

## Implications of the Observed Distributions of Very Long Period Comet Orbits

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With allowance for galactic perturbations and observational error, the observed distributions of sizes and orientations of very long period comets are consistent with a uniform distribution of comets within the Oort Cloud. © 1985 Academic Press, Inc.

Van Flandern (1977, 1978) has proposed that the very long period "new" comets originated only a few million years ago in the explosion of a very large planet in the vicinity of what is now the asteroid belt. The primary evidence is the nonrandom distribution of both sizes and orientations of the orbits of these comets. There is a sharp peak in the distribution of the inverse semimajor axes (see Fig. 1a), suggesting to him that most of these comets passed through their previous apparition at a common epoch (that of the destruction of the planet). The nonrandom distribution of the orientations is then taken to reflect the common point of origin of these comets.

Specifically, there is a suggestion of a nonuniformity of the distribution of the arcs traced out on the sky by these comets, producing a clustering of the projected intersections. If the orbits are integrated backward one revolution, taking into account the galactic potential, the clustering becomes more pronounced (see the figures in Van Flandern, 1977). Van Flandern found the point of maximum concentration (presumed to be the point of origin) to be at ecliptic longitude  $258^\circ$  and "close to the ecliptic." Longitude  $258^\circ$  and latitude  $0^\circ$  is at galactic coordinates  $l^{\text{II}} = 0^\circ$ ,  $b^{\text{II}} = +11^\circ$ , i.e., close to the galactic center, a point noted by Van Flandern but not elaborated upon. Ovenden and Byl (1978) have con-

firmed this clustering at the previous apparition, although they have not interpreted it to indicate the origin with a single event.

Recently, Byl (1983) has looked more closely at the effects of the galactic potential, and he has pointed out that one result is to raise the perihelia at previous and subsequent apparitions to the point that presently observable comets would be out of the inner Solar System at those times. Since there would not be many randomizing planetary perturbations, complete equipartition of the cometary orbital energies would not be possible. However, both Byl and Van Flandern seriously underestimated the degree of galactic perturbations and their effects, since they entirely neglected that part of the potential perpendicular to the galactic plane.

Consider a comet moving in a fixed coordinate system with the  $x$  axis pointing toward the galactic center at the present epoch, the  $y$  axis perpendicular and in the plane, and the  $z$  axis perpendicular to the plane. Assume that the comet motion is sufficiently limited that only the first terms in the tidal expansions of the accelerations are required. Let  $L$  be the angle from the  $x$  axis to the radial vector toward the galactic center at any instant. [ $L$  is  $180^\circ$  plus the galactic longitude of the Sun and thus has a period of about  $2 \times 10^9$  years. Specifically,  $L = 2.565 \times 10^{-8} \times (\text{year}-1950.0)$ ]. The equations of motion can then be written as

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$$\begin{aligned} \ddot{x} &= C_R[(1 + \cos(2L))x + \sin(2L)y] \\ \ddot{y} &= C_R[\sin(2L)x + (1 - \cos(2L))y] \quad (1) \\ \ddot{z} &= C_Z \cdot z. \end{aligned}$$

The coefficients have the following approximate values at the solar distance:

$$\begin{aligned} C_R &= 5 \times 10^{-16} \text{ years}^{-2} \\ C_Z &= -45 \times 10^{-16} \text{ years}^{-2}. \end{aligned}$$

[See Mihalas and Routly (1968) for a complete discussion of this potential and the terms in it.] The third, or  $z$ , equation is decoupled from the other two and has a coefficient that, while an order of magnitude greater than the radial one, is negative, producing a high-frequency oscillation on the radial motion. For a single cometary revolution, this is the dominant term and must be included.

The numerical experiment consisted of taking 155 comet orbits with initial osculating parabolic velocities at perihelion and covering the entire range of orbital elements that included perihelia within the inner Solar System, and numerically integrating forward one revolution those that were perturbed into long period elliptical orbits, using the above equations of motion. All orbits except those with perihelion directions close to the galactic radial had their perihelia raised to well outside the Solar System. Those comets with major axes and thus virtually all of their motion along the line from the galactic center experienced neither tangential nor  $z$  component perturbations of any significance, thus producing only small changes in their orbits (motion in high-eccentricity ellipses is essentially rectilinear, thus making the initial inclination irrelevant). The only comets that therefore returned to where they could be observed again were those that had closely aligned lines of intersection at their previous apparition. Independent of the initial distribution, presently observable comets would show a clustering of intersections toward the galactic center at their previous apparition,

making this a rather exotic form of observational selection.

As a secondary effect, the lack of equipartition of energy means that the distribution of inverse semimajor axes does not have to be uniform. Indeed, this distribution must almost inevitably be peaked. Consider, for example, the case in which the semimajor axes are uniformly distributed out to some limiting distance, which will be assumed to be a sharp cutoff (which is indeed the case, due to both the galactic tides and stellar encounters). Define  $x$  to be  $10^6/a$ , where  $a$  is the semimajor axis in AUs. If  $f(a)$  is the distribution in semimajor axes then, in this example,  $f(a) \sim a^2$  out to a limiting  $a_0$ . If  $F(x)$  is the corresponding distribution in  $x$ , then  $F(x) \sim f(a(x))(da/dx)$ , or  $F(x) \sim x^{-4}$  down to a limiting  $x_0$ . This distribution increases rapidly toward  $x_0$  and, after convolution with a normal error distribution for the observed values of  $x$ , becomes almost indistinguishable from the result by assuming all true  $x$ 's have the same value. A test was run by assuming all

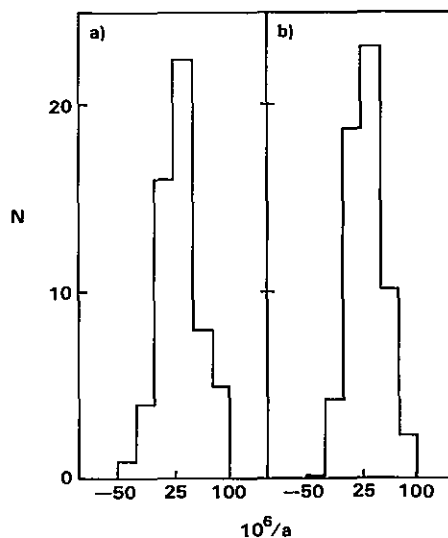


FIG. 1. (a) The observed distribution of  $10^6/a$  for those comets clearly classified as "new"; (b) the simulated distribution in the same parameter from the model in the text. The values of  $1/a_{orig}$  were taken from Marsden *et al.* (1978).

true  $x$ 's come from a single parent distribution that can be characterized by a single rms error, and by determining by trial-and-error this rms error and the  $x_0$  that best fit the observed distribution of  $x$ . The resulting predicted distribution, shown in Fig. 1b, results from an rms error in  $x$  of  $\pm 20$  ( $10^6$  AU $^{-1}$ ) and a limiting  $x_0$  of 25, corresponding to a limiting aphelion, or radius of the Oort Cloud, of 80,000 AU. Both of these estimates are quite consistent with other, quite independent results.

The conclusion, then, is that the observed distribution of elements of very long period comets can tell us virtually nothing about the complete distribution of elements at any previous epoch. In fact, the observed distribution is not inconsistent with a completely uniform distribution of comets within an Oort Cloud of radius 80,000 AU.

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