Modeling insurgency, counter-insurgency, and coalition strategies and operations

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Modeling insurgency, counter-insurgency, and coalition strategies and operations

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Abstract

We model insurgency and counter-insurgency (COIN) operations with a large-scale system of differential equations and a dynamically changing coalition network. We use these structures to analyze the components of leadership, promotion, recruitment, financial resources, operational techniques, network communications, coalition cooperation, logistics, security, intelligence, infrastructure development, humanitarian aid, and psychological warfare, with the goal of informing today's decision makers of the options available in COIN tactics, operations, and strategy. In modern conflicts, techniques of asymmetric warfare wreak havoc on the inflexible, regardless of technological or numerical advantage. In order to be more effective, the US military must improve its COIN capabilities and flexibility to match the adaptability and rapid time-scales of insurgent networks and terror cells. Our simulation model combines elements of traditional differential equation force-on-force modeling with modern social science modeling of networks, PSYOPs, and coalition cooperation to build a framework that can inform various levels of military decision makers in order to understand and improve COIN strategy. We show a test scenario of eight stages of COIN operation to demonstrate how the model behaves and how it could be used to decide on effective COIN resources and strategies.

Keywords

counter-insurgency, force-on-force model, differential equations, network model

1. The terrorism–counter-terrorism model

In modeling a terrorist (T) organization, we extend the basic differential equation model of Arney et al.[1,2] and the mathematical modeling concepts of Fox,[3] Gutfraind,[4] Udwadia,[5] Ganor,[6] and Committee on Organizational Modeling from Individuals to Societies[7] to track the populations (numbers) of several groups within the terrorist organization: senior leaders (l), junior leaders (j), outside supporters (o), bomb-makers (b), and foot soldiers (f). We simultaneously track and compute the intensity of several terrorism factors: financial support for the organization (m), intellectual level of the organization (i), public (in-country, local) support for the organization or cause (p), and world-wide support for the cause (w). These elements all factor into the overall strength of the terror organization (s). Next we consider the following counter-terrorism (CT) factors (all in upper case): public support for the CT effort (C), the cooperative coalition (multi-national/multi-agency) effort (CC), aggressiveness of direct CT operations (D), aggressiveness of intelligence gathering (G), aggressiveness of PSYOP/information distribution (P), aggressiveness of aid to the local public/host country government (A), aggressiveness of US/coalition logistics (L), and aggressiveness of US/coalition security (Y). Lastly, we combine CT measures to determine the overall strength of the CT operations (S). From a mathematical point of view we are embracing complexity by modeling 19 dependent factors with 19 equations and over 80 parameters, from a social science and military professional perspective we are including many, if not most, of the primary factors discussed for terrorism and CT operations, especially the

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perspectives taken in FM 3-24.[8] Even taking the complexity of our model into account, we recognize that our equations are a far cry from completely reflecting the intricate realities of T–CT operation. Our equations are dynamic and time dependent as we use time (t) as our one independent variable. Through numerical iteration, we are able to optimize the CT resource allocation (strategic aggressiveness) to determine an optimal counter-insurgency (COIN) strategy given the T and CT parameters and initial conditions of the insurgency (IN) and COIN.[1] After we develop the components of the model, we provide an eight-stage test scenario to demonstrate the behavior and utility of the model.

The following 19 items briefly explain the equations in our T–CT model. These equations relate the change of the various elements of the IN with the resources used and organizations involved in waging the COIN (see Arney et al.[1] for more details of this part of the model):

1. To track the rate of change of the number of senior leaders in the organization, we take six different processes into account: promotion of junior leaders (j), loss of senior leaders to internal factors (l), recruitment of senior leaders (o), rate that senior leaders are removed by CT measures (Dl), rate that senior leaders are removed by PSYOP (P) and the negative effects associated with US intelligence gathering (Gl) The resulting differential equation is

$$\frac{dl}{dt}=\alpha_1 j-\alpha_2 l-\alpha_3 Dl+\alpha_4 o-\alpha_5 Pl-\alpha_6 Gl \quad (1)$$

2. The equation for the rate of change of junior leaders is modeled in a similar manner as senior leaders, using: promotion of foot soldiers to junior leaders ($\beta_1 f$), the loss of junior leaders promoted to senior leaders ($\alpha_1 j$), loss of junior leaders to internal factors ($\beta_2 j$), junior leaders removed by CT measures (D), recruited junior leaders (o), bomb makers (b) promoted to junior leaders, loss by PSYOP (Pj) and loss through the results of US intelligence gathering (Gj). The resulting differential equation is

$$\frac{dj}{dt}=\beta_1 f-\alpha_1 j-\beta_2 j-\beta_3 Dj+\beta_4 o+\beta_5 b -\beta_6 Pj-\beta_7 Gj \quad (2)$$

3. The change in foot soldiers (f) is modeled through recruitment (o), internal factors ($\delta_2 f$), promotion loss to junior leaders ($\beta_1 f$) and bomb makers ($\sigma_1 f$), direct CT (D), and PSYOP (P). Therefore, the resulting differential equation is

$$\frac{df}{dt}=\delta_1 o-\delta_2 f-\delta_3 Df-\delta_4 Pf-\beta_1 f-\sigma_1 f \quad (3)$$

4. Bomb makers (b) are trained and promoted from the foot soldiers at a rate σ_1, the number of bomb makers lost to internal causes is proportional to σ_2, the rate promoted to junior leaders is $\beta_5 b$, the rate eliminated by CT measures is $\sigma_3 Db$ and removed through the consequences of intelligence gathering is computed by $\sigma_4 Gf$. The resulting differential equation is

$$\frac{db}{dt}=\sigma_1 f-\sigma_2 b-\sigma_3 Db-\sigma_4 Gb-\beta_5 b \quad (4)$$

5. The rate of change for outside supporters (o) is modeled from a proportional factor γ_1 of the strength (s) and reduction by the factor proportional γ_2 to the success of CT (S). We also include the proportional growth factors for the funding level $\gamma_3 m$ and the world-wide support $\gamma_4 w$ The resulting differential equation is

$$\frac{do}{dt}=\gamma_1 s-\gamma_2 S+\gamma_3 m+\gamma_4 w \quad (5)$$

6–8. The three equations for change in financial support (m), change in intellectual level (i) and change in public support (p) for the terror organization are all modeled similarly. The change for all three is positively proportional to: the current financial support (m), intellectual level (i), public support for the organization or cause (p), world-wide support (w), strength (s), and aggressiveness of CT operations (D). The change is negatively correlated with the aggressiveness of US PSYOP (P), aggressiveness of US aid to the public (A), aggressiveness of US logistics (L), and aggressiveness of US security (Y). The equations are

$$\frac{dm}{dt}=\varphi_1 m+\varphi_2 i+\varphi_3 p+\varphi_4 w+\varphi_5 s+\varphi_6 D -\varphi_7 P-\varphi_8 A-\varphi_9 L-\varphi_{10} Y \quad (6)$$

$$\frac{di}{dt}=\psi_1 m+\psi_2 i+\psi_3 p+\psi_4 w+\psi_5 s+\psi_6 D -\psi_7 P-\psi_8 A-\psi_9 L-\psi_{10} Y \quad (7)$$

$$\frac{dp}{dt}=\lambda_1 m+\lambda_2 i+\lambda_3 p+\lambda_4 w+\lambda_5 s+\lambda_6 D -\lambda_7 P-\lambda_8 A-\lambda_9 L-\lambda_{10} Y \quad (8)$$

9. The rate of change for the world-wide support for the T cause (w) is proportional to the world-wide support for the cause ($\tau_1 w$), the strength of the organization ($\tau_2 s$), and the aggressiveness of CT operations ($\tau_3 D$). It is negatively correlated to the overall success of CT ($\tau_4 S$). The equation is

$$\frac{dw}{dt}=\tau_1 w+\tau_2 s+\tau_3 D-\tau_4 S \quad (9)$$

10. The overall strength of the terror organization (s) is a weighted sum of the nine computed terrorism factors with a negative factor of public (US) support for the CT effort (C). Therefore, the resulting algebraic equation is

$$s = \theta_1 l + \theta_2 j + \theta_3 o + \theta_4 b + \theta_5 f + \theta_6 m + \theta_7 i + \theta_8 p + \theta_9 w - \theta_{10} C \tag{10}$$

11. The rate of change of the factor of public (US) support for the CT effort (C) is proportional to itself ($\Delta_1 C$), the security of the CT effort ($\Delta_2 Y$), and the success of the CT as ($\Delta_3 S$). It is negatively correlated to in-country support for the organization or cause ($\Delta_4 p$) and world-wide support for the cause ($\Delta_5 w$). The resulting differential equation is

$$\frac{dC}{dt} = \Delta_1 C + \Delta_2 Y + \Delta_3 S - \Delta_3 p - \Delta_5 w \tag{11}$$

12–17. These six equations provide the aggressiveness or resource allocations of the elements of the COIN effort determined by the strategy or tactical rules of engagement being employed by the COIN leadership. The six elements are the aggressiveness of direct (kinetic) CT operations (D), aggressiveness of intelligence gathering (G), aggressiveness of PSYOP/information distribution (P), aggressiveness of aid to the local nation (A), aggressiveness of CT logistics (L), and aggressiveness of CT security (Y). Each element is controlled as an input function in our model.

18. One of the key elements in CT/COIN success and a new component in our model is the Cooperation/Coalition factor (CC). We use a coalition network model with three subgroups of US (US forces and organizations, governmental and non-governmental), Host (host country forces and organizations), and World (world-wide forces and organizations, other countries forces and agencies, world-level non-governmental organizations, and UN organizations) to calculate the metrics to use in our COIN model. The network metrics we use are seven coalition network metrics of CCS, ICS_{US}, ECS_{US}, ICS_{World}, ECS_{World}, ICS_{Host}, ECS_{Host}, as well as the link density (LD). CC is computed as a weighted sum of these elements of the CT coalition network while also being proportional to the levels of aggressiveness of security (Y), aggressiveness of intelligence gathering (G), aggressiveness of PSYOP (P), aggressiveness of US aid (A), aggressiveness of CT logistics (L), number of nations in the coalition squared N^2, and number of total organizations in the coalition (M). The non-linear squared term for the number of nations is the key part of this measure showing the important nature of that aspect of Coalition strength. The CC factor is an influential component of our dynamic COIN model

$$CC = \eta_1 N^2 + \eta_2 M + \eta_3 LD + \eta_4 CCS + \eta_5 ICS_{US} + \eta_6 ECS_{US} + \eta_7 ICS_{Host} + \eta_8 ECS_{Host} + \eta_9 ICS_{World} + \eta_{10} ECS_{World} + \eta_{11} G + \eta_{12} P + \eta_{13} A + \eta_{14} L + \eta_{15} Y + \eta_{16} D \tag{12}$$

The impact of this factor is a key element of this paper. In a later section, we track the impact of the changing coalition network on our model in an eight-stage test scenario. It is through the variation in CC from the changing metrics of the coalition network that the COIN model is directed affected by the COIN coalition. The effect is one-way and the changes are made in set stages of time by the model operator. There are no automatic or dynamic changes to the coalition other than the ones the operator sets at each stage.

19. The strength of the CT effort (S) combines measures to determine the success of the CT as it affects the terrorist operations. It is inversely proportional to the square root of the strength of the terrorism (s) and proportional to the cooperation level (CC) and US public support for the CT effort (C).

$$S = \left(\frac{1}{\sqrt{s}}\right)(C + CC) \tag{13}$$

In order to visualize the inter-relationships between the 19 variables, we provide a relationship graph that shows how the 19 variables connect to each other through the 19 equations (Figure 1).

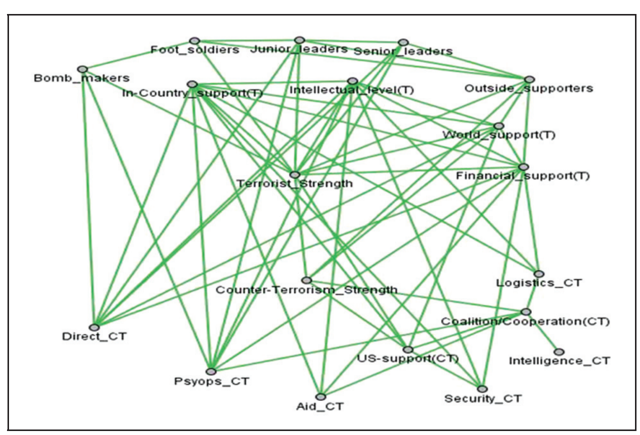

Figure 1. Relationship graph for the 19 variables in the model.

This model of 19 equations cannot replicate the complexity of a real T–CT struggle or the workings of any real terrorist cell or COIN organization. One constraining issue with our model is that many of our relationships are linear, while the real-world relationships, including components of terrorist cells and CT operations, are non-linear and complex. However, our model does attempt to include many of sources of strength of a terrorist organization and components of CT from real COIN operations as explained in FM 3-24.[8] In addition, the parameter values in the equations are difficult to estimate or calculate. As described by Arney et al.,[1] the process used is to tune the values in a balanced, unchanging baseline scenario. Our primary goals are to model the effects of different CT tactics and gain strategic insights in the T–CT struggle by adjusting the aggressiveness in direct kinetic CT operations, intelligence gathering, aid, security, logistics, and PSYOPs with an overall objective of winning the 'hearts and minds' of the people, specifically targeting increases in the ratio of the strength of the CT operations to the strength of the T operations. By optimizing these aggressiveness factors, we can determine the proper mix of resource allocation to produce the best CT strategy given realistic constraints on the CT resources.

2. Coalition effectiveness and collaboration

One of the most important aspects of COIN operation is the effectiveness of the coalition of organizations and agencies involved in the operation.[8-16] For the purposes of this simulation, we use a coalition network model that consists of three subgroups: US agencies (governmental and nongovernmental), host country organizations, and world-wide organizations (other countries forces and agencies, world-level nongovernmental organizations, and UN organizations).

COIN, IN, T and CT operations involve not only power, force, control, and other military-based components, but also diplomatic and nation-building elements of influence, politics, legitimacy, and service.[11-13]The agencies that work with the populace along with the military forces form the COIN/CT coalition that wages the COIN. FM 3-24[8] (p. 2.1) explains the roles these coalition partners play to succeed in COIN:

> 'Although military efforts are necessary and important, they are only effective if integrated into a comprehensive strategy employing all instruments of national power…. The integration of civilian and military efforts is crucial in COIN and must be focused on supporting the local population and the HN government. Political, social and economic programs are usually more valuable than conventional military operations as a means to address root causes of conflict and undermine an insurgency. In COIN, military personnel, diplomats, police, politicians, humanitarian aid workers, contractors, and local leaders are faced with making decisions and solving problems in a complex and acutely challenging environment.'

The coordination of effort and cooperation the coalition network is essential. The JP 3-24[9] explains:

> 'Unified action refers to the synchronization, coordination, and/or integration of military operations with the activities of governmental and nongovernmental entities to achieve unity of effort… The military contribution to COIN must be coordinated with the activities of USG (United States Government), interagency partners, IGOs (Intergovernmental Organizations), NGOs (Nongovernmental Organizations), regional organizations, the operations of multinational forces, and activities of various HN (Host Nation) agencies to be successful. … Successful interagency, IGO, and NGO coordination helps enable the USG to build international support, conserve resources, and conduct coherent operations that efficiently achieve shared goals.'

In summary, from page 2-1 of FM 3-24,[8]

> 'The preference in COIN is always to have civilians carry out civilian tasks. Civilian agencies of individuals with the greatest expertise for a given task should perform it – with special preference for legitimate local civil authorities… the preferred or ideal division of labor is frequently unattainable. The more violent the insurgency, the more unrealistic is this preferred division of labor.'

3. The coalition network model

In order to compute viable measurements for the effectiveness of the coalition, we represent the coalition with a network structure. We model the various organizations as nodes and the strength of the collaboration between the organizations as weighted edges. More precisely, the weights on the edges are the percentage of the perfect or desired collaboration between the two connecting organizations. Some organizations should maintain an intense collaboration with another organization because of the nature of their missions, whereas others may have little need to collaborate in COIN except to maintain communication of basic information. Therefore, in our model, a coalition network with perfect (ideal) collaboration is a completely connected graph with all its weighted links all set to 1 (or 100% effective collaboration). A completely dysfunctional coalition with no collaboration is modeled by a completely disconnected network graph.

Our network metrics seek to measure the strength of the coalition collaboration. There are several classic network metrics that measure cohesion of the network. A definition of network cohesion from the Committee on

Organizational Modeling from Individuals to Societies[7] explains that

> 'cohesion refers to the connectedness or structural integrity of a network, and it is often interpreted in terms of the network's potential for coordinating among its members.'

Sometimes average path distance is used as a cohesion measure or key nodes are determined through centrality measures, but network density appears to be more germane to the collaboration in our coalition network.

In our model, the coalition's collaboration strength (*CCS*) is the weighted density measure of the graph. For a undirected graph, the sums of all of the weighs of connecting edges (Σe_k, where k goes from 1 to Z, the total number of possible connections) are divided by the total possible connections of the graph $Z = (M)(M - 1)/2$, where M is the number of nodes in the graph or total number of agencies in the network. Subgroups of the overall coalition produce two collaboration measures, its own internal collaboration strength (*ICS*) measured by only taking into account the network of the subgroup and the external collaboration strength (*ECS*) by taking into account the sum of the weights of links between the subgroup and its complement.

The following is a simple example for a five-agency coalition network with two subgroups A and B. Subgroup A contains nodes 1, 2, 3 and Subgroup B has just two nodes 4 and 5. The graph is shown in Figure 2. We compute the five possible collaboration strengths (overall *CCS*, Group A *ICS*, Group A *ECS*, Group B *ICS*, and Group B *ECS*).

First we compute the *CCS* for the entire network. Since there are five nodes the total possible connections are (5)(4)/2 or 10. For this network shown in Figure 1, there are eight explicitly valued collaboration measures shown on the edges and two missing connections that will be implicitly scored as zero. This gives a link density of 0.8. Therefore, since the sum of all these collaborations is 3.8, the *CCS* is 3.8/10 or 0.38. For the three-node subgroup A, the internal collaboration strength (ICS_A) is $(0.7 + 0.6)/3$ or 0.43. The external collaboration strength (ECS_A) is the ratio of measures between the two subgroups (five explicit scores, $0.3 + 0.2 + 0.6 + 0.3 + 0.3 = 1.7$) to the total possible connections, 6. Therefore ECS_A is $1.7/6 = 0.283$. For the two-node subgroup B, the ICS_B is just the score of the one possible connection 0.8 and the ECS_B is the same as ECS_A since for networks with just two subgroups they have the same computation for the ECS. It should be noted that the network in Figure 1 has the same *CCS* metric of 0.38 as a completely connected network where all 10 collaborations have identical measures of 0.38.

We score the collaborations for another simple example network with three subgroups before we move to larger, more realistic networks for counter-insurgency coalitions. This time we have a seven-agency network with three subgroups Red, Blue, Green. Subgroup Red contains nodes 1, 2, 3, and subgroup Blue has just two nodes 4 and 5, and subgroup Green contains nodes 6 and 7. The coalition network is shown in Figure 3. We compute the seven collaboration strength scores (CCS, ICS_{Red}, ECS_{Red}, ICS_{Blue}, ECS_{Blue}, ICS_{Green}, ECS_{Green}).

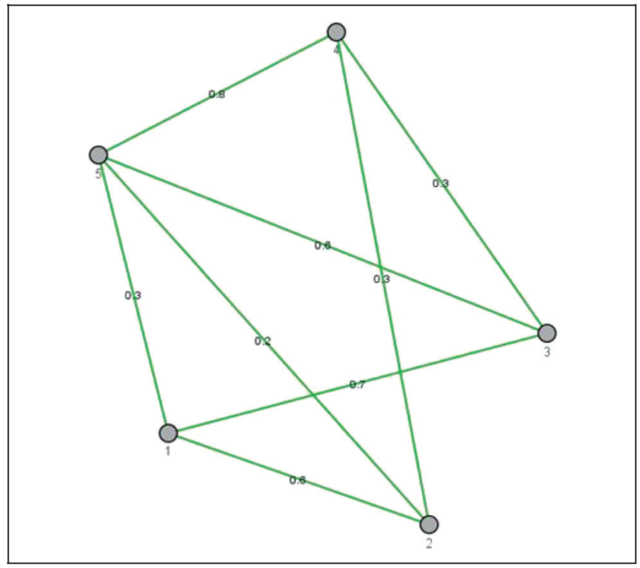

Figure 2. A coalition collaboration graph of five nodes (organizations) and eight edges (active collaborations).

For the coalition network in Figure 2, the *CCS* is computed as 4.8/21 = 0.23. For the Red subgroup, $ICS_{Red} = 1/3 = 0.33$ and $ECS_{Red} = 0.9/12 = 0.075$. For the Blue subgroup, $ICS_{Blue} = 0.7$ (the weight of one link) and $ECS_{Blue} = 2.0/10 = 0.20$. For the Green subgroup, $ICS_{Green} = 0$ (there is no link in subgroup Green) and $ECS_{Green} = 1.6/10 = 0.16$. The link density is 13/21 = 0.62.

We show another notional coalition network with a *CCS* of 0.23 in Figure 4. The network has all of its 7 nodes connected by 21 links at the same level of 0.23. In this case, all of the other subgroup measures, ICS_{Red}, ECS_{Red}, ICS_{Blue}, ECS_{Blue}, ICS_{Green}, ECS_{Green}, have the same value of 0.23. The link density of a fully connected graph is 1.0. All of these coalition metrics are normalized between 0 and 1, where a value of 1 indicates the highest metric value.

4. Using the network metrics in the COIN model

As we indicated earlier in the model description, one of the key elements in CT/COIN success and a major component in our model is the cooperation/coalition factor (*CC*). We use a coalition network model with three subgroups of

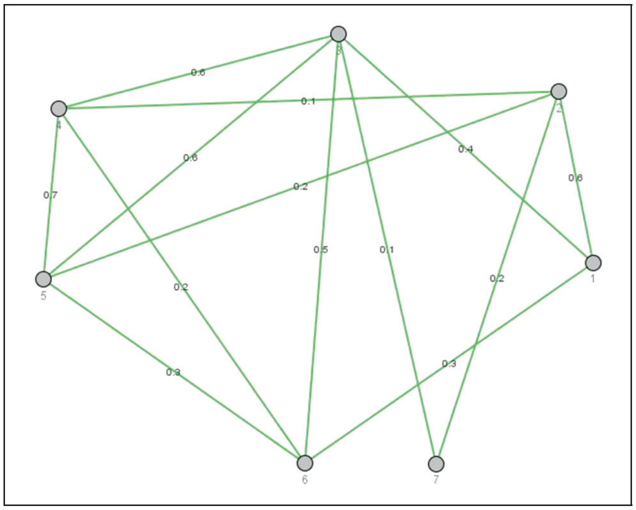

Figure 3. A coalition collaboration graph of seven nodes (organizations) and 13 edges (active collaborations).

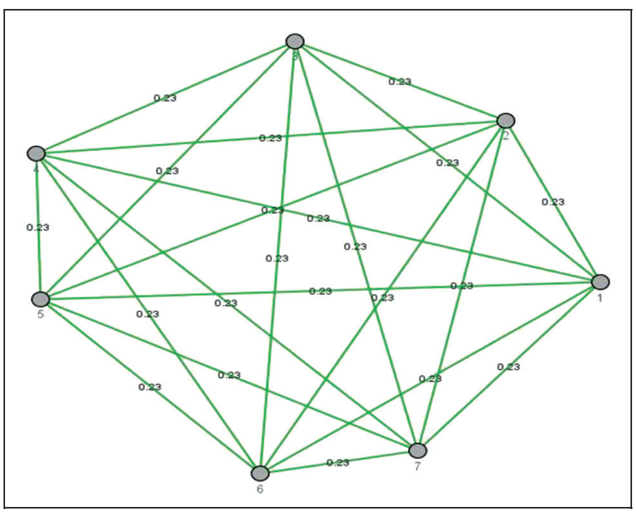

Figure 4. A completely connected coalition collaboration graph of 7 nodes (organizations) and 21 edges (active collaborations) all with the same collaboration measure of 0.23.

(1) US (US forces and organizations, military, governmental and non-governmental), (2) Host (host country forces and organizations), and (3) World (world-wide forces and organizations, other countries forces and agencies, world-level non-governmental organizations, and UN organizations) to calculate the metrics to use in our COIN model.

We use this scenario knowing that from the Executive Summary of JP 3-08:[11]

> 'Military operations must be coordinated with the activities of other agencies of the USG, IGOs, NGOs, regional organizations, the operations of foreign forces, and activities of various host nation (HN) agencies. Sometimes the joint force commander (JFC) draws on the capabilities of other organizations; sometimes the JFC provides capabilities to other organizations; and sometimes the JFC merely deconflicts his activities with those of others. Interagency coordination forges the vital link between the military and the diplomatic, informational, and economic instruments of power of the USG. Successful interagency, IGO, and NGO coordination enables the USG to build international support, conserve resources, and conduct coherent operations that efficiently achieve shared international goals.'

Department of Defense (DoD) agencies often take the lead and exert the most influence in COIN operations. One reason is because they often have the most personnel for the task. From FM 3-24, Chapter 2, we see that:[8]

> 'Military forces do apply their combat skills to fighting insurgents; however, they can and should be engaged in using their capabilities to meet the local populations' fundamental needs as well. Only regaining active and continued support of the HN government by the local population can deprive an insurgency of its power and appeal. The military forces' primary function in COIN is protecting the population. However, military force is not the sole means to provide security of defeat insurgents.'

All of these factors are considered as we built the scenario of this simulation.

The eight network metrics we use are the seven coalition network metrics of CCS, ICS_{US}, ECS_{US}, ICS_{World}, ECS_{World}, ICS_{Host}, ECS_{Host}, as well as the link density LD. CC is computed as a weighted sum of these seven elements of the CT coalition network while being proportional to the levels of aggressiveness of security (Y), aggressiveness of intelligence gathering (G), aggressiveness of PSYOPs (P), aggressiveness of US aid (A), aggressiveness of CT logistics (L), number of nations in the coalition squared N^2, and number of total organizations in the coalition M. The non-linear squared term for the number of nations is a critical component of this measure. This increased influence of number of nations shows the important nature of that aspect of coalition strength. As JP 3-16 indicates, more nations as members of a coalition can provide legitimacy, a united front, and a diversity of skills and perspectives. However, these strengths of a larger coalition are sometimes tempered by the dealing with various levels of commitment and resources.[12] The equation is thus:

$$\begin{aligned} CC = {} & \eta_1 N^2 + \eta_2 M + \eta_3 LD + \eta_4 CCS + \eta_5 ICS_{US} \\ & + \eta_6 ECS_{US} + \eta_7 ICS_{Host} + \eta_8 ECS_{Host} \\ & + \eta_9 ICS_{World} + \eta_{10} ECS_{World} + \eta_{11} G + \eta_{12} P \\ & + \eta_{13} A + \eta_{14} L + \eta_{15} Y + \eta_{16} D \end{aligned}$$

Therefore, this factor is dependent on the metrics of the dynamic coalition network, which makes it a viable and

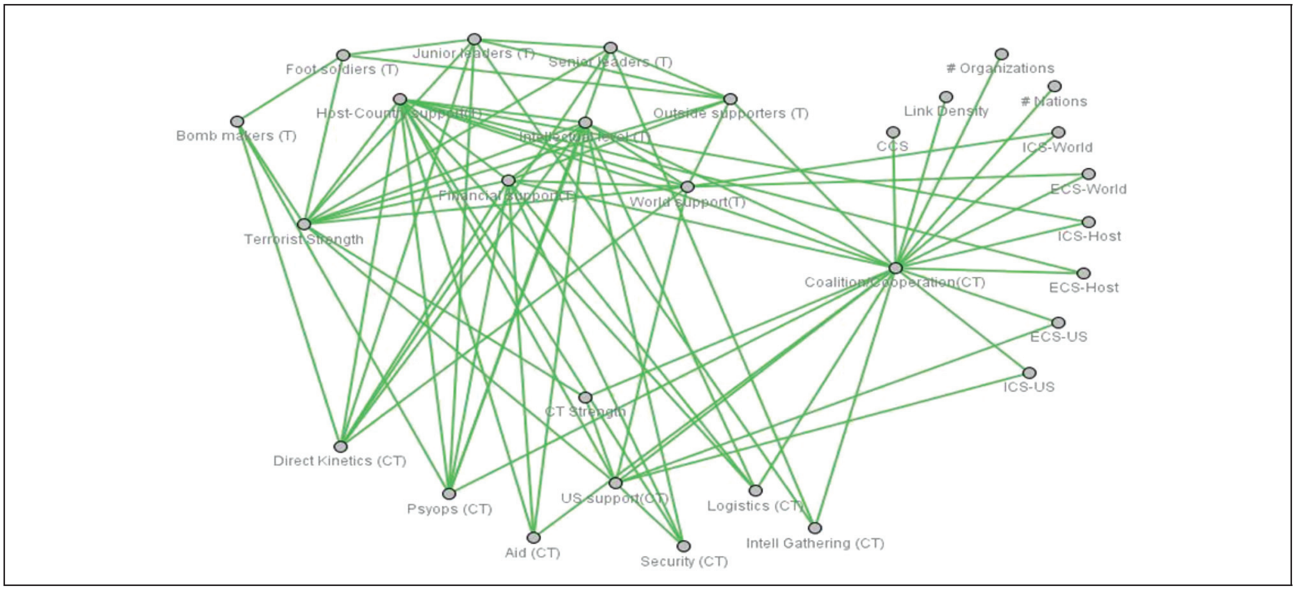

Figure 5. Relationship graph for the variables in the model including the coalition metrics

effective component of our dynamic COIN model. We will show the use of these metrics in the scenario simulation below.

As shown in the elements of the model, there are equations which contain factors (metrics) from the coalition network model. Several are influenced by the value of *CC*. These are: the outside supporters of the Insurgency (o), the financial support of the insurgency (m), the intellectual level of the insurgency (i), the host country public support for the insurgency (p), the world-wide public support for the insurgency (w), and the US public support for the COIN (C). Others are directly influenced by the subgroup metrics. These are: (1) the host country public support (p) calculation contains the elements ICS_{Host} and ECS_{Host}; (2) the world-wide support for the insurgency (w) is calculated from ICS_{World} and ECS_{World}; (3) the US public support for the COIN (C) is computed using the factors ICS_{US} and ECS_{US}. We report these values in the results of our simulation since they are the most directly affected by the collation metrics and dynamics. In order to understand the relationships, we update Figure 1 with a new relationship graph in Figure 5 and provide the definitions of the variables in Table 1.

5. COIN scenario simulation using coalition networks

To show the effects of the dynamics of the coalition network on the COIN model, we simulate an eight-stage scenario of coalition evolution. We begin with US forces arriving and forming a weak coalition with just the host nation in order to fight an insurgency. As the arrival of additional world organization and other allied forces begins, we simulate the increased numbers of parties involved and the increased strength of their collaborations internally (US, Host, and World) as well as the coalition as a whole. There are considerable obstacles to overcome when numerous agencies and forces from different countries and with different missions have to come together to collaborate in order to succeed. This is explained in JP 3-16:[12]

> 'In a parallel command structure, national forces essentially operate under their own doctrine and procedures within the guidelines determined by the strategic national guidance and are not significantly impacted by multinational influences. Under the integrated and lead nation command structures, more multinational involvement and interaction occurs… The basic challenge in multinational operations is the effective integration and synchronization of available assets toward the achievement of common objectives.'

The Executive Summary of JP 3-16 indicates that[12]

> 'nations form partnerships in both regional and worldwide patterns as they seek opportunities to promote their mutual national interests, ensure mutual security against real and perceived threats, conduct foreign humanitarian assistance operations, and engage in peace operations. Cultural, diplomatic, religious, psychological, economic, technological, and informational factors all influence and impact multinational operations and participation.'

We want to determine how this works in our simulation.

As the COIN wears on and the will of the coalition partners weakens, we simulate the withdrawal of organizations

Table 1. Definitions of COIN (left) and coalition collaboration (right) terms.

COIN variable	Definition	Coalition collaboration variable	Definition
l	Senior Leaders	*N*	Number of nations in the coalition
j	Junior Leaders	*M*	Number of organizations in the coalition
o	Outside Supporters	*Z*	Total number of possible connections in Coalition network
b	Bomb Makers	*LD*	Link Density
f	Foot Soldiers	*CCS*	Coalition's Collaboration Strength
m	Financial Support	ICS_{US}	Internal Collaboration Strength of the US organizations
i	Intellectual Level	ECS_{US}	External Collaboration Strength of the US organizations
p	Public Support for T	ICS_{World}	Internal Collaboration Strength of the World organizations
w	World-wide Support	ECS_{World}	External Collaboration Strength of the World organizations
s	Strength of the Terrorist Organization	ICS_{Host}	Internal Collaboration Strength of the Host Country organizations
C	Public Support for CT	ECS_{Host}	External Collaboration Strength of the Host Country organizations
CC	Cooperation/Coalition		
D	Aggressiveness of Direct CT Operations		
G	Aggressiveness of Intelligence Gathering Operations		
P	Aggressiveness of PSYOPs Operations		
A	Aggressiveness of aid to the public		
L	Aggressiveness of Logistics for CT		
Y	Aggressiveness of Security Operations		
S	Success/Strength of the CT Operations		
t	Time (independent variable)		

COIN, counter-insurgency; T, terrorism; CT, counter-terrorism.

and the lessening of commitments. Finally, we include the withdrawal of US forces and US resource reduction.

Since each stage affects the COIN results, we will show the graph of the coalition network model, the computed collaboration metrics, and the results of running the COIN model for the six-month duration at each stage. We start with an initial basic coalition formation and then implement changes to the coalition structure as the scenario unfolds. For this scenario we keep all six of the resource levels equal and constant at 0.83 until the last stage to run a balanced COIN strategy. All other values and parameters are held constant so we can isolate the effects of the changes in the coalition collaboration.

5.1. Stage 1. The initial coalition (nine nodes)

We start with the US forces arriving in a Host country to form a small, weakly connected coalition with several Host country organizations. This coalition has no elements outside those of the US and the Host country. The coalition of nine organizations is modeled by the nine-node network shown in Figure 6. We track the three subgroups US, Host, and World. Subgroup Host contains three nodes, subgroup World has no nodes, and subgroup US contains six nodes. From the collaboration weights, we compute the seven possible collaboration strength (*CS*) scores of CCS, ICS_{US}, ECS_{US}, ICS_{World}, ECS_{World}, ICS_{Host}, ECS_{Host}. The CCS is computed as 4.5/36 = 0.125. For the Host subgroup, $ICS_{Host} = 1.3/3 = 0.43$ and $ECS_{Host} = 0.6/18 = 0.03$. For the World subgroup, $ICS_{World} = \mathrm{ECS}_{\mathrm{World}} = 0$, since there are no World organizations in the coalition. For the US subgroup, $ICS_{US} = 2.6/15 = 0.173$ and $ECS_{US} = ECS_{Host} = 0.6/18 = 0.03$, since there are only two subgroups present in the network, the external collaboration scores must be the same. The link density (*LD*) is 12/36 = 0.33. We run the COIN model for 6 months to obtain the results shown in Table 2 along with the coalition metrics.

The collaboration scores show that the US and Host country do not yet collaborate very effectively. Since no World organizations are involved in the coalition, these metrics create only a weak collaboration influence that has little impact on the insurgency. In general, the T and CT values have little or no change during this build-up stage of the coalition.

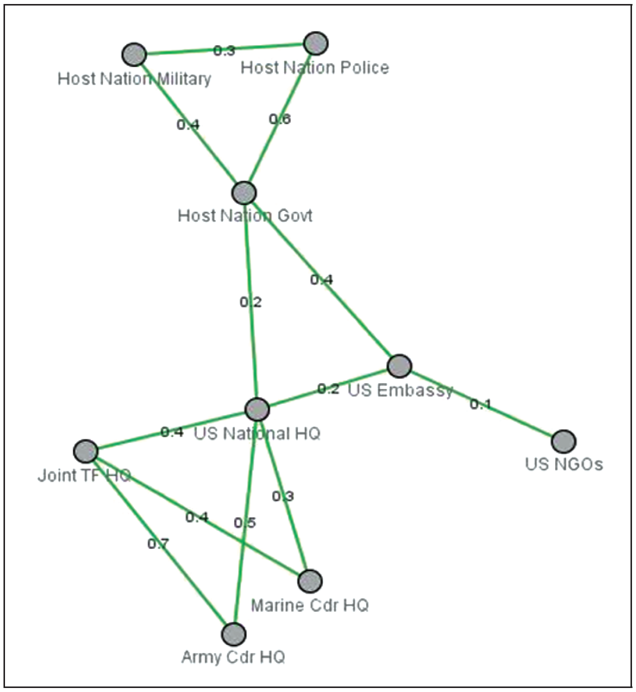

Figure 6. The coalition collaboration network for stage 1.

5.2. Stage 2. The coalition grows: world organizations and allied force arrives (16 nodes)

At this stage, the coalition has added more US forces (USAID), maintained the same basic Host nation involvement, and added one other allied country force (Allied Country Embassy and Allied Country Army) along with some UN and world-wide organizations (UN HQ, UN Relief Agencies, Worldwide NCOs, Red Cross). The model for this rudimentary coalition of 16 nodes is shown in Figure 7.

The weights of the collaborations for this coalition of 16 organizations are shown in Figure 5. You can see the increase in strength within the Host nation groups from stage 1 and the added link between the Army Cdr HQ and Marine Cdr HQ. The subgroup Host contains three nodes, subgroup World has six nodes , and subgroup US contains seven nodes. We compute the seven possible collaboration strength (*CS*) scores of *CCS*, ICS_{US}, ECS_{US}, ICS_{World}, ECS_{World}, ICS_{Host}, ECS_{Host}. The *CCS* is computed as 11.4/120 = 0.095. For the Host subgroup, ICS_{Host} = 1.4/3 = 0.47 and ECS_{Host} = 1.6/39 = 0.04. For the World subgroup, ICS_{World} = 2/15 = 0.133 and ECS_{World} = 2.6/60 = 0.043. For the US subgroup, ICS_{US} = 4.3/21 = 0.20 and ECS_{US} = 2.4/63 = 0.04. The *LD* is 27/120 = 0.23. We run the COIN model for 6 months to obtain the results shown in Table 3 along with the coalition metrics.

These results show that the collaboration has improved with a higher *CC* score, and the effects of the improvement have had only marginal impact on the T or CT element metrics. However, the *S/s* ratio has increased dramatically (45%) due to the increased strength *S* of the CT caused by a more potent coalition.

5.3. Stage 3. The coalition grows (47 nodes)

In this stage the coalition has grown substantially to 47 organizations and 5 countries, but they are sparsely linked with little collaboration across the 3 subgroups. One of the countries is involved diplomatically, but not militarily and contributes one node to the network ('Involved Country Embassy'). This coalition network is shown in Figure 8.

Subgroup Host contains 7 nodes, subgroup World has 17 nodes, and subgroup US contains 23 nodes. We show the collaboration and coalition metrics in Table 4. We run the COIN model for 6 months to obtain the results shown in Table 3.

The rapid growth in the coalition results a low *CCS* of 0.04, since the collaboration total is just 44.7 out of a possible of 1081. Also, the coalition has a *LD* of 123/1081 = 0.114. Just a little over 11% of the possible coordination links are even established by the coalition. As the coalition grows, the number of possible collaborative links grows exponentially with the number of organizations: keeping up with this growth is difficult to do immediately so many of the collaborations scores have decreased. However, the dramatically increased size of the coalition (5 counties and 47 organizations) and the growing strengths of the three subgroups has resulted in a large increase in the *CC* value.

Table 2 Results of stage 1.

Collaboration metrics for the stage 1 coalition (*N* = 2 (US and Host), *M* = 9)

US nodes	Host nodes	World nodes	*CCS*	ICS_{Host}	ECS_{Host}	ICS_{World}	ECS_{World}	ICS_{US}	ECS_{US}	*LD*
6	3	0	0.125	0.43	0.03	0	0	0.173	0.03	0.33

COIN model metrics for 6 months with stage 1 coalition

CC	*s*	*S*	*S/s* ratio	Change of *S/s*	% change in *o*	% change in *m*	% change in *i*	% change in *p*	% change in *w*	% change in *C*
0.4126	0.765	0.817	1.07	− 0.003	0	0.014	0.005	− 0.01	0.015	0.003

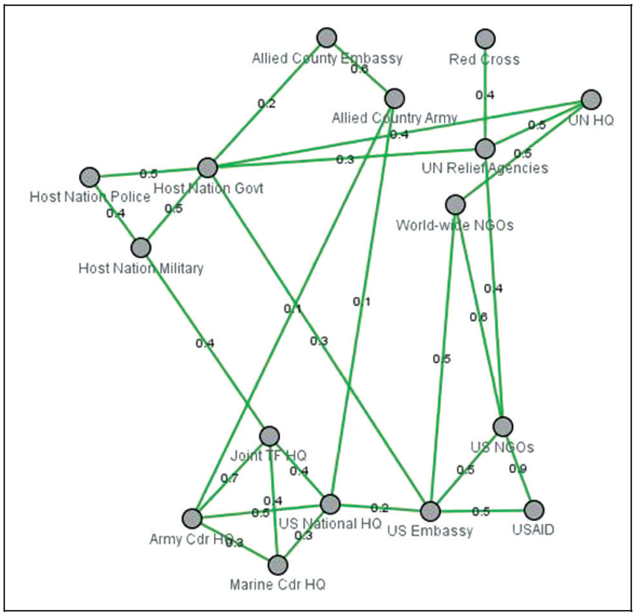

Figure 7. The coalition collaboration network for stage 2.

This increase in CC leads to small decreases in the IN measures and a large increase in the strength of the COIN S. The effect is that the S/s ratio doubles during this stage. The results in Table 4 show these mixed results as the coalition has established itself, even though it is not fully collaborative at this time.

5.4. Stage 4. The military forces coalesce and strengthen their collaborations (49 nodes)

In this stage the military forces are able to coordinate their work within and between the US, the three Allied counties, and Host nation. Only two new organizations enter the coalition in this stage. Since the involved country has now committed military forces, two new organizations ('Allied Country 3 HQ' and 'Allied Country 3 Army') are added and the previous organization from stage 3 'Involved Country Embassy' is now titled 'Allied Country 3 Embassy'. Most of the effort during this stage has been spent in strengthening existing military collaborations. This new stronger coalition network is shown in Figure 9.

We show the values of collaboration and coalition metrics in Table 5 as we run the COIN model for 6 months to obtain the results.

The increased cooperation of the military forces in the coalition results in a slightly increased CCS of 0.0615, since the collaboration total is 72.35 out of a possible of 1176 links. All eight collaboration metrics increased slightly from stage 3 as the coalition matures. Also, the coalition has a LD of 162/1176 = 0.14, which is also slightly higher LD than stage 3. The results in Table 5 show that CC and S have increased from stage 3, resulting in another slight S/s ratio increase. The COIN operation is starting to show its strength in the model and affect the IN elements, all of which are decreasing.

5.5. Stage 5. The relief and aid agencies coalesce and strengthen their collaborations (50 nodes)

In this stage the non-military agencies are able to coordinate their work. Only one new organization entered the coalition in this stage, the UN World Food Program. Most of the effort during this stage has been to strengthen existing collaborations. This new stronger coalition network is shown in Figure 10.

We show the collaboration and coalition metrics in Table 6. The LD is 198/1225 = 0.16. We run the COIN model for 6 months to obtain the results shown in Table 6.

These results show a maturing coalition. Once again, all of the collaboration metrics have increased during this stage. The model calculations produce a CC value of 1.8521 which in turn results in a higher S value and a slightly higher S/s ratio. The trend of success of the COIN continues as the coalition cooperation strength has adversely affected the IN.

5.6. Stage 6. The coalition connects (50 nodes)

During this stage the organizations that have been involved in the COIN have discovered each other and learned to

Table 3. Results of stage 2.

Collaboration metrics for the stage 2 coalition (N = 3, M = 16)										
US nodes	Host nodes	World nodes	CCS	ICS_{Host}	ECS_{Host}	ICS_{World}	ECS_{World}	ICS_{US}	ECS_{US}	LD
7	3	6	0.095	0.47	0.04	0.133	0.043	0.20	0.04	0.23
COIN model metrics for 6 months with stage 2 coalition										
CC	s	S	S/s ratio	% change of S/s	% change in o	% change in m	% change in i	% change in p	% change in w	% change in C
0.7372	0.767	1.189	1.549	0.45	0	0.001	−0.005	−0.01	0.01	0.05

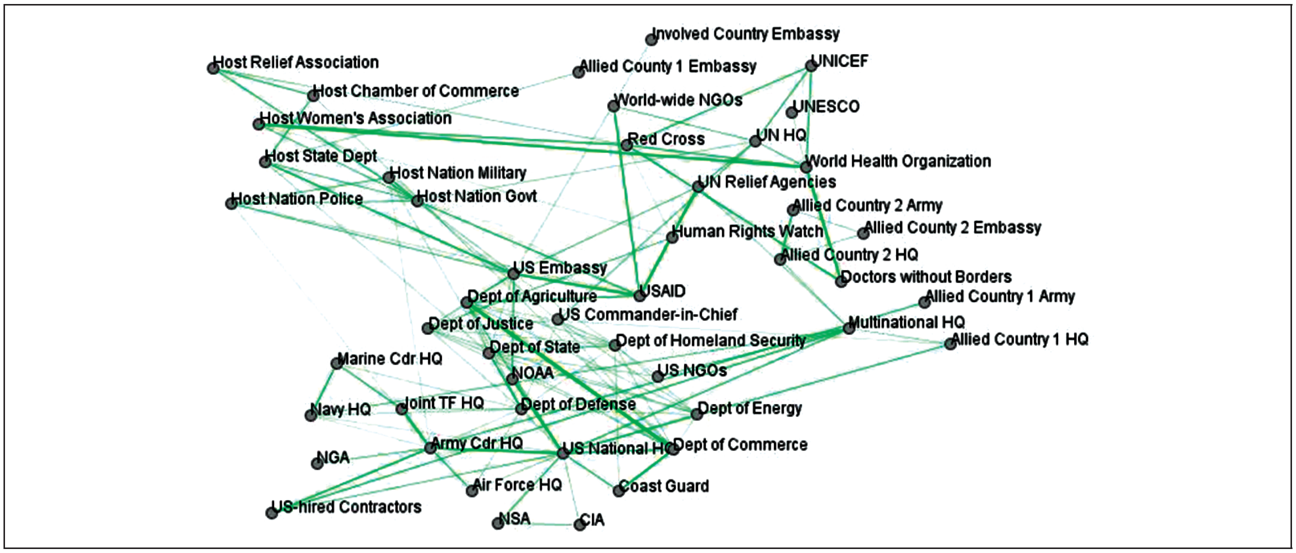

Figure 8. The coalition collaboration network for stage 3. The thickness of the edges indicate the collaboration weights between organizations.

Table 4. Results of stage 3.

Collaboration metrics for the stage 3 coalition (N = 5, M = 47)										
US nodes	Host nodes	World nodes	CCS	ICS_{Host}	ECS_{Host}	ICS_{World}	ECS_{World}	ICS_{US}	ECS_{US}	LD
23	7	17	0.04	0.18	0.02	0.05	0.015	0.09	0.017	0.114

COIN model metrics for 6 months with stage 3 coalition										
CC	S	S	S/s ratio	% change	% change in o	% change in m	% change in i	% change in p	% change in w	% change in C
1.7872	0.766	2.393	3.126	1.02	– 0.01	– 0.01	– 0.04	– 0.01	– 0.005	0.02

cooperate and collaborate, especially across the three subgroups of Host, World, and US. There are no new organizations, just stronger and more effective links to each other. The coalition is shown in Figure 11.

We show the collaboration and coalition metrics in Table 7. The LD is much higher at 337/1225 = 0.275. We run the COIN model for 6 months with this stronger coalition to obtain the results shown in Table 7.

These data show the results of a mature and strong coalition. All of the collaboration metrics have increased during this stage, some quite substantially. This produces a CC value of 1.9332 which in turn results in higher S value and S/s ratio. Even without additional resources for the operation or changes in strategy (the six COIN resources are still set at the balanced state of 0.83 for each), the COIN operation has become more successful because the coalition has strengthened.

5.7. Stage 7. Coalition weakens through attrition and decision (32 nodes)

As the COIN wears on and the will for the COIN weakens, the coalition collaboration dissipates as organizations withdraw or lessen their commitment. In this case, the three allies and their forces withdraw during this stage, the relief agencies cut back to a meager state of effectiveness, and the politics of the Host country weaken its organizations' commitment. The result is a smaller, leaner, shriveled coalition that is shown in Figure 12.

The leaner coalition still has a few strong collaborations to sustain itself, but its metrics given in Table 8 show its decline.

These results show a decaying coalition. All of the collaboration metrics have decreased during this stage, some quite substantially. This situation produces a CC value of

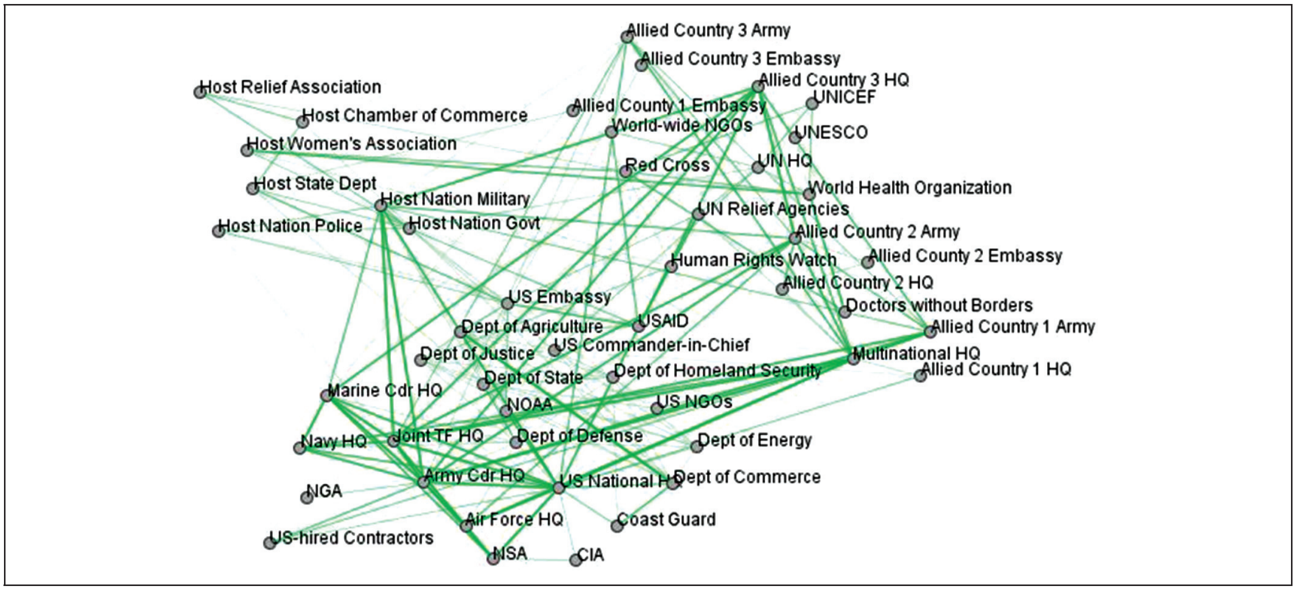

Figure 9. The coalition collaboration network for stage 4.

Table 5. Results of stage 4.

Collaboration metrics for the stage 4 coalition (N = 5, M = 49)										
US nodes	Host nodes	World nodes	CCS	ICS_{Host}	ECS_{Host}	ICS_{World}	ECS_{World}	ICS_{US}	ECS_{US}	LD
23	7	19	0.0615	0.18	0.04	0.07	0.04	0.13	0.035	0.14

COIN model metrics for 6 months with stage 4 coalition										
CC	S	S	S/s ratio	% change of S/s	% change in o	% change in m	% change in i	% change in p	% change in w	% change in C
1.8208	0.7613	2.449	3.222	0.03	−0.01	−0.01	−0.06	−0.005	−0.01	0.03

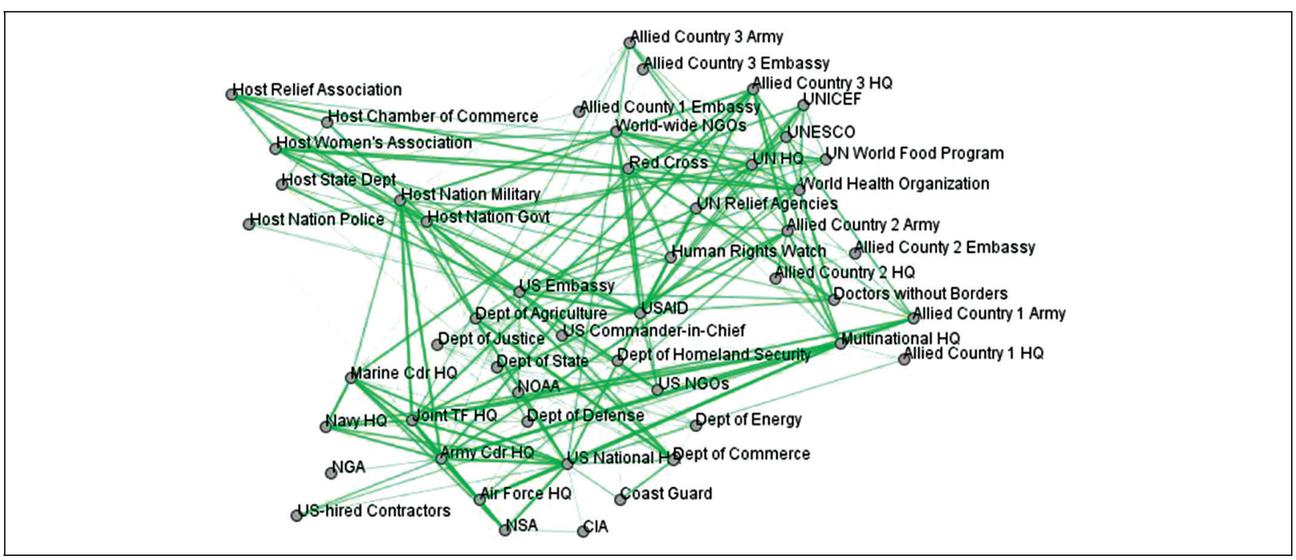

Figure 10. The coalition collaboration network for stage 5.

Table 6. Results of stage 5

Collaboration metrics for the stage 5 coalition (N = 5, M = 50)										
US nodes	Host nodes	World nodes	*CCS*	ICS_{Host}	ECS_{Host}	ICS_{World}	ECS_{World}	ICS_{US}	ECS_{US}	*LD*
23	7	20	0.09	0.25	0.07	0.125	0.06	0.14	0.06	0.16

COIN Model Metrics for 6 months with Stage 5 Coalition										
CC	*S*	*S*	*S/s* ratio	% change of *S/s*	% change in *o*	% change in *m*	% change in *i*	% change in *p*	% change in *w*	% change in *C*
1.8521	0.754	2.502	3.317	0.03	− 0.01	− 0.022	− 0.10	− 0.01	− 0.01	0.02

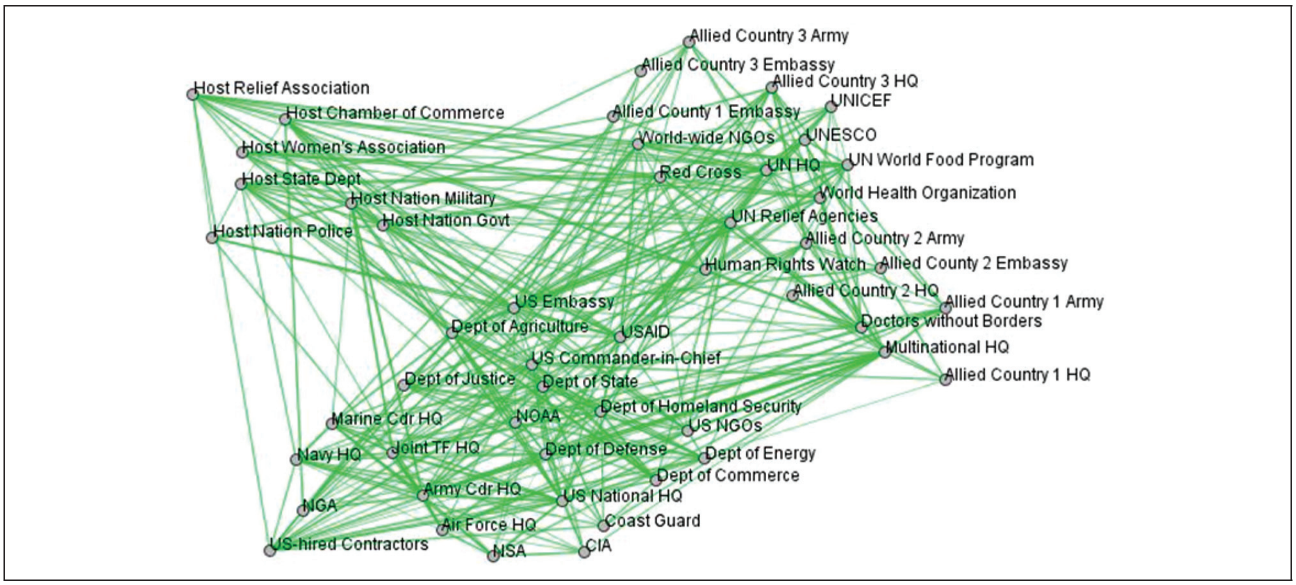

Figure 11. The coalition collaboration network for stage 6. One can visually see that the strength of the collaborations have increased from stages, 3, 4, and 5.

Table 7. Results of stage 6.

Collaboration metrics for the stage 6 coalition (N = 5, M = 50)										
US nodes	Host nodes	World nodes	*CCS*	ICS_{Host}	ECS_{Host}	ICS_{World}	ECS_{World}	ICS_{US}	ECS_{US}	*LD*
23	7	20	0.185	0.53	0.18	0.17	0.12	0.31	0.13	0.275

COIN model metrics for 6 months with stage 6 coalition										
CC	*s*	*S*	*S/s* ratio	% change of *S/s*	% change in *o*	% change in *m*	% change in *i*	% change in *p*	% change in *w*	% change in *C*
1.9332	0.748	2.610	3.489	0.05	− 0.02	− 0.02	− 0.13	− 0.02	− 0.02	0.01

0.6354, which in turn results in a much lower *S* value and a dramatic decrease of 56% in the *S/s* ratio. While the insurgency metrics only seem to hold their own for this stage, the tide has turned and the affects of a weakening coalition are beginning to show in the strengthening insurgency.

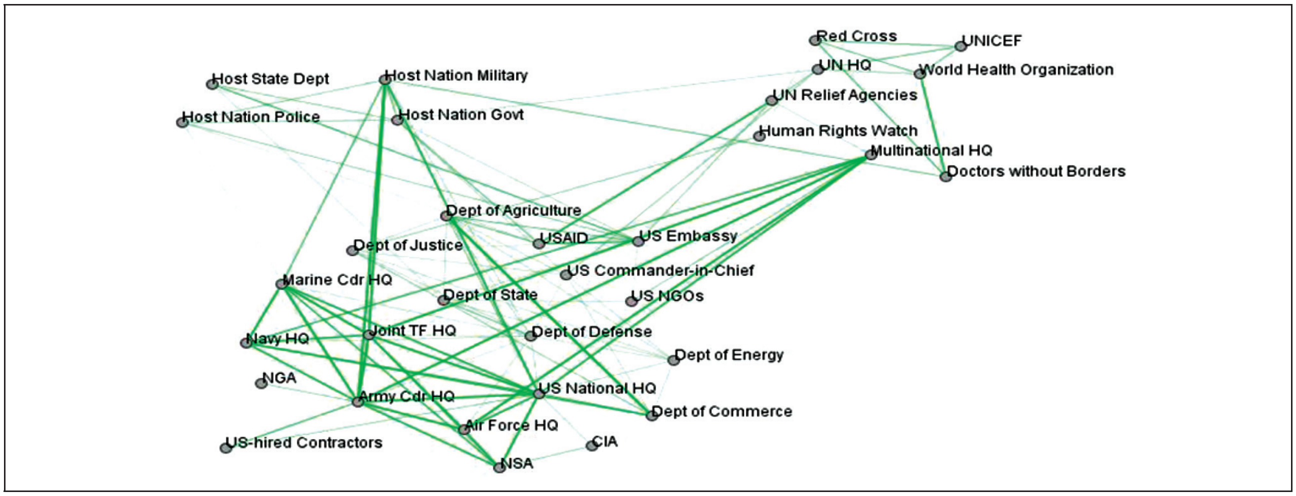

Figure 12. The coalition collaboration network for stage 7.

5.8. Stage 8. US forces withdraw and resources reduced (23 nodes)

As the COIN effort fades and the US forces withdraw, the coalition collaboration drops precipitously. In this case, the US forces maintain only a small training force and the COIN resources are cut to 10% of their previous levels, the relief agencies close or curtail many of their operations, and the Host country tries to pick up the slack. The result is a small coalition that is shown in Figure 13.

The coalition collaborations are minimal as shown in the metrics given in Table 9. The coalition is hung together by the Host country collaboration strength. A more detailed model of the endgame of insurgency wars that includes dynamical systems and graphs is found in McCormick et al.[16]

These results show just a ghost of the much stronger coalition of stages 3-6. All the collaboration metrics have decreased during this stage, some quite substantially. The CC value of 0.5012 results in lower COIN metrics of S and S/s ratio. As the counter-insurgence coalition shrinks and decays, the insurgency rebuilds itself.

6. Accumulated effects of the eight stages

As seen in each stage the *CC* and other coalition collaboration metrics directly affected the COIN metrics. In stages 1 and 2, as the coalition grew, there were only minor changes in the values of the insurgency strength. The coalition strength in stages 3–6, slowed the IN although the changes in the values were minimal, but it survived with enough strength to rebound in stages 7 and 8, as the coalition decayed and shrunk. We provide summaries of the total results of all eight stages (4 years of our COIN model simulation) in Table 10 and the graph of the metrics in Figure 14.

We have presented just one, rather generic, scenario to demonstrate the methodology and robustness of the model. More scenarios and tests are needed to understand fully the utility and limits of the model, to explore 'what-if' cases, to establish bounds and more precise methodology for determining the parameter values and calculating coalition network metrics, and to recommend changes or principles in COIN operations. One limitation of the model we are working to overcome is the lack of dynamic or

Table 8. Results of stage 7

Collaboration metrics for the stage 7 coalition (N = 2, M = 32)

US nodes	Host nodes	World nodes	*CCS*	ICS_{Host}	ECS_{Host}	ICS_{World}	ECS_{World}	ICS_{US}	ECS_{US}	*LD*
20	4	8	0.10	0.33	0.06	0.17	0.04	0.16	0.05	0.21

COIN model metrics for 6 months with stage 7 coalition

CC	*s*	*S*	*S/s* ratio	% change of *S/s*	% change in *o*	% change in *m*	% change in *i*	% change in *p*	% change in *w*	% change in *C*
0.6354	0.744	1.117	1.501	– 0.56	– 0.005	0	– 0.09	– 0.02	– 0.02	0.01

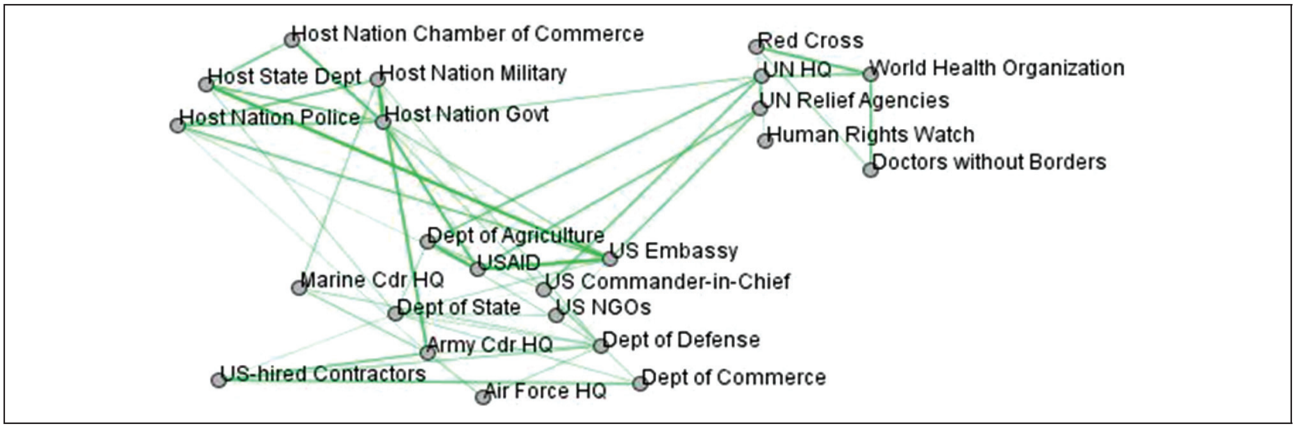

Figure 13. The coalition collaboration network for stage 8.

Table 9. Results of stage 8.

Collaboration metrics for the stage 8 coalition ($N = 2$, $M = 23$)										
US nodes	Host nodes	World nodes	*CCS*	ICS_{Host}	ECS_{Host}	ICS_{World}	ECS_{World}	ICS_{US}	ECS_{US}	*LD*
12	5	6	0.066	0.26	0.042	0.15	0.019	0.097	0.04	0.20
COIN model metrics for 6 months with stage 8 coalition										
CC	*s*	*S*	*S/s* ratio	% change of *S/s*	% change in *o*	% change in *m*	% change in *i*	% change in *p*	% change in *w*	% change in *C*
0.5012	0.748	0.962	1.285	−0.14	0	0.02	0.10	0.04	0.01	0

Table 10. Summary data for all eight stages.

% change *s*	% change *S*	% change *S/s*	% change *o*	% change *m*	% change *i*	% change *p*	% change *w*	% change *C*
−0.02	0.18	0.20	−0.05	−0.04	−0.30	−0.05	−0.04	0.10

continual coalition changes. Our model is currently restricted by the limitation that the coalition network changes only at set stages and not dynamically with the rest of the components in the model.

7. Conclusion

We have used this eight-stage notional scenario to simulate COIN operations in a dynamic environment to show the functionality of our coalition network model and the interfacing of the network model with the differential equations model. Many of the mathematical issues of combining large-scale networks and large systems of differential and algebraic equations are not yet known. However, we see this combination as potentially giving us better insights into understanding the complexity of COIN warfare and enabling decision makers to use the model to determine operational decisions, strategies and resource allocations for future coalition building and COIN operations. While more model testing and historical verification are needed, populating the model parameters with data for possible insurgencies can enable planners to make better and more thorough plans for coalition building and COIN operations. Our hybrid model (force-on-force, COIN factors, and coalition network model) also enables study of the most feared and possibly likely wars of the future: hybrid wars. As described by McCuen,[17] Cancian,[18] and Lacquement,[19] these full spectrum conflicts will involve many elements of COIN, CT and full force-on-force operations, basic elements of which are found in our model.

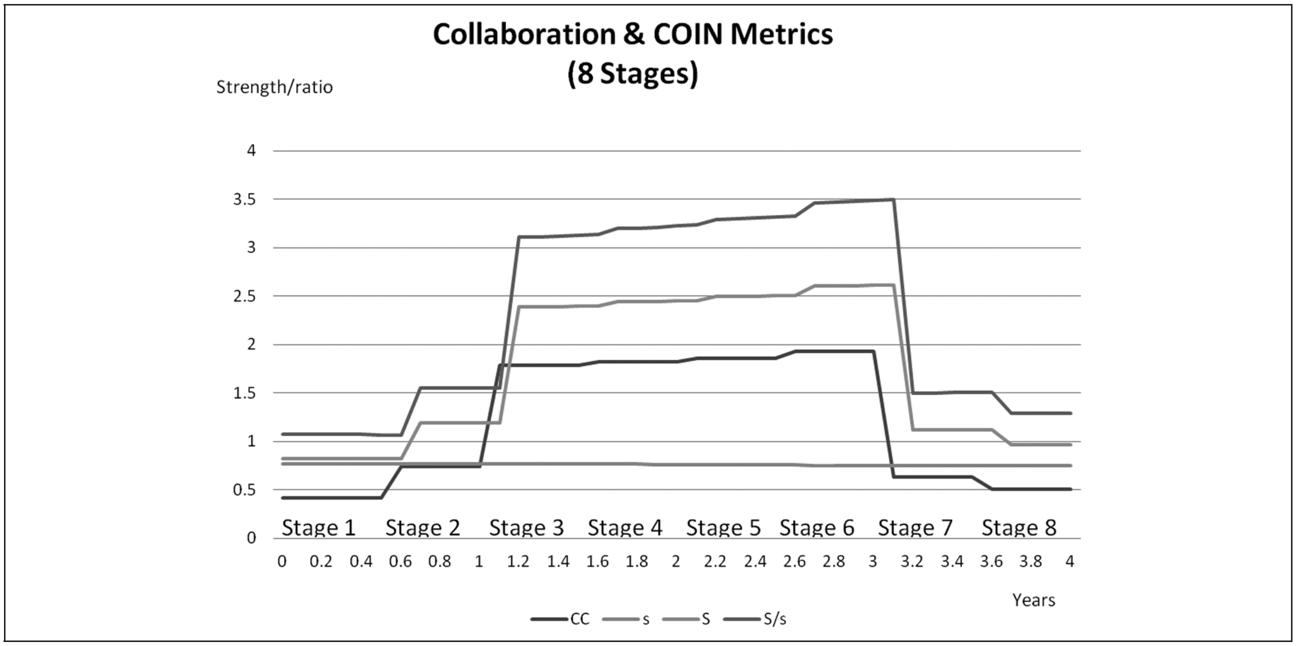

Figure 14. Graphs of the change of over the 4 years and eight stages.

While we have more to do to understand the dynamics of a coalition modeling and to model the significant elements of the coalition such as cohesion and centrality, we feel our model and this scenario experiment have laid a good foundation for future work.

Conflict of interest

The views expressed herein are those of the authors and do not reflect the position of the United States Military Academy, the US Department of the Army, or the US Department of Defense.

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