Coastal Engineering Technical Note

PRACTICAL CONSIDERATIONS IN LONGSHORE TRANSPORT RATE CALCULATIONS

PURPOSE: To present two methods for evaluating the reliability and accuracy of longshore sand transport rate calculations.

INTRODUCTION: Estimates of the longshore sand transport rate Q , the rate at which littoral material moves alongshore in the surf zone from currents produced by obliquely breaking waves, are often required in planning, design, and evaluation of various types of coastal projects. The widely-utilized CERC formula (SPM 1984) for estimating the potential longshore sand transport rate is based on the assumption that Q is proportional to the longshore component of energy flux in the surf zone, given by Equation 4-49 of the SPM (1984),

$$Q = \frac{K}{(\rho_s - \rho)\, g\, a'} P_{ls} \tag{1}$$

where K = a dimensionless empirical sand transport coefficient (K = 0.39 if significant breaking wave height is used to calculate P_{ls})
ρ_s = density of sand (quartz, ρ_s = 2,650 kg/m^3)
ρ = the density of water (seawater at 20° C, ρ = 1,025 kg/m^3)
g = acceleration due to gravity (g = 9.81 m/sec^2)
a' = ratio of the volume of solids to total volume, accounting for sand porosity (a' = 0.6)
P_{ls} = longshore wave energy flux factor

P_{ls} depends on wave conditions at breaking (SPM (1984) Equation 4-39),

$$P_{ls} = \frac{\rho g}{16} H_{sb}^2\, C_{gb} \sin 2\theta_b \tag{2}$$

where H_{sb} = significant wave height at breaking
C_{gb} = wave group speed at breaking
θ_b = angle breaking wave crest makes relative to the shoreline

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In shallow water,

$$C_{gb} = \sqrt{g d_b} \tag{3}$$

where d_b is the depth at breaking, usually assumed to be linearly related to the wave height at breaking,

$$H_b = \gamma \, d_b \tag{4}$$

in which $\gamma = 0.78$ is the wave breaking index.

Estimates of breaking wave heights and angles can be obtained from Littoral Environment Observation (LEO) data, or by transforming waves inshore to breaking from an offshore source such as wave gage or Wave Information Study (WIS) data (CETN-II-19). Any source of wave data has an inherent accuracy range which is dependent on the measuring device or observer's bias. Best estimates of Q result when many wave conditions representative of a typical year are weighted according to their likelihood of occurrence.

Two practical considerations in calculating potential longshore sand transport rates are presented. The first method describes a procedure for evaluating input wave conditions to determine if they exceed a threshold value for significant transport. Wave events below the threshold are eliminated, thereby reducing the number of calculations required. This procedure is most useful when a large number of wave conditions are evaluated, as in a coded routine or numerical model. The second method gives a measure of uncertainty in transport rate estimates based on the accuracy of input data. Both of these considerations are illustrated in a simple example problem, and result in more efficient and accurate longshore sand transport rate calculations, together with an appreciation of the limitations in the overall longshore sand transport rate estimation method.

THRESHOLD FOR SIGNIFICANT TRANSPORT:

Based on measurements of the longshore sand transport rate, waves, and currents obtained during two field experiments on a medium sand beach (median grain size $D_{50} = 0.2$ mm), a threshold for significant transport was empirically determined and expressed in terms of a "longshore discharge parameter," R , defined as

$$R = V X_b H_{sb} \tag{5}$$

where V = average longshore current speed in the surf zone (m/sec), and
X_b = average width of the surf zone (m).

The critical value of the longshore discharge parameter, R_c , below which significant sand transport did not occur, was empirically determined to be R_c = 3.71 m^3/sec (Kraus and Dean 1987; Kraus, Gingerich, and Rosati 1988).

The longshore current speed on an open coast can be expressed by an empirical relation (Komar and Inman 1970),

$$V = 1.35\ u_m \sin 2\theta_b \tag{6}$$

in which the maximum wave orbital velocity at breaking u_m is given by

$$u_m = 0.5\ H_{sb} \sqrt{\frac{g}{d_b}} \tag{7}$$

Assuming a plane beach profile ($X_b = d_b/\tan\beta = H_{sb}/(\gamma \tan\beta)$, where $\tan\beta$ is the bottom slope), and combining Equations (5), (6), and (7) gives

$$R = \frac{1.35}{2}\sqrt{\frac{g}{\gamma}}\frac{H_{sb}{}^{5/2}}{\tan\beta}\sin 2\theta_b \tag{8}$$

For values of $|R| \leq R_c$ = 3.71 m^3/sec , appreciable sand transport will not occur. Note that the functional form $H_{sb}{}^{5/2} \sin 2\theta_b$ is consistent with the P_{ls} factor (Equation (2)) appearing in the CERC formula.

For large wave data sets, the threshold parameter can be used to evaluate various input conditions and determine whether or not values of calculated sand transport will have engineering significance. Elimination of insignificant wave events can reduce computational time in numerical models.

ESTIMATING UNCERTAINTY IN Q :

Wave measurements and observations have associated uncertainties based on instrumentation accuracy and observer bias. Given that there are breaking wave height and wave angle uncertainty values ΔH_{sb} and $\Delta\theta_b$, respectively, an associated longshore transport uncertainty ΔQ can be calculated. Combining Equations (1), (2), (3), and (4)

$$Q \sim (H_{sb}{}^{5/2} \sin 2\theta_b) \tag{9}$$

An estimate of the uncertainty in the longshore transport rate can be evaluated by including the uncertainties in breaking wave height and angle:

$$Q \pm \Delta Q \sim (H_{sb} \pm \Delta H_{sb})^{5/2} \, sin2(\theta_b \pm \Delta\theta_b) \tag{10}$$

Assuming that the wave angle at breaking is small, and using the first two terms of a Taylor series expansion of Equation (10), the uncertainty in the longshore transport rate is estimated as

$$\Delta Q \sim Q(\pm \frac{\Delta\theta_b}{\theta_b} \pm \frac{5}{2} \frac{\Delta H_{sb}}{H_{sb}}) \tag{11}$$

The uncertainty in wave height is greatly amplified compared to the uncertainty in wave angle. For example, a 15 percent accuracy in wave height and 15 percent accuracy in wave angle result in 37.5- and 15-percent uncertainty contributions for height and angle, respectively, totalling a 52.5 percent uncertainty in Q .

***********************************EXAMPLE***********************************

Given: Calculate the potential longshore sand transport rate and associated uncertainty using the breaking wave conditions given in Table 1. The original wave height, period, and direction were obtained in deepwater with a gage at the site, and shoaled to breaking using linear theory. Each wave condition given in Table 1 is expected to occur equally during a typical year. Nearshore beach slope at the site is 1:30. Wave gage accuracy is estimated to be 10 percent of the measured wave height and 10 percent of the measured wave angle.

Table 1

Wave Conditions and Corresponding Sand Transport Rate Parameters

Wave Condition	H_{sb} (m)	θ_b (deg)	R (m^3/sec)	Q (m^3/year)	ΔQ (m^3/year)
1	0.43	-3.9	-1.2	0	0
2	1.00	-6.2	-15.4	-61,500	-/+21,500
3	1.25	2.2	9.6	38,400	+/-13,400
4	1.53	7.6	54.5	217,600	+/-76,200
5	0.78	-1.7	-2.3	0	0
6	0.60	-10.1	-6.9	-27,600	-/+9,700
7	2.20	4.1	73.5	2,939,000	+/-1,029,000
8	2.78	-11.3	-355.6	-1,419,000	-/+496,700
9	0.67	-1.5	-1.4	0	0
10	3.10	8.8	-367.4	-1,466,000	-/+513,200
Total:				220,900	+/-77,500

Solution:

A. Transport Rate Magnitude

Using Equation (8) to evaluate the discharge parameter, sand transport produced by wave conditions 1, 5, and 9 can be assumed to be insignificant since $|R|$ is less than $R_c = 3.71\ m^3/sec$. For example, wave condition 1 gives

$$|R| = \frac{1.35}{2}\sqrt{\frac{9.81}{0.78}}\,\frac{(0.43)^{5/2}}{1/30}\,|sin2(-3.9)|$$
$$= 1.2\ m^3/sec$$

Potential longshore sand transport rates for the remaining wave conditions can be evaluated by combining Equations (1), (2), (3), and (4) as follows

$$Q = \frac{(0.39)(1025)(9.81)}{(2650-1025)(9.81)(0.6)(16)}\sqrt{\frac{9.81}{0.78}}\ H_{sb}{}^{5/2} sin2\theta_b$$
$$= 0.091\ H_{sb}^{5/2}\sin 2\theta_b \quad (m^3/sec)$$
$$= 2.87\ 10^6\ H_{sb}^{5/2}\sin 2\theta_b \quad (m^3/year)$$

Calculated values of Q are presented in Table 1.

B. Uncertainty in Q

Assuming that the cited accuracy of the wave measurements in deep water and at breaking are similar, the uncertainty in Q can be estimated using $\Delta H_{sb}/H_{sb} = 0.10$ and $\Delta\theta_b/\theta_b = 0.10$ in Equation (11). For wave condition 2,

$$\Delta Q \sim \pm(-61{,}500)\ (0.1+5/2\ 0.1)$$

$$\sim \pm(-61{,}500)\ (0.35) \qquad (11)$$

$$\sim \pm\ 21{,}500\ m^3/year$$

Similarly, uncertainty estimates can be obtained for the other wave conditions, as shown in the right-hand column of Table 1. For this example, the estimated uncertainty (+/- 77,500 m^3/year) has a range spanning more than two-thirds the estimated net transport rate (220,900 m^3/year).

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ADDITIONAL INFORMATION: For additional information contact Dr. Julie D. Rosati, Coastal and Hydraulics Laboratory, at (251) 441-5535, Julie.D.Rosati@erdc.usace.army.mil.

REFERENCES: Komar, P.D. and Inman, D.L. 1970. "Longshore Sand Transport on Beaches," Journal of Geophvsical Research, 73(30), pp. 5914-5927.

Kraus, N.C. and Dean, J.L. 1987. "Longshore Sediment Transport Rate Distribution Measured by Trap," Proceedings, Coastal Sediments '87, American Society of Civil Engineers, pp. 881-896.

Kraus, N.C., Gingerich, K.J., and Rosati, J.D. 1988. "Toward an Improved Empirical Formula for Longshore Sand Transport," Proceedings, 20th Coastal Engineering Conference, American Society of Civil Engineers, pp. 1182-1196.

Shore Protection Manual. 1984. 4th ed., 2 vols, US Army Engineer Waterways Experiment Station, Coastal Engineering Research Center, US Government Printing Office, Washington, DC.

US Army Engineer Waterways Experiment Station, Coastal Engineering Research Center. 1989. "Estimating Potential Longshore Sand Transport Rates using WIS Data," CETN-II-19, Vicksburg, MS.