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Modification to the Joint Platform Allocation Tool (JPAT) to include additional warfighting functions

Aerial R&S Future

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14. ABSTRACT
This technical report investigates an update to the mathematical formulation employed by the Joint Platform Allocation Tool (JPAT) to include additional warfighting functions. This project directly addresses TRAC research requirements 3.2 under Future Aerial R&S Support. We demonstrate it is possible to account for additional warfighting functions while working within the current JPAT integer programming framework. The new formulation will not restrict use of any platform used in the previous implementation of JPAT, including the MQ1-C Gray Eagle. The implementation of this new formulation may require further refinement dependent on study requirements.

15. SUBJECT TERMS
Joint Platform Allocation Tool (JPAT), optimization, math modeling, integer programming, Army warfighting functions, knapsack constraints

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EXECUTIVE SUMMARY

This technical memorandum documents an update to the mathematical formulation employed by the Joint Platform Allocation Tool (JPAT) to account for additional warfighting functions. The previous JPAT model includes only intelligence requirements[10]. The capability to model additional warfighting functions will increase the use for the JPAT results.

This project directly addresses TRAC research requirements 3.2 under Future Aerial R&S Support [2]. This work, focused to support TRAC-FLVN in the Aerial R&S Study, is in line with TRAC research priorities.

In the current JPAT model, mission demands reflect only the commander's *intelligence* (INT) *requirements*. Including comprehensive commander requirements represented by the Army warfighting functions may add additional insight in answering research questions.

This technical memo supports and is a continuation of the work documented in Craparo et al [10]. The technical approach includes expanding the definition of requirements, including constraints and a means to combine discordant equipment.

We demonstrate it is possible to account for additional warfighting functions. Future work is required to identify how the behavior impacts runtime.

LIST OF ACRONYMS AND ABBREVIATIONS

INT	intelligence requirement
JPAT	Joint Platform Allocation Tool
JPAT (WF)	Joint Platform Allocation Tool, Warfighting functions Formulation
IPR	Interim Progress Report
MTRY	Monterey
TRAC	TRADOC Analysis Center
TRADOC	Training and Doctrine Command
WSMR	White Sands Missile Range

1. INTRODUCTION

1.1. Problem Description

The need exists to represent additional warfighting functions in the current Joint Platform Allocation Tool (JPAT). This project directly addresses TRAC research requirements 3.2 under Future Aerial R&S Support [2]. The current JPAT model includes only intelligence requirements. This project focuses on modifying the its formulation to use an augmented mission demand signal and create a feasible schedule. The capability to model additional warfighting functions will increase the use for the JPAT results.

1.2. Scope

Problem Statement. To expand the mathematical formulation employed by the JPAT to account for payloads and platforms beyond those required for aerial reconnaissance and surveillance.

Constraints. Constraints limit the team's options to conduct the project [13].

- Focus study on the MQ1-C Gray Eagle.
- Continue to work within the current JPAT integer programming framework.

Limitations. Limitations are a team's inability to investigate issues within the sponsor's bounds.

- Limited initial access to UNCLASSIFIED JPAT formulation or data at start of project.

Assumptions. Assumptions are specific statements that are taken as true in the absence of facts.

- There are no restrictions on total resource consumption, only instantaneous resource consumption.
- Maintain allocation periods similar to the current Aerial R&S model.
- The process of allocating equipment to additional mission demands will be similar to the current Aerial R&S model.
- Surrogate data is sufficient to represent additional demands.

1.3. Methodology

First, an UNCLASSIFIED version of JPAT was recovered from the the classified environment for research and collaboration purposes. With this version, analysis and potential solutions were developed for experimentation. The verification process determined the data requirements and initial constraints, limitations and assumptions of the new formulation. Figure 1.1 graphically shows the methodology used for this project.

1. Define the problem.
2. Recover a UNCLASSIFIED version of JPAT from the classified environment.
3. Identify potential solution for reformulation.
4. Update the model structure to include additional capabilities.
5. Experiment
6. Verification.
7. Complete Technical Memorandum.

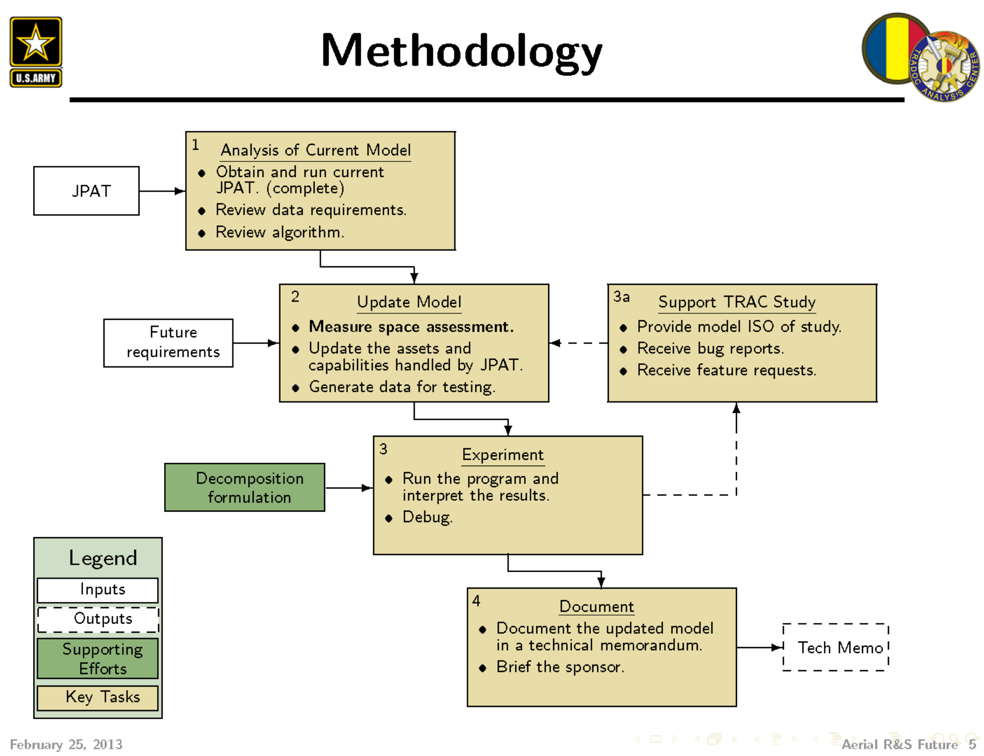

Figure 1.1: Methodology flowchart for TRAC Project 636, Modification to the Joint Platform Allocation Tool (JPAT) to Include Additional Warfighting Functions IPR, 20 May 2013 [9].

2. BACKGROUND

This technical memorandum supports and is a continuation of previous work on JPAT. See technical memo titled "Optimizing the Army's Aerial Reconnaissance and Surveillance Asset mix via the Joint Platform Allocation Tool (JPAT)" by Craparo, Smead and Tabacca for a complete background including a history of optimization models applied to military operations and a comprehensive account of the current Aerial R&S model in use [10].

2.1. Current Application of JPAT

JPAT, in the formulation documented in Craparo et al, is currently used to evaluate the strategic implications of cost, sensor performance, mission requirements, and production time lines to produce an optimal procurement and assignment schedule of aerial reconnaissance and surveillance assets [10]. The following are attributes of the current JPAT model that are not changed for the new formulation investigated in this document[3][4][5]. The current JPAT model includes established methods and techniques as documented in the reference section to this document[12][7][6][1][11].

2.1.1. Equipment

The basic organization of equipment is maintained from the current JPAT model[10]. Equipment includes both platforms and payloads. Platforms can carry a limited amount of payload equipment capable of fulfilling one or more requirements. A system is comprised of specified numbers of platforms and payloads. Figure 2.1 shows three examples of systems. Components of co-located systems can be combined to form various configurations; each consists of one platform and one or more equipment items. Figure 2.2 is an example of a configuration including platform P2 and equipment SN1 and SN3. JPAT assigns configurations to mission demands as appropriate based on their capability to meet mission requirements.

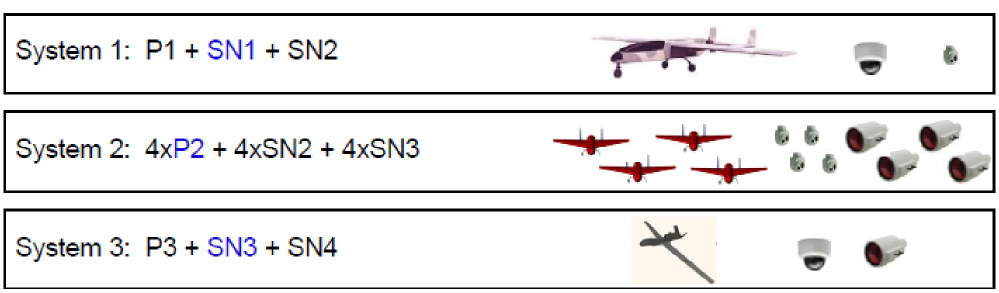

Figure 2.1: Three example systems. For example, System 2 is four P2 platforms with four SN2 and four SN3 items of available equipment to build configurations. Image from [10].

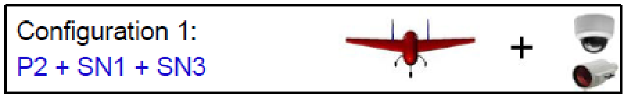

Figure 2.2: An example configuration consisting of equipment derived from multiple systems creating a single configuration. Image from [10].

2.1.2. Budget Considerations

JPAT models budgetary considerations such as costs associated with distributing, maintaining, and retiring systems. JPAT makes distribution and retirement decisions while adhering to a maximum budget constraint. The reformulation to support additional warfighting functions does not change the method of budget management.

2.1.3. Procurement, Transfer and Retirement

Assets are procured, constrained by production rates and budget. Systems are transferred between locations to meet high-priority mission demands. Systems are removed from the model only through retirement. The reformulation to support additional warfighting functions does not change the method of procurement, transfer and retirement.

2.1.4. Rolling Horizon

The rolling horizon heuristic reduces the computational burden of running the current JPAT. This technique reduces the problem into multiple time steps and identifies a heuristic policy through a sequential decision process. JPAT identifies a best policy for each time step in sequence, using the previous policy as a constant in the next time step. The reformulation to support additional warfighting functions maintains the rolling horizon approach.

2.1.5. Data Preprocessing

The preprocessing conducted for the current JPAT is still recommended for this formulation[10]. The first preprocessor aggregates mission demands with identical attributes. The second preprocessor uses information about the performance of the possible configurations-mission demand pairs to eliminate configuration-mission demand assignments that are provably unnecessary in an optimal solution.

3. JPAT with inclusion of Warfighting Functions

3.1. Purpose

We define JPAT with inclusion of Warfighting Functions as JPAT (WF) to avoid confusion between the modification and original formulation. JPAT (WF) investigates the inclusion of additional warfighting functions while maintaining the original JPAT purpose to determine the investment strategy and assignment of assets to mission demands that maximizes prioritized mission demand fulfillment.

3.2. Reformulation Considerations

The ability of a particular asset to fulfill a particular mission demand is based on requirements of the mission demand and the ability of the asset to satisfy these requirements. The current JPAT considers only *intelligence* (INT) *requirements*. An INT requirement is simply a required sensing task; for example, full motion video, signal intelligence, or radar. The reformulation considers a broader range of mission requirements as framed by the Army warfighting functions.

3.2.1. Army Warfighting Functions

JPAT's current use includes analysis based on mission demands of intelligence requirements. The mission demands for aerial platforms may not rely entirely on intelligence requirements. The warfighting functions serve as an intellectual organization of common critical functions in land operations and can function as the framework to extend the capabilities of JPAT. We use these other warfighting functions in Table 3.1 to identify other sources of mission demands.

Intelligence	Command and Control
Movement and Maneuver	Fires
Sustainment	Protection

Table 3.1: A warfighting function is a group of tasks and systems (people, organizations, information, and processes) united by a common purpose that commanders use to accomplish missions according to Army Doctrine Publication (ADP) 3-0, *Unified Land Operations* [8].

3.2.2. Expanded Scope of Mission Requirements

Incorporating additional mission demands for each warfighting functions does not require extensive modification to the formulation of JPAT. A change to the definition of a mission demand as described in Table 3.2 is necessary. This change of definition to include more than only intelligence missions requires a change in the data representing the expanded mission demands and equipment that can satisfy them.

Function	Requirement description
Intelligence	Tasks and systems that facilitate understanding the enemy, terrain, and civil considerations.
Command and control	Tasks and systems that conduct knowledge management and information management; conduct inform and influence activities; and conduct cyber electromagnetic activities.
Movement and maneuver	Tasks and systems that move and employ forces to achieve a position of relative advantage over the enemy and other threats. Direct fire and close combat are inherent in maneuver.
Fires	Tasks and systems that collect and coordinate the use of Army indirect fires, air and missile defense, and joint fires through the targeting process.
Sustainment	Tasks and systems that provide support and services to ensure freedom of action, extend operational reach, and prolong endurance.
Protection	Tasks and systems that preserve the force.

Table 3.2: Description of additional requirements and demand signals reflecting the full requirements of commanders pursuant to Army Doctrine Publication (ADP) 3-0, *Unified Land Operations* [8].

3.2.3. Additional Considerations to Equipment

The opportunity for conflict between items of equipment co-mounted on a single platform is expected to grow as more warfighting functions are considered. Sources of this conflict may be limited resources, such as power, or active interference, such as operating communications and jamming equipment simultaneously. These conflicts may not necessarily constrain the inclusion of this equipment into a configuration, but the employment while executing a mission. These conflicts are controlled and avoided by designating specific groups using an index $g \in GR$ and enumerating maximal groups of equipment that may be operated simultaneously. A group is maximal if no other piece of equipment can be added to it without violating at least one knapsack constraint. Only one group may gain value at any time instant during a mission. Figure 3.1 demonstrates a single configuration from Figure 2.2 and the two groups in that configuration.

3.3. JPAT (WF) Formulation

The following JPAT (WF) formulation focuses on modifications to the current JPAT model to include warfighting functions. See Craparo et al. for documentation of original JPAT formulation not covered in this memorandum [10]. This section discusses modifications to the precalculation

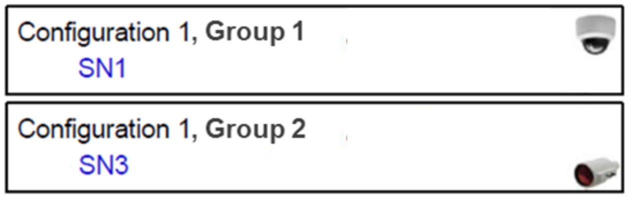

Figure 3.1: An example of the single configuration from Figure 2.2 as its component groups that identify what combinations of equipment can gain value simultaneously. This is a demonstration that Configuration 1 can gain value from either SN1 (Group 1) or SN3 (Group 2) during a mission at any time, but not both at the same time. Notice neither group includes a platform. We assume the platform is operating continuously during the mission and thus not necessary for inclusion into the groups.

3.3.1. Indices and Sets

$y,y' \in Y$	System y in set of all possible systems Y.
$c \in C$	Configuration c in set of all possible configurations C.
$e \in E$	Equipment item e (to include platforms and payloads) in set of all considered equipment E.
$g \in GR$	Group of equipment g as a subset of equipment e in a configuration c that may be operated simultaneously in the set of all groups (combinations) of equipment GR.
$(t,y,l,l') \in GP$	Identifies systems y eligible to transfer from location l to location l' at time t.
$(g,c) \in BL$	Identifies groups g in configuration c.
$(y,y') \in REP$	Identifies the system y' replacing a retiring system y.
$(i,g) \in SAT$	Identifies groups g containing equipment that satisfies requirement type i.
$l,l' \in L$	Location l and alias l' in set of all possible locations L.
$t,t' \in TIME$	Time step t and alias t' in set of all possible time steps T.
$m \in M$	Specific mission demand m in set of all mission demands M (later organized in set for time and place).
$i \in I$	Mission requirement types, previously only INT, now including additional warfighting requirements in set of all types of requirements I.
$r \in R$	Iterations in the rolling horizon model.
$t \in T(r) \subseteq TIME$	Time steps considered in an iteration r.
$t \in N \subseteq TIME$	Set of time steps at the beginning of a fiscal year.
$M(l)$	Set of mission demands residing in location l.
$l(m)$	Location of mission demand m (each mission demand resides in exactly one location).

Table 3.4: Indices and Sets for modification to JPAT (WF) to include additional warfighting functions.

3.3.1.1. Indices and Sets Discussion

The full table of indices and sets for JPAT (WF) is found in Table 3.7. JPAT (WF) determines systems y to purchase, transfer and retire in order to create configurations c of equipment e including platforms and payloads that can be turned on in groups g. At specific locations l and time steps t, mission demands m of different types i are matched with appropriate groups of equipment $(i,g) \in SAT$. Set GP describes systemsy e eligible to transfer from location l to location l' at time t: $(t,y,l,l') \in GP$. System y' can replace a retiring system y in the set $(y,y') \in REP$. The first preprocessor cross references mission demands and locations in sets $M(l)$ and $l(m)$ for computational efficiency. The rolling horizon approach requires an iteration counter $r \in R$ and time steps considered for an iteration $t \in T(r) \subseteq TIME$. New year time steps $t \in N \subseteq TIME$ are identified to enforce budgets constraints.

3.3.2. Input Data

$iq_{e,l}$	Initial quantity of equipment e in location l at time 0. [items]
$d_{t,m}$	Number of times mission demand m is present at time t. [occurrences]
$ok_{m,i,c}$	Number between 0 and 1 indicating the ability of configuration c to fulfill requirement type i in mission demand m. [unitless]
omc_e	Operation and maintenance (O&M) cost per month for equipment e. [$M]
pc_y	Procurement cost for system y. [$M]
rc_y	Retirement cost for system y. [$M]
$b_{t,y}$	Maximum budget for system y at time t. [$M]
$pr_{t,y}$	Maximum production rate of system y at time t. [items]
p_m	Number between 0 and 1 indicating the importance of mission demand m. [unitless]
$ec_{c,e}$	Number of equipment e in configuration c. [items]
$es_{y,e}$	Number of equipment e in system y. [items]
he_e	Hours available for transport and missions per time period for equipment. e, accounts for regular maintenance hours, etc. [hours]
hm_m	Hours required to perform mission demand m, not including equipment-specific setup and take down time. [hours]
$hi_{m,i}$	Hours required for requirement type i in mission demand m. [hours]
su_e	Hours to set up, take down, and maintain equipment e per assignment. [hours]
$ht_{e,y,l,l'}$	Hours required to transfer equipment e as part of system y from location l to location l'. Includes actual transit time as well as packing, unpacking, etc. [hours]
$sr_{m,c}$	Sorties required in order for configuration c to fully complete mission demand m. [sorties]
$maxdist_{t,y}$	Maximum number of system y that can be distributed as of time t. [items]
$mr_{t,y}$	Total number of system y that must be retired by time t. [items]
$intial_y$	Number of system y initially in theater. [items]
$BL_{g,c}$	Identifies which groups g are available in a configuration c. [binary]

$SAT_{i,g}$	Identifies which groups g are capable of gaining value from mission requirement types i. [binary]

Table 3.5: Input Data for modification to JPAT (WF) to include additional warfighting functions.

3.3.3. Input Data Discussion

Performance coefficients $ok_{m,i,c}$ are designed to account for variations in terrain and weather conditions. Production rates $pr_{t,y}$ account for equipment manufacture, unit standup, and deployment to the field. Equipment maintenance costs omc_e only include standard required maintenance and do not depend on usage. A description of input data is described in table form in Table 3.5. The extensive team of Army analysts and subject matter experts that contributed to the current JPAT input data as well as expected contribution for this proposed model is included in APPENDIX A. Previous data collected should be compatible yet will require augmentation. The augmentation will consider adding new mission demands and equipment, including their attributes identified by the input sets. Data collection is expected to be similar, although possibly specific to the warfighting function in question.

3.3.4. Positive Integer Variables

$G_{t,y,l,l'}$	Number of system y transferring from location l to location l' at time t.
$Z_{t,y,l}$	Number of system y retiring from location l at time t.
$D_{t,y,l}$	Number of system y distributed to location l at time t.

The positive integer variables track the magnitude of specific scheduled events during the implementation of JPAT. No positive integer variables were changed for the new formulation.

3.3.5. Binary Variables

$P_{t,c,l}$=1	if sufficient equipment is present to create configuration c at time t in location l; 0 otherwise.

The binary variables track the acceptability and inclusion of equipment into configurations and groups. No binary variables were changed for the new formulation.

3.3.6. Positive Variables

The positive variables track the magnitude of specific scheduled events during the implementation of JPAT. The positive variable $O_{t,m,c,g}$ is new for JPAT (WF). $O_{t,m,c,g}$ is the number of hours during time step t group g in configuration c is turned on for mission demand m. This allows control and tracking of group usage.

$X_{t,m,c,i}$	Number of hours configuration c is assigned to mission requirement type i for mission demand m at time t.
$S_{t,m,c}$	Number of sorties flown by configuration c against mission demand m at time t.
$Q_{t,e,l}$	Quantity of equipment e present in location l at time t.
B_t	Budget rolled over from previous time period at time t.
$O_{t,m,c,g}$	Number of hours during time step t group g in configuration c was on for mission demand m.

3.3.7. JPAT (WF) formulation

The JPAT (WF) formation is located in Figure 3.2.

3.3.8. Objective and Constraints

The objective function in Equation 3.1 maximizes mission demand coverage with respect to mission demand priority, mission demand to configuration performance, and time spent covering each intelligence requirement of a mission demand. These three factors contribute equally to the objective function value.

Constraint set 3.2 ensures that intelligence requirements are not over satisfied by the assigned configurations. Constraint sets 3.3 to 3.4 maintain a record of the quantity of each equipment type available in each location, beginning with the initial quantity 3.4 and updating the quantity based on system procurements, retirements, and transfers in subsequent time steps 3.3.

Constraint sets 3.5 to 3.9 ensure that configurations are employed appropriately based on equipment availability. Constraint set 3.5 forces $P_{t,c,l}$ to take on a value of zero if any piece of equipment require to construct configuration$c$ is not present in a sufficient quantity in location l at time t; otherwise, $P_{t,c,l}$ is allowed to take on a value of one. Constraint set 3.6 uses the variables $P_{t,c,l}$ to control the number of sorties flown by configuration c: if $P_{t,c,l}=$ 0, then configuration c cannot fly any sorties against any mission demands in location l at time t. Otherwise, configuration c can fly any number of sorties so long as it does not exceed the number of sorties required to completely satisfy the mission demand. Constraint set 3.7 ensures that the number of coverage hours recorded for mission requirement type i does not exceed the total number of hours a piece of equipment capable of covering mission requirement type i was operated for each configuration c, mission demand m, and time step t. Constraint set 3.8 ensures that the total time spent operating groups does not exceed the total hours flown by each configuration c supporting mission demand m at time t. Finally, constraint set 3.9 ensures that the hours spent fulfilling mission demands and transferring from one location to another do not exceed the “pool” of hours available for each equipment type.

Constraint sets 3.10 to 3.12 ensure that budgetary limitations are observed. Constraint set 3.10 calculates the monthly budget rollover B_t while accounting for equipment maintenance, system procurement, and system retirement costs. Because B_t is a

$$\max_{\substack{P,\ G,\ Z\\ D,\ S,\ X,\\ B,\ Q,}} \quad z = \sum_{(t,m,c,i):t\in T(r),d_{t,m}>0,hi_{m,i}>0} p_m ok_{m,i,c} \frac{X_{t,m,c,i}}{\sum_{i'} hi_{m,i'}} \tag{3.1}$$

$$\text{s.t.} \quad \sum_{c:ok_{m,i,c}>0} X_{t,m,c,i} \le hi_{m,i} d_{t,m} \qquad \forall\ t \in T(r), m, i : d_{t,m} > 0, hi_{m,i} > 0 \tag{3.2}$$

$$Q_{t,e,l} = Q_{t-1,e,l} + es_{y,e} \sum_y (D_{t,y,l} - Z_{t,y,l} + \sum_{l'} (G_{t,y,l',l} - G_{t,y,l,l'})) \qquad \forall\ t \in T(r), e, l : t > 1 \tag{3.3}$$

$$Q_{t=1,e,l} = iq_{e,l} \qquad \forall\ e, l \tag{3.4}$$

$$P_{t,c,l} \le \frac{Q_{t,e,l}}{ec_{c,e}} \qquad \forall\ t \in T(r), l, c, e : ec_{c,e} > 0, \exists m \in M(l) : d_{t,m} > 0 \tag{3.5}$$

$$S_{t,m,c} \le sr_{m,c} d_{t,m} P_{t,c,l(m)} \qquad \forall\ t \in T(r), m, c \tag{3.6}$$

$$X_{t,m,c,i} \le \sum_{g:(g,c)\in BL,\ (i,g)\in SAT} O_{t,m,c,g} \qquad \forall\ t \in T(r), m, c, i : ok_{m,i,c} > 0, hm_{m,i} > 0, d_{t,m} > 0 \tag{3.7}$$

$$\sum_g O_{t,m,c,g} \le \frac{hm_m S_{t,m,c}}{sr_{m,c}} \qquad \forall\ t \in T(r), m, c : \sum_i ok_{m,i,c} > 0, d_{t,m} > 0 \tag{3.8}$$

$$\sum_{y,l'} ht_{e,y,l,l'} G_{t,y,l,l'} + \sum_{c,m\in M(l)} ec_{c,e} \left(\frac{hm_m}{sr_{m,c}} + su_e \right) S_{t,m,c} \le he_e Q_{t,e,l} \qquad \forall\ t, e, l \tag{3.9}$$

$$B_t = B_{t-1} + \sum_y b_{t,y} - \sum_{y,l} \left(pc_y D_{t,y,l} + rc_y Z_{t,y,l} \right) - \sum_{e,l} omc_e Q_{t,e,l} \qquad \forall\ t \in T(r) \setminus N : t > 1 \tag{3.10}$$

$$\sum_{y,l} pc_y D_{t,y,l} + \sum_{y,l} rc_y Z_{t,y,l} + \sum_{e,l} omc_e Q_{t,e,l} \le \sum_y b_{t,y} \qquad \forall\ t \in T(r) \cap N \tag{3.11}$$

$$B_t = 0 \qquad \forall\ t \in T(r) \cap N \tag{3.12}$$

$$\sum_{l,t'\le t} D_{t',y,l} \le max_{t,y} \qquad \forall\ t \in T(r), y \tag{3.13}$$

$$\sum_{t'\le t, y:(y,y')\in REP} Z_{t',y,l} \ge \sum_{t'\le t} D_{t',y',l} \qquad \forall\ t \in T(r), l, y' : \exists y : (y, y') \in REP \tag{3.14}$$

$$P_{t,c,l} \in \{0, 1\} \qquad \forall\ t \in T(r), c, l \tag{3.15}$$

$$G_{t,y,l,l'} \in \mathbb{Z}^+ \qquad \forall\ (t, y, l, l') \in GP : t \in T(r) \tag{3.16}$$

$$Z_{t,y,l} \in \mathbb{Z}^+ \qquad \forall\ t \in T(r), y, l \tag{3.17}$$

$$D_{t,y,l} \in \mathbb{Z}^+ \qquad \forall\ t \in T(r), y, l \tag{3.18}$$

$$O_{t,m,c,g} \ge 0 \qquad \forall\ t, m, c, g \tag{3.19}$$

$$X_{t,m,c,i} \ge 0 \qquad \forall\ t \in T(r), m, c, i \tag{3.20}$$

$$S_{t,m,c} \ge 0 \qquad \forall\ t \in T(r), m, c \tag{3.21}$$

$$Q_{t,e,l} \ge 0 \qquad \forall\ t \in T(r), e, l \tag{3.22}$$

$$B_t \ge 0 \qquad \forall\ t \in T(r) \tag{3.23}$$

Figure 3.2: Formulation for JPAT (WF).

nonnegative variable, constraint set (10) ensures that the available budget is not exceeded on months that do not mark the beginning of a fiscal year. Likewise, constraint set 3.11 performs this function for months that do mark the beginning of a fiscal year, while constraint set 3.12 sets B_t to zero for months at the beginning of a fiscal year.

Constraint sets 3.13 to 3.14 control distribution and retirement of systems. Constraint set 3.13 ensures that the total number of system y distributed as of time t does not exceed the limits posed by system production rates and fielding restrictions. Constraint set 3.14 ensures that any system y that"upgrades" a system y is not distributed until its predecessor y is retired. Finally, constraint sets 3.15 to 3.23 declare variable types. Constraint set 3.19 is an addition to the current JPAT formulation defining the variable $O_{t,m,c,g}$.

3.4. Group Generation

There are already two preprocessing steps for the current JPAT. This section describes a third step for enumerating members of GR. The tractability of this modified formulation depends on the efficiency with which the members of GR can be enumerated. This is accomplished with an enumeration model that loops over all configurations and constructs groups for each, while checking to make sure its not duplicating the groups. A preliminary discussion of this model apears in [10].

3.4.1. Indices and Sets

$e \in E$	Equipment item e (to include platforms and payloads) in set of all considered equipment E.
$g \in GR$	Group of equipment g as a subset of equipment e in a configuration c that may be operated simultaneously in the set of all groups (combinations) of equipment GR.
$d \in D$	System y in set of all possible systems Y.
$i \in I$	Mission requirement types including additional warfighting requirements in set of all types of requirements I.
$r_{e,d}$	Equipment e resource consumption along dimension d.
$m \in M$	Specific mission demand m in set of all mission demands M (later organized in set for time and place).
$r \in R$	Iterations in the rolling horizon model.
$p_{e,i}$	Previous groups of equipment e generated in iteration i.

Table 3.7: Indices and Sets for modification to JPAT (WF) to include additional warfighting functions.

For preliminary modeling purposes, we assume that the physical limitations dictating membership in GR can be captured via knapsack constraints. The index d reflects knapsack dimensions (e.g., power), the parameter $r_{e,d}$ reflects equipment e resource consumption along dimension d (e.g., power consumed).

3.4.2. Input Data

Group enumeration has the same data requirements as JPAT (WF).

3.4.3. Variables

$Y_{e,d}$=1 if the equipment e violates any knapsack constraint d; 0 otherwise.
U_e=1 if the equipment e is present in current group under consideration; 0 otherwise.

3.4.4. New Group Generation Model formulation

The binary variable U_e reflects membership of e in g, and the parameter m_d reflects the resource availability along dimension d (e.g., power available on the platform). Set $u_{e,i}$ is an indicator variable equipment type e considered in iteration i. This methods creates the set *biglist(g,e)*, which describes which pieces of equipment belong to which groups, and the set *BL* $\ni$ *(g,c)*, which describes which groups belong to which configurations. Assume without loss of generality $m_d \leq \sum_e r_{e,d} \ \forall \ d$.

$$\max_{U,\ Y} \quad 0 \tag{3.24}$$

$$\text{s.t.} \quad \sum_e r_{e,d} U_e \leq m_d \qquad \forall \ d \tag{3.25}$$

$$U_e + \sum_d Y_{e,d} \geq 1 \qquad \forall \ e \tag{3.26}$$

$$Y_{e,d} \leq 1 + \frac{\sum_{e'} r_{e',d} U_{e'} + r_{e,d}(1 - U_e) - m_d}{\sum_{e'} r_{e',d}} \qquad \forall \ e, d \tag{3.27}$$

$$\sum_{e:p_{e,i}=0} U_e + \sum_{e:p_{e,i}=0} (1 - U_e) \geq 1 \qquad \forall \ i \tag{3.28}$$

$$U_e, Y_{e,d} \in \{0, 1\} \qquad \forall \ e, d \tag{3.29}$$

Constraint set in Equation 3.25 captures the physical constraints governing simultaneous equipment usages. Constraint set 3.26 ensures that the group generated is maximal; in particular, it ensures that each piece of equipment e is either selected for inclusion, or its selection would cause at least one knapsack constraint to be violated. Constraint set 3.27 ensures that the variables denoting violation of knapsack constraints are set correctly. Constraint set 3.28 ensures that the group generated differs from all previous groups $p_{e,i}$ of equipment e that have been generated in iteration i. Constraint set 3.29 declares variable types.

One can iteratively solve the New Group Generation Model to generate a new member of *GR*, given a set of existing members. Preliminary experimentation indicates that this formulation can be used to generate maximal groups of equipment within an acceptable amount of time. It is important to note that the inclusion of only maximal groups in the modified JPAT model reflects an implicit assumption that there are no restrictions on total resources consumption, only instantaneous resource consumption. While this assumption has been supported by subject matter experts, the authors recommend verifying it again should the model described in this section be used operationally.

4. RESULTS AND CONCLUSION

4.1. Verification of New Formulation

The results of this project and verification of the formulation are best explained with a discussion on a verification scenario.

4.1.1. Verification Scenario

The data for this verification scenario is located in APPENDIX B and under project 636 in the TRAC Knowledge Management System at the address trac/Projects/636/. Appendix Terms used in the current GAMS implementation of JPAT are maintained for ease of readability. In the following scenario, a mission requirement labeled 'INTTYPE' may not be an intelligence requirement type anymore. Table 4.1 contains the critical information defining the scenario.

Msn Req.	MD	Time step	Location	Duration (hrs)	SensorMatch	Performance
INTTYPE1	MD1	t2	LocationA	5	Sensor1	.1
INTTYPE2	MD1	t2	LocationA	3	Sensor2	1

Table 4.1: Fixed parameters for all verification scenarios.

There are two mission requirements in the same mission demand, time step and location. This scenario is also limited to a single system and configuration to allow the results to be replicated by hand. The two types of mission requirements represent separate warfighting functions with the same priority. The two items of equipment in the configuration each match only with a single corresponding requirement. These two items of equipment are in separate groups thus only one of the two items of equipment can be turned on at any time. Notice INTTYPE2 has a shorter duration than INTTYPE1, but its associated matching equipment has a higher performance, gaining more value per time unit collecting.

We demonstrate the mechanics and subtleties of the reformulation by changing a single parameter of hours the equipment is available and discussing the results.

4.1.2. Scenario 1, Long Mission Time

In the first situation, the system has 10 hours available to fly the entire mission. We see the optimal schedule in Table 4.2.

Time Step	MD	INT	HrsReqInt	TimeOnInt	%Covered
t2	MD1	INTTYPE1	5.00	5.00	100.00%
t2	MD1	INTTYPE2	3.00	3.00	100.00%

Table 4.2: Verification Scenario 1, Long Sensor Duration.

HrsReqInt are the total hours a mission requirement is available. TimeOnInt are the hours an item of equipment is actively collecting value from a mission requirement. We see the TimeOnInt equals the HrsReqInt for both requirements. Ten hours is enough time for a resulting optimal schedule to include both requirements for their entire duration. This schedule results in an objective value (using Equation 3.1) of 0.44. This is an upper limit to the amount of value available from these requirements.

4.1.3. Scenario 2, Medium Mission Time

In the second situation, the system has 5 hours available to fly the entire mission. We see the optimal schedule in Table 4.3.

Time Step	MD	INT	HrsReqInt	TimeOnInt	%Covered
t2	MD1	INTTYPE1	5.00	2.00	40.00%
t2	MD1	INTTYPE2	3.00	3.00	100.00%

Table 4.3: Verification Scenario 2, Scenario 2 Medium Mission Time.

We see the TimeOnInt is less than HrsReqInt for requirement INTTYPE1. Five hours is enough to cover all of INTTYPE2 with the sensor that has the higher performance, and only two hours of INTTYPE1. The resulting optimal schedule results in an objective value of 0.40.

4.1.4. Scenario 3, Short Flight Time

In the third situation, the system has 2 hours available to fly the entire mission. We see the optimal schedule in Table 4.4.

Time Step	MD	INT	HrsReqInt	TimeOnInt	%Covered
t2	MD1	INTTYPE1	5.00	0.00	0.00%
t2	MD1	INTTYPE2	3.00	2.00	100.00%

Table 4.4: Verification Scenario 3, Scenario 2 Short Mission Time.

We see the TimeOnInt is zero for INTTYPE1 and less than HrsReqInt for requirement INTTYPE2. All two hours cover INTTYPE2 with the sensor that has the higher performance. The resulting optimal schedule results in an objective value of 0.25.

4.1.5. Verification Results

We see how the new formulation properly restricts the use of the groups while maintaining the functionality of the original formulation. The results can be interpreted graphically in Figure 4.1.

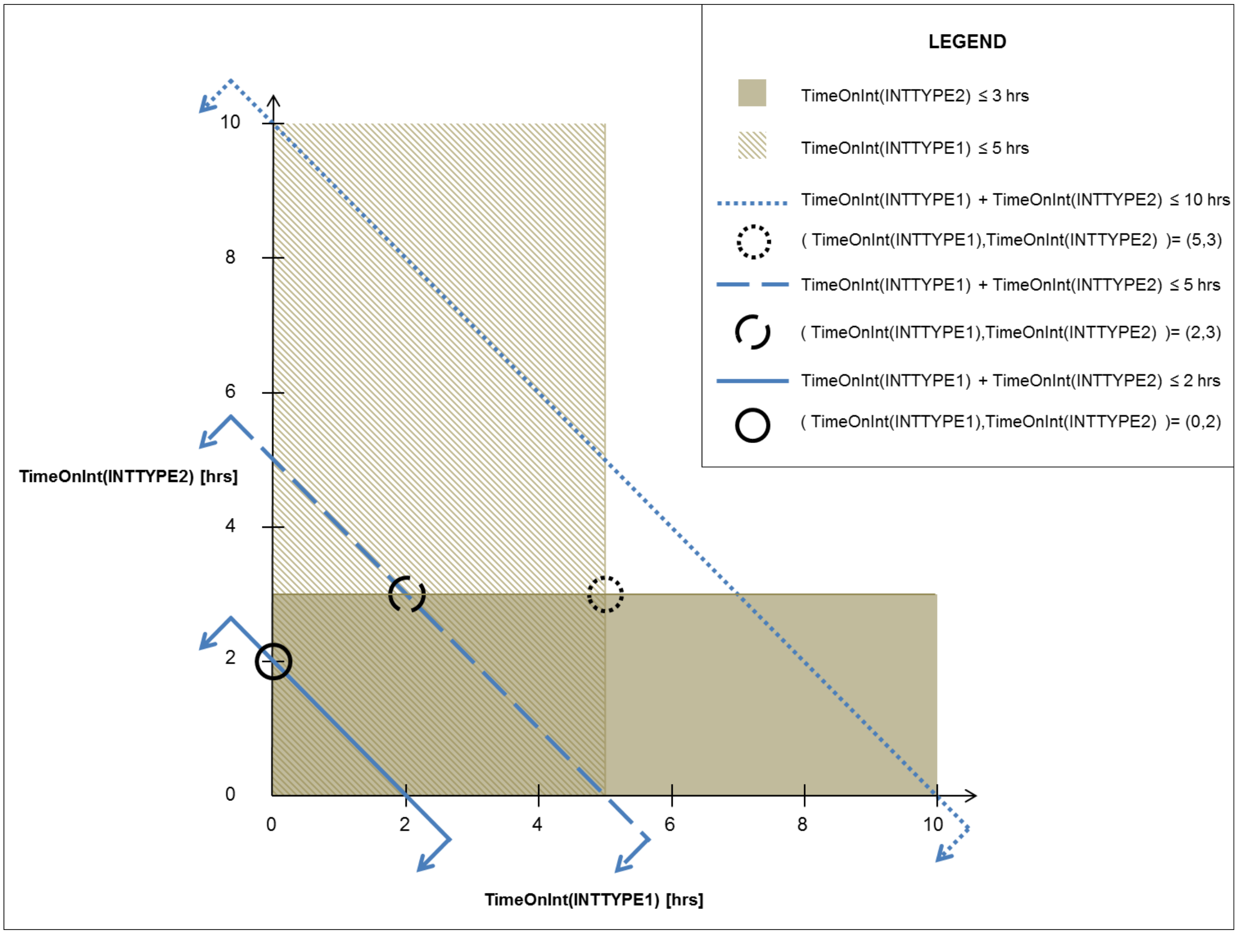

Figure 4.1: Graphical representation of verification scenario. The horizontal axis is time INTTYPE1 is available in hours and the vertical axis is time INTTYPE2 is available in hours. The dotted blue line represents a maximum of 10 hours of combined sensor usage. The objective function, using Equation 3.1, generally maximizes INTTYPE2 then INTTYPE1. This allows an optimal solution of 5 hours on INTTYPE1 and 3 hours on INTTYPE2 shown by a dotted circle. The dashed blue line represents a reduced maximum of 5 hours combined sensor usage. This allows an optimal solution of 2 hours on INTTYPE1 and 3 hours on INTTYPE2 shown by a dashed circle. The solid blue line represents a reduced maximum of 2 hours combined sensor usage. This allows an optimal solution of 2 hours on INTTYPE1 and 0 hours on INTTYPE2 shown by a solid circle.

4.2. Conclusion

We demonstrate it is possible to account for additional warfighting functions while working within the current JPAT integer programming framework. The new formulation will not restrict use of any platform used in the previous implementation of JPAT, including the MQ1-C Gray Eagle. The implementation of this new formulation may require further refinement dependent on study requirements. Future work is necessary to identify how the behavior impacts runtime.

APPENDIX A. Table of Data Sources

Input Data	Source(s)
Initial inventories ($initial_y$, $iq_{e,l}$)	Intelligence and Security Command (INSCOM)
System upper bounds ($upperbounds_y$, $mr_{t,y}$)	
System transfer times ($transdays_{y,l,l'}$)	
MD/INTEL durations (hm_m, $hi_{m,i}$)	Subject Matter Experts (SMEs) from the Centers of Excellence
MD priorities (p_m)	
MD locations ($l(m)$, $M(l)$)	
MD frequencies ($d_{t,m}$)	
Budget data ($b_{t,y}$)	Army Resource Management Office (G-8)
Production rates ($pr_{t,y}$)	Program Managers (PMs) for the aerial R&S systems
Cost data (pc_y, rc_y, omc_e)	
Equipment hours available (he_e)	
Equipment setup time (su_e)	
System configurations ($es_{y,e}$)	
Group belong in configurations ($BL_{g,c}$)	
Group satisfy requirements ($SAT_{i,g}$)	
Configuration data ($ok_{m,i,c}$, $ec_{c,e}$, $sr_{m,c}$)	Army Materiel Systems Analysis Activity (AMSAA)

Figure A.1: An extensive team of Army analysts and subject matter experts contributed to JPATs input data. The original JPAT sources and thereof can be found in Craparo et al [10].

APPENDIX B. Verification Input Data

The following tables describe the input files for the verification scenario. This data is located under project 636 in the TRAC Knowledge Management System at the address trac/Projects/636/.

Sensor1	2
Sensor2	2

Table B.1: Verification data file: WSMR_hrsavail_WF_MTRY.csv

MD1	INTTYPE1	c1	.1
MD1	INTTYPE2	c1	1

Table B.2: Verification data file: FLVN_oksmaller_WF_MTRY.csv

MD1	8

Table B.3: Verification data file: FLVN_hrsreq_WF_MTRY.csv

System2	System1

Table B.4: Verificatio data file: FLVN_replacements_WF_MTRY.csv

t2	MD1	1

Table B.5: Verification data file: FLVN_mdreducedm_WF_MTRY.csv

t4	System1	1

Table B.6: Verification data file: FLVN_retirements_WF_MTRY.csv

dummy	Sensor1	Sensor2
c1	1	1

Table B.7: Verification data file: AMSAA_equipconfig_WF_MTRY.csv

dummy	c1
group1	1
group2	1

Table B.8: Verification data file: MTRY_belong_WF_MTRY.csv

	INTTYPE1	INTTYPE2
group1	1	0
group2	0	1

Table B.9: Verification data file: MTRY_satisfy_WF_MTRY.csv

System1	1
System2	1

Table B.10: Verification data file: FLVN_upperbounds_WF_MTRY.csv

Sensor1	0
Sensor2	0

Table B.11: Verification data file: WSMR_setuptime_WF_MTRY.csv

t12	1
t24	1
t36	1
t48	1
t60	1
t72	1
t84	1
t96	1
t108	1
t120	1
t132	1
t144	1

Table B.12: Verification data file: WSMR_newyear_WF_MTRY.csv

System1	LocationA	LocationA	100

Table B.13: Verification data file: FLVN_transdays_WF_MTRY.csv

MD1	1

Table B.14: Verification data file: FLVN_priority_WF_MTRY.csv

dummy	APUC	Retirement
System1	0	0

Table B.15: Verification data file: WSMR_syscost_WF_MTRY.csv

Sensor1	0
Sensor2	0

Table B.16: Verification data file: WSMR_cost_WF_MTRY.csv

MD	c1
MD1	1

Table B.17: Verification data file: AMSAA_mult_WF_MTRY.csv

	INTTYPE1	INTTYPE2
Sensor1	1	0
Sensor2	0	1

Table B.18: Verification data file: AMSAA_cover_WF_MTRY.csv

MD1	LocationA

Table B.19: Verification data file: FLVN_mdloc_WF_MTRY.csv

dummy	LocationA
Sensor1	1
Sensor2	1

Table B.20: Verification data file: FLVN_iq_WF_MTRY.csv

dummy	Sensor1	Sensor2
System1	1	1

Table B.21: Verification data file: FLVN_equipsys_WF_MTRY.csv

dummy	INTTYPE1	INTTYPE2
MD1	5	3

Table B.22: Verification data file: FLVN_hrson_WF_MTRY.csv

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