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Tracking Evasive Objects via A Search Allocation Game

Huimin Chen, Dan Shen, Genshe Chen, Erik P. Blasch, and Khanh Pham

Abstract— This paper outlines a strategy for tracking evasive objects in discrete space using game theory to allocate sensor resources. One or more searchers have to allocate the effort among the discrete cells to maximize the object detection probability within a finite time horizon or minimize the expected search time to achieve the desired detection probability under a false alarm constraint. We review the standard formulations under a sequential decision setting for finding stationary objects. Then we consider both robust and optimal search strategies and extend the standard search problem to a two-person zero-sum search allocation game where the object wants to hide from the searcher and the object has incomplete information about the searcher's remaining search time. We discuss how the results affect the sensor management and mission planning for cooperative unmanned aerial vehicle (UAV) search tasks and provide simulation examples to show the effectiveness of the proposed method compared with random search strategy.

I. INTRODUCTION

One of the challenges in the constellation management of sensor platforms and in the path planning for tracking evasive objects is an associated search problem: for objects or threats that have not yet been identified, how to model the uncertainties in the operational field and allocate the sensing resources accordingly? The field of search theory addresses this problem from various aspects: the search space can be discrete or continuous; the object can be stationary or moving; the sensor can have single-look or multiple looks of the area at a particular time instant. For a comprehensive review, see [2].

A. Object Search Problem in Discrete Space

We consider a finite probability space $X = \{1, 2, ..., n\}$ with each point $i \in X$ being associated with a Wiener process $y_i(t)$. The n Wiener processes are independent with the same variance $\sigma^2 t$. If an object is in cell i, then $E[y_i(t)] = \mu t$ ($\mu > 0$). Otherwise, the mean of the process $y_i(t)$ is zero ($\mu = 0$). A searcher can only focus on one cell at any given time. Assume that the prior probability that cell i contains an object is $p_i(0)$, then a searcher looking at cell i from time 0 to t with measurement $y = y_i(t)$ will update its posterior probability that cell i containing the object using Bayes' rule

$$p_i(t|y) = \frac{p_i(0)}{p_i(0) + (1 - p_i(0))e^{\left(\frac{\mu}{2\sigma^2}\right)(\mu t - 2y)}} \quad (1)$$

In terms of the log-likelihood ratio

$$z_i(t) = \log\left(\frac{p_i(t|y)}{1 - p_i(t|y)}\right)$$

we have

$$z_i(t) = z_i(0) - \frac{\mu}{2\sigma^2}(\mu t - 2y) \quad (2)$$

Clearly, $z_i(t)$ is also a Wiener process with mean $(\mu^2/2\sigma^2)t$ if cell i contains an object and $-(\mu^2/2\sigma^2)t$ if cell i does not contain an object. The variance is $(\mu^2/\sigma^2)t$ in either case. The searcher needs to sequentially determine which cell to look at and for how long.

B. Related Works

Optimal search theory deals with the following generic problem: A single object is hidden in one of the n cells. Each cell can provide the searcher with prior probability of object presence as well as the detection and false alarm probabilities for a fully specified sensing action. The goal is to design a search policy that maximizes the probability of detecting the object at the end of the mission. The two-stage procedure was first proposed by Posner [8] where he considered using a radar to locate a satellite in the sky containing n cells. The optimality of greedy search which is to look at the cell with the largest log-likelihood ratio sequentially was proved in [11] and extended to the dynamical and multiple hypothesis testing cases in [3]. Connections to compressed sensing for acquiring sparse signals under energy constraint have been studied in [9], [1] where [9] showed an adaptive search policy for signals having a sparse representation in the search space while [1] provided an optimal two-stage procedure to recover sparse signals using a convex criterion.

The two-stage approach is mainly for a single searcher looking for a single stationary object. The k-stage approach is more appropriate for finding multiple objects in sparse locations using a team of searchers cooperatively. The search for intelligent object with dynamic mobility requires to study the search allocation game and dedicate the sensing resources in a not-too-greedy manner. A realistic mission may contain multiple objectives with conflicting interests in variable environments. Thus one needs to integrate various search policies for different situations into one coherent performance metric to study the effectiveness of the entire mission planning process. Such a performance metric has to be comprehensible and relatively easy to optimize. The

existing search theory does not have an immediate answer to such a requirement.

The rest of the paper is organized as follows. Section II presents the optimal search strategy for finding stationary objects. Section III discusses the robust search procedure when the object distribution is unknown. Section IV extends the existing search problem to a search-allocation game where the evasive object motion is modeled as a multi-stage search-and-hide game. Simulation examples are provided in Section V and concluding summary is in Section VI.

II. Optimal Search Strategy for Stationary Objects

Consider a searcher who starts with the most likely cell i and does not change to another cell unless $z_i(t)$ drops by δ/n for some small $\delta > 0$. When this decrease of the log-likelihood ratio occurs, the searcher switches to the most likely cell j (which used to be the second best cell to search). If there is only one object hiding in one of the n cells and the maximum allowable error probability is ε, then the searcher will make a decision that cell i contains the object once $z_i(t) > \tau(\varepsilon)$ and the threshold is chosen by

$$\tau(\varepsilon) = \log\left[\left(\frac{1-\varepsilon}{\varepsilon}\right)(n-1)\right]. \tag{3}$$

As $\delta \to 0$, the above search procedure becomes optimal in the sense of minimizing the expected search time to reach a decision with error probability no larger than ε [11]. Assume that the prior probability for cell i to contain an object is $\frac{1}{n}$ for $i = 1, ..., n$. If the searcher applies the optimal policy with possible switching frequency among different cells being arbitrarily high, then the expected search time under the optimal procedure is

$$
\begin{aligned}
T(\varepsilon) &= \frac{2\sigma^2}{\mu^2}\left\{(n-2)\left(\frac{n-1-n\varepsilon}{n-1}\right)+ \right. \\
&\quad \left. (1-2\varepsilon)\log\left[\left(\frac{1-\varepsilon}{\varepsilon}\right)(n-1)\right]\right\}
\end{aligned}
\tag{4}
$$

and we can see that the expected search time scales like $O\left(\frac{2\sigma^2}{\mu^2}\log\frac{1}{\varepsilon}\right)$ in the asymptotic regime.

A practical procedure to approximate the optimal search policy can be described as a two-stage approach [8]:

- Stage 1: Search each cell with a small fixed time $t_1(\varepsilon)$ to update the posterior probability for each cell.
- Stage 2: Search the cells in the order of decreasing posterior probabilities for time $t_2(\varepsilon)$ and declare the finding of object whenever the log-likelihood ratio exceeds $\tau(\varepsilon)$.

The two-stage approach has only twice of the expected search time by the optimal policy in the asymptotic regime and the same scaling law independent of n.

In a discrete-time setting, one assumes that the searcher has to spend at least T seconds in any cell and then decides whether to look at this cell for another T seconds or search a different cell. In this case, the searcher will always choose the cell with the largest log-likelihood ratio at the decision time. The searcher will declare the finding of object whenever the log-likelihood ratio exceeds $\tau(\varepsilon)$. If at the end of a T second search on a cell, the searcher has to quantify its belief on whether the cell contains an object or not, then the problem becomes a sequential decision with quantized input — instead of the actual log-likelihood ratio, only two quantized values of 0 and 1 are allowed. The optimal search policy remains to be a greedy one by focusing on the cell with the highest cumulative score at each step [7]. The quantization rule at each step is assumed to be fixed, which leads to the false alarm probability α and miss probability β. Since $p_i(0)$ is usually fairly small for any cell i, the most informative quantization rule should operate at the condition

$$\frac{\alpha_i}{1-\beta_i} = \frac{p_i(0)}{1-p_i(0)} \tag{5}$$

for any cell i.

Next, we consider another asymptotic regime where the number of cells n becomes very large and the number of objects hiding among the n cells increases sublinearly according to n^{1-c} where $c \in (0,1)$ is a constant scaling factor. If the searcher distributes its effort equally among the n cells, we will have the following simplified observation model

$$Y_i \sim \mathcal{N}(\mu_i, 1), i = 1, ..., n$$

where $\mu_i = 0$ if cell i does not contain any object while $\mu_i = \mu > 0$ if cell i contains an object. Note that μ can be interpreted as the normalized signal-to-noise ratio (SNR). We assume that μ scales like $O\left(\sqrt{2r\log n}\right)$ where r depends on the scaling factor c on how the number of objects grows as the number of cells increases. For any search procedure to declare as many objects as possible and maintain the false discovery rate to grow in a lower scaling law, we need to use two performance metrics to characterize the desired asymptotic property. Define the false discovery proportion (FDP) to be the number of incorrectly declared objects relative to the total number of object declarations. The non-discovery proportion (NDP) is defined as the number of objects missed by the searcher relative to the total number of no-object declarations. A searcher is said to be asymptotically efficient if both FDP and NDP approach zero as $n \to \infty$. Intuitively, the normalized SNR has to be high enough for the searcher to design an efficient search policy. In fact, if $r < c$, no searcher can be made asymptotically efficient. On the other hand, if $r > c$, then a searcher using coordinate-wise thresholding rule to declare the object on each cell is asymptotically efficient [5]. The interesting case lies at the boundary $r = c$ where the design of optimal search policy is highly related to sparse signal recovery and compressed sensing [4].

Consider a normalized observation model for cell i that allows multiple looks

$$Y_i^{(j)} = \sqrt{\phi_i^{(j)}}\mu_i + N_i^{(j)}, i = 1, ..., n, j = 1, ..., k \tag{6}$$

where $N_i^{(j)} \sim \mathcal{N}(0,1)$ is the additive white Gaussian noise and $\phi_i^{(j)}$ is related to the signal-to-noise ratio that has been

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dedicated in the j-th sensing action to cell i. Without loss of generality, we impose the total energy constraint for the whole search effort given by

$$\sum_{i,j} \phi_i^{(j)} \le E \qquad (7)$$

Note that setting

$$\phi_i^{(j)} = \frac{1}{k}, i = 1,...,n, j = 1,...,k$$

is equivalent to a single look for each cell with $\phi_i = \frac{E}{n}$ owing to the independence of the noises in the multiple looks and the total energy constraint.

We consider a sequential search procedure that takes the advantage of the multiple looks in the spirit of the two-stage method. We apply a portion of the energy to crudely search all cells; eliminate a fraction of the cells that appear least promising from further consideration; and iterate this procedure for several times, at each step searching only those cells retained from the previous step. The algorithm runs in the following manner.

- **Input:** Number of total stages k and energy budget $E^{(j)}$ for stage j such that $\sum_{j=1}^{k} E^{(j)} \le E$.
- **Initialization:** Index set of cells to be searched $I^{(1)} = \{1, 2, ..., n\}$.
- **Adaptive Sensing:** At stage j, search cell i with equal effort $\frac{E^{(j)}}{|I^{(j)}|}$ if $i \in I^{(j)}$ and obtain the measurement $Y_i^{(j)}$. Update the index set by $i \in I^{(j+1)}$ if $Y_i^{(j)} > 0$ for all the cells being searched.
- **Output:** The final index set $I^{(k)}$ which is very likely to contain most of the objects.

In order to retain the signal component at each stage, we need to allocate a large portion of sensing energy to the first step. Due to the sparsity of the objects, most cells will be eliminated in the subsequent search stages with reduced energy. One possible energy allocation design is to exponentially decrease the energy allocated on each cell from one stage to the next. For example, given a design parameter $d \in (0, 1)$, we have

$$E^{(j)} = \begin{cases} \frac{dE}{2}\left(1-\frac{d}{2}\right)^{j-1}, & j = 1,...,k-1 \\ E\left(1-\frac{d}{2}\right)^{k-1}, & j = k \end{cases} \qquad (8)$$

that satisfies $\sum_{j=1}^{k} E^{(j)} = E$. In this case, when $r > c/(2 - d)^{k-1}$, the k-stage procedure guarantees that FDP and NDP approach zero with probability one as $n \to \infty$. Note that when $r \to c$, we need at least

$$k = O\left(\frac{\log(\log n)}{\log(2-d)}\right) \qquad (9)$$

stages to reliably identify the sparse locations of the objects. The proof follows the ideas presented in [5] and is omitted due to page limit.

III. ROBUST SEARCH IN DISCRETE SPACE WITH UNKNOWN OBJECT DISTRIBUTION

Consider searching an object in one of n discrete cells where the probability of finding the object in cell i within the search time t given that the object is in cell i is denoted by $b(i,t)$. Note that $b(i,t)$ is often called the detection function for cell i and satisfies

$$\frac{d^2 b(i,t)}{dt^2} < 0, \left.\frac{db(i,t)}{dt}\right|_{t=0} > 0, \left.\frac{db(i,t)}{dt}\right|_{t=\infty} = 0 \quad (10)$$

A popularly used non-detection function $q_i(t) = 1 - b(i,t)$ is exponential

$$q_i(t) = e^{-\eta_i t}, i = 1,...,n \qquad (11)$$

where η_i is an indicator factor measuring how effective a unit resource is in the i-th cell for detecting the object. If the total search time is T, then we are interested in how much effort in terms of search time t_i should be allocated to cell i in order to maximize the overall object detection probability. Of course this depends on the prior probability p_i that the object is in cell i. Given any probability distribution $\{p_i\}_{i=1}^n$, the optimal search strategy can be written as

$$J = \max_{\{t_i\}} \sum_{i=1}^{n} p_i b(i,t_i)$$

$$\text{subject to} \sum_{i=1}^{n} t_i \le T$$

and it has a unique solution for the case of $p_i > 0$ given by

$$t_i = c\frac{\log(p_i \eta_i)}{\eta_i}, i = 1,...,n \qquad (12)$$

where c is the normalizing constant given by

$$c = \frac{T}{\sum_{i=1}^{n} \log(p_i \eta_i)/\eta_i} \qquad (13)$$

However, when the true distribution $\{p_i\}$ is unknown while the searcher assumes a different distribution $\{\bar{p}_i\}$ to derive the optimal search procedure, the resulting detection probability becomes

$$\bar{J} = \sum_{i=1}^{n} p_i(1 - e^{-\eta_i \bar{t}_i}) \qquad (14)$$

where the search time for cell i is given by

$$\bar{t}_i = \frac{\log(\bar{p}_i \eta_i)/\eta_i}{\sum_{i=1}^{n} \log(\bar{p}_i \eta_i)/\eta_i} T \qquad (15)$$

It is clear that $\bar{J} \le J$. Without knowing $\{p_i\}$, one can choose $\{\bar{p}_i\}$ to minimize $J - \bar{J}$ for all possible distributions $\{p_i\}$. In the worst case, assuming that $\eta_1 = \eta_2 = \cdots = \eta_n = 1$, we have

$$J - \bar{J} = e^{-T/n} \sum_{i=1}^{n} \left[\left(\prod_{i=1}^{n} \bar{p}_i\right)^{1/n} p_i - \left(\prod_{i=1}^{n} p_i\right)^{1/n} \bar{p}_i \right] / \bar{p}_i \qquad (16)$$

The performance gap increases as n increases. This indicates that the robust solution can be significantly worse than the optimal solution when knowing $\{p_i\}$. Thus when the evasive

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object has certain level of intelligence to select its $\{p_i\}$, it will have the incentive to do so in order to minimize the detection probability of the searcher. We will discuss an alternative formulation of the search problem via search-allocation game in the next section.

IV. EVASIVE OBJECT SEARCH VIA SEARCH-ALLOCATION GAME

When objects can move among those cells being searched, the problem becomes a search allocation game where two players, a searcher and an evader, join the game. At the initial time, the searcher has a total energy constraint E. Using this total energy, the searcher has to allocate resources in the search space to detect the evader. The evader has an initial energy e_0. The evader can move in the search space under energy constraint as well as some other factors to be described next. The strategies and information sets of the players, the payoff function and the process of the game are as follows.

- At the beginning of time k, the searcher obtains the information about the evader's position, say, cell i, and his residual energy. At the same time, the evader is informed on the searcher's residual budget.
- Then the evader makes a decision to move from the cell i to its neighborhood cells $N(i)$ in a probabilistic manner. Specifically, he will spend $e(i,j)$ to move from cell i to cell j assuming that $e(i,j) > 0$ if $i \neq j$ and $e(i,i) = 0$.
- The searcher allocates his resources based on his hypothesized estimate of the cell that the evader moves to. However, this allocation has to take his residual energy into account.
- If the evader is in cell i and the x amount of resource is allocated there, then the searcher can detect the evader with probability $1 - q_i(x)$ where the non-detection function $q_i(x)$ is monotonously decreasing in x. Typically, we can model the miss probability by

$$q_i(x) = e^{-\eta_i x} \qquad (17)$$

where η_i is an indicator on how effective a unit resource is in the i-th cell for detecting the evader. When the searcher detects the evader, he receives payoff 1 and the evader loses the same amount. At this moment, the game is terminated.

- Unless the detection occurs at time k, the game will proceed to the next stage $k-1$ until it reaches $k = 0$.

The above formulation is clearly a multi-stage zero-sum stochastic game. A general stochastic game may be played forever; however, it terminates with probability one under the assumption that it has positive probability of termination at any stage and the value of the game is uniquely determined [10].

Let p_i be the probability that the evader chooses cell i for his hiding location. Let c_i be the budget that it costs to distribute a unit resource in cell i. Let η_i be the effectivity that unit resource has on the detection of evader in cell i. Let ϕ_i be the search resources allocated to cell i. Let ξ_i be

the value of the game in the state that evader is in cell i and the searcher allocates ϕ_i resource on it with the criterion of miss probability. In one-stage game, we have the following minimax problem.

$$\min_{\{\phi_i\}} \max_{\{p_i\}} \sum_{i \in \mathscr{A}} p_i \xi_i e^{-\eta_i \phi_i} \qquad (18)$$

subject to

$$p_i \geq 0, \sum_{i \in \mathscr{A}} p_i = 1 \qquad (19)$$

$$\phi_i \geq 0, \sum_{i \in \mathscr{A}} c_i \phi_i \leq E \qquad (20)$$

This minimax problem has a unique solution given by the following water-filling procedure.

$$\phi_i = \frac{1}{\eta_i} \left[\log \frac{\xi_i}{\rho} \right]^+ \qquad (21)$$

$$p_i = \begin{cases} \frac{c_i / \eta_i}{\sum_{j, \rho \leq \xi_j} c_j / \eta_j}, & \rho \leq \xi_i \\ 0, & \rho > \xi_i \end{cases} \qquad (22)$$

where $[x]^+ = \max\{x, 0\}$ and ρ is determined by the following equation

$$\sum_{i \in \mathscr{A}} \frac{c_i}{\eta_i} \left[\log \frac{\xi_i}{\rho} \right]^+ = E \qquad (23)$$

We can apply the above result of the single-stage game recursively to the multi-stage game and the solution becomes an iterative water-filling procedure. Note that the above game theoretic formulation assumes that the evader's position is exposed to the searcher at every stage, which is clearly a disadvantage to the evader. A more challenging problem would be that the evader's position is only revealed to the searcher at the initial time. Then it becomes a standard pursuit-evasion game where only long-term strategies for both players are meaningful in the analysis [6], [10].

V. SIMULATION STUDY

To demonstrate the effectiveness of the optimal strategy for the search-allocation game, we implemented the resource allocation method in an intelligence, surveillance, and reconnaissance (ISR) scenario where the searcher needs to find a moving object with prior information that it may hide near one of the bridges (Fig. 1a). We first divided the search space into 10×10 cells and assigned the relevant parameters based on the terrain feature and importance of each cell. Since η_i is an indicator on how effective a unit resource is in the i-th cell for detecting the evader, we assigned the value for each cell as shown in Fig. 1c. Note that on each cell, value 1 means the least effective; value 3 indicates the most effective (in this case, the cell is on the river), and 2 is an average effective level without complex urban buildings. The value c_i indicates the searcher's cost for allocating unit resource on the i-th cell. It will be 1 if cell i is on river and 2 when including urban environment with buildings. The searcher's cost to assign unit resource on each cell is shown in Fig. 1d. The game value ξ represents the importance of the i-th cell. Since the searcher's top level objective is to capture two bridges, the

a. terrain of the search area

b. the game value for each cell

c. effectivity of unit resource for each cell

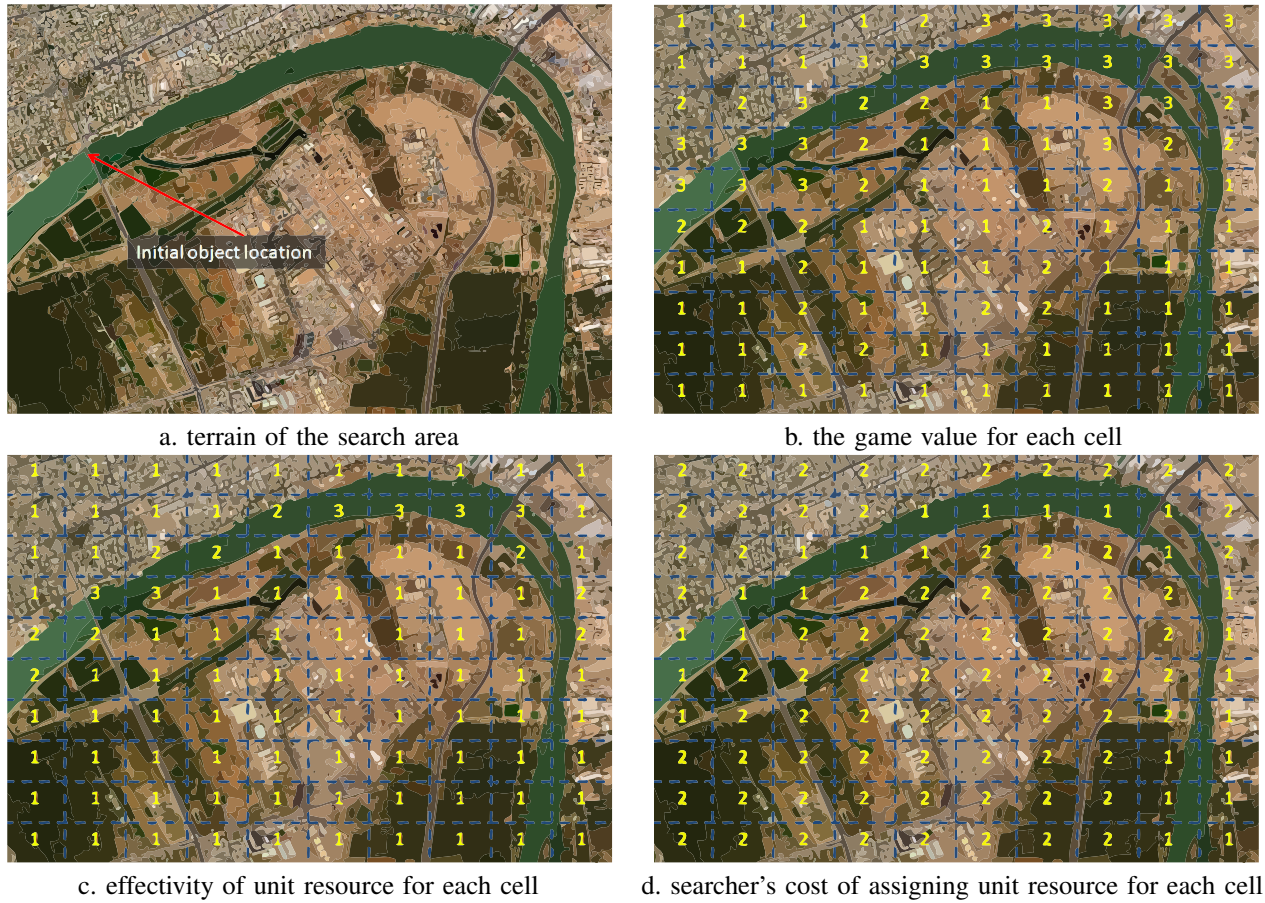d. searcher's cost of assigning unit resource for each cell

Fig. 1. An illustrative scenario for searching an evasive object

cells near two bridges have relatively larger values. The value for each cell is shown in Fig. 1b.

To evaluate the performance of the proposed search strategy (denoted by game search), we compare it with the random search method where cells are randomly selected. The probability of selecting each cell is based on its importance. At the end of each stage, the searcher will either declare the acquisition of the evader in a particular cell or continue allocating his energy to cells until running out of the budget. For a fixed budget E, we performed 1000 Monte Carlo simulations for each algorithm and estimated the detection probability where evasive object applies the optimal strategy to the minimax game in each stage. The estimated detection probabilities after playing 20 stages are shown in Fig. 2. It is expected that the detection probability will increase as the searcher has more budget to allocate to the search area. However, we observe some fluctuations in both curves using game search and random search due to inadequate Monte Carlo runs. Nevertheless, game search outperforms random search in all cases for E ranging from 5 to 50. Fig. 3 compares the detection probability at the end of each stage when the searcher has the budget $E = 5$. The miss probability using game search is 54.1%. This means that there is 54.1% chance that the searcher can not find the object after using all of the budget. Note that the miss probability is 65% using random search under the same condition. We

can also see that the probability of detecting the object in early stages using game search is usually much higher than that using random search. For different budget constraint, we summarized the comparative results for the first five stages in Table 1. It is clear that game search outperforms random search in the following two aspects: 1) it yields larger detection probability in each stage for the first five stages; and 2) it also has larger detection probability for extended stages so that the overall miss probability is smaller than that using random search. This is mainly due to the intelligent behavior of the evasive object: it has a tendency to hide to a cell where the searcher needs to allocate more resource in order to make a detection. This confirms the theoretical analysis that the searcher's expected game value can not increase in any stage by deviating from the optimal search strategy derived from the search-allocation game.

VI. DISCUSSION AND CONCLUSIONS

We considered the search problem in a sequential decision setting where the searcher has to determine which cell to perform the sensing action at any given time based on the measurements accumulated so far. For a single stationary object located in one of the n cells, the two-stage approach has close-to-optimal performance in terms of the minimum expected time to declare the object location with a given error rate. For acquiring a few objects in sparse locations,

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TABLE I

COMPARISONS OF DETECTION PROBABILITIES

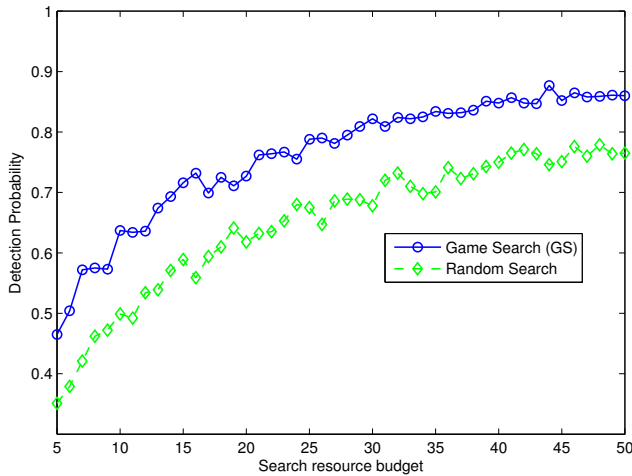| Search Budget	Search Algorithm	Detection Probabilities						
		Stage 1	Stage 2	Stage 3	stage 4	Stage 5	First 5	Total
$E = 5$	Random	6.5%	6.4%	4.7%	3.0%	1.7%	22.3%	34.4%
	Game	10.6%	11.5%	9.0%	10.0%	4.8%	45.9%	49.4%
$E = 10$	Random	9.5%	10.3%	6.5%	6.3%	3.5%	36.1%	51.6%
	Game	12.3%	14.3%	9.5%	9.3%	5.9%	51.3%	62.3%
$E = 20$	Random	10.7%	10.7%	8.5%	7.3%	5.6%	42.8%	60.3%
	Game	16.1%	15.5%	7.8%	6.6%	5.6%	51.6%	75.9%
$E = 40$	Random	11.6%	12.5%	10.5%	7.6%	7.5%	49.7%	75.4%
	Game	18.7%	15.6%	9.7%	8.5%	6.3%	58.8%	83.4%

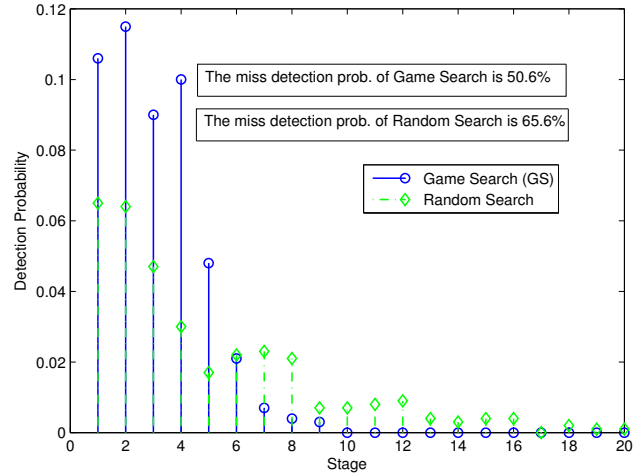Fig. 2. Comparison of detection probabilities with different resource constraints

Fig. 3. Comparison of detection probabilities at each stage when $E = 5$

the k-stage procedure ensures the false discovery proportion (FDP) and non-discovery proportion (NDP) approach zero with probability one in the asymptotic regime, which meets the best scaling law of object sparsity. When the object can hide from cell to cell during different stages of the search procedure, the search allocation game in the two-player zero-sum complete-information setting has a unique minimax solution corresponding to an iterative water-filling procedure by allocating the sensing effort to those cells with relatively larger detection probabilities. A simulated ISR example demonstrated the effectiveness of using the minimax solution to the search-allocation game for acquiring the evasive object. The minimax solution significantly outperforms the random search method in terms of the probability of detecting the evasive object in the repeated game.

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