

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing this collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden to Department of Defense, Washington Headquarters Services, Directorate for Information Operations and Reports (0704-0188), 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to any penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number. **PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ADDRESS.**

1. REPORT DATE (DD-MM-YYYY)		2. REPORT TYPE	3. DATES COVERED (From - To)		
4. TITLE AND SUBTITLE			5a. CONTRACT NUMBER		
			5b. GRANT NUMBER		
			5c. PROGRAM ELEMENT NUMBER		
6. AUTHOR(S)			5d. PROJECT NUMBER		
			5e. TASK NUMBER		
			5f. WORK UNIT NUMBER		
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)			8. PERFORMING ORGANIZATION REPORT NUMBER		
9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)			10. SPONSOR/MONITOR'S ACRONYM(S)		
			11. SPONSOR/MONITOR'S REPORT NUMBER(S)		
12. DISTRIBUTION / AVAILABILITY STATEMENT					
13. SUPPLEMENTARY NOTES					
14. ABSTRACT					
15. SUBJECT TERMS					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			19b. TELEPHONE NUMBER (include area code)

Multi-scale Uncertainty Propagation in Dynamical Systems

Final Report
FA9550-10-1-0143

Bassam Bamieh, Mustafa Khammash and Igor Mezic

Department of Mechanical Engineering
University of California at Santa Barbara

Clarence Rowley

Mechanical & Aerospace Engineering
Princeton University

August 8, 2013

Abstract

Theoretical and computational methods to analyze and control the dynamic behavior of complex systems under uncertainty were investigated. Compressive Polynomial Chaos Expansions were used to circumvent the large-scale difficulties common in other Polynomial Chaos expansions. In the area of Koopman and Dynamic Mode Decomposition Analysis, stable and efficient computational techniques were developed that address a suite of problems, from Ergodic Quotient computations to complex turbulent flow characterizations. This resulted in a Koopman mode theory that rigorously unifies a number of seemingly distinct concepts advanced in fluid dynamics. Using the setting of stochastic structured uncertainty, a purely input-output theory of systems with time-varying stochastic parameters was developed. New mean-square stability tests were discovered with two important features, computational complexity that scales with number of uncertainties rather than with state dimension, and the ability to handle correlated uncertainty. Distributed control design in large-scale stochastic networks was studied. In the limit of large system size, surprising dimensionality dependencies and phase transition phenomena were discovered in the optimal control design problem itself.

1 Summary

The original aim of this project was the development of a framework that accelerates the computational investigation and control of complex physical models under uncertainty. The underlying principle was not the development of a single methodology that is equally applicable to any uncertain dynamical system. Such overarching methodologies, while general in applicability, can always be shown to have deficiencies in certain examples. The underlying principle was rather to build from the ground up, by using specific methods on certain classes of problems, and develop abstractions and generalizations when appropriate. This bottom-up approach appears to be more natural and effective in developing powerful methodologies that are applicable to problems of interest to the scientific and engineering communities. Significant progress has been made in four thrust areas. These are Compressive Polynomial Chaos (CPC) Expansions, Koopman/Dynamic Mode Decomposition (DMD) techniques, dynamical Structured Stochastic Uncertainty, and phase transitions in large-scale stochastic networks. A common feature of these results are new algorithms suitable for large-scale problems.

Compressive Polynomial Chaos is a technique developed in the course of this project [1]. It is a novel computationally tractable technique for computing the coefficients of polynomial chaos expansions based on ℓ^1 and convex optimization. The approach can be applied to problems with a large number of random variables and uses a modest number of Monte Carlo simulations while avoiding model manipulations. This technique has also been applied to stochastic model predictive control [2] where the computationally expensive step of cost function evaluation was performed using the CPC expansion.

In the area of Koopman/DMD analysis, a theory was developed that describes a sense in which DMD approximates the action of the Koopman operator on an appropriate Krylov subspace. It was shown that Koopman mode theory unifies and provides a rigorous background for a number of different concepts that have been advanced in fluid mechanics [3, 4, 5], including Global Mode Analysis, triple decomposition and Dynamic Mode Decomposition. In addition, important relationships have been discovered between Koopman, DMD and Fourier analyses. Using only data as snapshots of a vector-valued observable, it was shown that DMD algorithms have several advantages over the Discrete Fourier Transform (DFT). Variants of the Balanced POD algorithm have been developed that in one instance dramatically reduce the required number of snapshots of the state, and in another uses the restriction of the state to the controllable subspace which often has much smaller dimension in control problems with limited actuation [6, 7, 8, 9, 10, 11].

In dynamical systems with structured stochastic uncertainty new necessary and sufficient conditions for mean square stability have been obtained that are tractable for large-scale systems. These conditions were obtained through the development of a purely input-output theory of dynamic systems with structured stochastic perturbations [12]. The loss of mean square stability has recently been shown to indicate the emergence of complex behavior in networked dynamical systems, and these conditions are thus very relevant for apriori predictions (i.e. pre-simulations) of the emergence of such behavior. These results include necessary

and sufficient conditions for the physically important case of correlated uncertainties, which has previously been unresolved.

Distributed control in stochastic networks is an area of intense current research. The central question of performance limitations due to network structure was studied in the both the general setting as well as the setting of vehicular formations and consensus-like dynamical networks. For large-scale networks, asymptotic results that quantify the limits of achievable performance were obtained [13, 14, 15]. In particular, a clear answer to the question of why the so-called vehicular platoons problem is inherently difficult for large systems was obtained, in that it is spatially a one dimensional problem. These results made strong contact with the problems of harmonic solids and order-disorder transitions in statistical mechanics. This result is one of few in the distributed control literature that directly connect limits of performance statements with network topology, a significant question in this field.

2 Personnel Supported

Aside from the four co-PIs, the following postdoctoral and graduate researchers worked on, and were supported or partially supported by this project.

Postdoctoral Researchers

- Bryan Eisenhower (UCSB)
- Alice Hubenko (UCSB)
- Paul Kauffman (UCSB)
- Dirk Martin (Princeton)
- Stacy Patterson (UCSB)

Graduate Student Researchers

- Carla Bahri (Princeton)
- Marko Budisic (UCSB)
- Mitchel Craun (UCSB)
- Anthony DeGennaro (Princeton)
- Jonathan Epperlein (UCSB)
- Ryan Mohr (UCSB)
- Gunjan Thakur (UCSB)

References

- [1] L. Fagiano and M. Khammash, “Simulation of stochastic systems via polynomial chaos expansions and convex optimization,” *Physical Review E*, vol. 86, no. 3, p. 036702, 2012.
- [2] —, “Nonlinear stochastic model predictive control via regularized polynomial chaos expansions,” in *Decision and Control (CDC), 2012 IEEE 51st Annual Conference on*. IEEE, 2012, pp. 142–147.
- [3] M. Budišić and I. Mezić, “Geometry of the ergodic quotient reveals coherent structures in flows,” *Physica D: Nonlinear Phenomena*, vol. 241, no. 15, pp. 1255–1269, 2012.
- [4] M. Budišić, R. Mohr, and I. Mezić, “Applied koopmanism,” *Chaos: An Interdisciplinary Journal of Nonlinear Science*, vol. 22, no. 4, pp. 047 510–047 510, 2012.
- [5] K. Chen, J. Tu, and C. Rowley, “Variants of dynamic mode decomposition: Boundary condition, koopman, and fourier analyses,” *J Nonlinear Science, online first*, April 2012.
- [6] J. H. Tu and C. W. Rowley, “An improved algorithm for balanced pod through an analytic treatment of impulse response tails,” *Journal of Computational Physics*, vol. 231, no. 16, pp. 5317–5333, 2012.
- [7] K. K. Chen and C. W. Rowley, “H 2 optimal actuator and sensor placement in the linearised complex ginzburg-landau system,” *Journal of Fluid Mechanics*, vol. 681, pp. 241–260, 2011.
- [8] J. H. Tu, J. Griffin, A. Hart, C. W. Rowley, L. N. Cattafesta III, and L. S. Ukeiley, “Integration of non-time-resolved piv and time-resolved velocity point sensors for dynamic estimation of velocity fields,” *Experiments in Fluids*, vol. 54, no. 2, pp. 1–20, 2013.
- [9] D. Luchtenburg and C. Rowley, “Model reduction using snapshot-based realizations,” *Submitted to SIAM J Control and Optimization*, Feb. 2012.
- [10] K. Chen and C. Rowley, “Normalized coprime robust stability and performance guarantees for reduced-order controllers,” *IEEE Transactions on Automatic Control*, vol. 58, no. 4, pp. 1068–1073, 2013.
- [11] L. E. Padilla, G. K. Vallis, and C. W. Rowley, “Probabilistic estimates of transient climate sensitivity subject to uncertainty in forcing and natural variability,” *Journal of Climate*, vol. 24, no. 21, pp. 5521–5537, 2011.
- [12] B. Bamieh, “Structured stochastic uncertainty,” in *Communication, Control, and Computing (Allerton), 2012 50th Annual Allerton Conference on*. IEEE, 2012, pp. 1498–1503.

- [13] B. Bamieh, M. R. Jovanovic, P. Mitra, and S. Patterson, “Coherence in large-scale networks: Dimension-dependent limitations of local feedback,” *Automatic Control, IEEE Transactions on*, vol. 57, no. 9, pp. 2235–2249, 2012.
- [14] S. Patterson and B. Bamieh, “Network coherence in fractal graphs,” in *Proceedings of the IEEE Conf. on Decision and Control (CDC)*, 2011.
- [15] S. Patterson, M. P. Wittie, K. Almeroth, and B. Bamieh, “Network optimization with dynamic demands and link prices,” in *Communication, Control, and Computing (Allerton), 2012 50th Annual Allerton Conference on*. IEEE, 2012, pp. 120–127.