Coastal Engineering Technical Note

IRREGULAR WAVE ORBITAL VELOCITIES AT COASTAL STRUCTURES

Purpose: The purpose of this note is to describe an available PC program for estimating near-bottom water particle velocities which contribute to the problem of scour at and around various coastal structures (including vertical walls, sloping revetments, and rubble-mound structures).

Background: Undermining of the toes of coastal structures which may occur as a result of scour hole development is a serious problem. The undermining causes damage to structures and often to the buildings/beaches/roads they were designed to protect. Coastal engineers have long recognized the consequences of scour at the base of these structures, and expensive toe protection schemes often have been implemented. In instances where appreciable scour already has occurred, a common solution has been to fill the scour hole with stone or other suitable material. Scour potential at the toe of coastal structures can be assessed if the magnitude of the wave orbital velocities can be estimated. An estimate of irregular wave near bottom velocities is possible using a new technique developed at CERC. Empirical relationships for the reflection coefficient and the reflection phase that result from irregular waves being partially reflected by coastal structures can be used in conjunction with a theoretical expression (Hughes 1992) to estimate the horizontal and vertical root-mean-squared, rms, water velocities anywhere in the water column seaward of the structure (particularly near the bottom in the vicinity of the toe). The theoretical formulations are given below:

$$u_{rms}^2 = \sum_{n=1}^{\infty} \left(\frac{gk_n}{\sigma_n}\right)^2 \left[\frac{\cosh^2 k_n(h+z)}{\cosh^2 k_n h}\right] \left(1-2K_{r_n}\cos(2k_n x+\theta_n)+K_{r_n}^2\right) \frac{a_n^2}{2} \tag{1}$$

and

$$w_{rms}^2 = \sum_{n=1}^{\infty} \left(\frac{gk_n}{\sigma_n}\right)^2 \left[\frac{\sinh^2 k_n(h+z)}{\cosh^2 k_n h}\right] \left(1+2K_{r_n}\cos(2k_n x+\theta_n)+K_{r_n}^2\right) \frac{a_n^2}{2} \tag{2}$$

The above equations represent the approximation of the water particle velocities under random waves by the linear superposition (summation) of many linear components, where the subscript n refers to the nth individual component. Also,

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u_{rms}	--	root-mean-squared horizontal water velocity
w_{rms}	--	root-mean-squared vertical water velocity
z	--	vertical coordinate with z=0 at the SWL and z = -h at the bottom
x	--	horizontal coordinate with x=0 at structure toe
h	--	water depth
g	--	gravitational acceleration
k_n	--	wave number of the nth linear component
σ_n	--	angular wave frequency of the nth linear component
a_n	--	incident amplitude of the nth linear component
K_{rn}	--	reflection coefficient of the nth linear component
θ_n	--	phase shift of the nth linear component due to reflection

These equations were developed assuming nonbreaking irregular waves propagating over a flat bottom normally incident to the reflective structure. The rms-velocities can be estimated at any position (x,z) given the incident wave spectrum, water depth, and the reflection and phase functions.

Reflection Phase Relationship

Analysis of flume test results using monochromatic waves imply a relationship for phase in terms of an Irribarren-type parameter, ξ_{h_n}, namely

$$\xi_{h_n} = \frac{\sqrt{\dfrac{h}{gT_n^{\,2}}}}{\tan\beta} \tag{3}$$

where T_n is the wave period of the nth component and $\tan\beta$ is the slope of the seaward face of the structure (β is the angle the slope makes relative to horizontal). The empirical (best-fit) equation derived from this analysis is given for both smooth and rubble-mound structures by

$$\theta_n = 4\pi\left(1 - 2.20\,\xi_{h_n}^{1.15}\right) \tag{4}$$

where θ_n has units of radians, as shown in Figure 1. This particular form of the phase equation was chosen so that $\theta_n \rightarrow 4\pi$ as $\xi_h \rightarrow 0$, one of the limiting cases for complete reflection. (Note: Irregular waves cannot be used to determine the reflection phase relationship due to the 2π cyclic nature of the phases.)

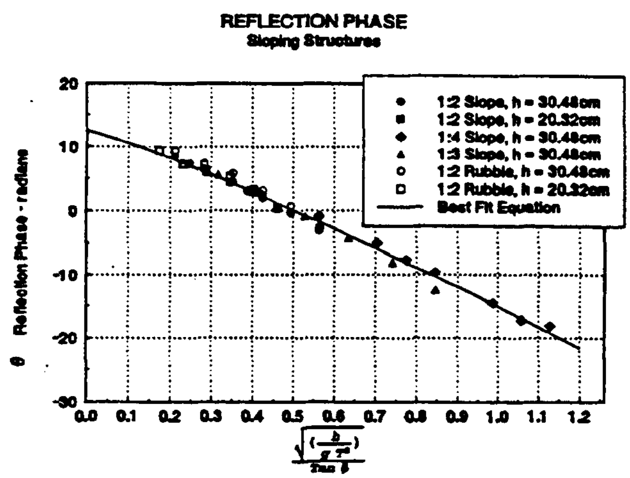


Figure 1. Reflection phase relationship for smooth and rubble mound structures for non-breaking waves.

Reflection Coefficient Relationship

Estimates of rms-velocity are for irregular waves, so data from irregular wave tests were used to establish the reflection coefficient relationship. Analysis of data obtained from irregular wave conditions covering 3 water depths, 3 peak wave periods, smooth plywood slopes of 1:2, 1:3, and 1:4, and a rubble-mound slope of 1:2, produced the relationships shown in Figures 2 and 3. Once again, the Irribarren-type parameter, ξ_h, proved to be a good nondimensional parameter. The data shown on Figure 2 yielded a best-fit equation given by

$$K_{r_n} = \frac{0.1176}{0.1176+\xi_{h_n}^{2.6}} \qquad (Smooth\ Structures) \tag{5}$$

which is shown as the solid line on Figure 2, whereas the data in Figure 3 produced the following relationship:

$$K_{r_n} = \frac{0.1415}{0.1415+\xi_{h_n}^{0.80372}} \qquad (Rubble\text{-}Mound\ Structures) \tag{6}$$

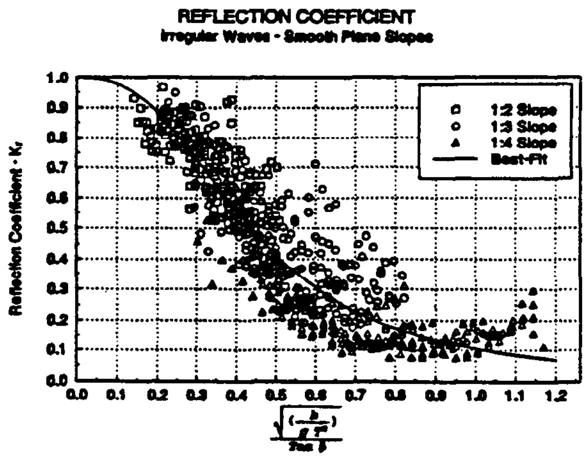


Figure 2. Reflection coefficient relationship for smooth plane sloping structures.

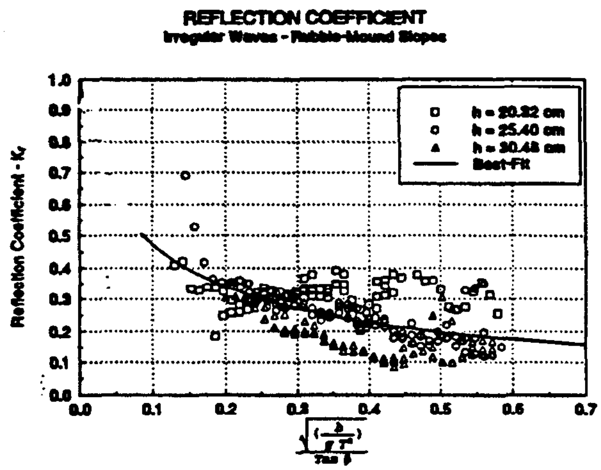


Figure 3. Reflection coefficient relationship for rubble-mound structures.

The above empirical expressions for reflection coefficient (Eqns. 5 and 6) and reflection phase (Eqn. 4) have been programmed into a PC SHAREWARE program that solves Eqns. 1 and 2 for given input incident wave spectra.

Using this SHAREWARE program, estimates were made of the horizontal and vertical rms-velocities at mid-depth using the resolved incident spectra from laboratory tests mentioned above. These estimates were compared to actual measurements of irregular wave rms-velocities as shown for two cases in Figure 4. The structure toe is situated at position $x=0$, and this represents the closest distance to the structure for which estimates can be made. Note that the location of the maximum u_{rms} velocity occurs at different positions relative to the structure toe for the two different wave conditions. This illustrates the importance of including the phase relationship in estimates of potential scour at the toes of structures.

The following sections provide details about the PC-computer SHAREWARE program that allows the user to estimate the rms horizontal and vertical velocities using the above described methodology. Included is the procedure for using the SHAREWARE and generating program output.

PC Requirements

The present version (Ver 2.00) of the code has been compiled to execute on a PC with a math coprocessor. It can be used immediately on systems which have a math coprocessor, VGA graphics card, and a HP LaserJet III installed on the LPT1 port (only necessary to get plot hardcopies). Different PC setups will need to install the program. Instructions for installing the program are very straightforward and are included with the software.

Executing the Program

The program is started by typing **RMSVEL <return>** . The program first examines the current directory for a default file named RMSVEL.CFG which contains the parameter defaults from the last time the program was run. If this file is found, the default parameters are read into the program. If the file does not exist, hard-wired defaults are used. At the end of the program, the file RMSVEL.CFG is created and user-modified defaults are written into the file for use the next time. This feature saves time during repeated application of the program where only one or two of the parameters are altered.

User Input Section

The first screen (Figure 5) is typical of the interactive screens which prompt the user for input or decisions required to run the program[1]. This screen lists the assumptions inherent in the analysis performed by the program. The next screen presents the first option for the user - which type of irregular wave condition to use as input. There are two basic options for the input incident wave spectrum:

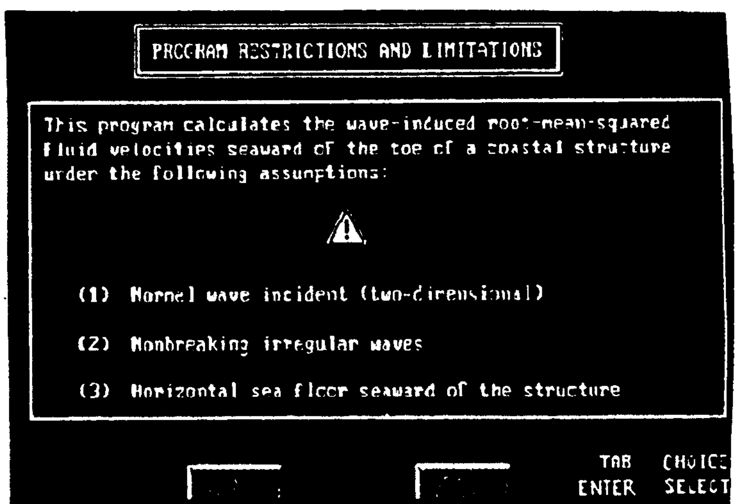
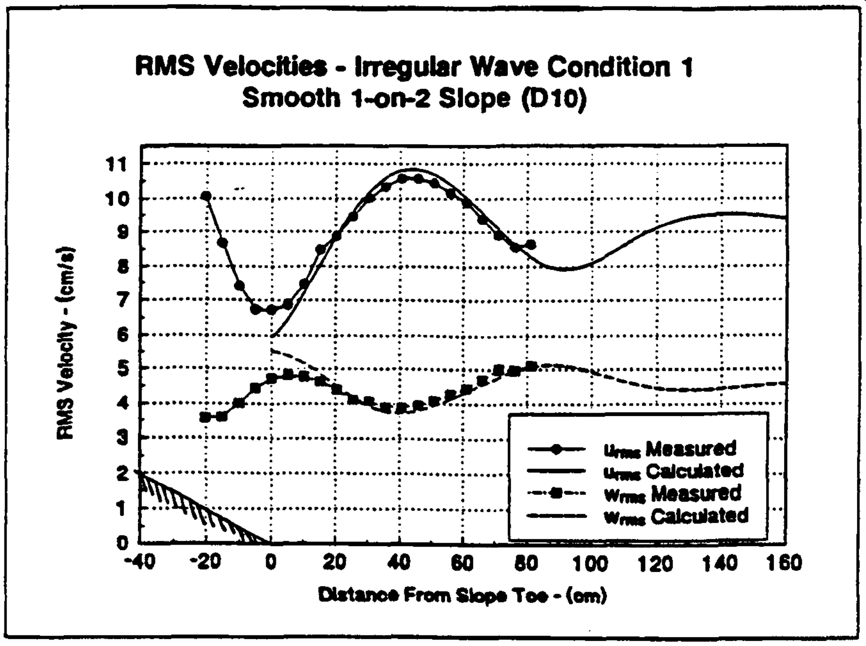


Figure 5. Initial interactive screen displayed in Shareware program.

[1] Note: Help screens are normally available by pressing the F1 key.



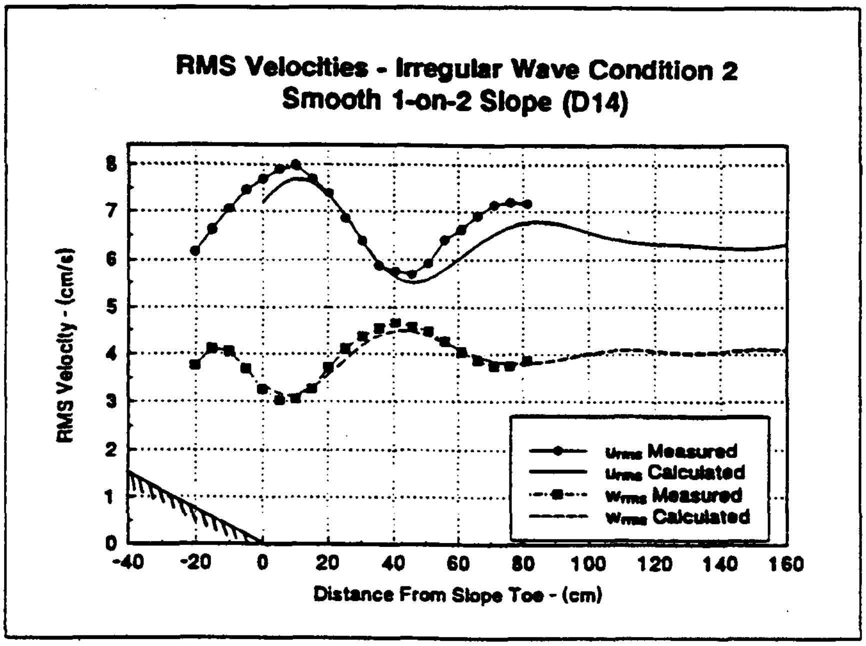


Figure 4. Comparison of predicted values to measured values of u_{rms} and w_{rms}, for two different conditions measured in the laboratory.

1) TMA Spectrum Option.

The TMA option will probably see the most use by the field offices. This option generates an incident TMA spectrum, which is a spectral representation of waves which field experience has shown to exist with specified conditions of depth, peak frequency, and windspeed. The TMA spectrum is generated internally by the program, using the user-specified parameters H_{mo}, T_p, and γ. H_{mo} is known as the zero moment wave height, and is the energy-based definition of significant wave height, H_s. In deep water, H_{mo} is approximately equivalent to H_s, but the two can be quite different as waves shoal (Hughes and Fowler, 1990). T_p is the peak spectral period and γ is a spectral peakedness parameter typically ranging between 1 and 7. The default γ is 3.3, which will generate a spectrum typical of that associated with the widely known Joint North Sea Wave Program (JONSWAP) data. The user also supplies a name for the output file.

2) User-Supplied Spectrum Option.

The user-specified spectrum option allows input of a measured (or otherwise specified) incident spectrum for use in the calculation. The following options are presently supported:

Generic: Input file contains values of frequency (Hz) and continuous energy density (m^2-s).

RAF: These are special files created using CERC's laboratory reflection analysis procedure on Laser Doppler Velocimeter (LDV) data. They are obtained as output from the program REFLECT.

GAF: These are also special files created using CERC's PC-based GODA analysis program PCGODA.

For each of these cases, reading of the input files continues until an End-of-File, <EOF>, mark is encountered.

All of the above options create an output file having the same name as the incident spectrum input file, but having an ".RMS" extension. For program evaluation purposes, an input file named D10.RAF is provided with the shareware program for use with the second option.

Existing Output Filenames

If the program detects that the selected (TMA option) or generated (User-selected option) output filename already exists in the current directory, a screen will appear which offers following choices:

Overwrite the existing file
Rename the existing file
Enter a new output filename
Exit the program

Input Parameter Specification

The final set of options are the same irrespective of what type of incident spectrum is chosen to produce the velocities. A menu is offered giving the user options to select:

Water Depth: Depth at the toe of the structure (in meters).

Solution Depth: Depth at which velocities are calculated (in meters). This entry should be the distance down from the SWL in meters. For example, the bottom would be the same as the depth. It is necessary to include a negative. Also, attempts to enter a value greater than the depth will not be accepted.

Horizontal Increment: The program first calculates the wavelength associated with the peak period, then divides the distance of three of these wavelengths into equal length increments at which solutions will be computed. The default for the number of horizontal increments is 200, but the user may input any value within the range of 1 - 200. Units for horizontal distance are meters.

Structure Slope: Structure slopes are entered as fractions representing rise/run. Because the mildest slope represented in the experimental data was 1/4 (slope = 0.25), acceptable slope values range between $0.25 < \tan \beta < \infty$. The ∞ condition represents case of a vertical wall.

<u>Structure Type</u>: Structure type can be either smooth slope or rubble-mound slope. The empirical relationship for rubble-mound slopes is based on a slope of 1:2, but is thought that reasonable estimates will be produced for other typical rubble-mound slopes.

<u>NOTE</u>: To input the incident spectrum contained in file D10.RAF, the following parameters are associated with this wave condition:

Depth: $h = 0.305$ m
Slope: $\tan \beta = 0.5$

Calculation Section

After receiving correct user input, the program first calculates the continuous incident wave spectrum, then offers the user the option of plotting results on the screen. The user can zoom the plot, or print the plot if the program is correctly installed for the printer. An example of a plot calculated for a TMA spectrum generated using $H_{mo} = 1.20$ m, $T_p = 10.0$ sec, and $\gamma = 3.3$, is shown below in Figure 6.

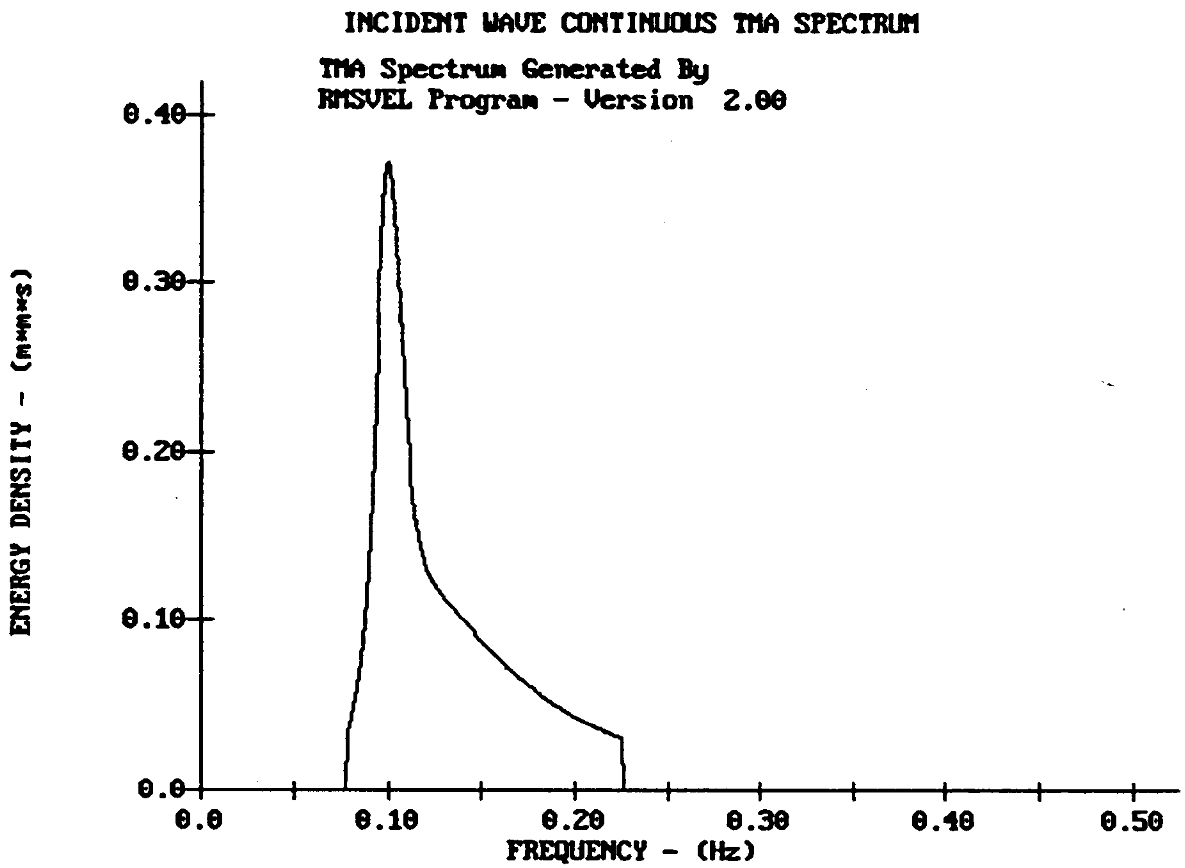


Figure 6. Example of TMA Spectrum.

The next step is the calculation of the rms-velocities at each of the 200 horizontal locations. A counter on the screen lets you know how the computation is progressing. Once the computing is completed, the spatial variation of rms-velocities as a function of distance from the toe of the structure can be viewed on the screen and/or printed. Examples of rms-velocity plots associated with the inputted TMA spectrum are shown in Figures 7 and 8. Figure 6 presents results for a mid-depth location and Figure 8 presents calculated velocities at the bottom. Note that vertical velocity is zero near the bottom.

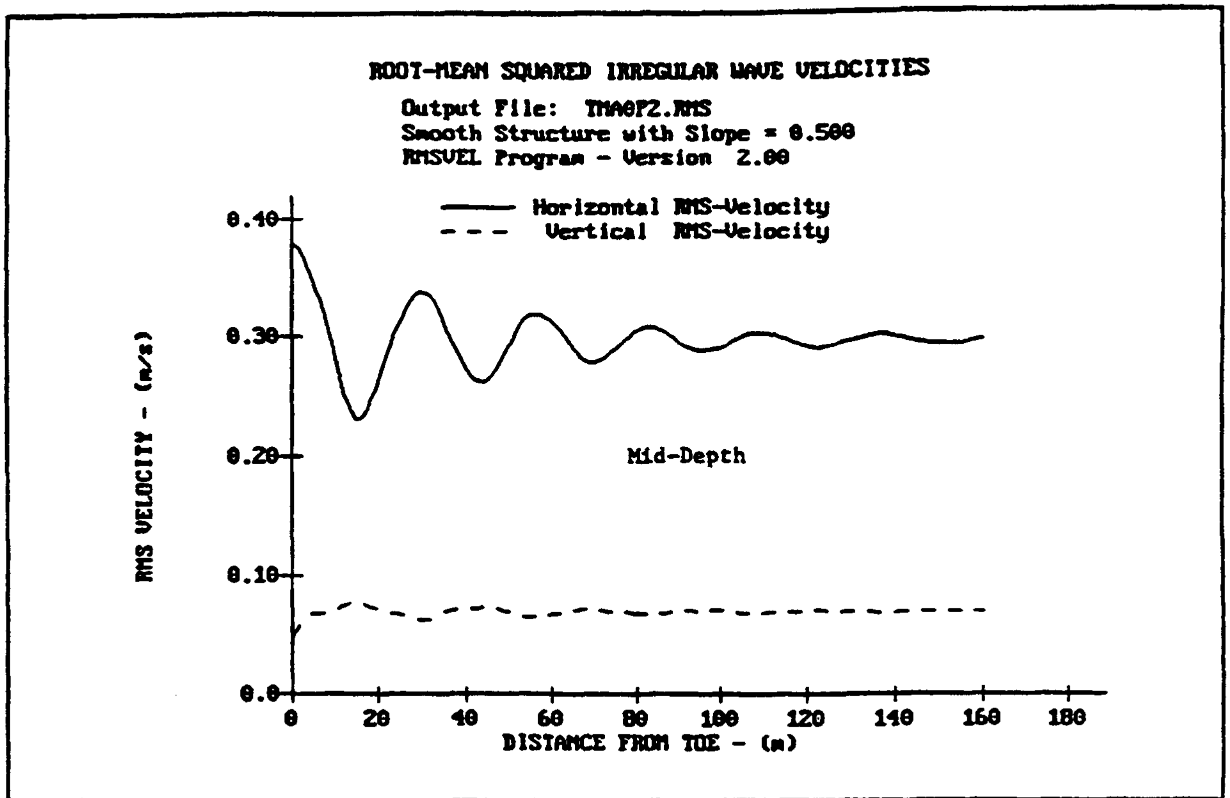


Figure 7. Plot showing calculated velocities for mid-depth condition.

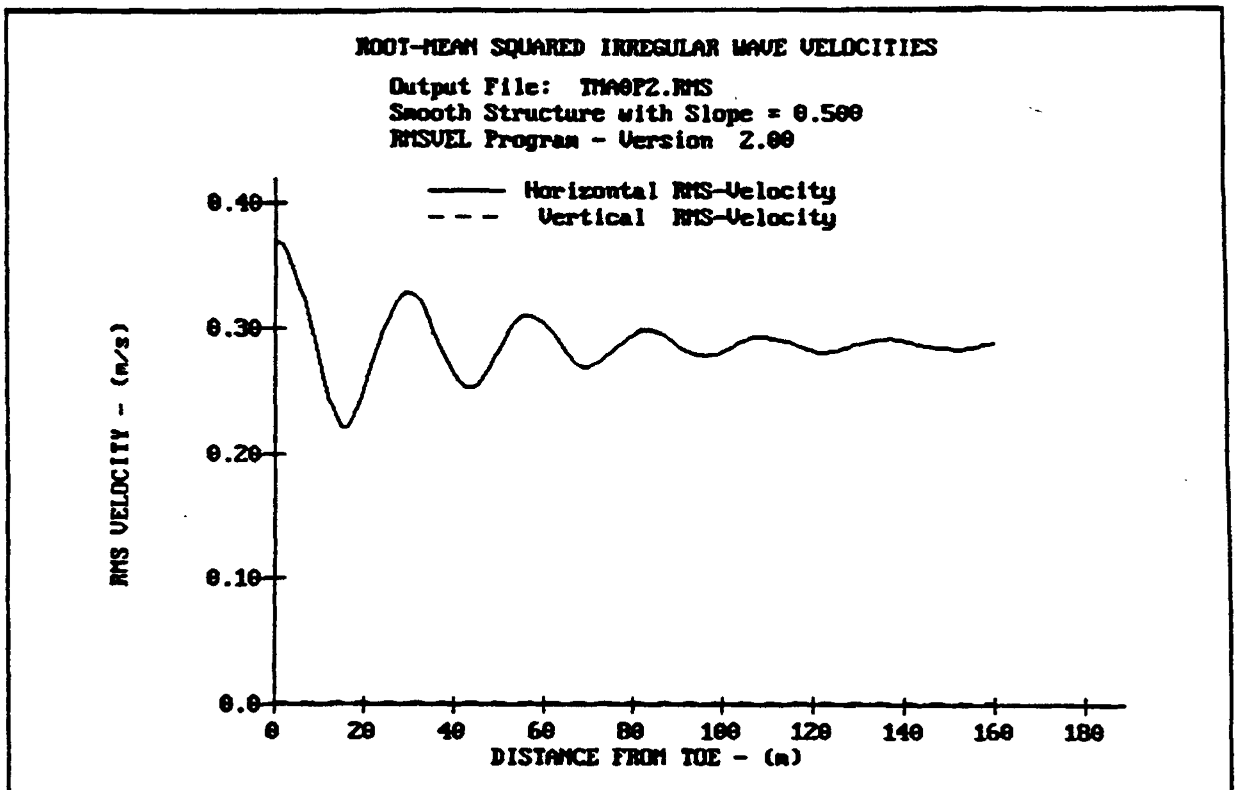


Figure 8. Plot showing calculated velocities for near bottom location.

Output Section

After leaving the plot option, a final screen is displayed showing the parameters for this calculation. This same information is written to the output file, along with the 200 values of horizontal and vertical rms-velocity. A truncated sample of the ASCII output file is shown below.

```
ESTIMATION OF ROOT-MEAN-SQUARED IRREGULAR WAVE VELOCITIES
=====> Program-Generated TMA Spectrum with GAMMA =  3.30
       Version  1.00 Analysis Performed on: 07/07/93 at 10:26:14

=====> USER-SPECIFIED PARAMETERS:
       Incident Significant Wave Height (Hmo)....[  1.2000] m
       Incident Peak Spectral Wave Period (Tp)...[ 10.0000] s
       Water Depth (h)...........................[  5.0000] m
       Velocity Solution Depth (z)...............[ -4.9000] m
       Structure Slope (rise/run)................[  0.5000]
       Structure Type............................[ Smooth ]

=====> MAXIMUM RMS-VELOCITIES
       Maximum Urms =   0.6133 m/s at Location, X =   0.0000
       Maximum Wrms =   0.0068 m/s at Location, X =  16.2398
```

X (m)	Urms (m/s)	Wrms (m/s)
0.000	0.6133	0.0055
1.015	0.6032	0.0056
2.030	0.5903	0.0057
3.045	0.5749	0.0058
4.060	0.5571	0.0060
5.075	0.5375	0.0061
6.090	0.5165	0.0063
7.105	0.4950	0.0065
8.120	0.4736	0.0066
9.135	0.4532	0.0067
10.150	0.4344	0.0067
11.165	0.4177	0.0067
12.180	0.4035	0.0067
13.195	0.3922	0.0068
14.210	0.3842	0.0068
15.225	0.3799	0.0068
16.240	0.3796	0.0068
17.255	0.3832	0.0068
18.270	0.3907	0.0068
19.285	0.4016	0.0067
20.300	0.4150	0.0066
21.315	0.4302	0.0065
22.330	0.4464	0.0064
23.345	0.4628	0.0063
24.360	0.4788	0.0062
25.375	0.4941	0.0061

Near-Bottom

SUMMARY: Engineering approximations for the rms horizontal and vertical velocities in front of coastal structures can be estimated in terms of structure slope, water depth, and incident wave spectrum. The formulations in this technical note are based on linear wave theory and should provide the practicing coastal engineer with useful tools for calculating both vertical and horizontal velocities under irregular waves at a coastal structure. These formulations have been verified in the laboratory but have not yet been verified with field data. However, the estimates they provide should be sufficiently accurate so long as wave nonlinearities are minor. For nonlinear wave conditions (breaking waves), these estimates must be considered preliminary.

ADDITIONAL INFORMATION: For additional information contact Dr. Jimmy E. Fowler of the Coastal and Hydraulics Laboratory.

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