectrum of two signals is similarly

\[
S_{12}(f) = \lim_{T \to \infty} (1/T) E[\hat{s}_1^*(f, T)\hat{s}_2(f, T)]
\]  

where \* denotes a complex conjugate. A convenient, real-valued method of measuring the amplitude of the cross spectrum is the coherence, given by

\[
\text{coherence} = \frac{S_{12}(f)}{G_1(f)G_2(f)}
\]
\[ \gamma^2(f) = \frac{|S_{12}(f)|^2}{S_{11}(f)S_{22}(f)} \]  

(3)

The coherence is essentially a frequency-dependent cross correlation of the signals. The normalization factor is the maximum value that the cross spectrum can achieve. Thus, normalizing by this value yields coherence values from 0 to 1, with 0 meaning no correlation between the signals and 1 signifying perfect correlation.

The phase spectrum can then be found by

\[ \Theta(f) = \arctan \left( \frac{\Im[S_{12}]}{\Re[S_{12}]} \right) \]  

(4)

where \(\Im[]\) and \(\Re[]\) represent the real and imaginary components, respectively.

The time delay associated with the phase spectrum is given by

\[ \tau(f) = \frac{\Theta(f)}{2\pi f} \]  

(5)

An arbitrary configuration of four probes and incident planar waves is sketched in Figure 6a. Rotating the coordinate system by \(\Psi\), the angle between the wave propagation direction and the \(\epsilon\) axis, transforms the arrangement to that shown in Figure 6b. Here, \(\epsilon\) and \(\eta\) are the axial and circumferential surface coordinates projected onto the \(x - y\) plane. This method is valid for regions of small local surface curvature.

![Diagram of sensor layout and coordinate system](image-url)

(a) Arbitrary sensor layout with planar wave  
(b) Rotated coordinate system

Figure 6: Notional sensor layout for cross-spectral analysis

The relation between points in the original coordinate system \((\epsilon, \eta)\) and the rotated system \((\epsilon', \eta')\) is given by

\[ \eta' = \eta \cos \Psi - \epsilon \sin \Psi \]  

(6)

\[ \epsilon' = \eta \sin \Psi + \epsilon \cos \Psi \]  

(7)

The time delay between the arrival of a particular phase surface at sensors \(a\) and \(b\) is given by

\[ c_r \tau_{12} = \epsilon'_{1} - \epsilon'_{2} \]  

(8)

where \(c_r\) is the phase speed and \(\tau_{12}\) is the time delay between a particular phase surface arriving at sensors 1 and 2. Two unique sensor pair sets \(ab\) and \(cd\) are chosen. Here, \(a, b, c,\) and \(d\) can be any of the four sensors, subject to the constraints that \(a \neq b\) and \(c \neq d\). Using Equations 7 and 8 for both sensor pairs and solving the resultant system of equations, the propagation angle, \(\Psi\) and phase speed, \(c_r\) can be found from

\[ \Psi = \arctan \left( \frac{\tau_{ab}(\epsilon_d - \epsilon_c) - \tau_{cd}(\epsilon_b - \epsilon_a)}{-\tau_{ab}(\eta_d - \eta_c) + \tau_{cd}(\eta_b - \eta_a)} \right) \]  

(9)

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and

\[ c_r = \frac{\sin \Psi (\eta_a - \eta_b) + \cos \Psi (\epsilon_a - \epsilon_b)}{\tau_{ab}} \]  

(10)

**IV.C. Traveling Crossflow Wave Properties**

Using Equations 9 and 10 and the pressure traces from sensors 1, 3, and 5, the wave angle and phase velocity of the traveling crossflow waves were calculated for \( Re=6.8-9.0 \times 10^6/\text{m} \). Typical plots of PSD, coherence, phase delay, and time delay for \( Re=8.5 \times 10^6/\text{m} \) are shown in Figure 7.

![Figure 7: Typical PSD, \( \gamma^2 \), \( \Theta \), and \( \tau \) for \( Re=8.5 \times 10^6/\text{m} \)]

Figure 8 shows \( \Psi \) and \( c_r \) for \( Re=6.8-9.0 \times 10^6/\text{m} \). Additionally, a few computational data points for traveling crossflow are shown. For portions of the spectra with low coherence (\( \lesssim 0.2 \)), there are little or no correlated disturbances common among the sensors, and derived quantities such as \( \Psi \) and \( c_r \) are meaningless. The coherence was typically greatest for \( 25 \leq f \leq 80 \text{ kHz} \) and insignificant outside this band. Thus, \( \Psi \) and \( c_r \) are shown only for this frequency band.

Here, \( \Psi \) is the direction of propagation of wavefronts with respect to the \( x \) axis. Thus, for a wave angle of \( \Psi=70^\circ \), lines tangent to the wavefronts are inclined 20° with respect to the \( x \) axis, and propagate in an outward direction inclined 20° downstream of the \( y \) axis. As shown in Figure 8a, good agreement among the various Reynolds numbers and with the computational curve is seen for \( 6.8 \leq Re \leq 8.5 \times 10^6/\text{m} \). For the two highest Reynolds numbers shown, \( Re=8.9 \) and \( 9.0 \times 10^6/\text{m} \), the curves diverge significantly from those in good agreement with each other. This attributed to nonlinear processes becoming significant at these higher Reynolds numbers. Evaluation of the cross spectrum assumes linear processes. When nonlinear

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effects become significant, this assumption is no longer valid and the resultant quantities lose significance. The observed nonlinearities are further investigated and discussed in Section VI.

In the range of 30-55 kHz, the experimentally calculated values of $\Psi$ for $Re=6.8-8.5 \times 10^6/m$ match both the values and slope of the computations. Above 55 kHz, the computational values of $\Psi$ depart from the nearly linear trend with frequency exhibited for lower frequencies. Here, the computational values depart from the experimental wave angles by as much as 15%. In the range of 30-55 kHz, the computationally calculated values of $c_r$ vary from 35% at 30 kHz and decrease to about 15% below the experimental values at 55 kHz. For frequencies above 55 kHz, the difference increases to about 25%.

For both $\Psi$ and $c_r$, the curves for $Re=6.8-8.5 \times 10^6/m$ show good agreement among themselves, even though they differ from the computational values. It is somewhat surprising that the wave angles over a range of Reynolds numbers and disturbance frequency agree so well with the computational values, while the experimental $c_r$ is markedly higher than the computation for all frequencies and Reynolds numbers.

The reason for this discrepancy is unknown, but is possibly due to experimental uncertainties such as sensor hole locations and the fact that sensors do not provide point measurements, but rather an integrated value over a finite area. For both $\Psi$ and $c_r$, the curves for $Re=6.8 \times 10^6/m$ show a considerable amount of scatter. This is because the surface pressure signature of the traveling crossflow waves is very small at this low Reynolds number, resulting in low coherence and more scatter in the measured values of $\Psi$ and $c_r$.

Neither $\Psi$ nor $c_r$ seem to exhibit any major dependence on Reynolds number for for $Re=6.8-8.5 \times 10^6/m$. It does appear that $\Psi$ becomes slightly more sensitive to frequency as $Re$ increases. The lack of any significant dependence on Reynolds number is possibly due to the fairly narrow Reynolds number range for which nonlinear processes are absent and for which the crossflow waves are of measureable amplitude, $Re=6.8-8.5 \times 10^6/m$. This is a fairly modest increase in $Re$ of only 24%. If the boundary layer thickness scales with $1/\sqrt{Re}$, this is a decrease in boundary layer thickness of only 10%. If $\Psi$ and $c_r$ scale with the boundary layer thickness, changes in these quantities could be overwhelmed by the uncertainties of the measurements.

V. Repeatibility at $Re=8.1E6/m$

Repeatability was checked with duplicate runs for all Reynolds numbers. For $Re=8.1 \times 10^6/m$, 11 separate runs were performed. The results are shown in Figure 9.

Figure 9a shows PSDs for 11 runs at $Re=8.1 \times 10^6/m$. Excellent agreement is shown for the peak frequency of about 45-47 kHz. However, the root-mean-square (RMS) amplitude of the disturbance varies by about a factor of 2.8. Runs 25, 29, 32, 33, and 36 were completed over several days with various other runs at different conditions in between them. Runs 37-42 were completed consecutively on the same day with a 40-60 minute delay between runs. There is no obvious trend for the RMS amplitudes of the PSDs for runs 25, 29, 32, 33, and 36. Runs 37-42 exhibit a clear trend, however. Each subsequent run shows a marked increase in peak amplitude. Over the span of those 6 runs, the amplitude increased by a factor of about 1.9.

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The exact cause of this apparent amplitude increase is unknown.

It is possible that the change is due to the increasing temperature of the model throughout the course of a day. With each run, the model temperature increases due to convective heating while the air is flowing over the model. It is generally time prohibitive to allow the model to return to room temperature before each subsequent run. The increase in model surface temperature from the beginning of Run 37 to the end of Run 42 is estimated to be on the order of 10-15 K, as measured in Reference 55. It is possible that this temperature increase modifies the calibration of the pressure sensors, causes the model to expand altering the fit of the sensors in the holes and thus the sensor output, or that the increased surface temperature destabilizes the traveling crossflow instability, leading to larger amplitudes.

The Kulite sensors are quoted as being temperature compensated in the range from 299 to 353 K. Per Kulite specifications, in this range, the thermal zero shift is ±1% full scale per 56 K temperature change. The thermal sensitivity change is ±1% per 56 K temperature change. For a 15 K temperature increase, the expected thermal zero shift is about 300 Pa. The calibration slope would change by about 25 Pa/V. The resultant change in the PSD amplitudes from the change in the calibration curve due to a 15 K temperature increase is estimated to be less than 1%. It seems unlikely that shifts in the Kulite calibration due to temperature changes can account for the observed amplitude variations.

It would be somewhat surprising if such modest temperature changes, about 5%, would destabilize the traveling crossflow instability to this degree, a shift in N-factor of about N=ln(2.8/1)=1.0. However, Malik et al.15 found that a large surface temperature increase of about 100 K increased the computed stationary crossflow N-factor by about 10 at Mach 4-5. Scaling these results linearly to the present data suggests an N-factor increase of 1.5.
Although this result suggests that increased surface temperature destabilizes the crossflow instability, the surface temperatures observed by Malik et al. were an order of magnitude larger than those observed during the present experiments. Eppink\textsuperscript{56} found that a small surface temperature increase of 3.9 K at Mach 0.24 increased the N-factor by 0.32. This N-factor change represents an amplitude increase of 38%. The experimentally measured amplitude change was 23%. The findings of Malik and Eppink suggest that the observed PSD amplitude increase in the present experiments may be due to destabilization of the boundary layer by the increased surface temperature. Investigation into this effect is ongoing.

Figure 9b shows values of $\Theta_{31}$, $\Theta_{35}$, and $\Theta_{51}$ for all 11 runs at $Re=8.1\times10^{6}/m$. Here, the subscripted numbers refer to the sensors of interest (e.g. $\Theta_{31}$ refers to the phase difference between the signals for sensors 3 and 1). In this case, the band of curves are colored to correspond to the phase delay between two particular sensors. The standard deviation of each $\Theta$ band can be used to give an estimate of experimental uncertainty. Figures 9c and 9d show values of $\Psi$ and $c_r$, as well as the computational data points, for all 11 runs. As shown, both $\Psi$ and $c_r$ are very repeatable. Excellent agreement with the computation is shown for $\Psi$ for frequencies between 30-55 kHz. The agreement is not as good for higher frequencies, but still within about 15% of the computations. The agreement in $c_r$ is not quite as good, but is still close. Table 2 shows the mean, standard deviation, and range of $\Theta$, $\Psi$, and $c_r$ for the 11 runs. These statistics will be used to quantify uncertainty in the measurements. Despite the large range in PSD amplitude, no such variation is evident in $\Theta$, $\Psi$, or $c_r$. There is also no discernible trend in these variables with increasing model temperature. Evidently, if the surface temperature is destabilizing the boundary layer, it does not impact the frequency, phase speed, or wave angle of the traveling crossflow waves.

The bispectrum will average to 0 if the phase of $X$ is random with respect to the phase of $X_1$ and $X_2$. That is,\[ f_1 \pm f_2 = f_3 \] (11)\[ \angle f_1 \pm \angle f_2 = \angle f_3 \] (12)

where $\angle$ denotes the phase.

The bispectrum is defined as
\[ B(f_1, f_2) = E[X(f_1)X(f_2)X^*(f_1 + f_2)] = \left| \frac{1}{N} \sum_{k=1}^{N} X_k(f_1)X_k(f_2)X_k^*(f_1 + f_2) \right| \] (13)

where $X$ denotes the Fourier Transform of a time series $x(t)$, $X^*$ is the complex conjugate of $X$, and $N$ is the number of windows into which the time series is divided. Using this technique, it becomes clear that the bispectrum will average to 0 if the phase of $X^*(f_1 + f_2)$ is random with respect to the phase of $X(f_1)$ and $X(f_2)$. If it is quadratically coupled, however, the mean, and thus the bispectrum, will be nonzero.

The bicoherence is a normalized version of the bispectrum and quantifies the extent of quadratic phase coupling. The normalizing factor is maximized when $f_1$, $f_2$ and $f_3$ are perfectly phase coupled. The normalizing factor used in this analysis was

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Table 2: Statistics for 11 repeat runs at $Re=8.1\times10^{6}/m$
\[ \frac{1}{N} \sum_{k=1}^{N} |X_k(f_1)X_k(f_2)X_k^*(f_1 + f_2)| \]  

By computing the absolute value before averaging, phase information is removed, effectively setting \( \angle f_1 \), \( \angle f_2 \), and \( \angle f_3 \) identically equal to 0 and enforcing perfect phase coupling for all frequency combinations. The bicoherence,

\[ b^2(f_1, f_2) = \frac{1}{N} \sum_{k=1}^{N} X_k(f_1)X_k(f_2)X_k^*(f_1 + f_2) \]

then assumes a value from 0 to 1, with 0 indicating no nonlinear quadratic phase coupling, and 1 indicating complete quadratic phase coupling. References 40, 57–63 discuss the theoretical foundation and some applications of bispectral analysis.

VI.B. Nonlinear Wave Coupling

The bispectral analysis was applied to the traveling crossflow data. Figure 10 shows contour plots of bicoherence for Re=8.9, 9.0, 9.7, and 9.8×10^6/m for sensor 5. For Reynolds numbers outside this range, the value of the bicoherence everywhere is small, indicating little phase coupling and low levels of nonlinear interactions. For reference, the PSD of the signal is plotted in red above the bicoherence. Units are not given and amplitudes are scaled in each case to fit within the plot area.

In all cases, the range of bicoherence displayed is from 0.20-0.40. Additionally, the bicoherence is symmetric about the \( f_2 = f_1 \) line, demarcated by a black diagonal line. Non-zero values of bicoherence along this line indicate wave self-interaction and the stimulation of harmonic modes. Since the bicoherence is symmetric about the \( f_2 = f_1 \) line, only values below this line are shown. Additionally, the Kulite sensors have a strong resonance at about \( f = 280 \) kHz. Frequency summations greater than 200 kHz are thus not shown.

For Re=8.9×10^6/m, low levels of phase coupling are evident. Disturbances with frequencies ranging from 50-100 kHz show some phase locking with frequencies from 50-90 kHz in a broad sense, including some self-interaction. The peak bicoherence occurs for \((f_1, f_2) = (75, 70) \) kHz, although the PSD does not have an apparent peak at \( f_1 + f_2 = 145 \) kHz. The elevated levels of bicoherence for Re=8.9×10^6/m comport well with the divergence of the \( \Psi \) and \( c_r \) curves for Re=8.9 and 9.0×10^6/m in Figure 8, which were attributed to the onset of nonlinear processes.

As the Reynolds number increases to 9.0×10^6/m, the bicoherence shows interactions for the same frequencies as the Re=8.9×10^6/m case, but with elevated bicoherence. The main interactions appear to be for \( f_1 = 62 \) kHz and \( 62 < f_2 < 138 \) kHz. The corresponding PSD shows slightly elevated power for frequencies from 100-200 kHz, likely due to these broadband nonlinear interactions. This interaction is somewhat surprising since the main traveling crossflow peak is at about 45 kHz, which shows no phase locking with any frequencies.

The Re=9.7×10^6/m condition exhibits the highest levels of bicoherence, though the maximum value is only about 0.42. Significant phase-coupling continues among \( f_1 = 62 \) kHz and \( 62 < f_2 < 138 \) kHz disturbances is evident, contributing to both general spectral broadening as well as a peak in the PSD at 130 kHz. The PSD shows a marked disturbance at 62 kHz distinct from the main traveling crossflow peak. Additionally, a small peak in the PSD at about 7-10 kHz has developed and appears to be phase coupled with several other frequency bands, including the 62 kHz disturbance.

As Re is further increased to Re=9.8×10^6/m, major distortions in the PSD continue, and the bicoherence amplitudes fall significantly. This likely indicates that the flow is transitional, and that phase-coupled disturbance relationships are destroyed by the random phases of the transitional boundary layer. The secondary peak at 62 kHz is more pronounced relative to the main traveling crossflow peak at 45 kHz for Re=9.8×10^6/m. Additionally, significant spectral broadening is observed in the PSD. It appears that the wide bands of nonlinear interactions observed in the bicoherence for Re=9.0 and 9.7×10^6/m transfer energy into the higher frequencies of the spectrum, while the low-frequency interactions, observed primarily for Re=9.7×10^6/m, contribute to the low-frequency spectral broadening.

For higher Reynolds numbers (not shown), the bicoherence levels fall further, indicating a turbulent boundary layer. The significance of the maximum value that the bicoherence achieved, 0.42, is unclear. Kimmel and Kendall\cite{62} cite a bicoherence value of 0.4 as being “high.” They report this as a high value.
Figure 10: Bicoherence and PSD for Re=8.9-9.8×10^6/m for sensor 5
because their signal-to-noise ratio was near unity. The signal-to-noise is considerably better in the present experiments. Chokani\textsuperscript{57} observed bicoherence levels as high as 0.9 for second mode waves.

The lack of any significant nonlinear interactions involving the primary traveling crossflow wave at 45 kHz suggests that the traveling mode may not be the primary instability mechanism responsible for transition under quiet conditions. It may be that, as expected in quiet freestream environments, the stationary mode is ultimately responsible for transition. The cause of the secondary peak that appears in the PSDs for Re=9.7 and 9.8×10\textsuperscript{6}/m is unclear, though it may be a secondary instability of the stationary modes.

### VII. Noisy Flow

The freestream noise level was increased to “conventional” levels and a Reynolds number sweep from Re=0.4-5.5×10\textsuperscript{6}/m was performed. Traveling crossflow waves were expected since they were observed at the lower “quiet” freestream noise level.

Figure 11 shows PSDs for all noisy Reynolds numbers. Two quiet PSDs are also included for reference. The quiet Re=8.3×10\textsuperscript{6}/m condition represents laminar flow with large traveling crossflow waves. The quiet Re=12.4×10\textsuperscript{6}/m case demonstrates a turbulent model boundary layer with quiet freestream flow. A PSD of the sensor output with quiescent air at vacuum, representative of the electronic noise floor, is also shown.

![Figure 11: PSD for noisy flow, Re=0.4-5.5×10\textsuperscript{6}/m, sensor 3](image)

The Re=0.4×10\textsuperscript{6}/m condition represents the lowest Reynolds number at which the BAM6QT can operate. The spectrum for this condition is likely laminar since the power drops quickly from its initial value at 0 kHz to the electronic noise floor for frequencies greater than about 40 kHz. The highest Reynolds number condition, Re=5.5×10\textsuperscript{6}/m appears to be nearly turbulent since it approaches the quiet, Re=12.4×10\textsuperscript{6}/m turbulent spectrum.

Although spectra with elevated freestream noise are shown spanning from fully laminar to fully turbulent conditions, there is no evidence of the prominent 40-50 kHz traveling waves seen with quiet flow. This lack of traveling crossflow waves is surprising.

The initial amplitudes of traveling crossflow waves are determined by vortical disturbances in the incoming flow.\textsuperscript{36} With elevated freestream noise, the freestream Reynolds number is considerably below that for which traveling crossflow waves were measured in quiet flow. It is possible that the traveling crossflow growth rates at these reduced freestream Reynolds numbers are so low that traveling crossflow waves do not
amplify sufficiently to be measured prior to boundary layer transition. Additionally, the freestream vortical disturbances at these reduced Reynolds numbers may be of lower amplitude than for the higher Reynolds number quiet cases, giving the traveling crossflow a lower initial amplitude. The absence of observable traveling crossflow waves in noisy flow while they are prominent in quiet flow is different from what has been observed for the second mode instability on a circular cone. In this case, second mode waves are seen in both noisy and quiet flow at the same Reynolds number in Purdue’s tunnel. However, the amplitude of the second-mode waves is about 2 orders of magnitude lower for quiet flow than for noisy flow.\(^{64}\)

This explanation accounts for the lack of measured traveling crossflow waves, but does not account for the transition mechanism. It seems evident that at this reduced Reynolds number and increased freestream noise, the dominant transition mechanism is not the traveling crossflow instability. In a previous study,\(^{50}\) stationary crossflow vortices were seen in oil flow for noisy flow at higher Reynolds numbers than those in the present experiments, but never in the TSP images. This behavior implies that the stationary crossflow modes in noisy flow do not reach the amplitudes that they do in quiet flow. Under noisy flow, transition is not caused by traveling crossflow instabilities. Under noisy flow, transition appears to be the result of broadband growth of lower-frequency disturbances. The precise nature of the lower-frequency disturbances is uncertain and remains under investigation.

**VIII. Summary and Conclusions**

A traveling crossflow instability was clearly measured for the HIFiRE-5 in quiet flow using surface-mounted pressure sensors. The frequency, phase speed, and wave angle were measured and were all in good agreement with computations. The peak frequency at $Re=8.3\times10^6/m$ shows good repeatability. The amplitudes of the PSDs was shown to increase over the course of a day. This increase is thought to be due to the increasing model surface temperature with each subsequent run. It is possible that the increased surface temperature destabilizes the traveling crossflow instability.

The traveling crossflow waves were observed to undergo a period of what appears to be primarily linear growth over a Reynolds number range from about $6.8-8.5\times10^6/m$ for sensor 5. Bispectral analysis showed some nonlinear interactions for $Re=8.9-9.8\times10^6/m$, but not at frequencies corresponding to the peak traveling crossflow frequency. Higher Reynolds numbers then led to transition onset and turbulent flow. For the freestream Reynolds numbers showing no nonlinear processes, wave angle and phase speed matched the computations well over a range of frequencies. When nonlinear effects became significant, both wave angle and phase speed diverged considerably from the values observed for lower Reynolds numbers. The lack of nonlinear interactions involving the peak traveling crossflow frequency suggests that, although traveling crossflow disturbances are prominent in the surface pressure spectrum, they may not play a dominant role in the transition process.

No traveling crossflow waves were observed in noisy flow. This is thought to be due to a smaller area of the model being unstable to the traveling crossflow instability at the lower Reynolds numbers for which the model boundary layer was laminar with noisy flow. The dominant instability mechanism for low Reynolds number noisy flow is unknown, but analysis is ongoing.

**Acknowledgments**

The authors are grateful to Prof. Steven Schneider of Purdue University for donating time in the BAM6QT for these experiments. Prof. Schneider’s research group was very helpful in supporting these tests. Thanks are also due to Dr. Thomas Juliano for his assistance, providing valuable insight into the experiments and the model based on his previous experience, as well as for several fruitful discussions of the data. Dr. Meelan Choudhari of NASA LaRC performed the computations. This work was supported by Dr. John Schmisseur of AFOSR/RTE.

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