Acoustic Transparency of Non-homogeneous Plates (with repeating inclusions) using Periodic Structures Methodology

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Abstract: The paper addresses a class of problems for determining the acoustic interaction of time harmonic plane waves impinging upon submerged elastic plates (fluid backed or void backed), that are of infinite in extent. The plates can have repeated equally spaced inclusions (voids, solids, ribs etc.), and also can be constructed from orthotropic and/or isotropic layers that can have a frequency dependent loss factor for the modulus data (e.g. $E = E'(1+i\eta_E)$, and $\nu = \nu'(1+i\eta_\nu)$, where η_E and η_ν are the corresponding loss factors for Young's modulus and Poisson's ratio for example. Since the plates are infinite in extent, the issue of dealing with the infinite domain of fluid must be dealt with and is treated through the application of PML (Perfectly Matched Layers) boundary conditions, for absorbing the reflected acoustic waves and transmitted acoustic waves when the plates are fluid backed. The physical quantities of interest are typically the reflected acoustic waves and transmitted acoustic waves, however detailed information of how the elastic stress waves propagate through and around the inclusions are often of interest as well.
Keywords: Fluid Structure Interaction, Acoustic Transparency

1. Introduction

The paper treats a general class of problems as encountered in structural acoustics, involving a submerged elastic plate that is subject to an incident harmonic plane wave of the form $p_{inc}(x,y,z,t) = p_o e^{i(-xk_x - yk_y - zk_z + \omega t)}$ Eq(1) ,where{k_x k_y k_z }are the acoustic wave number components defining the direction $\vec{n}$ of propagation (i.e. $\vec{n} = \vec{i}k_x + \vec{j}k_y + \vec{k}k_z$) and ω is the frequency in rad./sec . The thrust of this paper is to compute the *acoustic transparency* $|p_{tr}/p_o|$ and *acoustic reflection* $|p_{rf}/p_o|$, which are defined as the fractions of the incident pressure p_o that are transmitted and reflected from the plate respectively. Further, we are interested in the case where the plate is not necessarily homogenous in the plane of the plate, but rather has some sort of *repeated anomaly*, such as the model configuration shown in Fig. 1 .

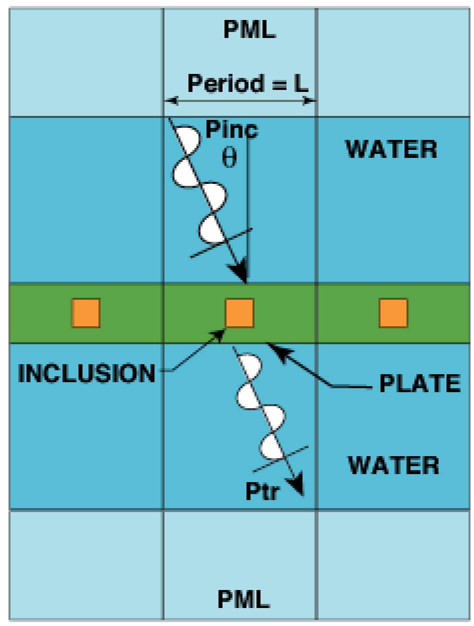

Figure 1.. Typical Repeated inclusion Submerged Structure Model Subject to Incident Plane Wave (with PML Wave Absorbing Layers)

The paper will pass through a sequence of increasingly difficult problems which illustrate the application of the periodic boundary condition, starting with:

(a) a simple free field block of fluid (e.g. Fig. 2)

(b) a homogenous submerged elastic plate with no inclusions, (e.g. Fig 1 case but with the softer surrounding matrix material set equal to the stiffer inclusion material)

(c) a submerged plate with periodic inclusions for the problem type shown in Fig. 1 .

Each of these problems has a specific point to illustrate, namely: *problem (a)* shows the enforcement of the periodic boundary condition in the cleanest form (no fluid structure interaction to deal with), and employs the perfect (exact in this case) impedance boundary condition to terminate the far field; *problem (b)* illustrates the setup for fluid structure interaction

Report Documentation Page

Form Approved
OMB No. 0704-0188

1. REPORT DATE **OCT 2007**	2. REPORT TYPE	3. DATES COVERED **00-00-2007 to 00-00-2007**

4. TITLE AND SUBTITLE **Acoustic Transparency of Non-homogeneous Plates (with repeating inclusions) using Periodic Structures Methodology**	5a. CONTRACT NUMBER 5b. GRANT NUMBER 5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)	5d. PROJECT NUMBER 5e. TASK NUMBER 5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **Naval Undersea Warfare Center,Newport Division,Newport,RI,02841-1708**	8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSOR/MONITOR’S ACRONYM(S) 11. SPONSOR/MONITOR’S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited

13. SUPPLEMENTARY NOTES
Presented at the 2007 COMSOL Users Conference Boston, 4-6 Oct, Newton. MA

14. ABSTRACT
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15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT **unclassified**	b. ABSTRACT **unclassified**	c. THIS PAGE **unclassified**	**Same as Report (SAR)**	**42**	

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std Z39-18

boundary conditions, absorbing radiation boundaries, and the issue of using the scattered pressure formulation, along with validation against an exact solution, and finally *problem (c)* solves the problem of main interest, where inclusions are distributed in a periodic pattern.

1.1 Past Work

The motivation for this work stems from the need to evaluate the acoustic transparency of various kinds of submerged plates and shells, and in particular, when acoustic sensors are placed behind these structures and their performance depends on the strength of the transmitted signal $|p_{tr}/p_o|$.The closer this ratio is to 1.0, the better the transparency and therefore the better the performance. There is a related problem of target strength (but will not be covered herein), where it is desired to keep the reflected pressure $|p_{rf}/p_o|$ small, usually through energy absorption via large loss factors in the material elastic constants. For either the transparency or target strength application, the methodology addressed herein can be used to attack these problems. References [1-4] treat the general issue of acoustic transparency from a materials point of view and references [5-7] employ three FEM based computer codes (NASTRAN, ATILA, and STARS3D respectively) that have been used to solve for the acoustic response of submerged periodic structures.

2. Governing Equations

The governing equations for the total pressure p in the acoustic domain and displacement vector, u_j, in the solid domain (for $e^{i\omega t}$ time harmonic type response) are given by:

$$c^2(\partial^2 p/\partial x^2 + \partial^2 p/\partial y^2 + \partial^2 p/\partial z^2) + \omega^2 p = 0$$

$$c_d{}^2 \frac{\partial}{\partial x_j}\left(\frac{\partial u_i}{\partial x_i}\right) + c_s{}^2 \frac{\partial}{\partial x_i}\left(\frac{\partial u_j}{\partial x_i} - \frac{\partial u_i}{\partial x_j}\right) + \omega^2 u_j = 0$$

where c, c_d and c_s are the acoustic wave speed, complex solid dilatational wave speed and complex solid shear wave speed respectively. The live index j takes on values j=1,2,3 and repeated indices sum over 1 to 3. We write the governing equations in this form, in order to: (a) illustrate the similarity of the form of the solid and acoustic media, and (b) illustrate how wave speeds appear as primary coefficients in the harmonic field equations. The concept of acoustic transparency in the simple case of say a normal incident plane wave, Eq(1), impinging on a homogenous elastic plate without loss, is treated in Ref [8]. The normal incident wave will completely pass through the plate if the impedance ratio $\overline{Z} = \rho_s c_d / \rho c$ =1.0, where ρ and ρ_s are the fluid and solid plate mass density respectively. Therefore when selecting material parameters for a plate, where acoustic transparency is desired, one usually strives to make the plate materials have impedance ratios in the neighborhood of 1.0 . In the case of non normal incidence, and/or the case where the matrix material of the plate has elastic inclusions imbedded within the plate and/or has nonzero loss factors, other kinds of waves are present in the dynamic response, and then having $\overline{Z} \approx 1.0$ to achieve transparency is only used as a guideline.

2.1 Elastic Material Constants

The complex wave speeds can be expressed in terms of primed real wave speeds and loss factors in the form:

$$c_d^2 = (c'_d)^2(1 + i\eta_d)$$

$$c_s^2 = (c'_s)^2(1 + i\eta_s) \qquad \text{Eqs(2a)}$$

Where η_d and η_s are the dilatational and shear loss factors. Complex wave speeds can also be expressed in terms of complex E modulus and complex Poisson's ratio ν in the form:

$$c_d^2 = \frac{(1-\nu)E}{\rho_s(1-2\nu)(1+\nu)} \quad c_s^2 = \frac{0.5E}{\rho_s(1+\nu)} \qquad \text{Eqs(2b)}$$

Say we are given the two real primed wave speeds and two associated loss factors η_d and η_s (which are typically different for rubber like materials). Upon equating the corresponding like complex wave speeds of Eqs(2a) Eqs(2b), we can easily solve for complex E and complex ν in terms of c'_d, c'_s, η_d and η_s . In COMSOL, the isotropic elastic data is entered via E and ν, where one can enter the complex E and ν in the form E=E' + i* E" and ν=ν' + i* ν". The COMSOL program has a feature in the *acoustics module* that permits the user to work with the scattered (reflected) pressure formulation, p_s, rather than the total p formulation. However the formulation used here still uses the structure of the total pressure equations, where the loading of the incident field enters via the boundary loading at the incident side fluid structure interface. We start by substituting $p = p_{inc} + p_s$ into the original time harmonic pressure pde to get:

$$c^2(\partial^2 p_s/\partial x^2 + \partial^2 p_s/\partial y^2 + \partial^2 p_s/\partial z^2) + \omega^2 p_s = 0$$

and since p_{inc} is a solution to the original homogenous pde, it drops out after the substitution, leaving the new form of the pde in terms of p_s which has the same appearance as the original.

2.2 Fluid Solid Interface Boundary Conditions

The p_{inc} term enters the problem via the fluid structure interface. On the solid interface, (p_{inc}+ p_s)= the total pressure is applied, and similarly at the fluid interface, both the interface plate acceleration and incident fluid acceleration are loaded. For example:

On incident side interface:

inward normal acceleration fluid loading:
nx_acpn*u_tt_acpn+ny_acpn*v_tt_acpn+p_i_acpr*i*(nx_acpn*kx+ny_acpn*ky)/rhow
normal pressure loading
-p*ny_acpn-p*nx_acpn-p_i_acpr*ny_acpn-p_i_acpr*nx_acpn

On transmitted side interface:

inward normal acceleration fluid loading:
nx_acpn*u_tt_acpn+ny_acpn*v_tt_acpn
normal pressure loading
-p*ny_acpn-p*nx_acpn

Where p_i_acpr = Eq(1) without $e^{i\omega t}$; rhow = ρ
p_o=1.0, $k_x=-k_o\sin(\theta)$, $k_y=k_o\cos(\theta)$, $k_z=0$ $k_o=\omega/c$.

2.3 Radiation Boundary Conditions

The mesh termination at the top of the incident side fluid and bottom of the transmitted side fluid must include some sort of radiation absorbing boundary condition. Three types were considered:

a) COMSOL's built in plane wave radiation absorber (don't need *acoustics module*)

b) COMSOL's built in PML (Perfectly Matched Layer), as indicated by the extra PML zones shown in Fig.1 above (need *acoustics module*).

c) User defined Impedance ($Z=p_s/v_n$) radiation condition , where for plane waves impinging at angle θ (see Fig. 1), on one or more completely submerged homogenous elastic layers and using Snell's law, ref.[9], we have $Z=\rho_f\, c/\cos(\theta)$ as an exact radiation boundary condition at both incident side mesh termination and transmitted side termination. This same impedance condition can be used for the non-homogenous plate (e.g. with inclusions), however in this case, the condition is approximate and the degree of success depends on how close the solution field is to a plane wave structure, at the fluid domain truncation cut (*don't need acoustics module*).

2.4 Periodic Boundary Conditions

Here we are concerned about the boundary condition at the left cut vertical faces and right cut vertical faces (line x=0 and line x=L in Fig. 1. The response at the left cut say, p_{lcut}, and response at the right cut p_{rcut}, is not known in advance, however we do know a relationship between them, namely:
$p_{rcut} = p_{lcut}e^{ik_xL}$, therefore p_{lcut} and p_{rcut} are not independent unknowns. This type of condition is used in refs.[5-7] for example, and will not elaborate on them further. Applying this condition in COMSOL is awkward and not intuitive. There are two issues: a) when applying the periodic boundary involving a complex multiplier condition, the user applies variable p on the left and $p\,e^{-ik_xL}$ on the right (the conjugate of e^{ik_xL}) and b) when using the solve parameters *advanced settings*, one ***must*** (according to COMSOL staff), check the box engaging "Use Hermitian transpose of constraint matrix).
The treatment for the application of the periodic boundary condition for the solid follows exactly along similar lines, (where upon defining displacement components as $u\equiv u_1$ and $v\equiv u_2$) we have $u_{rcut} = u_{lcut}e^{ik_xL}$ and $v_{rcut} = v_{lcut}e^{ik_xL}$ with $\{u_{lcut}, v_{lcut}, u_{rcut}, v_{rcut}\}$ as the left and right cut displacement component values, analogous to the pressure left and right cut values. The displacements are enforced similar to enforcing p, but instead in terms of COMSOL variables {u,v}. Thus {u,v} are applied at the left cut, and $\{u\, e^{ik_xL}, v\, e^{ik_xL}\}$are applied at the right cut.

2.5 Mesh sizing

For harmonic steady state problems, the mesh size is set according to the shortest wave length expected during the event. For example, if N_{ew} is the number of elements/wave length required for accurate modeling and C_{min} is the slowest wave speed, and f_{max} the largest frequency experienced in a frequency sweep, then the mesh can be sized with $\Delta_{min} = C_{min} / (N_{ew} f_{max})$, (e.g. N_{ew} =6 for quadratic element shape functions and N_{ew} = 10 for linear element shape functions). When rubber like materials are employed, the shear wave speed, c'_s, is typically the smallest and governs the mesh size needed in the solid.

2.6 Post Processing Pressure Fields

When the scattered formulation is employed as used in this paper, while plotting the pressure on the incident side, the basic COMSOL variable p_t_acpr will actually represent the scattered component, p_s, even though it is labeled as total in the post processing output list. The actual total can be obtained (if needed) by simply adding back the incident pressure, Eq(1), (without $e^{i\omega t}$). The pressure on the transmitted side is also post-processed via COMSOL via the variable p_t_acpr.

When the plate is homogenous, the phase of the reflected and transmitted pressure vary with space, however the magnitudes of these fields are constant over the domain. Therefore, one can sample $|p_{tr}/p_o|$, or $|p_{rf}/p_o|$ anywhere in the field to get a representative value of the acoustic transparency and reflectivity. However in the case of the presence of a repeated inclusion, like in the Fig.1 model, one needs a strategy for computing the transmitted and reflected pressure. One simple measure would be to compute integrated pressure at the far field mesh boundary (but not inside the PML zone if present), thus getting:

$$|\frac{p_{tr}}{p_o}|_{avg} = \frac{1}{L}\int_0^L \left|\frac{p(x)_{tr}}{p_o}\right| dx$$

$$|\frac{p_{tr}}{p_o}|_{avg} = \frac{1}{L}\int_0^L \left|\frac{p(x)_{rf}}{p_o}\right| dx \qquad \text{Eqs(3)}$$

where integration along x is at y=constant (the fluid mesh boundary).

Next, we consider an alternate method to process a representative transparency pressure in a variable spatial field, namely by computing the power flow across the y=constant cut boundary, and then convert the power into an equivalent pressure we shall call $\tilde{p} \equiv$"*pseudo pressure*".

The power flow can be computed by integrating the work done over one time cycle, (e.g. as in Ref.[10]), and then integrating that power/area result over the top (or bottom) boundary cut surface of the finite element mesh, getting

$\Pi = \int_0^L \frac{1}{2}\mathrm{Re}(pv_n{}^*)dx$,where v^*_n denotes the complex conjugate of the velocity normal to the surface . The power of the incident wave is given by $\Pi_{inc} = (1/2)p_o^2 L\cos(\theta)/(c\rho_f)$ and is used to form the normalized power ratio for transmitted and reflected fields, which we defined as

$$\overline{\Pi}_{tr} = \Pi_{tr}/\Pi_{inc} \text{ and } \overline{\Pi}_{rf} = \Pi_{rf}/\Pi_{inc} . \qquad \text{Eqs(4)}$$

These expressions can be used to compute the *pseudo pressure ratio* $|\tilde{p}_{tr}|/p_o$, by equating the normalized power $\overline{\Pi}_{tr}$, to the normalized power in a pseudo plane wave (i.e. $\tilde{\overline{\Pi}}_{tr} = (1/2)\tilde{p}_{tr}^2 L\cos(\theta)/(c\rho_f)/\Pi_{inc}$), and solving for the transmitted pseudo pressure (and similarly the reflected value), we obtain

$$|\tilde{p}_{tr}|/p_o = \sqrt{\overline{\Pi}_{tr}} \quad |\tilde{p}_{rf}|/p_o = \sqrt{\overline{\Pi}_{rf}} . \qquad \text{Eqs(5)}$$

When the plate is homogenous, the reflected and transmitted waves are plane with constant amplitudes, then Eqs(3) and Eqs(5) yield the same value for normalized pressure.

The power relations of Eqs(4) have an alternate use, namely for checking the consistency of the FEM solutions. Power levels can't be created greater than the input normalized incident power level of $\overline{\Pi}_{inc}$=1, hence the following inequality must be met:

$$\overline{\Pi}_{tr} + \overline{\Pi}_{rf} \le \overline{\Pi}_{inc} = 1.0 . \qquad \text{Eq(6)}$$

The strict equality sign is used when all the plate loss factors are zero. When the plate loss factors are not zero, the inequality is used due to material energy dissipation, where Eq(5) then infers that the transmitted plus reflected normalized power is less than the normalized free field incident power (i.e. 1.0). When computing power with Eq(4) at the fluid-PML interface, the v_n quantities must be computed with one sided shape function data (in the regular acoustic side), by using the ***down***(vy_acpr) option for the reflected side fluid and ***up***(vy_acpr) option for the transmitted side fluid. Otherwise accurate surface velocities will not be obtained.

3. Applications

Here we give examples of the three problem types (a,b,c) outlined in the introduction section.

3.1 Free Field Propagation Through a Simple Fluid Block

The purpose of this demonstration is to illustrate the enforcement of the periodic boundary conditions, with out the additional complications of PML absorbers, and fluid structure interaction boundary conditions. The model is shown in Fig. 2, where the bottom is driven with Eq(1) (without $e^{i\omega t}$ since it is suppressed throughout the harmonic solution), with k_z=0 and $k_o=\omega/c$, where positive k_x and k_y values send the wave in the direction shown in Fig. 2.

In this special example we work directly with total pressure p, since there is no issue of reflections in the solution. Further, the model is terminated with the perfect impedance absorber $Z=\rho_f c/\cos(\theta)$, so there is no need for PML absorbers here.

The left and right boundary conditions are treated by two methods:

(i) a direct Dirichlet type b.c. where the left and right cuts are directly driven with Eq(1) at x=0 for the left cut and with Eq(1) with x=L at the right cut (without $e^{i\omega t}$).

(ii) a periodic boundary condition was employed to the left and right vertical cuts, and since there is no characteristic period length, L can be set as anything, thus a nominal L=1.0" was selected. The left and right cuts were driven by the method explained in section 2.4, with strict attention to the issues (a) and (b) mentioned therein.

A 1"x 6" block of fluid was modeled, and driven at a frequency f=10000 Hz, at an angle θ=60º. The water wave speed was c=60000 in/sec and the mass density ρ_f=.000096 sec-lbf/in^4 and the mesh size was set via section 2.5 .

The solutions for the (i) and (ii) methods of driving the vertical faces were the same, thus validating the procedure for enforcing the periodic boundary condition. The solution for the (ii) case is shown in Fig.3 and perfectly checks out against the exact solution (i.e. the |p| magnitude should be 1.0 in the entire field, and the direction of wave propagation should be along straight lines that are at angles θ =60^o to the y axis, where it is noted that these lines are

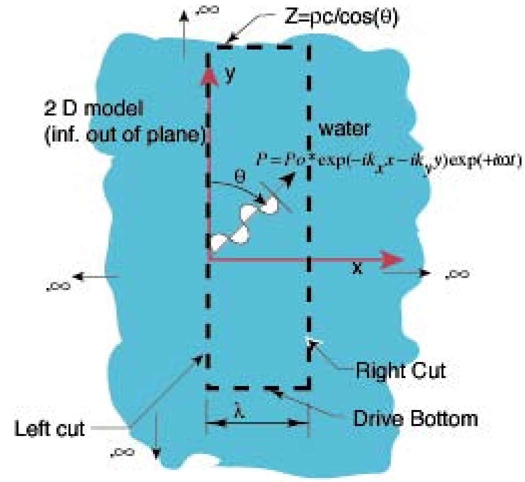

Figure 2. FEM model Zone (dashed) for Free Field Propagating Plane Wave Traveling θ Degrees to the y Axis.

perpendicular to the lines of constant phase wave fronts. This clean simple demo validates the procedure for enforcing the periodic boundary condition.

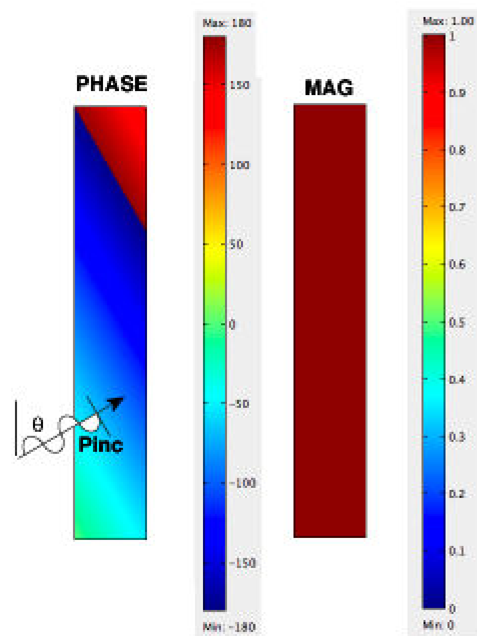

Figure 3. FEM Pressure Magnitude and Phase for Free Field Propagating Plane Wave Traveling θ =60 º to the y Axis.

3.2 Homogenous Submerged Elastic Plate with No Inclusions (with and without loss)

This example is selected in order to validate the solution in the more complicated case, where in addition to the periodic boundary condition, the issue of dealing with an elastic plate (with and without loss) is treated. The model setup is shown in Fig. 1, except the softer surrounding matrix material set equal to the stiffer inclusion material), thus ending up with a single (all orange) material plate.

Since the elastic layer is now simply flat and homogenous, we can directly compare the FEM solutions to an exact solution of the same problem taken from Ref.[9] . The model consist of a 5.5" wide x 6" long front and back side fluid, truncated with an optional PML zone of 5.5"x 4" . The water properties are the same as the previous problem and the drive frequency is f=11,000 Hz. The solid properties have a dilatational wave speed magnitude in the general vicinity of water, but still has a substantial shear modulus, hence a substantial shear wave speed. More specifically, the for the plate we have: $c'_d =$ 80000 in/sec, c'_s =20000 in/sec, η_d =0.01, η_s = 0.10 , and ρ_s=.00011 sec-lbf/in^4, where upon using the relations explained in section 2.1, corresponds to direct COMSOL domain input E= 129,090 +i*126250 and ν=0.46665 –i *0.031999 . This material has an impedance mismatch ratio of $\bar{Z} = \rho_s c_d / \rho c$ =1.52 , which is well away from the ideal 1.0 and therefore it is expected that unwanted reflections appear as will be shown later in Fig.4 . When entering this modulus and Poisson's ratio data, we skip COMSOL's built in loss mechanism input which assigns the same loss (unwanted here) to both dilatation and shear. Instead, we enter the data directly as *MATLAB like* syntax entries. The frequency is held constant and swept over a range of angles of incidence $0 \leq \boldsymbol{\theta} \leq 60^{o}$. The same problem was solved for three radiation boundary absorbers described in section 2.3 and FEM solutions are compared to the exact solution (for transmitted pressure, reflected pressure and Power). The simpler Eqs(3) were used to process pressure across the mesh truncating fluids faces, and Eqs(4) for the power. The comparisons in Fig 4. are excellent for both the PML and the Z=ρ_fc/cos(θ) absorbers. Note: in all plots, "MPL" labels imply PML absorbers.

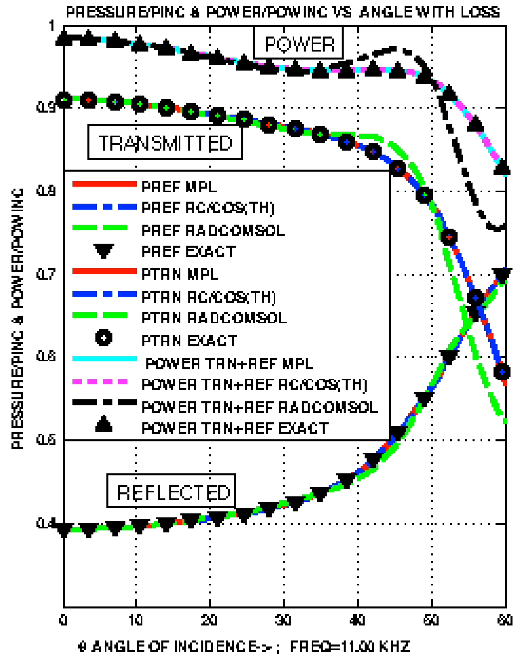

Figure 4. Exact vs. FEM Solution for Three Different Radiation Boundary Conditions (With Loss , f=11 KHz).

The same problem was solved for no loss in the solid (setting the imaginary parts of E and

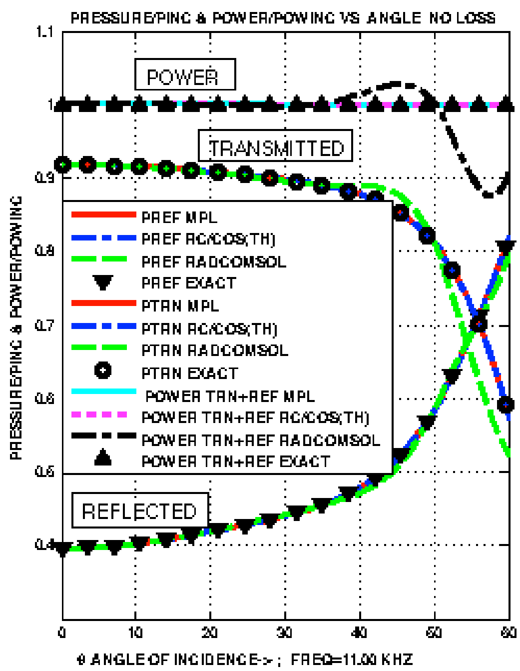

Figure 5. Exact vs. FEM Solution for Three Different Radiation Boundary Conditions (With No Loss, f=11 KHz).

ν equal to zero), where the results are shown in Fig. 5 . We note that since the loss is zero, the total power, via Eq(6), should be conserved (i.e. be equal 1.0 all angles). However as in the previous plot, the power computed with the COMSOL plane wave absorber, drifts off the exact solution of 1.0 for $30^o \leq \theta \leq 60^o$.

3.3 Non-homogenous Submerged Elastic Plate With Periodic Inclusions (with and without loss)

The model is the same as the Fig.1 sketch (drawn to scale), except the matrix material is a rubber like material, and a centered 1”x 1” inclusion made of the same material as the previous homogenous plate example. The matrix material (green), is given as: $c'_d =$ 65000. in/sec, c'_s =10000., η_d =0.03, $\eta_s = 0.30$, ρ_s=.000105 sec-lbf/in^4, which upon using the relations explained in section 2.1, corresponds to direct COMSOL domain input E= 31265 +i*9302.8 and ν=0.4878 –i ∗0.0033502. This matrix material has an impedance mismatch ratio of $\overline{Z}_{matrix} = \rho_s c_d / \rho c$ =1.18 , which is near the ideal 1.0, however the stiffer inclusions ($\overline{Z}_{inclusion}$=1.52) are expected to somewhat degrade the overall transparency performance.

3.3.1 Baseline case: The resulting solution is shown in Fig 6, where note that results using

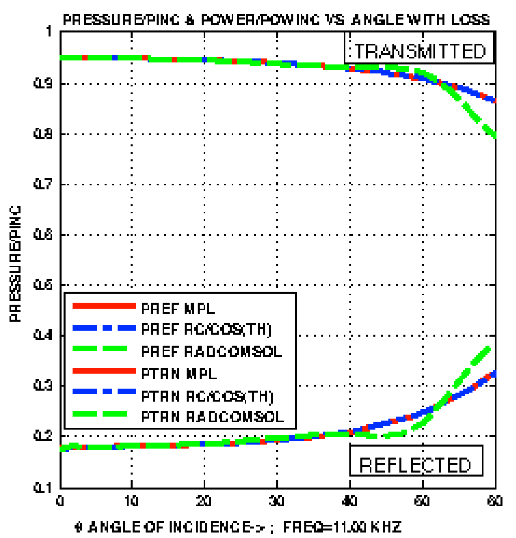

Figure 6. FEM Solution for Three Different Radiation Boundary Cond. (With Loss, f=11 KHz)

the PML and Z impedance absorbers are in good agreement, however the COMSOL plane wave absorber solution drifts away for $30^o \leq \theta \leq 60^o$.

Next we consider the same solution, except the loss terms are turned off for both the matrix material and the inclusion. The resulting solution is shown in Fig. 7, where this no loss case is included in order to illustrate how the Eq(6) power inequality checks the solution accuracy. Thus for the COMSOL built in plane wave absorber, the power sum does not always add up to 1.0, whereas with both the PML and the $Z=\rho_f c/\cos(\theta)$ absorbers, it does nicely add up to 1.0 across the entire angle sweep, even when the solution encounters the spikes in the response.

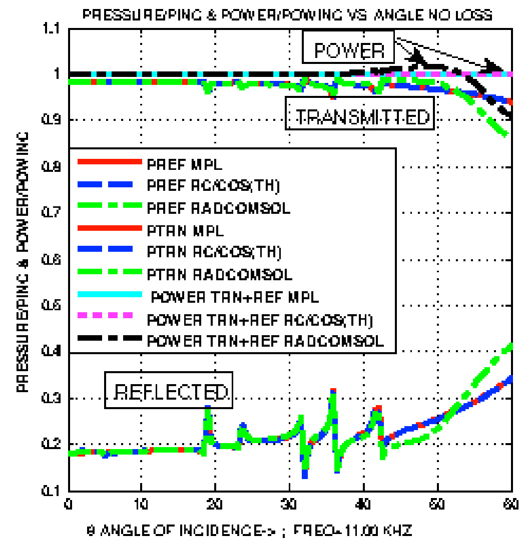

Figure 7. FEM Solution for Three Different Radiation Boundary Cond. (No Loss, f=11 KHz).

The color contour plots in Fig. 8, illustrate how the transmitted and reflected pressures vary with the spatial x,y field ,and how phase wave fronts exist in the transmitted side and reflected side within the fluid domain at $\theta = 60^o$.

3.3.2 Variation of Baseline Case for three Inclusion Stiffnesses:

In this demonstration, we illustrate how the transmitted pressure is affected by the stiffness of the inclusion. The stiffnesses are changed via the solid dilatational wave speeds. The *without inclusion* case is run first for comparison purposes, where the entire plate is made of the matrix material (red curve solution). This is used as a yardstick reference, for measuring the

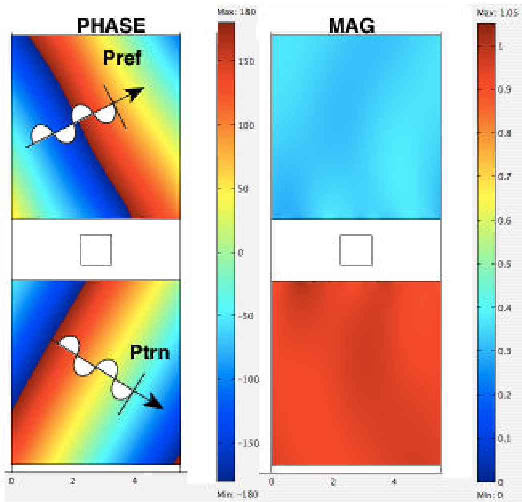

Figure 8. FEM Amplitude and Phase Solution for PML Radiation Boundary Conditions (With No Loss, f=11 KHz, at θ= 60°).

without inclusion solution against the *with inclusion* solution. The blue curve represents the *with inclusion* base line case for the case, c'_d = 80000 in/sec (Fig.6 repeat), and the green curve for the stiffest inclusion with c'_d = 160,000 in/sec (keeping all other material parameters the same as the baseline case). The sequence of inclusion stiffnesses for the (Fig.9 red blue green curves) are $\overline{Z}$ ={1.0, 1.18, 3.05}. As expected, Fig.9, illustrates that the stiffer the inclusion material, the worst the acoustic transparency.

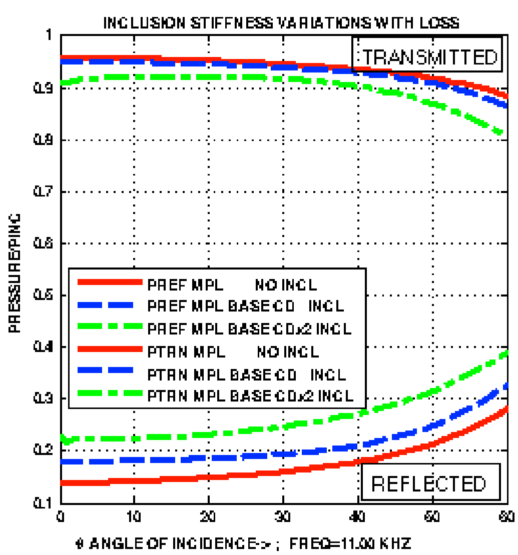

Figure 9. Variation of Baseline Case for Three Inclusion Stiffnesses (With Loss, f=11 KHz).

3.3.3 Variation of Baseline Case for Two Fluid Domain Lengths and Two Pressure Post Processing Techniques:

In this example, we examine the effect of the fluid length on the accuracy of the FEM solution for the baseline case without loss and also the effect of average vs. pseudo pressure post-processing as shown in Fig.10. We purposely zero out the loss portion of the material constants, since there is a richer profile of response vs. θ in order to better examine the

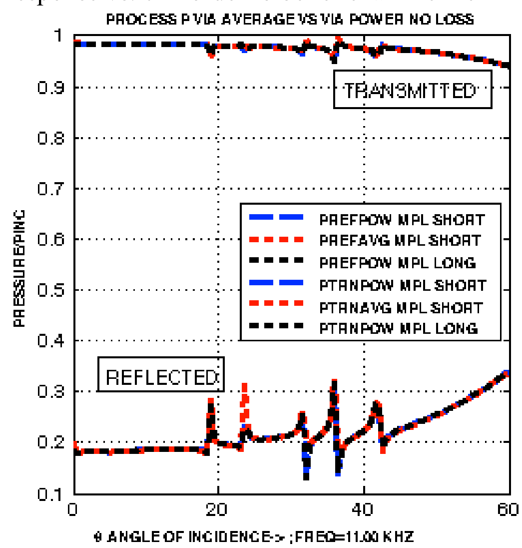

Figure 10. Variation of Baseline Case for Two Fluid Lengths and Two Post-processing Methods (No Loss, PML absorber at f=11 KHz).

pressure post-processing techniques under more severe spatial distribution conditions. The curves labeled MPL LONG refer to the 6" length front and back side fluid domains, and the curves labeled MPL SHORT refer to a 0.60" length front and back side acoustic domain. The detailed pressure contours (not shown here), indicated that the shorter fluid domain cuts are now in the near field, where the solution pressure amplitude vary with x more substantially than the long model, and thereby provide a good opportunity to compare average pressure, Eq(3), vs. pseudo power based pressure ,Eq(5), post-processing. The results in Fig. 10, illustrate the pseudo pressure of the short model and the long model produce the same measure of transmitted pressure and reflected pressure, implying one can employ a shorter model (and hence less DOF in the event core memory is an issue for higher frequency models). However, the average pressure from Eq(3), (dashed red) of the shorter model reflected pressure does not completely track the corresponding long model pseudo pressure, missing at peak $\theta=24^{o}$ and valleys $\theta=32^{o}$, 37^{o} . This illustrates that the pseudo pressure post-processing is a more general procedure for evaluating the transparency and reflectivity of panel type materials with inclusions, and that it produces results for short length models that are consistent with the longer (larger DOF) models.

Finally we consider the same long-short model comparison, except now the loss factors are turned on for both materials, resulting in the solution shown in Fig. 11. Unlike the previous Fig.10 no loss case, we have essentially complete agreement, across the entire θ sweep, between the LONG, SHORT model solutions, and also agreement between the method of post-processing. The one exception is a relatively small reflected pressure blip (red dashed curve), near normal incidence using Eq(3) pressure averaging.

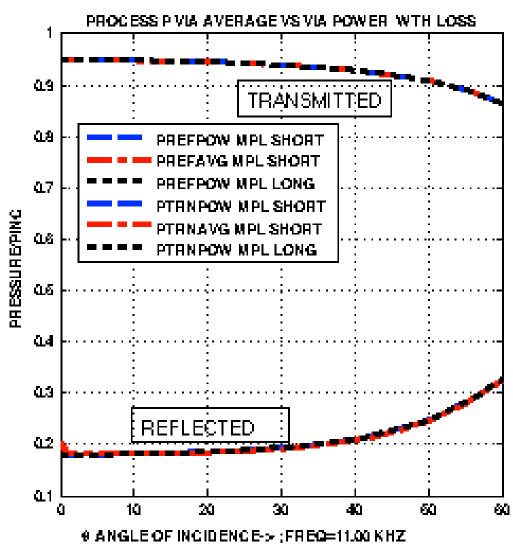

Figure 11. Variation of Baseline Case for Two Fluid Lengths and Two Post-processing Techniques (With Loss, PML Absorber at f=11 KHz).

4. Conclusions

The results in this paper illustrate how COMSOL could be used to solve for the transmitted and reflected pressure in totally submerged plates, having periodically spaced inclusions. Although we have emphasized the issue of transparency with simple inclusions, the methodology could be applied to ribbed plates, where the periodically spaced ribs would play the same role as the different material inclusions considered herein. Also we have shown another method for post-processing a variable spatial distributed pressure field, by computing a *pseudo pressure*, derived from the amount of power flowing across the cuts in the fluid domain outer boundaries. The power flow calculations also provide a measure ,via Eq(6), of the accuracy of the FEM model solution . Violations of Eq(6), provide a red flag that something is wrong with the model (e.g. coarse mesh, bad absorbing boundary, some unknown user error).

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9. Acknowledgements

This work was partially sponsored by the Naval Undersea Warfare Centers In-house Laboratory Independent Research Program. The author would also acknowledge the model set up help received from COMSOL staff members Gaozhu Peng and Vineet Dravid .

Presented at the COMSOL Users Conference 2007 Boston

Acoustic Transparency of Non-homogeneous Plates (with repeating inclusions) using Periodic Structures Methodology

PRESENTED BY PI:

Anthony J. Kalinowski

NUWC CODE 70

OCT 5 2007

NAVAL UNDERSEA WARFARE CENTER

NEWPORT RHODE ISLAND

PAPER OVERVIEW

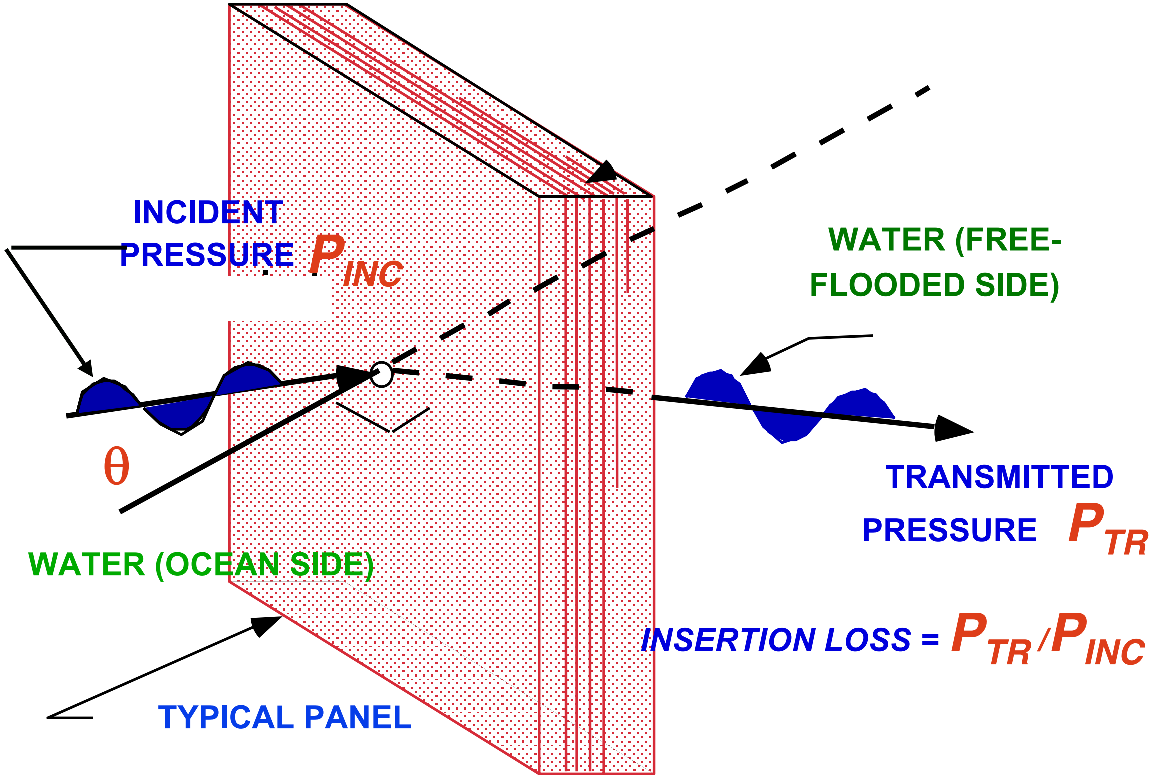
- **The paper addresses a class of problems for determining the acoustic interaction of $e^{i\omega t}$ time harmonic plane waves impinging upon submerged elastic plates (fluid backed or void backed), that are of infinite in extent.**

- **The plates can have repeated equally spaced inclusions (voids, solids, ribs etc.), and also can be constructed from orthotropic and/or isotropic layers**

- **Materials can have a frequency dependent loss factor for the modulus data (e.g. $E= E'(1+i\eta_E)$, and $\upsilon= \upsilon'(1+i\eta_\upsilon)$, where η_E and η_υ are the corresponding loss factors for Young's modulus and Poisson's ratio for example.**

- **THE *ACOUSTIC TRANSPARENCY* : I.E. INSERTION LOSS=MAG(P_{TR}/P_{INC}) , is the main physical quantity of interest**

- **Transparency is of great interest when sonar devices operate behind protective dome enclosures (" acoustic windows")**



DEFINITION OF ACOUSTIC WINDOW KEY PARAMETERS

PAPER OVERVIEW cont.

- **Since the plates (and surrounding fluid) are infinite in extent, the issue of dealing with the infinite domain of fluid must be dealt with and is treated through the application of wave absorbing boundary conditions for absorbing the reflected and transmitted acoustic waves**

- **Three absorbing boundary conditions are examined**
  (1) COMSOL's PML (Perfectly Matched Layers) boundary conditions
  (2) Impedance boundary condition
  (3) COMSOL's plane wave absorbing boundary condition

- **Only a unit cell of the repeating structure, including a small part of the surrounding fluid domain has to be modeled, by applying the the Bloch-Floquet theorem based boundary condition in the the direction of the repeat pattern**

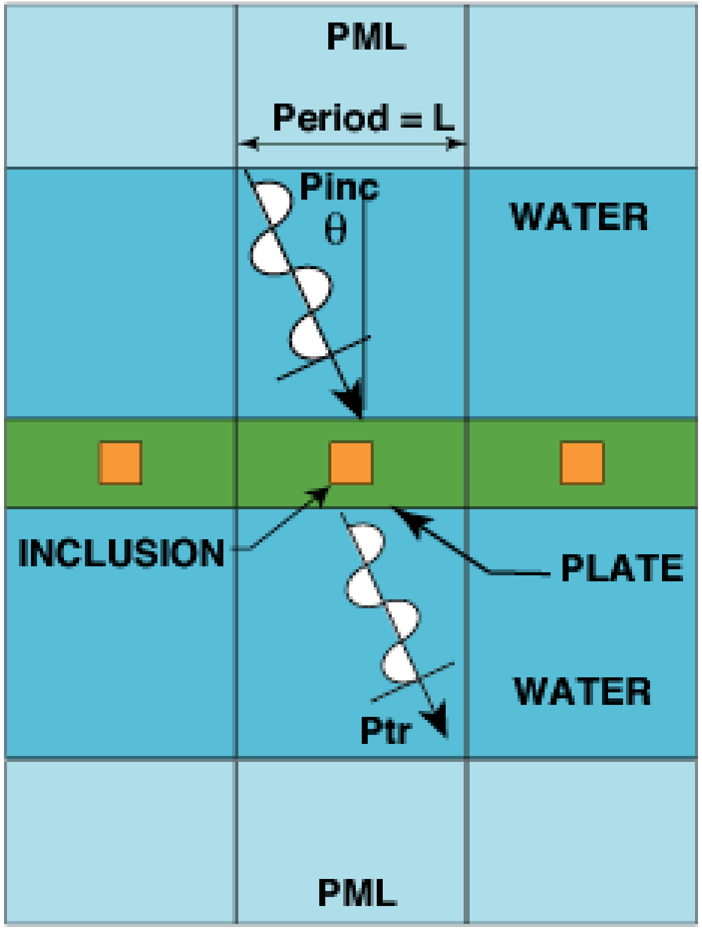


FIG. 1_Typical Repeated inclusion Submerged Structure Model Subject to Incident Plane Wave (with PML Wave Absorbing Layers)

Past Work on <u>repeated discontinuities using periodic structures</u>

- *Ref-1* ***"Plates with regular stiffening in acoustic media: Vibration and radiation", Mead D.J., 1990, J. Acoustic Society of America, 88 (1), July 1990.***

This important paper treats plates and cylindrical shells with regular and identical stiffening that constitute "spatially periodic structures". Specially convenient methods of vibration analysis are considered for these cases, some of which are suitable for the inclusion of the effects of fluid loading from adjacent acoustic media.

The paper outlines the nature of free wave motion in periodic structures stiffened either in one or two orthogonal directions. In our application, we could view the stiffened sections say due to ribs for example, as unstiffened -or-weakened stiffness sectors, like the failed region L_F we illustrated earlier.

Past Work on repeated discontinuity using periodic structures cont.

- ***Ref-2 "Steady state solutions to dynamically loaded periodic structures",Kalinowski A.J. 1979, NASA Conference Publication 2131, Eight NASTRAN User's Colloquium.***

This was an early application of periodic structures applied to acoustic scattering and transmission of acoustic structure interaction problems having one way periodicity based early Mead papers predating his general Ref-29 paper. DMAP matrix operations in the NASTRAN FEM code were used for performing matrix manipulations needed to extract acoustic solutions. The beauty of this approach is that one can interact with the inner workings of the NASTRAN FEM code without having to have access to the source code. This is a rare feature not readily found in today's large scale FEM codes.

Past Work on *repeated discontinuity using periodic structures cont.*

- ***Ref-3* *"Analysis of the scattering of a plane acoustic wave by a periodic elastic structure using the finite element method: Application to compliant tube gratings", Hennion A.C., Bossut R. and Decarpigny J.N., 1990, J. Acoustic Society of America, 87 (5), May 1990.***

A mathematical model has been developed to analyze the scattering of plane acoustic waves from an infinite, uniform, plane grating of compliant tubes. It relies upon the FEM method and uses the ATILA FEM code (see Ref 30). Only a unit cell of the repeating structure, including a small part of the surrounding fluid domain has to be modeled, by relying on the Bloch-Floquet theorem, and the effects of the remaining fluid domain are accounted for by matching the pressure field in the finite element model with the simple plane wave expansions of the on going and outgoing waves.

Past Work on repeated discontinuity using periodic structures cont.

- ***Ref-4*** ***"Analysis of the scattering of a plane acoustic wave by a doubly periodic structure using the finite element method: Application to Alberich anechoic coatings", Hlady-Hennion A.C., andDecarpigny J.N., 1991,Dec, J. Acoustic Society of America, 90 (6).***

Ref-5"Computation of Acoustic Transmission Loss Through Doubly-Periodic 3D Elastic Panels", Dey S. and Shirron J., Proceedings of IMECE 2006 ASME 2006 International Mechanical Engineering Congress & Exposition,

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TECHNICAL ISSUES

- **Any signal processing and acoustic beam forming operations strongly depend on the incident wave front remaining relatively undistorted in the presence of a protective window**
- **Guide to Perfect Transparency: Let the (solid material Impedance) $(\rho c_D)_{sol} \approx (\rho c)_{wat}$ (water impedance): ρ=density ; c=sound speed , with zero solid shear modulus G**
- **…NOT POSSIBLE! SOLID SHEAR MODULUS "G" IS ALWAYS >0**
- **Therefore different kinds of structural waves can be excited during the fluid-solid interaction**

Governing Equations:

We treat a general class of problems as encountered in structural acoustics, involving a submerged elastic plate that is subject to an incident harmonic plane wave of the form

$$p_{inc}(x,y,z,t) = p_o e^{i(-xk_x - yk_y - zk_z + \omega t)} \qquad \textbf{Eq(1)}$$

Where $\{k_x\ k_y\ k_z\}$ are the acoustic wave number components defining the direction of propagation

$$\vec{n} = \vec{i}\,k_x + \vec{j}k_y + \vec{k}k_z\,)$$

and ω is the frequency in rad./sec .

The thrust of this paper is to compute the acoustic transparency $|p_{tr}/p_o|$ and acoustic reflection $|\,p_{rf}/p_o|$

Governing Equations cont.:

The governing equations for the total pressure p in the acoustic domain and displacement vector u_j, in the solid domain (for $e^{i\omega t}$ time harmonic response) are given by:

$$c^2(\partial^2 p/\partial x^2 + \partial^2 p/\partial y^2 + \partial^2 p/\partial z^2) + \omega^2 p = 0$$

and

$$c_d^{\,2}\frac{\partial}{\partial x_j}\left(\frac{\partial u_i}{\partial x_i}\right) + c_s^{\,2}\frac{\partial}{\partial x_i}\left(\frac{\partial u_j}{\partial x_i} - \frac{\partial u_i}{\partial x_j}\right) + \omega^2 u_j = 0$$

where c, c_d and c_s are the acoustic wave speed, complex solid dilatational wave speed and complex solid shear wave speed respectively.

The live index j takes on values j=1,2,3 and repeated indices sum over 1 to 3.

MATERIAL CONSTANTS WITH LOSS FACTORS:

- **The complex wave speeds can be expressed in terms of primed real wave speeds and loss factors in the form:**

$$c_d^2 = (c'_d)^2(1 + i\eta_d)$$

$$c_s^2 = (c'_s)^2(1 + i\eta_s)$$

Where η_d and η_s are the dilatational and shear loss factors.

- **Complex wave speeds can also be expressed in terms of complex E modulus and complex Poisson's ratio ν as :**

$$c_d^2 = \frac{(1-\nu)E}{\rho_s(1-2\nu)(1+\nu)} \quad c_s^2 = \frac{0.5E}{\rho_s(1+\nu)}$$

- **Upon equating the corresponding like complex wave speeds we can easily solve for complex E and complex ν in terms of c'_D ,c'_S , η_d and η_s . In COMSOL, the isotropic elastic data is entered via E and ν (in MATLAB syntax format) ,**
- **Thus one can directly enter the complex E and ν in the form**

E= E' + i* E" and ν= ν' + i* ν".

Governing Equations cont. (SCATTERED FORMULATION):

- COMSOL has a feature in the *acoustics module* that permits working with the scattered (reflected) pressure formulation, p_s, rather than the total pressure p formulation

- However the formulation used here still uses the structure of the total pressure equations, where the loading of the incident field now enters via the boundary loading at the incident side fluid structure interface.

- We start by substituting $p = p_{inc} + p_s$ into the original total form time harmonic pressure pde to get:

$$c^2(\partial^2 p_s / \partial x^2 + \partial^2 p_s / \partial y^2 + \partial^2 p_s / \partial z^2) + \omega^2 p_s = 0$$

- Since p_{inc} is a solution to the original homogenous pde, it drops out after the substitution, leaving the new form of the pde in terms of p_s which has the same appearance as the original in terms of total p.

Governing Equations cont. (BOUNDARY CONDITIONS):

- ***On the solid interface, $(p_{inc} + p_s)$= the total pressure is applied, and similarly at the fluid interface, both the interface plate acceleration and incident fluid acceleration are loaded.***

- ***On incident side interface:***

inward normal acceleration fluid loading:

nx_acpn*u_tt_acpn+ny_acpn*v_tt_acpn+p_i_acpr*i*(nx_acpn*kx+ny_acpn*ky)/rhow

normal pressure loading:

-p*ny_acpn-p*nx_acpn-p_i_acpr*ny_acpn-p_i_acpr*nx_acpn

On transmitted side interface:

inward normal acceleration fluid loading:

nx_acpn*u_tt_acpn+ny_acpn*v_tt_acpn

normal pressure loading:

-p*ny_acpn-p*nx_acpn

Where p_i_acpr = Eq(1) DRIVER without $e^{i\omega t}$; rhow = ρ_o=1.0, $k_x=-k_o\sin(\theta)$, $k_y=k_o\cos(\theta)$, $k_z=0$ $k_o=\omega/c$.

Governing Equations cont. (RADIATION BOUNDARY CONDITION):

The mesh termination at the end of the fluid domain must include some sort of radiation absorbing boundary condition.

a) COMSOL's built in plane wave radiation absorber (don't need acoustics module)

b) COMSOL's built in PML (Perfectly Matched Layer), as indicated by the extra PML zones shown in Fig.1 above (do need acoustics module).

c) User defined Impedance ($Z=p_s/v_n$) radiation condition , where for plane waves impinging at angle θ, on one or more completely submerged homogenous elastic layers and using Snell's law, we have $Z= \rho_f c/\cos(\theta)$ as a radiation boundary condition at both incident side mesh termination and transmitted side termination. (don't need acoustics module).

Governing Equations cont. (FLUID PERIODIC BOUNDARY CONDITIONS):

- ***Consider next the boundary condition at the left cut vertical faces and right cut vertical faces (line x=0 and line x=L in Fig. 1.***

- ***The response at the left cut say, p_{lcut} , and response at the right cut p_{rcut} , is not known in advance, however we do know a relationship between them, namely:***

$$p_{rcut} = p_{lcut} e^{ik_x L}$$

- **Therefore p_{lcut} and p_{rcut} are not independent unknowns. This is a "*Bloch-Floquet theorem*" type condition and is is used in refs.[5-7] .**

- **Applying this condition in COMSOL is not intuitive. There are two quirky issues:**

a) when applying the periodic boundary involving this complex multiplier condition, the user applies variable *p* on the left cut and p*exp(-ik$_x$L) on the right cut (the conjugate of p*exp(+ik$_x$L))

b) when using the solve parameters advanced settings, one must (according to COMSOL staff), check the box engaging "Use Hermitian transpose of constraint matrix).

Governing Equations cont. (SOLID PERIODIC BOUNDARY CONDITIONS):

- **The treatment for the application of the periodic boundary condition for the solid follows exactly along similar lines, (where upon defining displacement components as u≡u_1 and v≡u_2) we have**

$$u_{rcut} = u_{lcut} e^{ik_x L} \quad \text{and} \quad \mathrm{v}_{rcut} = \mathrm{v}_{lcut} e^{ik_x L}$$

$$\text{with} \quad \{ u_{lcut}, \mathrm{v}_{lcut}, u_{rcut}, \mathrm{v}_{rcut} \}$$

as the left and right cut displacement component values, analogous to the pressure left and right cut values.

- **The displacements are enforced similar to enforcing p, but instead in terms of COMSOL variables {u,v}.**

- **Thus {u,v} are applied at the left cut, and {u*exp(ik$_x$L, v*exp(ik$_x$L} are applied at the right cut.**

Post Processing Pressure Fields

- **For the scattered formulation employed here, on the incident side, the basic COMSOL variable p_t_acpr will actually represent the scattered component, p_s , even though it is labeled as total in the post processing output list.**

- **The actual total can be obtained (if needed) by simply adding back the incident pressure, Eq(1), (without $e^{i\omega t}$).**

- ***in the case of the presence of a repeated inclusion, like in the Fig.1 model, one needs a strategy for computing the transmitted and reflected pressure.***

- ***One simple measure would be to compute integrated average pressure at the far field mesh boundary (but not inside the PML zone if present), thus getting:***

$$\left|\frac{p_{tr}}{p_o}\right|_{avg} = \frac{1}{L}\int_0^L \left|\frac{p(x)_{tr}}{p_o}\right| dx \quad \left|\frac{p_{tr}}{p_o}\right|_{avg} = \frac{1}{L}\int_0^L \left|\frac{p(x)_{rf}}{p_o}\right| dx$$

Post Processing Pressure Fields (continued)

- **Next, consider an alternate method to process a representative transparency pressure in a variable spatial field, namely by computing the power flow across the y=constant cut boundary**

- **The power flow can be computed by integrating the work done over one time cycle, and then integrating that power/area result over the top (or bottom) boundary cut surface getting**

$$\Pi = \int_0^L \frac{1}{2} \mathrm{Re}(\mathrm{pv_n}^*)dx$$

- **After normalizing by the incident wave power, can be used to compute the "*pseudo pressure ratio*", by equating the normalized power , to the normalized power in a pseudo plane wave (i.e.), and solving for the transmitted pseudo pressure (and similarly the reflected value), we obtain:**

$$|\tilde{p}_{tr}|/p_o = \sqrt{\overline{\Pi}_{tr}} \quad |\tilde{p}_{rf}|/p_o = \sqrt{\overline{\Pi}_{rf}}$$

- ***The power relations can be used for checking the consistency of the FEM solutions. Power levels can't be created greater than the input normalized incident power level hence***

$$\overline{\Pi}_{tr} + \overline{\Pi}_{rf} \leq \overline{\Pi}_{inc} = 1.0$$

Scope of Applications:

We pass through a sequence of increasingly difficult problems which illustrate the application of the periodic boundary condition, starting with:

(a) a simple free field block of fluid (no plate)

(b) a homogenous submerged elastic plate but with no periodic inclusions

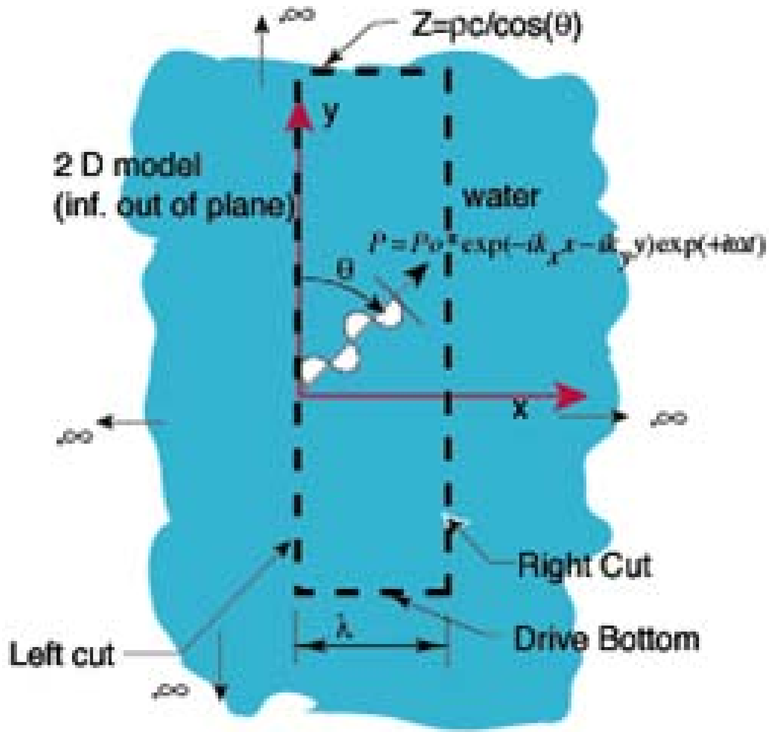
(c) a submerged plate with periodic inclusions

for the problem type shown in Fig. 1 .

Free Field Propagation Through a Simple Fluid Block

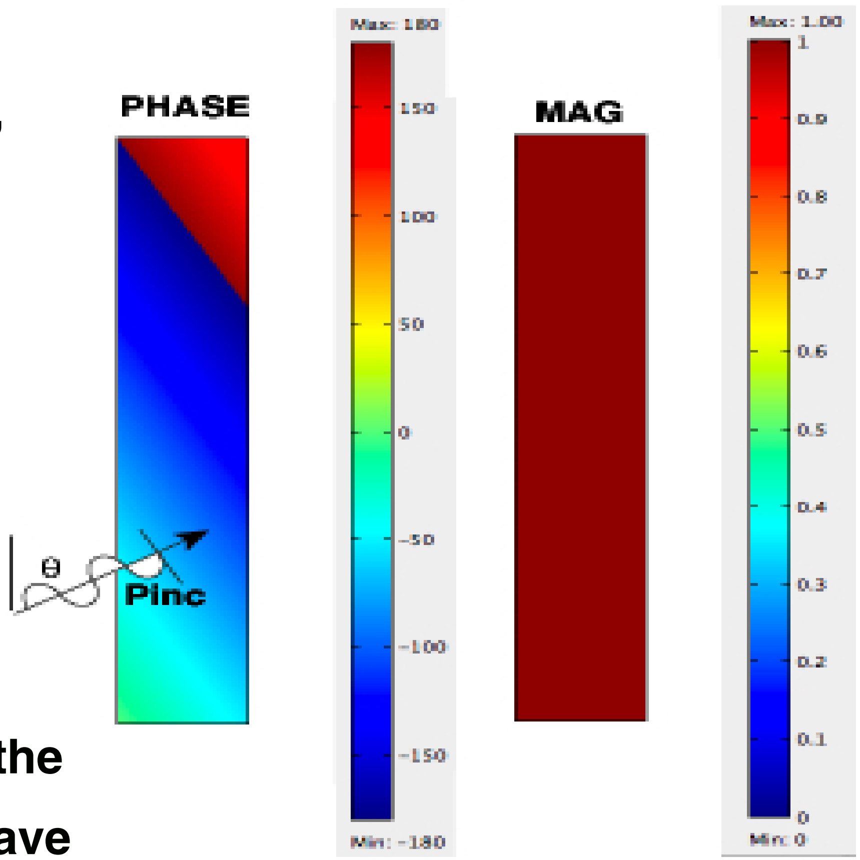
- **The purpose of this demonstration is to illustrate the enforcement of the periodic boundary conditions, <u>with out</u> the additional complications of PML absorbers, and fluid structure interaction boundary conditions.**

- **The model is shown here, where the bottom is driven with Free Field Plane wave pressure Eq(1) with k_z=0 and k_o=ω/c, where positive k_x and k_y values send the wave in the direction shown**



Free Field Propagation Through a Simple Fluid Block (Cont.)

- The solution is shown here and perfectly checks out against the exact solution.
- The |p| magnitude should be 1.0 everywhere, and the direction of wave propagation should be along straight lines that are at angles $\theta = 60^o$ to the y axis



- Where it is noted that these wave direction lines are perpendicular to the lines of constant phase wave fronts

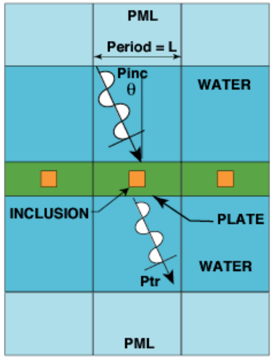
Plane Wave Propagation Through a Simple Flat Plate

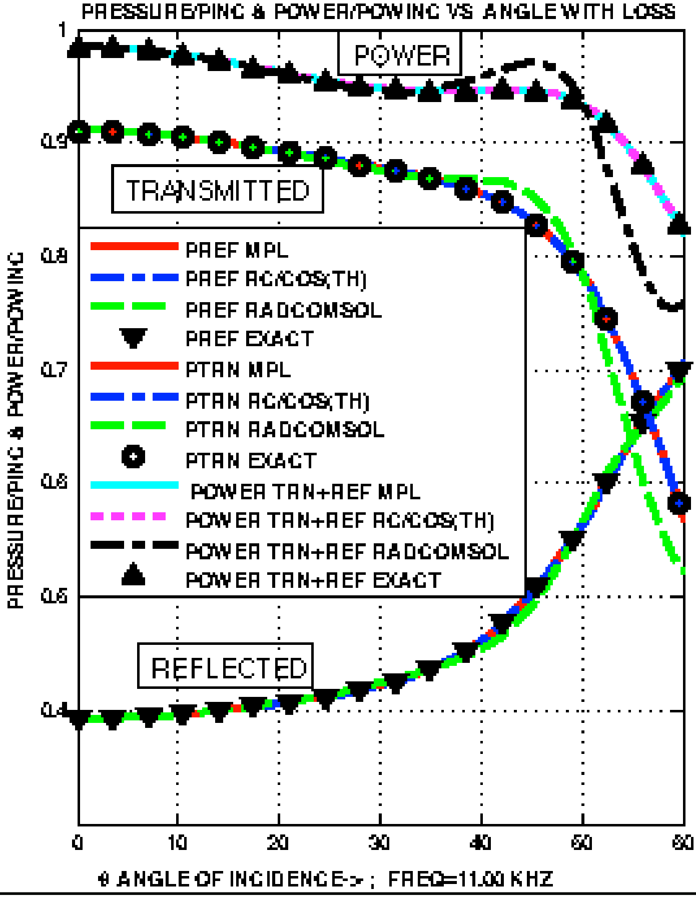
- The purpose of this example is to validate acoustic-structure interaction ***with loss*** with the enforcement of the periodic boundary conditions, with 3 different radiation boundary conditions, in a case where we have the exact solution.

- The model is shown below.

  except the inclusion and matrix material of the plate are equal

- This is basically a uniform plate where there is no characteristic x direction length , so L=any

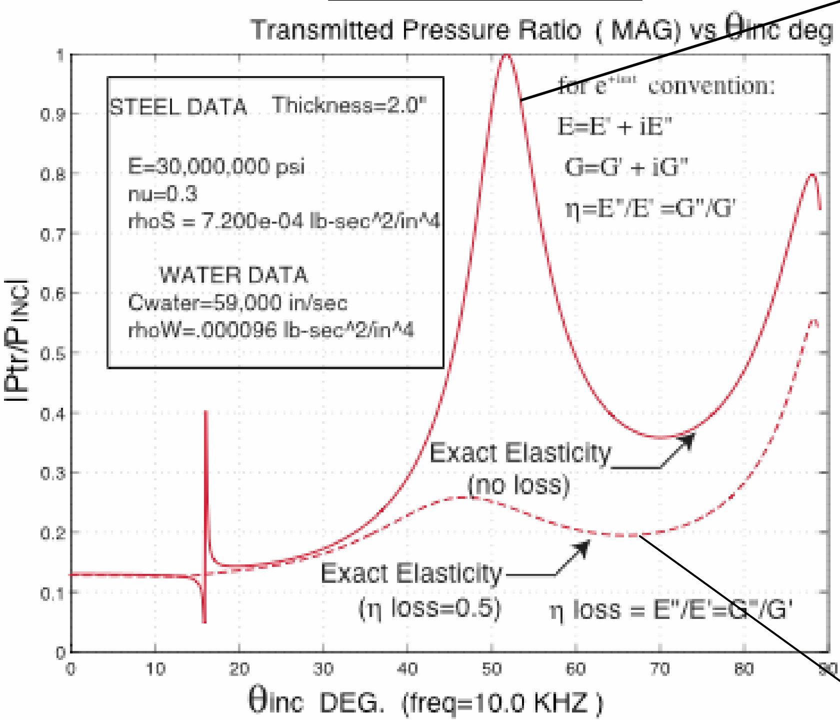




STARS3-D VALIDATION (EXACT VS FEM)
FOR TRANSMITTED PRESSURE
(WITH LOSS FACTOR AND NO LOSS FACTOR)

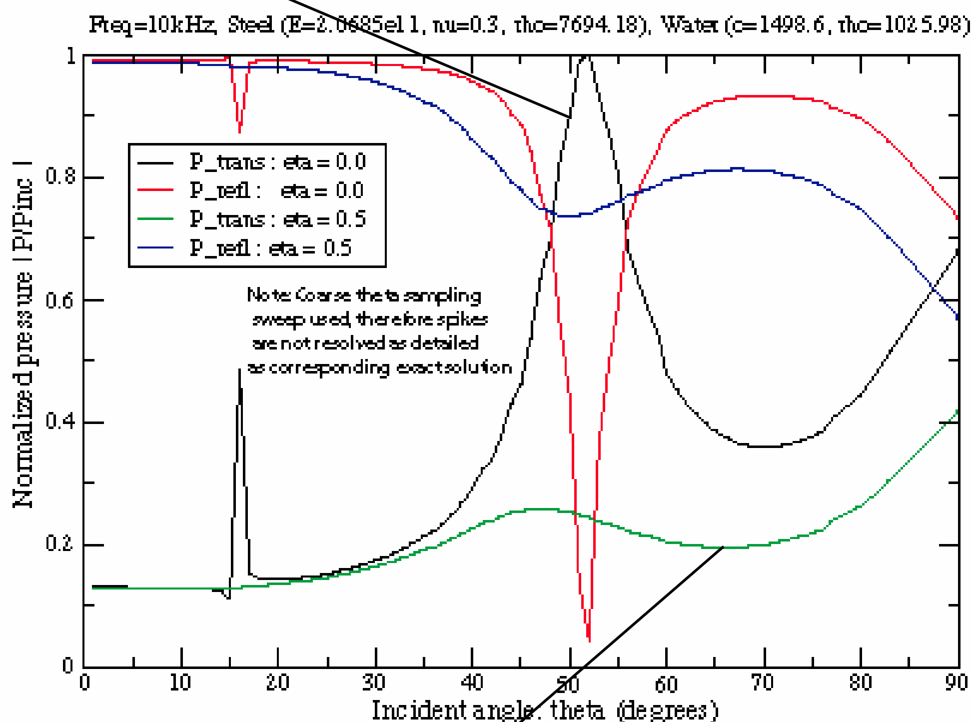
COMPARE

EXACT SOLUTION



Damping (Lossy plate) vs No Damping Elasticity
Solution for Submerged 2" Steel Plate

STARS3-D SOLUTION

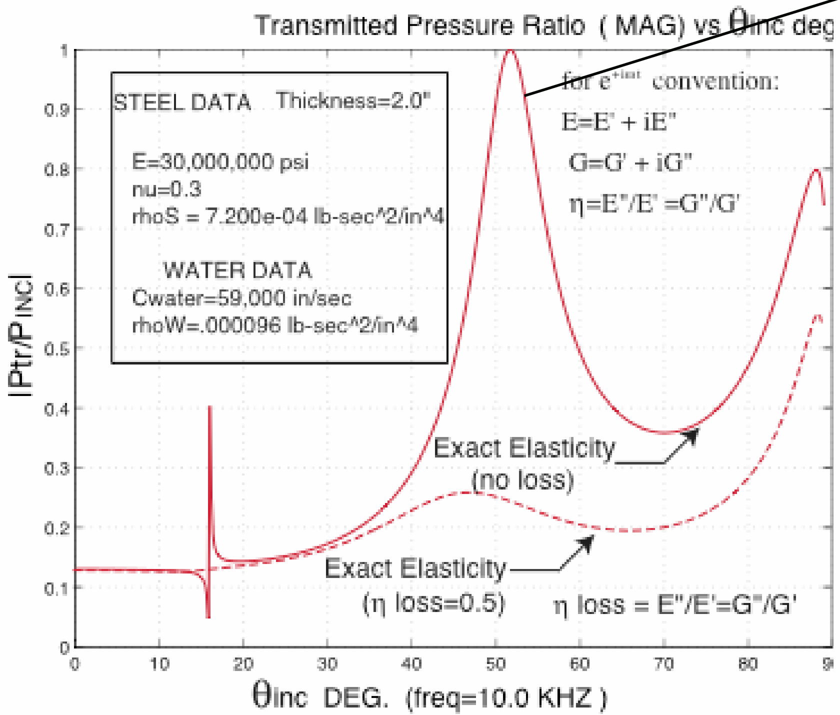


COMPARE

COMSOL VALIDATION (EXACT VS FEM)
FOR TRANSMITTED PRESSURE
(NO LOSS FACTOR CASE)

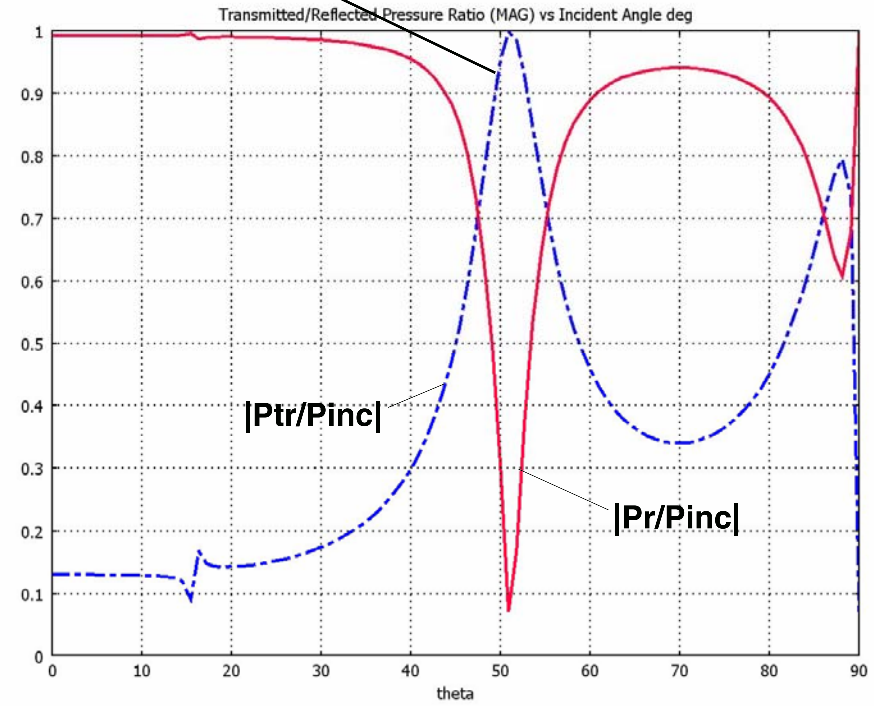
COMPARE

EXACT SOLUTION



Damping (Lossy plate) vs No Damping Elasticity
Solution for Submerged 2" Steel Plate

COMSOL SOLUTION

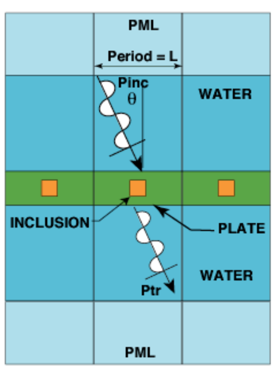


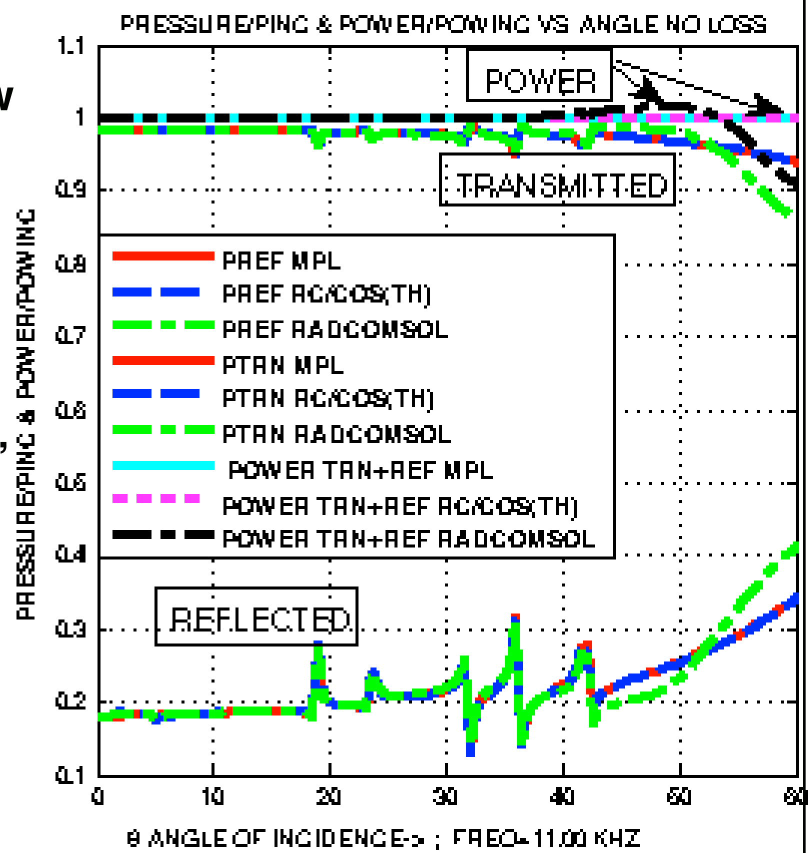
Propagation Through a Flat Plate With Periodic Inclusions

- **The purpose of this example is to demonstrate acoustic-structure interaction for plates with periodic inclusions, with 3 different radiation boundary conditions, for the *no loss model*.**

- **The FEM model is shown below except the inclusion and matrix plate material are not equal**

- **This is non-homogenous plate has a characteristic period in the x direction length , of L=5.0”**

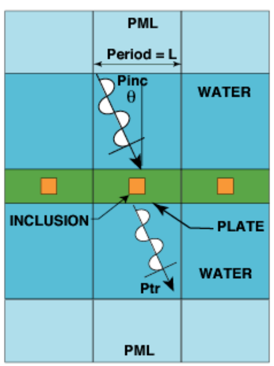


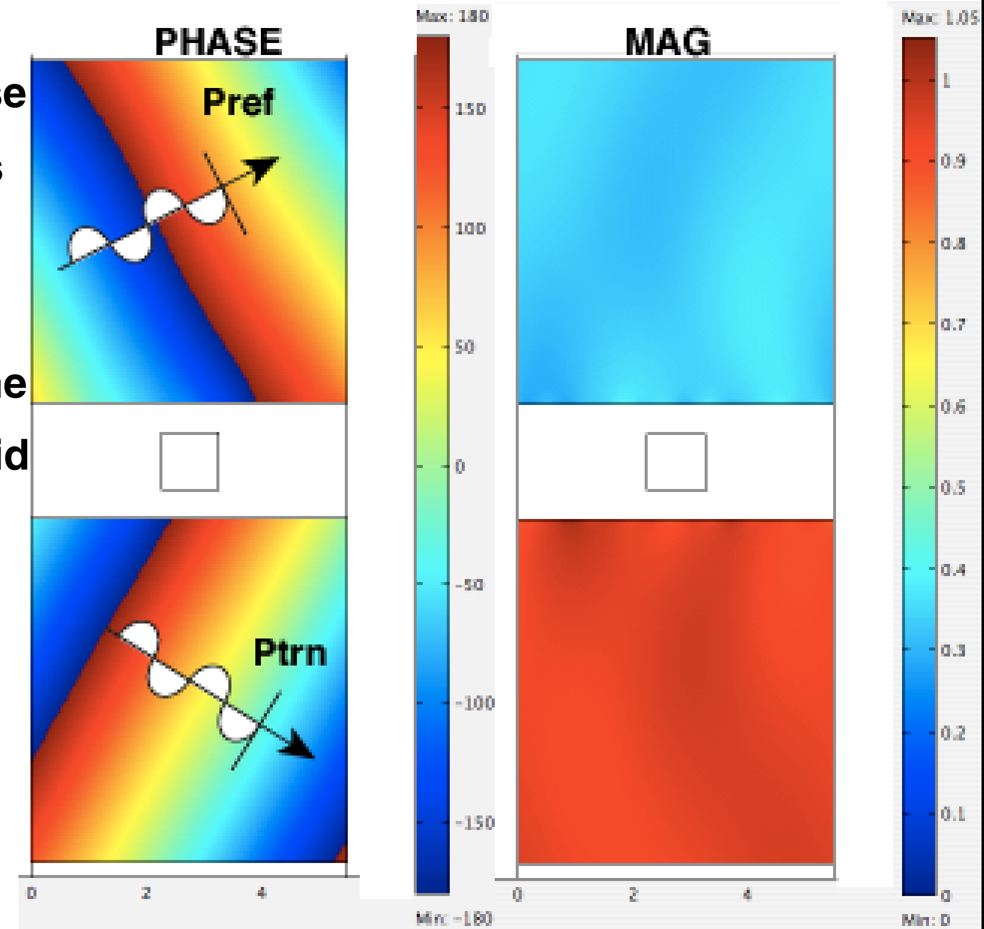


Propagation Through a Flat Plate With Periodic Inclusions

- **Here we show the amplitude and phase of the spatial field distributions (no loss model) in the front and back side fluid**

- **Lines of constant phase illustrate the wave fronts**
- **The magnitude plots illustrate the expected Non-uniform nature of the pressure field at the solid fluid interface**



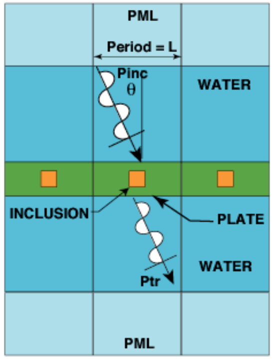


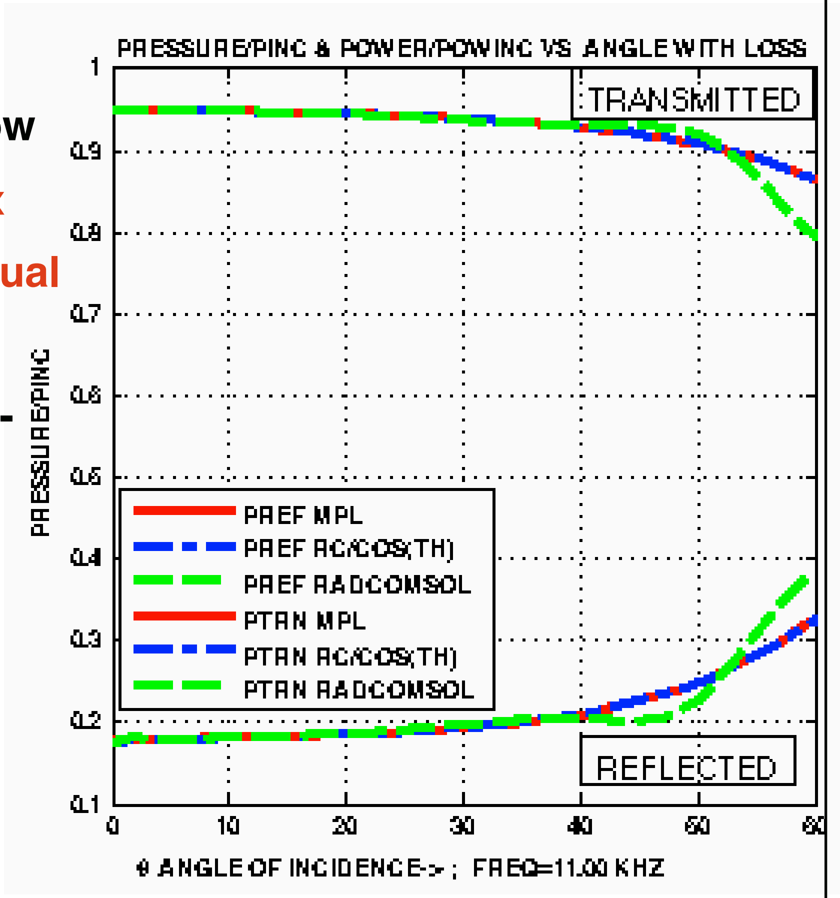
Propagation Through a Flat Plate With Periodic Inclusions

- **The purpose of this example is to demonstrate acoustic-structure interaction for plates with periodic inclusions, with 3 different radiation boundary conditions, for the *with loss model*.**

- **The FEM model is shown below where the inclusion and matrix material of the plate are not equal**

- **Note that the sharp peaks are now smoothed through the introduction of loss (damping)**

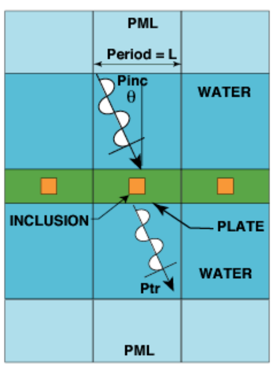


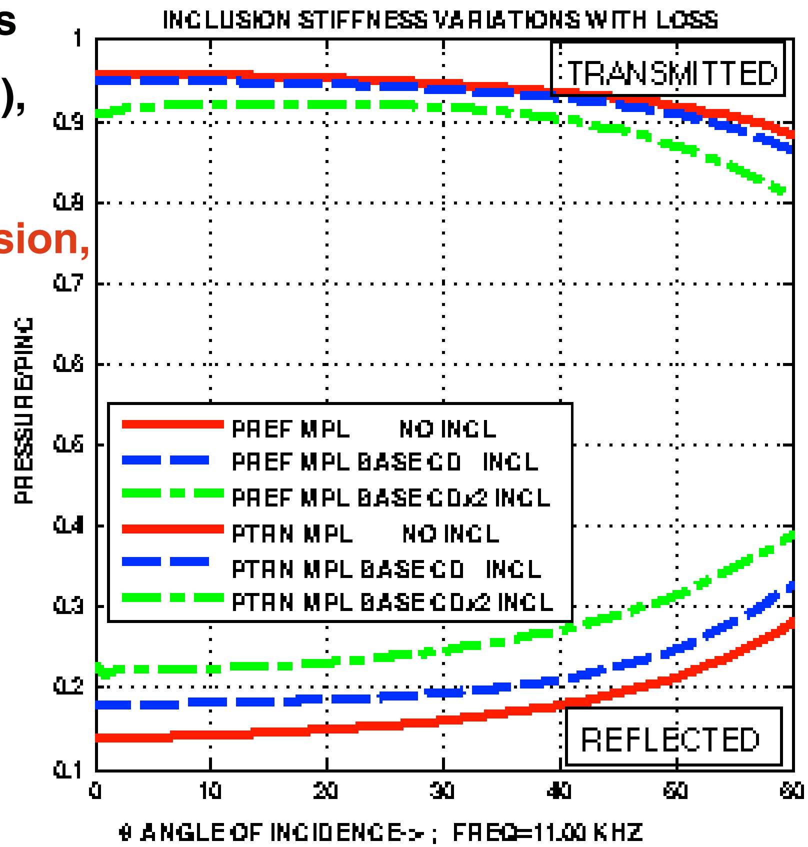


Propagation Through a Flat Plate With Periodic Inclusions

- **The purpose of this example is to demonstrate acoustic-structure interaction for plates with periodic inclusions, for a *variation in the inclusion stiffness (with loss)***

- **3 different inclusion stiffnesses (i.e. via wave speed c_D, changes), are compared**
- **The stiffer (bigger c_D) the inclusion, the worse the transparency, and the greater the reflection**

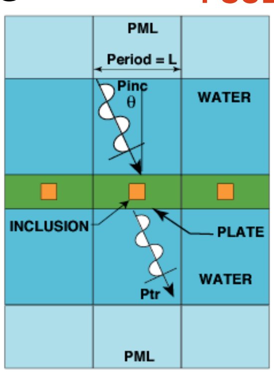


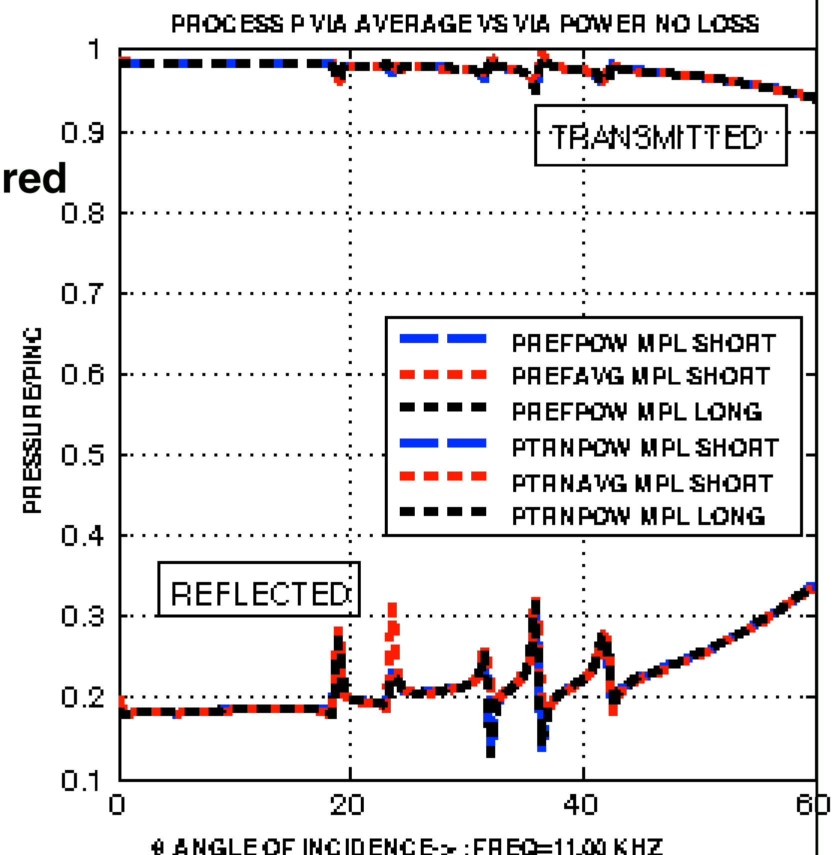


Propagation Through a Flat Plate With Periodic Inclusions

- **The purpose of this example is to demonstrate acoustic-structure interaction for plates with periodic inclusions, for a *<u>variation in the fluid length</u>* *<u>&post processing of p</u>***

- **2 different fluid model lengths "Long" (inset below) vs. "Short" (i.e. Long/10) are compared**
- **2 different pressure measures are compared, P_{AVG} vs P_{PSUDO}**
- **Short model P_{PSUDO} consistent with Long model P_{PSUDO}**





THE END