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15. SUBJECT TERMS
Kalman filter, linear dynamic system, target tracking, false information injection, bias

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SYSTEM STATE ESTIMATION IN THE PRESENCE OF FALSE INFORMATION INJECTION

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ABSTRACT

The problem of system state estimation in the presence of an adversary is investigated for linear dynamic systems. It is assumed that the adversary injects additive false information into the sensor measurement. The impact of the false information on the Kalman filter's estimation performance is analyzed for a general dynamic system. To be concrete, a target tracking system has been used as an example. In such a system, if the false information is injected only once, the effect of the false information on the Kalman filter proves to be diminishing over time, even when the Kalman filter is unaware of the false information injection. The convergence rate as a function of the maneuvering index is analyzed. If the false information is repeatedly injected into the system, the induced estimation error proves to reach a finite steady state. Numerical examples are presented to support the theoretical results.

Index Terms— Kalman filter, linear dynamic system, target tracking, false information injection, bias

1. INTRODUCTION

System state estimation in the presence of adversary that injects false information into sensor readings is an important problem with wide application areas, such as target tracking with compromised sensor data, and secure monitoring of dynamic electric power systems. This topic has attracted considerable attention and interest recently [1–4]. In [1], the authors showed that in some cases, an adversary can introduce arbitrary errors in state estimates without being detected. The close relationship between these attacks and power system observability was discussed in [2], where both the adversary's attack strategies and the control center's attack detection algorithms have been proposed. False data attacks on electricity market have also been investigated in [3] and [4].

In this paper, for a linear dynamic system, we analyze the impact of the injected false information on the Kalman filter's state estimation performance over time, which has received little attention in literature. Some related publications exist, where the problem of sensor bias estimation and compensation for target tracking has been addressed. Interested readers are referred to [5] and references therein for details. Note that these previous works focus on bias estimation and compensation. Instead of estimating and removing the bias, in this paper we concentrate on analyzing the effects of the bias on state estimation performance over time. In particular, two cases are considered where false information is injected at a single time instant, and it is injected continuously into the system, respectively. Through theoretical derivations, we show that even under the assumption that the Kalman filter is unaware of the false information injection, the impact of a single false injection converges to zero asymptotically. Although it is known that the Kalman filter has a forgetting property, to the best of our knowledge, the effect of false information injection has not yet been rigorously investigated in literature, especially for tracking applications. The convergence rate will be characterized by both the eigenvalues and the determinant of the bias gain matrix, which will be derived later in the paper. In addition, the case of repeated false information injection is considered. It is shown that the Kalman filter cannot forget the false information if it is continuously injected. However, the extra estimation error due to injected false information does reach a finite steady state asymptotically.

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2. SYSTEM MODEL

Let us consider the following discrete-time linear dynamic system,

$$\mathbf{x}_{k+1} = \mathbf{F}_k \mathbf{x}_k + \mathbf{G}_k \mathbf{u}_k + \mathbf{v}_k \tag{1}$$

where $\mathbf{x}_k$ is the state vector, $\mathbf{u}_k$ is a known input vector, and $\mathbf{v}_k$ is the sequence of zero-mean white Gaussian process noise with covariance $E[\mathbf{v}_k \mathbf{v}_k^T] = \mathbf{Q}_k$. The measurement equation is

$$\mathbf{z}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{w}_k \tag{2}$$

with $\mathbf{w}_k$ the sequence of zero-mean white Gaussian measurement noise with covariance $E[\mathbf{w}_k \mathbf{w}_k^T] = \mathbf{R}_k$. The matrices $\mathbf{F}_k$, $\mathbf{G}_k$, $\mathbf{H}_k$, $\mathbf{Q}_k$, and $\mathbf{R}_k$ are assumed known. For such a linear and Gaussian dynamic system, the Kalman filter is the optimal state estimator.

In this paper, we assume that a bias $\mathbf{b}_k$ is intentionally injected by the adversary into the measurement. Therefore, the measurement equation (2) becomes

$$\mathbf{z}'_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{w}_k + \mathbf{b}_k = \mathbf{z}_k + \mathbf{b}_k \tag{3}$$

where $\mathbf{b}_k$ is either an unknown constant or a random variable (r.v.) independent of $\{\mathbf{v}_k\}$ and $\{\mathbf{w}_k\}$.

3. IMPACTS OF FALSE INFORMATION INJECTION

3.1. General Linear Dynamic Systems

In this paper, it is assumed that the Kalman filter is not aware of the presence of the false information (bias). We consider two cases. In the first case, the false information $\mathbf{b}$ is only injected once into the system at time K. We call this particular case single false information injection. In the second case, the false information is continuously injected into the system at and after time K and we name this case continuous false information injection.

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3.1.1. Single False Information Injection

In the single injection case, the extra state estimation error due to the single bias injection is derived and provided in the following lemma.

Lemma 1. *The extra state estimation error at time $K+N$ due to the single bias $\mathbf{b}$ injected at time K is $\left(\prod_{i=0}^{N-1}\mathbf{B}_{K+N-i}\right)\mathbf{W}_K\mathbf{b}$, where*

$$\mathbf{B}_k \triangleq (\mathbf{I}-\mathbf{W}_k\mathbf{H}_k)\,\mathbf{F}_{k-1} \tag{4}$$

and $\mathbf{W}_K$ is the Kalman filter gain [6] at time K.

Proof Sketches: At time K, the updated state estimate $\hat{\mathbf{x}}_{K|K}$ is

$$\hat{\mathbf{x}}_{K|K} = \hat{\mathbf{x}}_{K|K-1} + \mathbf{W}_K(\mathbf{z}_K - \hat{\mathbf{z}}_{K|K-1}) \tag{5}$$

where $\hat{\mathbf{x}}_{K|K-1}$ and $\hat{\mathbf{z}}_{K|K-1}$ are the predicted state estimate and predicted measurement respectively. Now, in the presence of the additive bias, the updated state estimate $\hat{\mathbf{x}}'_{K|K}$ is

$$\hat{\mathbf{x}}'_{K|K} = \hat{\mathbf{x}}_{K|K} + \mathbf{W}_K\mathbf{b} \tag{6}$$

At time $K+1$, the predicted state estimate is

$$\hat{\mathbf{x}}'_{K+1|K} = \mathbf{F}_K\hat{\mathbf{x}}'_{K|K} + \mathbf{G}_K\mathbf{u}_K = \hat{\mathbf{x}}_{K+1|K} + \mathbf{F}_K\mathbf{W}_K\mathbf{b} \tag{7}$$

and the predicted measurement is

$$\hat{\mathbf{z}}'_{K+1|K} = \mathbf{H}_{K+1}\hat{\mathbf{x}}'_{K+1|K} = \hat{\mathbf{z}}_{K+1|K} + \mathbf{H}_{K+1}\mathbf{F}_K\mathbf{W}_K\mathbf{b} \tag{8}$$

Combining (7) and (8), we have

$$\begin{aligned}\hat{\mathbf{x}}'_{K+1|K+1} &= \hat{\mathbf{x}}'_{K+1|K} + \mathbf{W}_{K+1}(\mathbf{z}_{K+1} - \hat{\mathbf{z}}'_{K+1|K}) \\ &= \hat{\mathbf{x}}_{K+1|K+1} + (\mathbf{I}-\mathbf{W}_{K+1}\mathbf{H}_{K+1})\mathbf{F}_K\mathbf{W}_K\mathbf{b}\end{aligned} \tag{9}$$

Repeating this process for N times, finally we have

$$\hat{\mathbf{x}}'_{K+N|K+N} = \hat{\mathbf{x}}_{K+N|K+N} + \left(\prod_{i=0}^{N-1}\mathbf{B}_{K+N-i}\right)\mathbf{W}_K\mathbf{b} \tag{10}$$

Q.E.D.

From Lemma 1 and its proof, it can be shown that the evolution of the extra estimation error $\boldsymbol{\nu}_k$ due to the bias injected to the system at time K can be modeled by a linear equation:

$$\boldsymbol{\nu}_{k+1} = \mathbf{B}_k\boldsymbol{\nu}_k \quad k = K,\ K+1,\cdots \tag{11}$$

where $\mathbf{B}_k$ is the bias gain matrix defined in (4) and $\boldsymbol{\nu}_K = \mathbf{W}_K\mathbf{b}$ is the initial condition for the system. It is important to study the stability of the linear system defined in (11).

3.1.2. Continuous False Information Injection

In the continuous injection case, the extra estimation error at a particular time instant $K+N$ due to the bias is provided in the following lemma. Its proof is similar to that of Lemma 1, but is more involved, which is skipped here for brevity.

Lemma 2. *The extra state estimation error at time $K+N$ due to bias $\mathbf{b}_k$ injected at and after time K is*

$$\sum_{m=0}^{N}\left(\prod_{i=0}^{m-1}\mathbf{B}_{K+N-i}\right)\mathbf{W}_{K+N-m}\mathbf{b}_{K+N-m} \tag{12}$$

where $\mathbf{B}_{K+N-i}$ is defined in (4), and $\prod_{i=0}^{-1}\mathbf{B}_{K+N-i} = \mathbf{I}$ is an identity matrix.

3.2. Linear Time Invariant Systems

In general, it is difficult to analyze the stability of a time-varying system. In this subsection, we focus on linear time invariant (LTI) systems with constant $\mathbf{F}$, $\mathbf{H}$, $\mathbf{Q}$, and $\mathbf{R}$ matrices. To further simplify the problem, we assume that the Kalman filter has reached its steady state at time K, so that its gain becomes a constant too, and $\mathbf{W}_k = \mathbf{W}, \forall k \geq K$.

Given these assumptions, in the single injection case, the system defined in (11) becomes a LTI system, and it is well known that the system is asymptotically stable if all the eigenvalues of $\mathbf{B} = (\mathbf{I}-\mathbf{WH})\,\mathbf{F}$ have a modulus smaller than one. In an asymptotically stable system, for an arbitrary initial condition $\mathbf{Wb}$, as $k \to \infty$, $\boldsymbol{\nu}_k \to 0$. In the continuous injection case, we study a case where the false information $\mathbf{b}$ is repeatedly injected at and after time K. It is important to investigate the asymptotic behavior of the extra estimation error $\left(\sum_{m=0}^{N}\mathbf{B}^m\right)\mathbf{Wb}$, as $N \to \infty$. To be concrete, in the next section, we study a target tracking example.

4. TARGET TRACKING EXAMPLE

4.1. Direct Discrete-Time Kinematic Models

We assume that the target moves in a 1-dimensional space according to a discrete white noise acceleration model [6], which can still be modeled by the plant and measurement equations given in (1) and (2). In such a system, the state is defined as $\mathbf{x_k} = [\xi_k \ \ \dot{\xi}_k]^T$, where ξ_k and $\dot{\xi}_k$ denote the target's position and velocity at time k respectively. The input $\mathbf{u}_k$ is a zero sequence. The state transition matrix is

$$\mathbf{F} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \tag{13}$$

where T is the time between measurements. The process noise is $\mathbf{v}_k = \boldsymbol{\Gamma}v_k$, where v_k is a zero mean white acceleration sequence, with variance σ_v^2, and the vector gain multiplying the scalar process noise is given by $\boldsymbol{\Gamma}^T = \left[T^2/2 \ \ T\right]$. The covariance matrix of the process noise is therefore $\mathbf{Q} = \sigma_v^2\boldsymbol{\Gamma}\boldsymbol{\Gamma}^T$. It is assumed that only position measurements are available, so that $\mathbf{H} = [1 \ \ 0]$. The measurement noise process is zero mean white with variance, σ_w^2.

4.2. Analytical Results for Single Injection Case

Here we assume that the bias is injected only once into $\mathbf{z}_K$ by an adversary at time K when the Kalman filter has already reached its steady state. It is shown theoretically in this paper that for such a tracking system, the impact of the bias on the Kalman filter is diminishing over time. Mathematically, this result is summarized in the following theorem.

Theorem 1. *The two eigenvalues (κ_1 and κ_2) of the bias gain matrix $\mathbf{B}$ are inside the open unit disk.*

Proof Sketches: The steady state Kalman filter gain for such a system has been derived [6], which is $\mathbf{W} = [\alpha \ \ \beta/T]^T$, where

$$\alpha = -\frac{1}{8}\left(\lambda^2 + 8\lambda - (\lambda+4)\sqrt{\lambda^2+8\lambda}\right) \tag{14}$$

$$\beta = \frac{1}{4}\left(\lambda^2 + 4\lambda - \lambda\sqrt{\lambda^2+8\lambda}\right). \tag{15}$$

$\lambda \triangleq \sigma_v T^2/\sigma_w$ is the target maneuvering index, which is proportional to the ratio of the motion uncertainty and the observation uncertainty. It is easy to show that

$$\begin{aligned} \mathbf{B} &= (\mathbf{I} - \mathbf{W}\mathbf{H})\mathbf{F} \\ &= \begin{bmatrix} 1-\alpha & (1-\alpha)T \\ -\frac{\beta}{T} & 1-\beta \end{bmatrix} \end{aligned} \tag{16}$$

and that matrix $\mathbf{B}$ has two eigenvalues

$$\kappa_1,\ \kappa_2 = \frac{2-\alpha-\beta \pm \sqrt{(\alpha+\beta)^2 - 4\beta}}{2} \tag{17}$$

We consider two cases where $\lambda < 8$ and $\lambda > 8$, respectively. In both cases, we can show that $|\kappa_1| < 1$ and $|\kappa_2| < 1$.

Q.E.D.

From Theorem 1 and its proof, we can see that the bias has a diminishing impact on the tracking performance of the Kalman filter, which asymptotically converges to zero, since all the eigenvalues of $\mathbf{B}$ are inside the open unit disc. Further, it is useful to investigate the relationship between the eigenvalues of $\mathbf{B}$ and the target maneuvering index λ. In a stable LTI system, it is the eigenvalue with the largest modulus that determines the rate of convergence. The relationship between the largest eigenvalue magnitude and λ is provided in the following theorem, whose proof is skipped for brevity.

Theorem 2. *The largest eigenvalue modulus of the bias gain matrix* $\mathbf{B}$ *is a monotonically decreasing function of* λ *when* $\lambda < 8$*; otherwise, it is a monotonically increasing function.*

Using results obtained in the proofs of Theorems 1 and 2, it is easy to show the following theorem.

Theorem 3. $\det(\mathbf{B}) = 1 - \alpha$ *and* $\det(\mathbf{B}^N) = (1-\alpha)^N$, *both of which are monotonically decreasing functions of* λ.

Theorem 2 shows that if the target maneuvering index is smaller than 8, then a larger λ leads to faster convergence; otherwise when $\lambda > 8$, a larger λ leads to slower convergence. If we consider the determinant of $\mathbf{B}$, Theorem 3 shows that $\det(\mathbf{B}^N)$ decays exponentially, with a faster convergence when λ increases.

4.3. Analytical Results for Continuous Injection Case

In this subsection, let us assume that a constant bias $\mathbf{b}$ is repeatedly injected into the system at and after time K. To analyze the asymptotic behavior of the extra estimation error due to the continuous bias injection, we need to investigate $\sum_{i=0}^{N} \mathbf{B}^i$. First, let us denote $\mathbf{Q}$ as a square matrix whose columns are the eigenvectors of $\mathbf{B}$. Then through eigenvalue decomposition, and using properties of the eigenvalues derived in Subsection 4.2, we give the following theorem without proof.

Theorem 4. *As* $N \to \infty$,

$$\sum_{i=0}^{\infty} \mathbf{B}^i = \mathbf{Q} \begin{bmatrix} \frac{1}{1-\kappa_1} & 0 \\ 0 & \frac{1}{1-\kappa_2} \end{bmatrix} \mathbf{Q}^{-1} \tag{18}$$

and

$$\det\left(\sum_{i=0}^{\infty} \mathbf{B}^i\right) = \frac{1}{\beta} \tag{19}$$

Theorem 4 indicates that as $N \to \infty$, the extra state estimation error due to the repeated injection of the false information $\mathbf{b}$ reaches a steady state,

$$\mathbf{Q} \begin{bmatrix} \frac{1}{1-\kappa_1} & 0 \\ 0 & \frac{1}{1-\kappa_2} \end{bmatrix} \mathbf{Q}^{-1} \mathbf{W}_K \mathbf{b} \tag{20}$$

Since it can be shown that β is a monotonically increasing function of λ, as λ increases, $\det\left(\sum_{i=0}^{\infty} \mathbf{B}^i\right)$ decreases.

5. NUMERICAL RESULTS

In this section, numerical results are presented for both the single injection case and the continuous injection case.

5.1. Single False Information Injection

The parameters used in the simulations are $T = 1$ and $\sigma_w^2 = 1$. The injected bias b is taken as a zero mean Gaussian r.v. with variance σ_b^2. The effect of the bias injection on the Kalman filter is measured by the time required for the MSE to become consistent with the nominal covariance matrix once again, using a chi-squared test. The sum of the normalized MSE over N_m Monte-Carlo runs is given by

$$q_k = \sum_{i=1}^{N_m} \left[\hat{\mathbf{x}}_{k|k}^i - \mathbf{x}_k^i\right]^T \mathbf{P}_{k|k}^{-1} \left[\hat{\mathbf{x}}_{k|k}^i - \mathbf{x}_k^i\right] \tag{21}$$

where at time k, $\mathbf{P}_{k|k}$ is the state covariance matrix, and $\hat{\mathbf{x}}_{k|k}^i$ is the state estimate, and $\mathbf{x}_k^i$ is the true state, during the ith Monte-Carlo run. It could be shown that q_k is a chi-squared r.v. with $2N_m$ degrees of freedom, in the absence of bias. The chi-squared test compares q_k with a threshold η. When $q_k < \eta$, it is considered that the MSE of $\hat{\mathbf{x}}_{k|k}$ is consistent with $\mathbf{P}_{k|k}$. η is determined by solving the following equation, $\Pr\{q_k \leq \eta\} = 1 - \epsilon$, where ϵ is a small number. Here we run $N_m = 1000$ Monte-Carlo runs, and set $\epsilon = 10^{-4}$. Correspondingly, the threshold used in the test is $\eta = 2243.81$.

An example is shown in Fig. 1, where a single random bias is injected at time $K = 50$. It can be seen that the impact of the bias only lasts for several time steps and then becomes negligible. The time required to return to the normal state is shown in Table 1

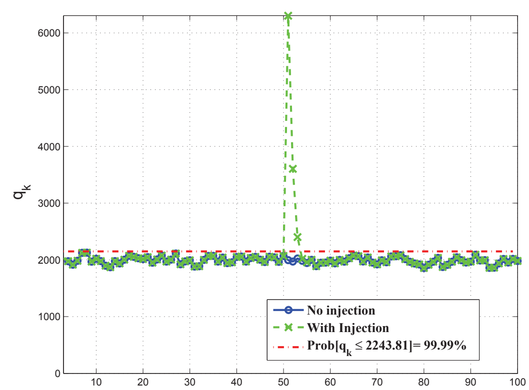

Fig. 1. Single Injection Case: $\sigma_b^2 = 5$, $\sigma_v^2 = 2$.

for varying λ and σ_b^2. It is clear that when $\lambda < 8$, as λ increases, the time to return to the normal state decreases; when $\lambda > 8$, as

	σ_b^2						
λ	1	5	10	20	50	100	1000
2	2	3	3	3	3	3	5
4	2	2	2	2	2	3	4
6	2	2	2	3	3	3	3
8	2	2	3	3	3	4	4
10	2	2	3	3	4	4	5
12	2	3	3	4	5	4	6
14	2	3	3	4	5	5	7

Table 1. Time required to return to normal state

λ increases, the time to return to the normal state increases. This is exactly as predicted by Theorem 2. Also, we can observe that a random bias with a larger variance σ_b^2 has a longer-lasting impact on the Kalman filter. But eventually, the filter will return to the normal state.

5.2. Continuous False Information Injection

In this subsection, it is assumed that a constant false information $\mathbf{b}$ is repeatedly injected in the measurement at and after time $K = 50$. The normalized MSE q_k is plotted as a function of time k in Fig. 2. We observe that the estimation error due to the repeated injected bias asymptotically reaches a finite steady state as shown in Theorem 4. A similar phenomenon is observed for the case where an independent and identically distributed (i.i.d.) random bias sequence is continuously injected to the system. The corresponding numerical example is skipped due to limited space. Note that for the random bias sequence case, the theoretical result is very difficult to obtain.

Fig. 3 shows the normalized MSE at time $k = 200$ for different b and λ values. It is clear that as λ increases, the MSE decreases, as indicated in Theorem 4. Further, we observe that the larger the bias b is, the larger the steady state value will be.

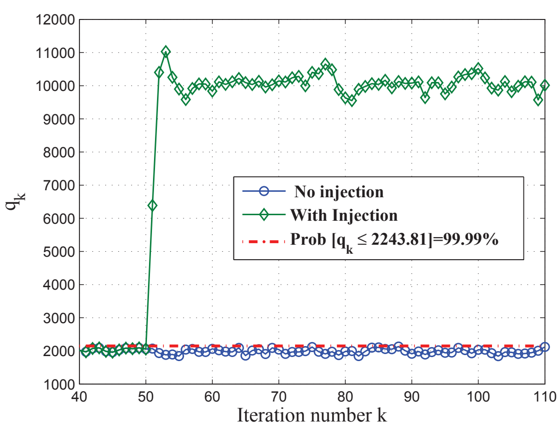

Fig. 2. Continuous Injection Case: $b^2 = 5$ and $\sigma_v^2 = 4$.

6. CONCLUSIONS

In this paper, for a linear system, we have studied the problem of state estimation in the presence of additive false information, which is injected by an adversary into the sensor measurement. For a general linear system, the impact of the false information on the Kalman filter's estimation performance over time was derived. Further, to be concrete, an example target tracking system has been provided, where the effect of the false information on the Kalman filter was investigated. In the case of single false information injection, it has been shown that the system that models the bias evolution is stable, and the impact of the false information decays exponentially. The convergence rate was also analyzed in terms of both the largest eigenvalue modulus and the determinant of the bias gain matrix. In the case of continuous constant false information injection, the finite steady state of the extra estimation error due to bias injection has been derived, and its relationship with the target maneuvering index was investigated.

In the future, the more challenging case where the adversary injects coordinated random false information over multiple sensors and over time will be explored.

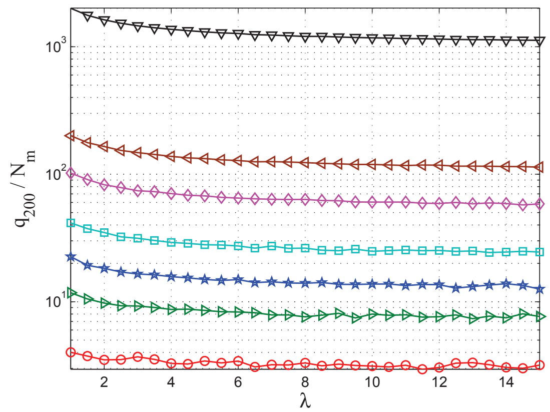

Fig. 3. Continuous Injection Case: The curves from top to bottom correspond to: $b^2 = 1000$, 100, 50, 20, 10, 5, 1.

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