Analyzing the Performance of Lined and Unlined Simplified Cylindrical Cloaks

Jeffrey S. McGuirk*, Peter J. Collins
Air Force Institute of Technology
2950 Hobson Way, WPAFB, OH 45433, Jeffrey.McGuirk@afit.edu

Abstract: The performance of simplified cylindrical cloaks with various material parameters was investigated. The performance metric was the overall scattering width of the cloak with various objects in the hidden region. COMSOL was used to simulate three cloaks with different material parameters to determine the total field in the simulation domain. The fields were then transformed to the far zone and scattering widths were determined. For all cloaks simulated in this effort, a PEC-lined cloak had equal or better performance compared to an unlined cloak no matter what type of object was placed in the hidden region. We also found that as the simplified cloaks' material parameters become more like ideal parameters, the performances of PEC-lined and unlined cloaks converges.

Keywords: Electromagnetic cloaks, electromagnetic scattering

1. Introduction

The use of electromagnetic cloaks presents a new tool for RF designers to use when reducing the scattering cross section of objects. Electromagnetic cloaks are complex anisotropic, spatially varying materials whose constitutive parameters can be determined based on a coordinate transformation [1, 2]. Electromagnetic energy interacts with the cloak such that energy is effectively guided around the cloak's hidden region, emerging from the cloaking structure with the same amplitude and phase as it would have if the energy were propagating in free space. Therefore, a scattering body can be placed within a cloak's hidden region and is effectively "invisible" to the electromagnetic energy.

Due to recent complementary advances in material manufacturing and efficient numerical analysis tools, there has been extensive analysis of cloaks in the academic literature. Ray tracing has shown both a spherical and cylindrical cloak

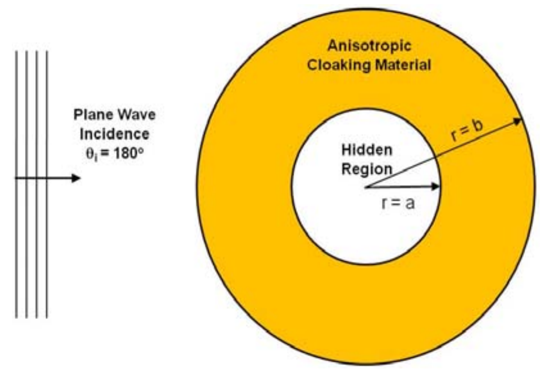

Figure 1. The geometry for a cylindrical cloak.

successfully guide energy around a hidden region with the electromagnetic fields emerging from the cloak unperturbed [3]. A Mie scattering model was used to analyze the interactions of waves with the spherical cloak, with the results clearly showing the cloak behavior was perfect if the cloak had the ideal constitutive parameters [4]. An analysis on the cylindrical cloak which matched boundary conditions at the cloak interfaces resulted in similar findings [5]. Finite element simulations of a variety of cloaking structures and geometries show the fields are guided around each cloaks' hidden region [6-9], while a finite-difference time-domain simulation of the cylindrical cloak found the same [10]. Therefore, there is agreement amongst the community that cloaks whose constitutive parameters are derived using the method in [1] result in a region of space that is completely shielded from electromagnetic radiation, and that the cloaking structure in no way perturbs incident energy. Thus, an object placed in a cloak's hidden region is effectively rendered "invisible."

2. Ideal Cylindrical Cloak

The material parameters for an ideal cylindrical cloak can be found using the method in [1] and are shown below.

Report Documentation Page

Form Approved
OMB No. 0704-0188

1. REPORT DATE **OCT 2005**	2. REPORT TYPE	3. DATES COVERED **00-00-2005 to 00-00-2005**

4. TITLE AND SUBTITLE **Analyzing the Performance of Lined and Unlined Simplified Cylindrical Cloaks**	5a. CONTRACT NUMBER
	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)	5d. PROJECT NUMBER
	5e. TASK NUMBER
	5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **Air Force Institute of Technology,2950 Hobson Way,Wright Patterson AFB,OH,45433**	8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSOR/MONITOR'S ACRONYM(S)
	11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited

13. SUPPLEMENTARY NOTES
Presented at the 2008 COMSOL Conference Boston, 9-11 Oct, Boston, MA

14. ABSTRACT
The performance of simplified cylindrical cloaks with various material parameters was investigated. The performance metric was the overall scattering width of the cloak with various objects in the hidden region. COMSOL was used to simulate three cloaks with different material parameters to determine the total field in the simulation domain. The fields were then transformed to the far zone and scattering widths were determined. For all cloaks simulated in this effort, a PEC-lined cloak had equal or better performance compared to an unlined cloak no matter what type of object was placed in the hidden region. We also found that as the simplified cloaks' material parameters become more like ideal parameters, the performances of PEC-lined and unlined cloaks converges.

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT **unclassified**	b. ABSTRACT **unclassified**	c. THIS PAGE **unclassified**	**Same as Report (SAR)**	**6**	

(1) $\mu_r = \frac{r-a}{r}\;\; \mu_\theta = \frac{r}{r-a}\;\; \mu_z = \frac{r-a}{r}\left(\frac{b}{b-a}\right)^2$

(2) $\varepsilon_r = \frac{r-a}{r}\;\; \varepsilon_\theta = \frac{r}{r-a}\;\; \varepsilon_z = \frac{r-a}{r}\left(\frac{b}{b-a}\right)^2$

Note the permittivity and permeability are identical. The variables *a* and *b* are the radii of the cloak's inner and outer boundaries as shown in Figure 1. For all simulations in this work, a = 1 m and b = 2 m. Also, plane wave incidence is assumed with the wave travelling in the positive *x* direction. Finally, we assume transverse magnetic (TM) waves (the electric field is *z*-directed) with a wavelength of 1 m. Note at $r = a$, μ_r and ε_z equal zero while μ_θ goes to infinity. The singular behavior of the ideal parameter values at $r = a$ makes it difficult to get accurate simulation results because the ideal parameter values cannot be used. Studies have shown significant degradation in cloak performance when even slight variations from ideal conditions are introduced [5,11]. This is evident in Figure 2. The ideal cylindrical cloak parameters were used for this simulation. As discussed in Section 1, ideal parameters should result in perfect cloak performance. However, there is clearly an electric field in the hidden region. Granted, the value seems to be quite close to zero. However, this illustrates how cloaking performance is degraded when the ideal parameter set is slightly changed. The parameter set is changed because COMSOL cannot perform the simulations when the ideal values are used at $r = a$ due to singularities in the governing equations; therefore, error is introduced. This degraded performance not only results in a field being transmitted into the hidden region, there is also a resulting scattered field. This can be seen by transforming the scattered field on the simulation boundary to the far zone. As an example, we placed a perfect electrically conducting (PEC) cylinder of radius $r = a$ in the cloak's hidden region and compared the scattering width to both the scattering width of an uncloaked PEC cylinder and the scattering width for an empty simulation domain. Results are shown in Figure 3. The blue line is the scattering width of an uncloaked PEC cylinder of radius $r = a$. The green line is the scattering width of a perfectly cloaked PEC cylinder of the same radius. The

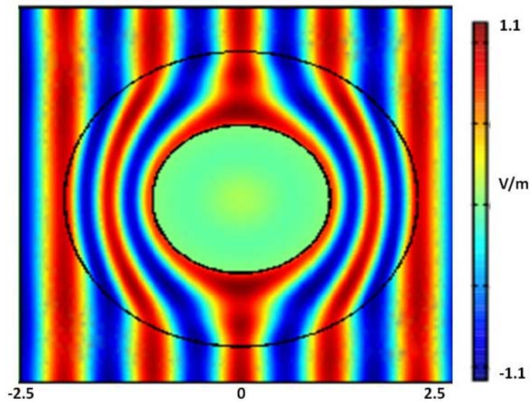

Figure 2. Perfect cloak behavior due to simulation error.

black line is the scattering width of an empty domain. One would expect the scattering width of the “perfectly” cloaked PEC cylinder to be similar to that if no objects were placed in the simulation domain. This is not the case due to the approximation of the material parameters along the boundary $r = a$. It has been shown that simplified parameter sets with extreme values at $r = a$ also have reduced performance due to discretizations necessary to perform the simulation. Therefore, an accurate simulation will require high fidelity meshing in the region $r = a$. This will be examined further in the following sections.

3. Simplified Material Parameters for a Cylindrical Cloak

Due to the singular nature of the material parameters, ideal cloaks are impossible to manufacture. Therefore, simplifications must be made to the constitutive parameter set. There have been many different sets of simplified parameters developed for a cylindrical cloak. For an assumed TM incident electromagnetic field, Maxwell's equations can be used to develop the following governing wave equation.

(3) $$\frac{1}{\varepsilon_z r}\left[\frac{\partial}{\partial r}\left(\frac{r}{\mu_\theta}\frac{\partial E_z}{\partial r}\right)\right] + \frac{1}{\varepsilon_z r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\mu_r}\frac{\partial E_z}{\partial \theta}\right) + k_o^2 E_z = 0$$

If we assume all constitutive parameters are θ-invariant but not necessarily r-invariant, the general wave equation can be expanded analytically to

$$\frac{1}{\varepsilon_z \mu_\theta}\frac{\partial^2 E_z}{\partial r^2}+\left[\frac{1}{\varepsilon_z \mu_\theta}\frac{1}{r}-\frac{\mu_\theta'}{\varepsilon_z \mu_\theta^2}\right]\frac{\partial E_z}{\partial r}+\frac{1}{\varepsilon_z \mu_r}\frac{1}{r^2}\frac{\partial^2 E_z}{\partial \theta^2}+k_o^2 E_z=0 \tag{4}$$

Note ' implies differentiation with respect to r. When one uses the ideal cylindrical cloak's material parameters shown in Eqs. (1) and (2) in the general wave equation shown in Eq. (4), the result is the wave equation for TM^z fields in an ideal cylindrical cloak.

$$\left(\frac{b-a}{b}\right)^2\frac{\partial^2 E_z}{\partial r^2}+\left(\frac{b-a}{b}\right)^2\frac{1}{r-a}\frac{\partial E_z}{\partial r}+\left(\frac{b-a}{b}\right)^2\left(\frac{1}{r-a}\right)^2\frac{\partial^2 E_z}{\partial \theta^2}+k_o^2 E_z=0 \tag{5}$$

When comparing Eqs. (4) and (5), we find the following constraints on the ideal cylindrical cloak's material parameters [12].

$$\frac{1}{\varepsilon_z \mu_\theta}=\left(\frac{b-a}{b}\right)^2 \tag{6}$$

$$\frac{1}{\varepsilon_z \mu_r}=\left(\frac{b-a}{b}\right)^2\left(\frac{r}{r-a}\right)^2 \tag{7}$$

$$\frac{1}{\varepsilon_z \mu_\theta}\frac{1}{r}-\frac{\mu_\theta'}{\varepsilon_z \mu_\theta^2}=\frac{1}{r-a}\left(\frac{b-a}{b}\right)^2 \tag{8}$$

Simplified parameter sets were developed using the constraint equations shown in Eqs. (6) – (8). A first set of simplified parameters was developed such that ε_z, μ_r, and μ_θ satisfied Eqs. (6) and (7) [13,14]. A short-coming of these initial parameter sets was the presence of a strong reflected field due to the impedance mismatch at the cloaks' outer boundaries. Improved sets of material parameters were developed such that the impedance at the cloak's outer boundary matched that of free space [15-17]. In terms of overall scattering width performance, the simplified set of material parameters put forth in [17] has been shown to

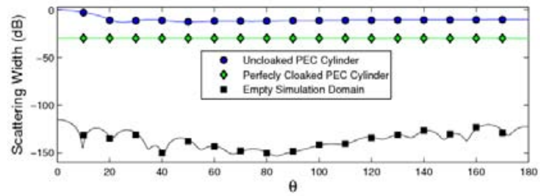

Figure 3. Scattering width degradation due to simulation approximations.

best approach the performance of the ideal cloak. However, all material parameters are spatially varying, making it much more difficult to currently manufacture.

4. Comparing Lined and Unlined Simplified Cloaks

As mentioned in Section 1, any deviation from the ideal cloak parameters results in degradation in performance. The hidden region is no longer completely shielded from electromagnetic energy. Objects placed inside the hidden region will scatter energy. The overall scattered field is a function of the geometry of the object placed within the hidden region in addition to possible resonant effects due to the cavity. A way to prevent energy from penetrating into the simplified cloaks' hidden regions is to line the cloaks with a PEC. This results in a geometry such that a scattering cylinder of radius $r = a$ is being cloaked. An object placed within the PEC cylinder won't have any additional impact on the overall scattered field. However, it is not clear the impact the PEC-liner has on cloak performance.

We wished to compare the scattering width reduction capabilities of three PEC-lined cylindrical cloaks with the performance of the same unlined cloaks with various objects in their hidden regions. We placed three different PEC objects into the unlined cloaks' hidden regions. The objects were a cylinder of radius $r = a/2$, a square with side length $d = a$, and an elongated diamond with length $l = a$ and height $h = a/4$. These are shown in Figure 4. As a baseline, we determined the scattering width of each object. These are shown in Figure 5. We chose these three objects because each has different scattering properties. The cylinder has a strong forward scattering component with moderate scattering at the remaining observation angles.

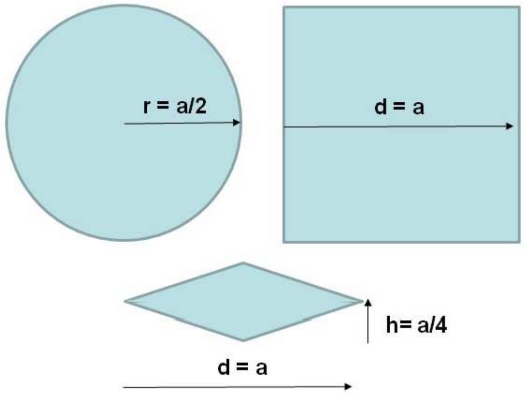

Figure 4. PEC objects placed in the cloaks' hidden regions

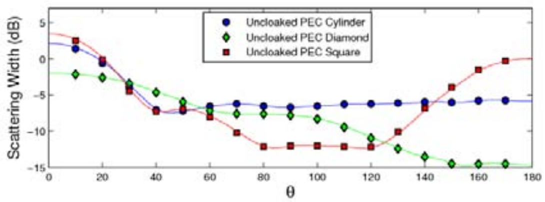

Figure 5. Baseline scattering widths for the PEC objects.

The square has strong forward and backscattering but a reduced signature when observation angles are near 90°. Finally, the diamond has a very small scattering width in the backscattering region. The variation in the signatures for these three targets allowed us to see the impact the liner has on overall cloak performance. The results for the three different cloaks are discussed below.

4.1 Performance Results for Cloak # 1

The first cloak we used has a parameter set that was first put forth in [16]. The parameters are shown below.

$$\text{(9)}\quad \mu_r = \left(\frac{r-a}{r}\right)\left(\frac{b}{b-a}\right)^2 \mu_\theta = \frac{b}{b-a} \varepsilon_z = \frac{b}{b-a}$$

Note how $\mu_r = 0$ at $r = a$. In order to get an accurate simulation, we needed to decrease the element size near the cloak's inner boundary. In order to minimize the error due to the material parameter values at $r = a$, we created an annular subdomain with outer radius $r = a + \delta$. We let δ = 0.005 m and set the maximum element size to 0.001 m. The restrictions result in a mesh with

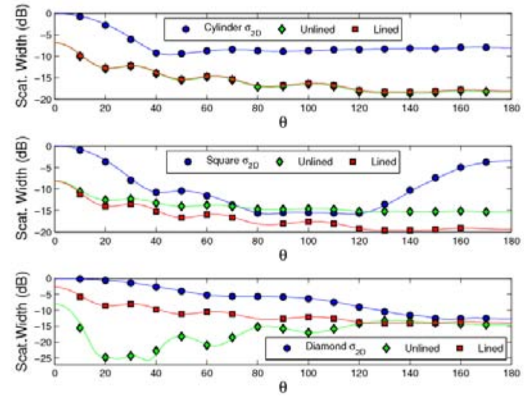

Figure 6. Scattering width comparisons for cloak #1.

on the order of 1,000,000 degrees of freedom, depending on the object that is placed in the hidden region. We then cloaked each object using the simplified cloak with constitutive parameters shown in Eq. (9). The scattering width results are shown in Figure 6. Note each graph is normalized by the maximum scattering width of the uncloaked object. For this particular cloak, we found the PEC-lined cloak had a similar performance to the unlined cloak when the scattering object was the cylinder with radius $r = a/2$. For the square PEC, the PEC-lined cloak had better performance than the unlined cloak for all observation angles. Additionally, for θ = 80° – 120°, the unlined cloak resulted in a higher signature than the uncloaked PEC square. Finally, for the PEC diamond, the unlined cloak had a much better performance than the lined cloak for observation angles less than 100°. Above 100°, the lined and unlined cloak performances are very similar.

4.2 Performance Results for Cloak # 2

The second cloak we used has a parameter set developed in [17]. The parameters are found by noting the ideal value for μ_θ can be approximated using the first N terms of a Taylor series. Specifically

$$\text{(10)}\qquad \mu_\theta \cong \sum_{n=0}^{N} \frac{f^n(b)}{n!}(r-b)$$

Using the approximation for μ_θ defined in Eq. (10), the parameters μ_r and ε_z are solved for using Eqs. (6) and (7). For this cloak, we used

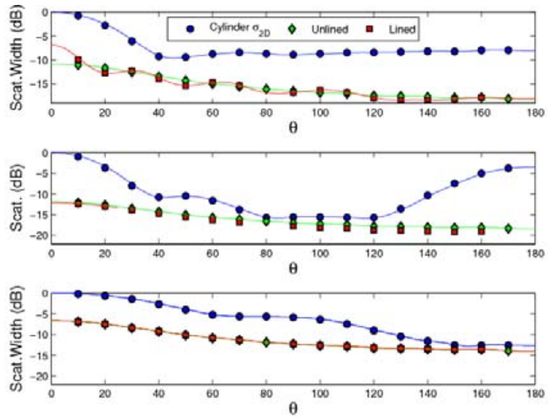

Figure 7. Scattering width comparisons for cloak #2.

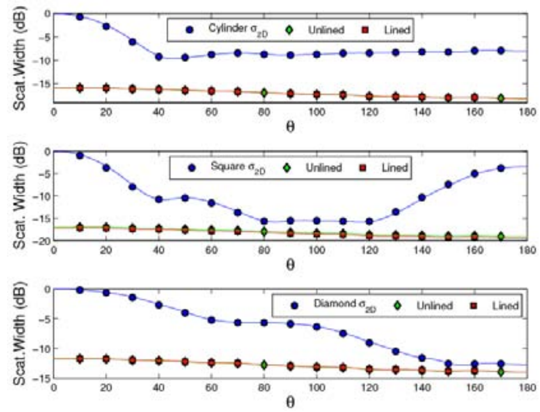

Figure 8. Scattering width comparisons for cloak #3.

the first 10 terms of the Taylor series to approximate μ_θ. The analysis then proceeded as stated in Section 4.1. The results for the various scattering widths are shown in Figure 7.

For this cloak, the only area where the unlined cloak performance exceeds that of the lined cloak was when the scattering width of the cloaked PEC cylinder of radius $r = a/2$ was compared to the lined cloak performance for observation angles less than 10^o. Otherwise the lined cloak performance equals the performance of the unlined cloak. There is slightly better performance of the lined cloak compared to the unlined cloak for the square PEC in the hidden region for observation angles 40^o; however, the improvement is one dB at best.

4.3 Performance Results for Cloak # 3

The final cloak we used had parameters values determined as described in Section 4.2. However, instead of using only the first ten terms in the Taylor series approximation of μ_θ, we now used N = 50. The results for the various scattering widths are shown in Figure 8.

5. Analysis of Results

As shown by the graphs in Figures 6 – 8, the performances of the unlined cloaks rarely are better than the performance of lined cloaks for all cloaks used in this study. The only time the unlined cloak outperformed the lined cloak was when the diamond geometry was placed in the first cloak's hidden region. For the second and third cloaks, there was very little difference in the scattering widths of a PEC-lined cloak compared to the scattering width of an unlined cloak with different objects in the hidden region. The performance of the unlined cloak we used in Section 4.1 was much more dependent on the object in the hidden region than the cloaks used in Sections 4.2 and 4.3. This is because the second and third cloaks are better approximations of the ideal cloak than the first cloak. A better approximation of the ideal cloak will have less energy transmitted into the cloak's hidden region, thereby making the objects placed within the hidden region less likely to impact the overall scattered field. This is illustrated in Figure 9 which shows the magnitude of the total field in the empty hidden regions of the first and third cloaks. Note how the first cloak (on the top) has a much stronger field in its hidden region than the third cloak (bottom). A stronger field transmitted into the region $r < a$ will likely result in objects in that same region having a larger impact on the cloak's overall response.

6. Conclusions

We have found PEC-lined cloaks typically will have better performance in terms of scattering width reduction than unlined cloaks no matter what objects are placed in the hidden region. However, as a cloak's simplified parameters stray further away from the ideal ones, there can be significant variation in the overall scattered field due to objects in the hidden region. This is due to the fact as a cloak's parameters become less and less ideal, more energy is transmitted into the hidden region, resulting in field interaction with the "hidden" object.

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8. Acknowledgements

The views expressed in this article are those of the authors and do not reflect the official policy or position of the Air Force, Department of Defense, or the U.S. Government.

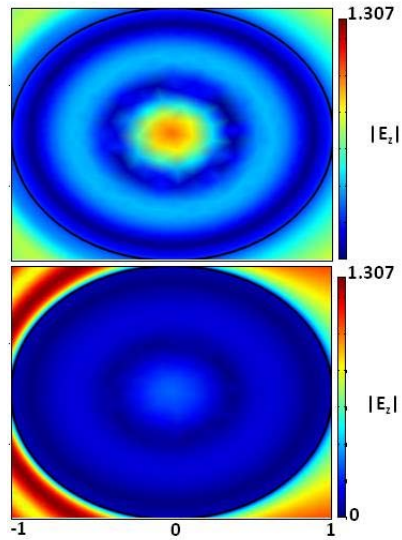

Figure 9. Hidden region fields for cloaks 1 and 3.