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Tracking with Converted Position and Doppler Measurements*

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In many target tracking applications, estimation of target position and velocity is performed in Cartesian coordinates. Use of Cartesian coordinates for estimation stands in contrast to the measurements, which are traditionally the range, azimuth and elevation measurements of the spherical coordinate system. It has been shown in previous works that the classical nonlinear transformation from spherical to Cartesian coordinates introduces a bias in the position measurement. Various means to negate this bias have been proposed. In many active sonar and radar applications, the sensor also provides a Doppler, or equivalently range rate, measurement. Use of Doppler in the estimation process has also been proposed by various authors. First, the previously proposed unbiased conversions are evaluated in dynamic situations, where the performance of the tracking filter is affected by the correlation between the filter gains and the errors in the converted position measurements. Following this, the "decorrelated unbiased converted measurement" approach is presented and shown to be superior to the previous approaches. Second, an unbiased conversion is derived for Doppler measurements from a moving platform.

Keywords: Kalman filter, converted measurement, unbiased, target tracking, nonlinear, range rate, Doppler

1. INTRODUCTION

In some estimation problems, it is advantageous to perform tracking in a different coordinate system than the measurements. In these cases, the measurements require some form of conversion prior to use in tracking. If this conversion process results in a biased converted measurement, degradation in tracking performance occurs. Use of unbiased measurement conversion is preferred for these cases. In this paper previous work on unbiased converted measurements is evaluated and expanded for improved performance. The resulting conversion technique is then applied to two estimation problems, (1) the Converted Measurement Kalman Filter (CMKF) and (2) range rate estimation from a moving platform. Measurement conversion bias and estimation bias are examined in Section 2 along with evaluation of conversion techniques. Section 3 evaluates the impact of the conversion techniques on tracking performance in a CMKF. Section 4 develops an unbiased range rate measurement conversion, and evaluates the result in a range rate estimator.

1.1 Converted Measurement Kalman Filter (CMKF)

The Converted Measurement Kalman Filter (CMKF) is a technique that converts polar or spherical measurements to Cartesian coordinates for tracking. Measurements of range, r, and bearing, α, are related to the (relative) Cartesian coordinates according to:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} r\cos\alpha \\ r\sin\alpha \end{bmatrix} \tag{1}$$

The conversion to Cartesian coordinates has the advantage of allowing for the use of a linear Kalman Filter. A disadvantage is that the conversion process is biased [5, 6], violating a premise of the Kalman filter that the measurement errors are unbiased. A second concern is that the converted measurement noise variance is a function of the true range and bearing to the target. Since this is not known in practice, it is approximated using the measured range and bearing. It has been identified that this technique leads to correlation between the measurement error and the Kalman gain, resulting in an estimation bias [7, 3].

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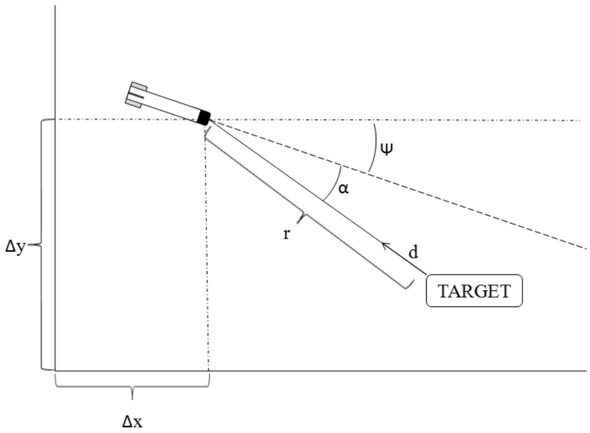

Figure 1. Illustration of target tracking from a moving platform. The seeker has a known position and heading in the Cartesian tracking coordinate system. Relative range, r, and bearing, α, to the target are measured using active sonar or radar, converted to relative position, and translated to the tracking coordinate system using the seeker position, Δx and Δy, and heading, ψ.

1.2 Range rate estimation from a moving platform

Estimation of range rate using active sonar or radar by taking advantage of the Doppler effect can be advantageous for improved target state estimation, track association and for discriminating targets from stationary clutter. When estimating range rate from a moving platform it is necessary to nullify the effect of the platform speed. This nullification process suffers from a similar bias problem as the position measurement conversion.

Figure 1 provides an illustration of estimation from a moving platform. Measurements of relative range, r, and bearing, α, are related to the relative Cartesian coordinates as in (1). The relative position is translated into a reference coordinate system for tracking using the seeker position and heading, ψ, both of which are assumed known.

$$\begin{bmatrix} x_{ref} \\ y_{ref} \end{bmatrix} = \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} \tag{2}$$

Also available is the additional measurement of range rate through the Doppler effect using the known transmit frequency, f_t, speed of the transmitted signal, c, and receive frequency, f_r. The Doppler imparted by the velocity of the own ship must be nullified based its known speed, V, and estimated bearing α.

$$d = \frac{(f_r - f_t)\,c}{2f_t} - V\cos\alpha \tag{3}$$

2. MEASUREMENT CONVERSION TECHNIQUES

Common to the polar to Cartesian conversion and the Doppler nullification equations is a trigonometric function of a random variable. An analysis of polar to Cartesian conversion is therefore instructive for both the CMKF and for range rate estimation. There are various challenges in the conversion process. One challenge is that the conventional conversion process (1) is biased. A second is that the estimation of the converted measurement error covariance requires the true range and bearing, unavailable in practice. The third is that the correlation between this covariance estimate and the measurement error results in an estimation bias when the converted measurement is used in tracking.

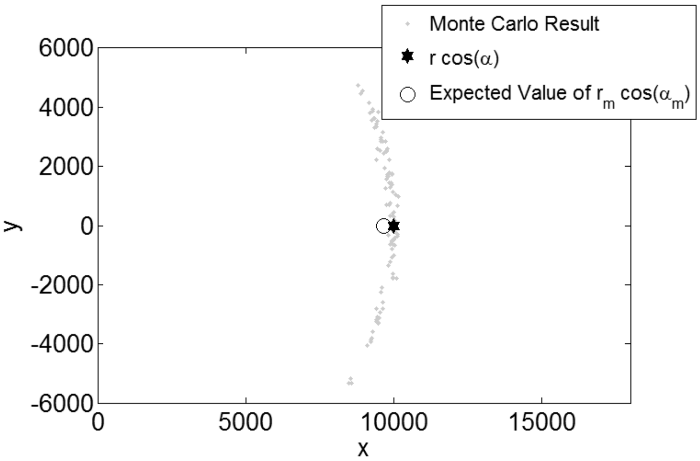

Figure 2. Illustration of bias imparted by a trigonometric function of a random variable from 100 Monte-Carlo runs using a range of 10,000 and bearing of 0°, with the sensor at the origin. The range and bearing measurement noises used were uncorrelated Gaussian with $\sigma_r = 100$ and $\sigma_\alpha = 15°$ respectively.

2.1 Evaluation of the conversion bias

The conventional conversion (1) introduces a bias in the mean of the converted measurement [6]. Figure 2 shows this bias graphically using Monte-Carlo simulation. The bias can be found by taking the expectation of the converted range and bearing measurements [6], r_m and α_m. Assume the range measurement noise, w_r, and bearing measurement noise, w_α, are uncorrelated, zero mean, and Gaussian with standard deviations of σ_r and σ_α respectively; the bias is as follows:

$$E\left[r_m \cos\alpha_m\right] = E\left[(r + w_r)\cos\left(\alpha + w_\alpha\right)\right] = e^{-\sigma_\alpha^2/2} r\cos\alpha \tag{4}$$

$$E\left[r_m \sin\alpha_m\right] = E\left[(r + w_r)\sin\left(\alpha + w_\alpha\right)\right] = e^{-\sigma_\alpha^2/2} r\sin\alpha \tag{5}$$

As seen in Figure 2, and confirmed by (4) and (5), the bias is along the true bearing to the target with a magnitude of $r\left(e^{\sigma_\alpha^2/2} - 1\right)$. To compensate for the bias, previous authors proposed an Unbiased Converted Measurement (UCM) [6]:

$$x_m^{\mathrm{UCM}} = e^{\sigma_\alpha^2/2} r_m \cos\alpha_m \tag{6}$$

$$y_m^{\mathrm{UCM}} = e^{\sigma_\alpha^2/2} r_m \sin\alpha_m \tag{7}$$

Monte-Carlo simulation verifies that the UCM is unbiased for all bearing angles. However, due to the fact that the distribution of the converted measurement is non-Gaussian, the UCM may not be the maximum likelihood estimate. Figure 3 shows the results of Monte-Carlo simulation for various bearing angles.

2.2 Estimation of the covariance

The true measurement covariance of the UCM is given by:

$$R_{true}^{11} = \frac{1}{2}\left(r^2 + \sigma_r^2\right)\left[1 + \cos(2\alpha)\exp\left(-2\sigma_\alpha^2\right)\right]\exp\left(\sigma_\alpha^2\right) - r^2\cos^2\alpha \tag{8}$$

$$R_{true}^{22} = \frac{1}{2}\left(r^2 + \sigma_r^2\right)\left[1 - \cos(2\alpha)\exp\left(-2\sigma_\alpha^2\right)\right]\exp\left(\sigma_\alpha^2\right) - r^2\sin^2\alpha \tag{9}$$

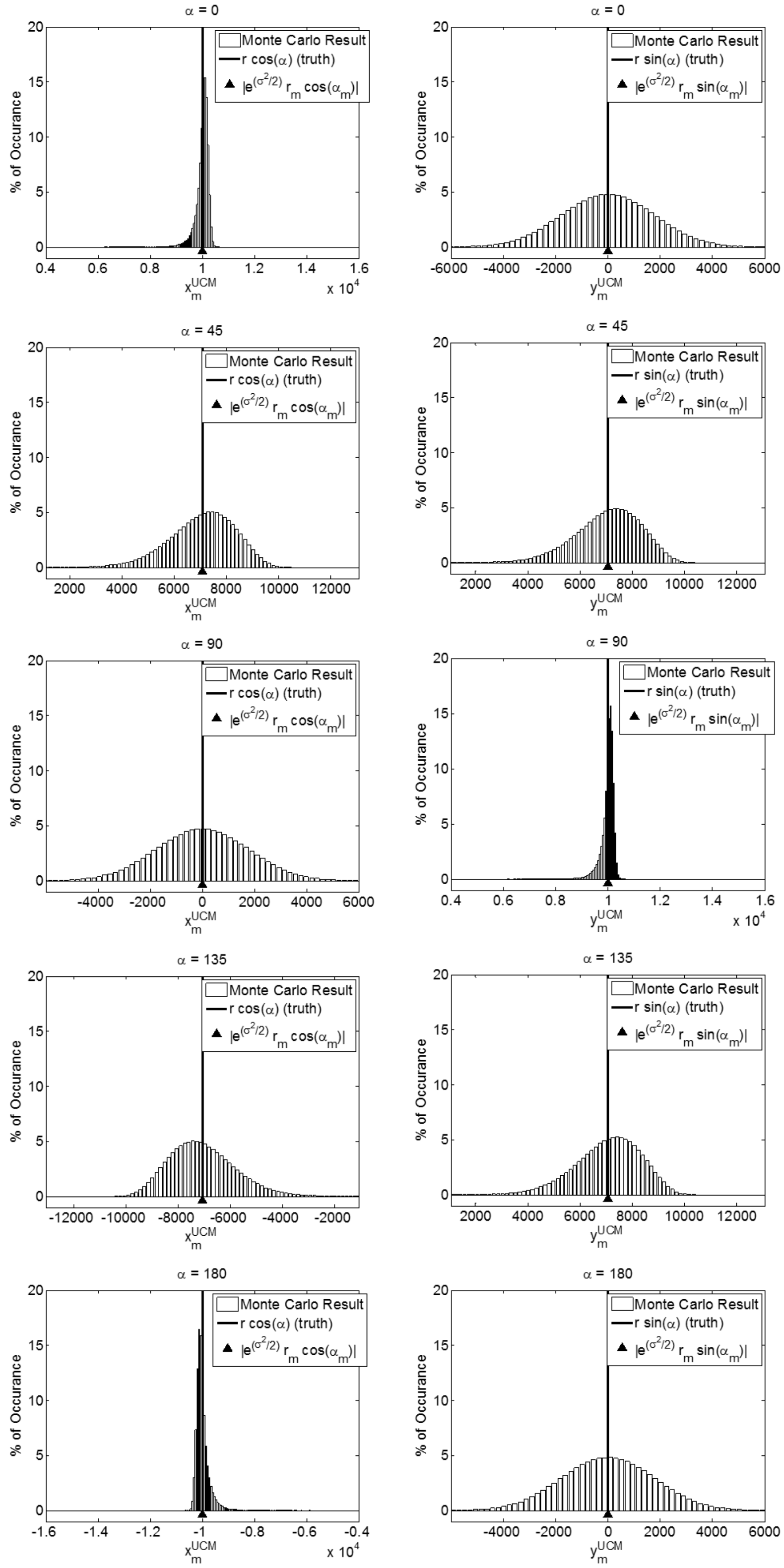

Figure 3. Histogram of 10^6 Monte-Carlo runs using a range of 10,000 and bearing of 0° to 180°. The range and bearing measurement noises used were uncorrelated Gaussian with $\sigma_r = 100$ and $\sigma_\alpha = 10^\circ$ respectively. The mean value of the Monte-Carlo result and the ground truth are plotted for comparison. In all cases, the UCM was unbiased.

$$R_{true}^{12} = \frac{1}{2}\left(r^2+\sigma_r^2\right)\left[\sin(2\alpha)\exp\left(-2\sigma_\alpha^2\right)\right]\exp\left(\sigma_\alpha^2\right) - r^2\cos\left(\alpha\right)\sin\left(\alpha\right) \tag{10}$$

Since the true covariance requires the true range and bearing, it cannot be calculated in practice. Two approaches have been proposed to approximate the covariance using the measurements. The UCM approach evaluates the covariance at the measurements [6], namely,

$$R_{\text{UCM}}^{11} = \frac{1}{2}\left(r_m^2+\sigma_r^2\right)\left[1+\cos(2\alpha_m)\exp\left(-2\sigma_\alpha^2\right)\right] + \left[\exp\left(\sigma_\alpha^2\right)-2\right]r_m^2\cos^2\alpha_m \tag{11}$$

$$R_{\text{UCM}}^{22} = \frac{1}{2}\left(r_m^2+\sigma_r^2\right)\left[1-\cos(2\alpha_m)\exp\left(-2\sigma_\alpha^2\right)\right] + \left[\exp\left(\sigma_\alpha^2\right)-2\right]r_m^2\sin^2\alpha_m \tag{12}$$

$$R_{\text{UCM}}^{12} = \frac{1}{2}\left(r_m^2+\sigma_r^2\right)\left[\sin(2\alpha_m)\exp\left(-2\sigma_\alpha^2\right)\right] + \left[\exp\left(\sigma_\alpha^2\right)-2\right]r_m^2\cos\alpha_m\sin\alpha_m \tag{13}$$

It can be seen that the measurement conversion (6) and (7) is derived by conditioning on the true range and bearing, while the error covariance (11) - (13) is derived by conditioning on the measurements. This incompatibility was pointed out by previous authors along with a modified unbiased conversion method [2]. The resulting Modified Unbiased Converted Measurement (MUCM), shown below, resolves the incompatibility, but results in a biased estimate.

$$x_m^{\text{MUCM}} = e^{-\sigma_\alpha^2/2} r_m\cos\alpha_m \tag{14}$$

$$y_m^{\text{MUCM}} = e^{-\sigma_\alpha^2/2} r_m\sin\alpha_m \tag{15}$$

$$R_{\text{MUCM}}^{11} = \frac{1}{2}\left(r_m^2+\sigma_r^2\right)\left[1+\cos(2\alpha_m)\exp\left(-2\sigma_\alpha^2\right)\right] - \exp\left(-\sigma_\alpha^2\right)r_m^2\cos^2\alpha_m \tag{16}$$

$$R_{\text{MUCM}}^{22} = \frac{1}{2}\left(r_m^2+\sigma_r^2\right)\left[1-\cos(2\alpha_m)\exp\left(-2\sigma_\alpha^2\right)\right] - \exp\left(-\sigma_\alpha^2\right)r_m^2\sin^2\alpha_m \tag{17}$$

$$R_{\text{MUCM}}^{12} = \frac{1}{2}\left(r_m^2+\sigma_r^2\right)\left[\sin(2\alpha_m)\exp\left(-2\sigma_\alpha^2\right)\right] - \exp\left(-\sigma_\alpha^2\right)r_m^2\cos\alpha_m\sin\alpha_m \tag{18}$$

A comparison of the UCM and MUCM conversion is instructive. The advantage of the UCM conversion is that it is unbiased, an essential attribute in state estimation. The MUCM conversion, however, results in a lower mean squared error. Both conversion techniques use a multiplicative term. The multiplicative term that results in the smallest expected square error can be derived using a factor η as follows:

$$x_m^{\text{MMSE}} = \eta r_m\cos\left(\alpha_m\right) \tag{19}$$

$$y_m^{\text{MMSE}} = \eta r_m\sin\left(\alpha_m\right) \tag{20}$$

The expected squared error

$$E\left[\left[\eta(r+w_r)\cos\left(\alpha+w_\alpha\right)-(r)\cos\left(\alpha\right)\right]^2 + \left[\eta(r+w_r)\sin\left(\alpha+w_\alpha\right)-(r)\sin\left(\alpha\right)\right]^2\right] \tag{21}$$

is then

$$\eta^2(r+\sigma_r^2) - 2\eta r^2 e^{-\sigma_\alpha^2/2} + r^2 \tag{22}$$

The minimizing term, shown below, requires knowledge of the true range. This term is bounded by the MUCM scaling term. Therefore, the mean square error of the MUCM conversion is always less than that of the UCM conversion.

$$\eta = \frac{r^2}{r^2+\sigma_r^2}e^{-\sigma_\alpha^2/2} \tag{23}$$

Both UCM and MUCM approaches are statistically consistent, based on the evaluation of the Normalized Error Squared (NES). The individual components, however, are inconsistent in certain geometries. In particular, the covariance estimate along the true bearing is overestimated, resulting in an NES less than one. Figure 4 shows the total NES and the individual components' NES. The overestimation of the covariance is exhibited for small angles in the x component and near $90°$ in the y component.

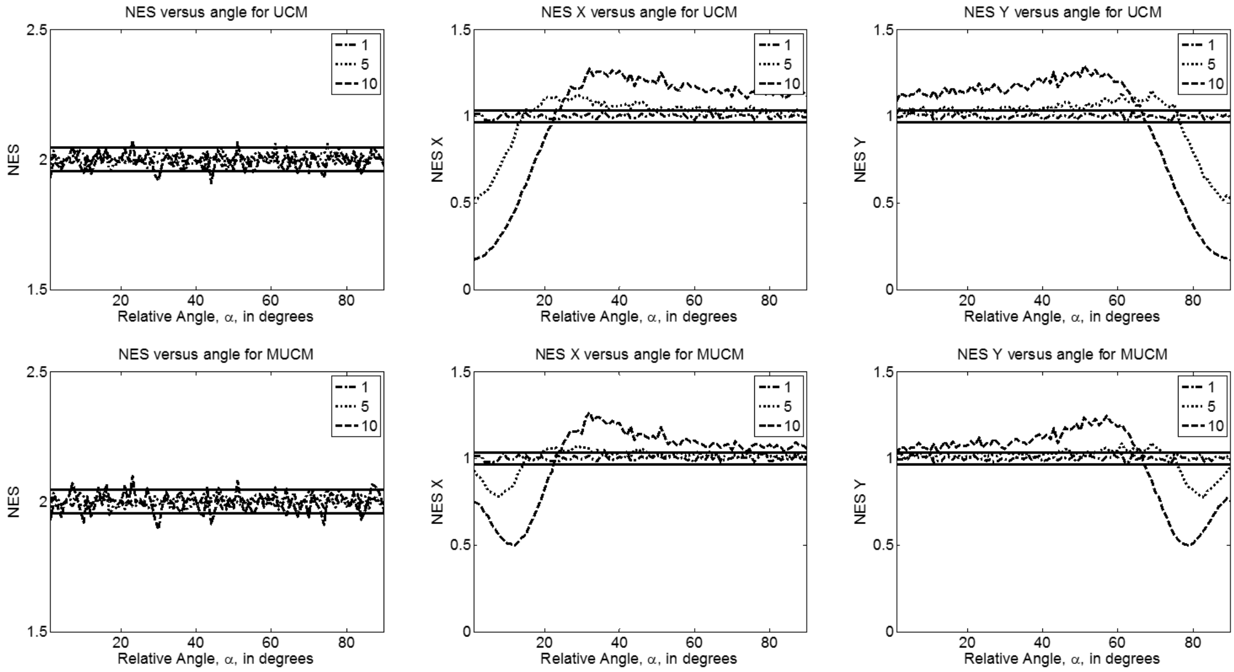

Figure 4. Normalized Error Squared (NES) for the UCM and MUCM conversion methods from 10^4 Monte-Carlo runs using a range of 10,000 and bearings of 0° to 90°. The range and bearing measurement noises used were uncorrelated Gaussian with $\sigma_r = 100$ and $\sigma_\alpha = 1^\circ$, 5° and 10°. Plots include chi-square 0.99 probability bounds.

2.3 Evaluation of the estimation bias

Both the UCM and MUCM conversions utilize the measurement to estimate the converted measurement error covariance. As a result, *the estimate of the covariance becomes correlated with the measurement noise*, leading to a *biased Kalman filter gain*. This issue results in biased estimates [7, 3]. In order to analyze this phenomenon, a position estimator for a static target using converted measurements is considered. For this simple case, consider the results of a batch linear least squares estimator (LLSE). The result is an average of the converted measurements, weighted by the inverse of the converted measurement noise covariance.

As Figure 5 shows, estimation bias due to the above correlation is a problem common to the UCM and MUCM conversion. The bias is along the line of sight to the target. It is interesting to note that the estimation bias relative to ground truth is the smallest for the MUCM technique. This is due to the fact MUCM conversion bias and the estimation bias are in opposite directions. This perhaps explains why MUCM conversion has outperformed UCM conversion in tracking simulations. While the MUCM conversion bias and estimation bias seem to have a symbiotic relationship, using a biased conversion is not ideal for recursive estimation.

Since the end goal is to use the converted measurements in a CMKF, the conversion bias and estimation bias are problematic. During the estimation process the position estimate will be initialized with the conversion bias (in the MUCM case) and transition to the estimation bias over time. If used for tracking, this results in degraded velocity estimates. While performance in a CMKF that estimates velocity (and possibly acceleration and turn rate) is most pertinent, target motion can obfuscate the underlying performance of the conversion process. Therefore, a simple CMKF that estimates position only, without process noise, is first considered. Simulation results, shown in Figure 6, show the conversion bias of the MUCM, and the estimation bias of the filters using UCM and MUCM. The UCM based estimator is initialized without bias, but becomes biased over time. The MUCM estimator is initialized with a conversion bias, but the bias is reduced over time due to the estimation bias.

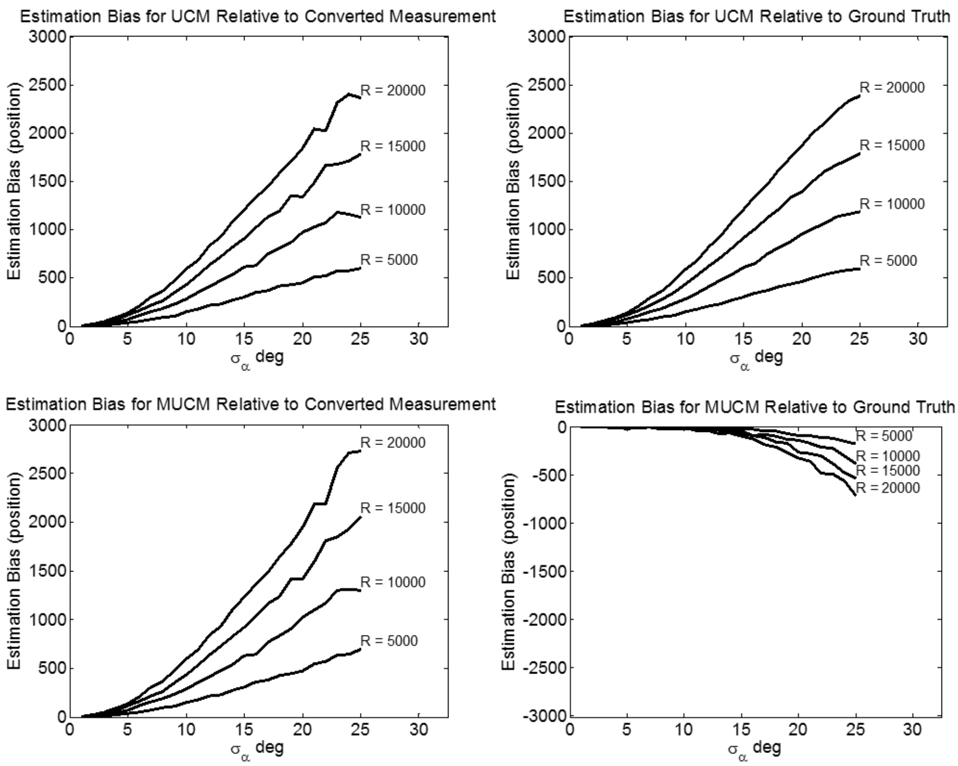

Figure 5. Position estimation bias versus σ_α for a 5000 sample batch LLSE using UCM and MUCM conversion methods with ranges of 5,000; 10,000; 15,000 and 20,000. For all cases $\sigma_r = 0.01$*range. Correlation between the measurement error and the converted measurement covariance estimate causes the LLSE to be biased.

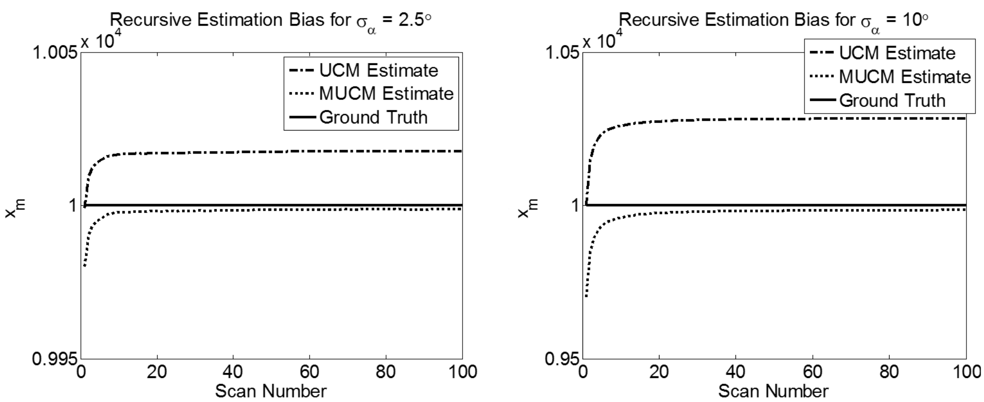

Figure 6. Recursive estimation bias for the CMKF using UCM and MUCM conversion methods from 1000 Monte-Carlo runs using a range of 10,000 and bearing of 0°. The range and bearing measurement noises used were uncorrelated Gaussian with $\sigma_r = 100$ and $\sigma_\alpha = 2.5°$ and $10°$.

2.4 Proposed new conversion technique

To overcome the disadvantages of the CMKF using the UCM and MUCM conversion techniques, an improvement to the UCM conversion is proposed. This method will be referred to as the Decorrelated Unbiased Measurement Conversion (DUCM). The design goals used in the development of the technique were:

1. Use the unbiased measurement conversion
2. Avoid correlation of the converted measurement covariance estimate and the measurement noise to preclude estimation bias
3. Provide minimum mean square error estimates

To achieve the first goal, the UCM measurement conversion is used:

$$x_m^{\text{DUCM}} = x_m^{\text{UCM}} = e^{\sigma_\alpha^2/2} r_m \cos\alpha_m \tag{24}$$

$$y_m^{\text{DUCM}} = y_m^{\text{UCM}} = e^{\sigma_\alpha^2/2} r_m \sin\alpha_m \tag{25}$$

To decorrelate the estimation of the measurement covariance from the measurement noise, an approach similar to previous work is used [7]. While conditioning on the previous measurement has been proposed [7], the approach used here is to condition on the previous estimate (i.e., use one-step predictions). Various techniques could be used for approximating the estimate conditioned covariance; for example use of the Unscented Transform has been proposed [8]. The technique chosen here is a linearization of tracked covariance, ignoring correlation between range and bearing errors, namely,

$$\begin{aligned} R_{\text{DUCM}}^{11} = \tfrac{1}{2}\left(r_m^2 + \sigma_r^2 + \sigma_{r,trk}^2\right)\left[1 + \cos(2\alpha_{trk})\exp\left(-2\sigma_\alpha^2\right)\exp\left(-2\sigma_{\alpha,trk}^2\right)\right]\exp\left(\sigma_\alpha^2\right) \\ -\tfrac{1}{2}\left(r_m^2 + \sigma_{r,trk}^2\right)\left[1 + \cos(2\alpha_{trk})\exp\left(-2\sigma_{\alpha,trk}^2\right)\right] \end{aligned} \tag{26}$$

$$\begin{aligned} R_{\text{DUCM}}^{22} = \tfrac{1}{2}\left(r_m^2 + \sigma_r^2 + \sigma_{r,trk}^2\right)\left[1 - \cos(2\alpha_{trk})\exp\left(-2\sigma_\alpha^2\right)\exp\left(-2\sigma_{\alpha,trk}^2\right)\right]\exp\left(\sigma_\alpha^2\right) \\ -\tfrac{1}{2}\left(r_m^2 + \sigma_{r,trk}^2\right)\left[1 - \cos(2\alpha_{trk})\exp\left(-2\sigma_{\alpha,trk}^2\right)\right] \end{aligned} \tag{27}$$

$$\begin{aligned} R_{\text{DUCM}}^{12} = \tfrac{1}{2}\left(r_m^2 + \sigma_r^2 + \sigma_{r,trk}^2\right)\left[\sin(2\alpha_{trk})\exp\left(-2\sigma_\alpha^2\right)\exp\left(-2\sigma_{\alpha,trk}^2\right)\right]\exp\left(\sigma_\alpha^2\right) \\ -\tfrac{1}{2}\left(r_m^2 + \sigma_{r,trk}^2\right)\left[\sin(2\alpha_{trk})\exp\left(-2\sigma_{\alpha,trk}^2\right)\right] \end{aligned} \tag{28}$$

The range and bearing standard deviation is approximated based on the track covariance, P, as

$$\sigma_{\alpha,trk}^2 = \frac{P_{x,x} y_{trk}^2 - 2P_{x,y} y_{trk} x_{trk} + P_{y,y} x_{trk}^2}{\left(x_{trk}^2 + y_{trk}^2\right)^2} \tag{29}$$

$$\sigma_{r,trk}^2 = \frac{P_{x,x} x_{trk}^2 - 2P_{x,y} + P_{y,y} y_{trk}^2}{x_{trk}^2 + y_{trk}^2} \tag{30}$$

Evaluation of the NES, as seen in Figure 7, shows that this approach is statistically consistent. However, similarly to the UCM and MUCM techniques, the individual components are inconsistent in certain geometries. The covariance estimate along the true bearing is overestimated, resulting in an NES less than one.

Evaluation of the recursive estimation bias, as seen in Figure 8, shows that, compared to Figure 6, the CMKF using the DUCM has a bias that is negligible when compared to using the UCM and MUCM techniques.

While the DUCM technique has the advantage of unbiased conversion and negligible estimation bias, based on the arguments of (23), it will have a larger mean square error than the MUCM technique for the initial conversion and first few recursive estimates. To overcome this issue, the output of the estimator can be scaled by an approximation of the MMSE scaling factor. This scaling factor converts the unbiased estimate into an approximate MMSE estimate. The scaling factor, given by,

$$\eta_{\text{DUCM}} = e^{-\sigma_{\alpha,trk}^2} \tag{31}$$

should be applied for output only. The evaluation of MSE performance is shown in Figure 9. The CMKF-DUCM method is equal to or an improvement over the other techniques at each scan.

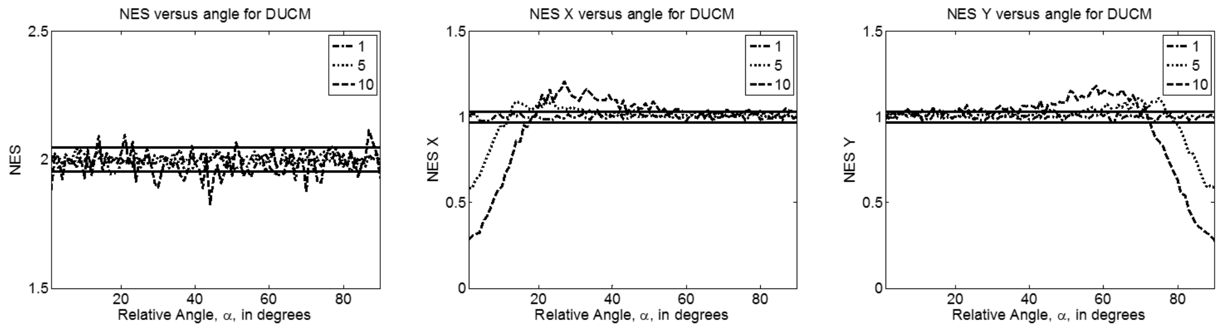

Figure 7. Normalized Error Squared (NES) for the DUCM conversion method from 10^4 Monte-Carlo runs using a range of 10,000 and bearing of 0° to 90°. The range and bearing measurement noises used were uncorrelated Gaussian with $\sigma_r = 100$ and $\sigma_\alpha = 1°$, 5° and 10°. Plots include chi-square 0.99 probability bounds.

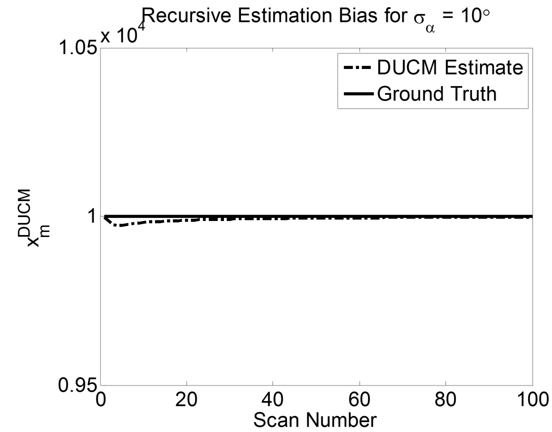

Figure 8. Recursive estimation bias for the CMKF using the DUCM conversion method from 1000 Monte-Carlo runs using a range of 10,000 and bearing of 0°. The range and bearing measurement noises used were uncorrelated Gaussian with $\sigma_r = 100$ and $\sigma_\alpha = 10°$.

3. APPLICATION TO CONVERTED MEASUREMENT TRACKING

For the static estimation case, results indicate that the CMKF using the DUCM technique has improved performance over the ones using the UCM or MUCM technique. Performance in a dynamic scenario is a more pertinent test. To evaluate the realistic tracking performance, the general approach of previous works is adopted. The target's initial x and y positions are taken from independent draws from a Gaussian distribution with mean 10,000 ft and standard deviation of 100 ft. Target speed is taken from a Gaussian distribution with a mean of 40 ft/s and a standard deviation of 10 ft/s. Target heading is taken from a uniform distribution. The target follows a constant velocity track and is estimated using a nearly constant velocity tracker. One point initialization of the tracker is used with an initial velocity estimate of 0 ft/sec and standard deviation of 30 ft/s in each component.

3.1 Mean Square Error performance

Evaluation of tracking performance indicates that the CMKF using the DUCM technique outperforms the CMKF using UCM or MUCM in position MSE. CMKF-DUCM velocity MSE slightly underperforms during the initial scans, but has the best performance for later scans. While CMKF-DUCM did not exhibit estimation bias for position, some estimation bias was exhibited in the velocity estimates. Figures 10 and 11 show the MSE comparison of the three techniques.

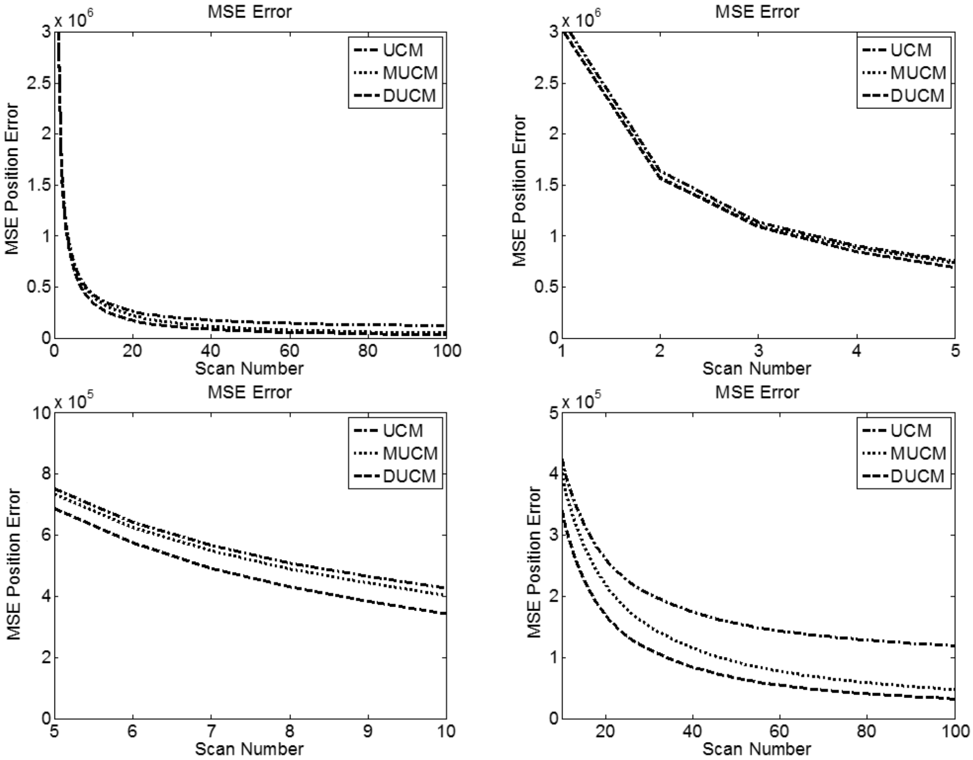

Figure 9. Static CMKF Mean Square Error comparison for the UCM, MUCM and DUCM conversion methods from 1000 Monte-Carlo runs using a range of 10,000 and bearing of 0°. The range and bearing measurement noises used were uncorrelated Gaussian with $\sigma_r = 100$ and $\sigma_\alpha = 10°$. The first sub-figure shows the results of 100 scans, while the other three sub-figures are zoomed figures of scans 1 to 5, 5 to 10 and 10 to 100.

3.2 Average Normalized Estimation Error Squared (ANEES) performance

To ensure credibility of the DUCM method, the ANEES performance is examined. The ANEES scaled to the state dimension, n, is [1]:

$$\text{ANEES} = \frac{1}{Nn}\sum_{i=1}^{N} \tilde{X}_i^T P_i^{-1} \tilde{X}_i \tag{32}$$

where $\tilde{X}_i$ is the estimation error and P_i is the error covariance for trial i. The ANEES of a consistent estimator should be close to 1. Figure 12 shows that the DUCM approach is the most consistent based on the ANEES.

4. APPLICATION TO RANGE RATE ESTIMATION FROM A MOVING PLATFORM

The results of Section 2 can also be applied to the problem of own Doppler nullification. The conventional technique used to estimate range rate is to nullify the effect of own-ship motion using (3). This conversion results in a biased measurement through the same mechanism as the polar to Cartesian conversion process. This bias can be avoided using a form of the decorrelated unbiased measurement conversion.

4.1 Evaluation of the Doppler conversion bias

The bias of the conventional technique can be found by taking the expected value of the converted frequency and bearing measurements, $f_{r,m}$ and α_m. Assuming the received frequency measurement noise, w_f, and bearing

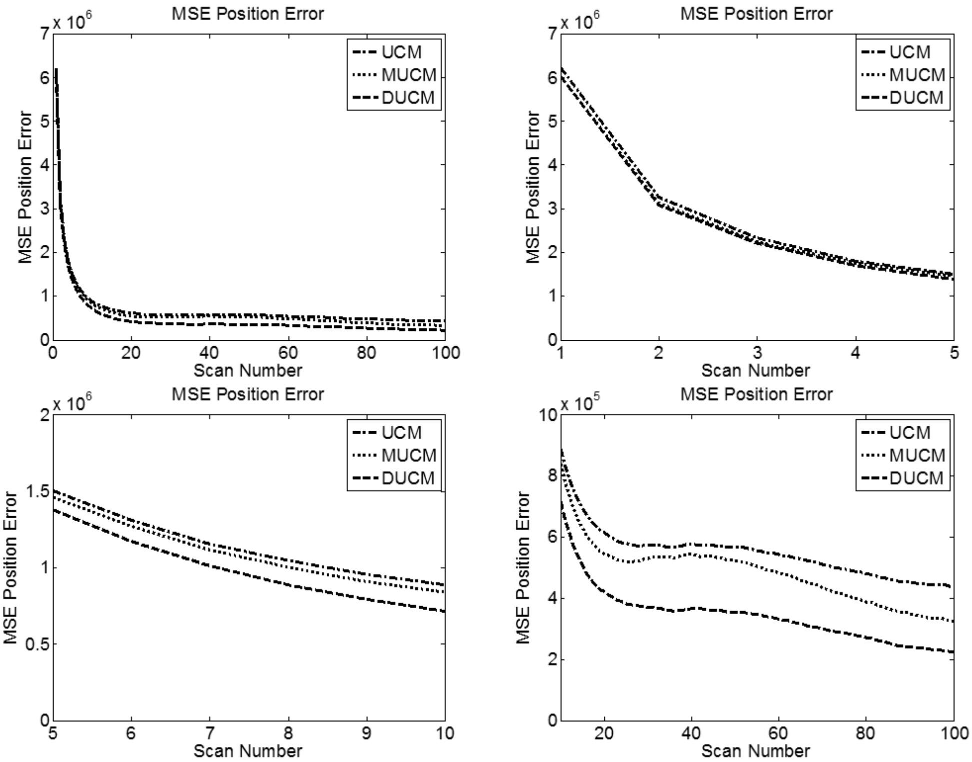

Figure 10. CMKF Position Mean Square Error comparison for the UCM, MUCM and DUCM conversion methods from 10,000 Monte-Carlo runs. The range and bearing measurement noises used were uncorrelated Gaussian with $\sigma_r = 100$ and $\sigma_\alpha = 10°$. The first sub-figure shows the results of 100 scans, while the other three sub-figures are zoomed figures of scans 1 to 5, 5 to 10 and 10 to 100.

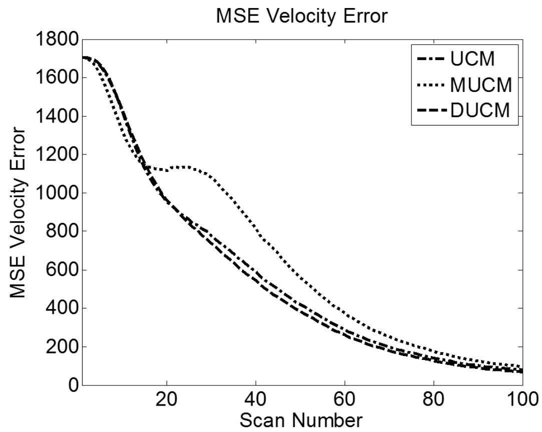

Figure 11. CMKF Velocity Mean Square Error comparison for the UCM, MUCM and DUCM conversion methods from 10,000 Monte-Carlo runs. The range and bearing measurement noises used were uncorrelated Gaussian with $\sigma_r = 100$ and $\sigma_\alpha = 10°$.

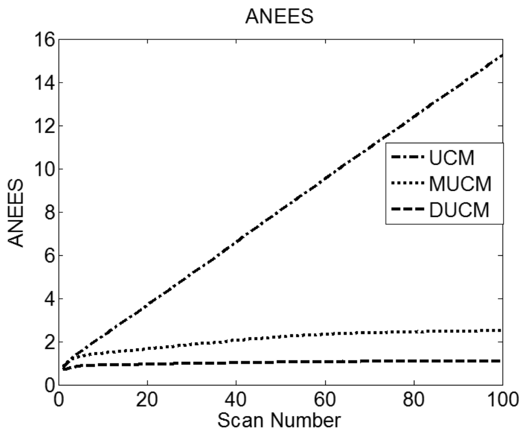

Figure 12. ANEES comparison of CMKF with the UCM, MUCM and DUCM conversion methods from 10,000 Monte-Carlo runs. The range and bearing measurement noises used were uncorrelated Gaussian with $\sigma_r = 100$ and $\sigma_\alpha = 10°$.

measurement noise, w_α, are uncorrelated, zero mean, and Gaussian with standard deviations of σ_f and σ_α respectively; the expected value is

$$E\left[\frac{(f_{r,m} - f_t)\,c}{2f_t} - V\cos(\alpha_m)\right] = E\left[\frac{(f_r + w_f - f_t)\,c}{2f_t} - V\cos(\alpha + w_\alpha)\right] = \frac{(f_r - f_t)\,c}{2f_t} - V\cos(\alpha)\,e^{-\sigma_\alpha^2/2} \quad (33)$$

resulting in the following bias

$$V\cos(\alpha)\left(1 - e^{-\sigma_\alpha^2/2}\right). \quad (34)$$

The corresponding mean squared error is

$$R = \frac{\sigma_f^2 c^2}{4f_t^2} + V^2\left[\frac{1}{2}\left(1 + \cos(2\alpha)e^{-2\sigma_\alpha^2}\right) + \cos^2(\alpha)\left(1 - 2e^{-\sigma_\alpha^2/2}\right)\right] \quad (35)$$

4.2 Bias significance

The significance of the bias depends on the characteristics of the radar or sonar system being employed. Key features include the velocity of the platform and the operating frequency relative to the speed of the sound (i.e. the wavelength). The ability of the system to estimate frequency and bearing is also important. Examining the Cramer-Rao Lower Bound (CRLB) for bearing and frequency estimation is useful for understanding the types of systems in which the bearing estimation error is more significant than the frequency estimation error. For the geometry of Figure 1, the CRLB definition for bearing estimation shows that bearing error is a function of signal to noise ratio, wavelength, array length and true bearing, α [4]. The ability to estimate bearing degrades as bearing increases. The CRLB definition for frequency estimation shows that the ability to estimate frequency is a function of data record length and signal to noise ratio [4]. For a realization of a signal with a given signal to noise ratio, the size of the array is a determining factor in the bias significance. Overall, the bias is most significant for high speed platforms with small arrays. Figure 13 shows the bias significance, defined as the bias divided by the square of the variance, for a conceptual sonar system operating at 50 kHz.

4.3 Unbiased range rate estimation

Using the approach for polar to Cartesian coordinate conversion, the standard and unbiased range rate conversion is shown in the following equations

$$d_m^{\mathrm{STANDARD}} = \frac{(f_{r,m} - f_t)\,c}{2f_t} - V\cos\alpha_m \quad (36)$$

$$d_m^{\mathrm{DUCM}} = \frac{(f_{r,m} - f_t)\,c}{2f_t} - V\cos(\alpha_m)\,e^{-\sigma_\alpha^2/2} \quad (37)$$

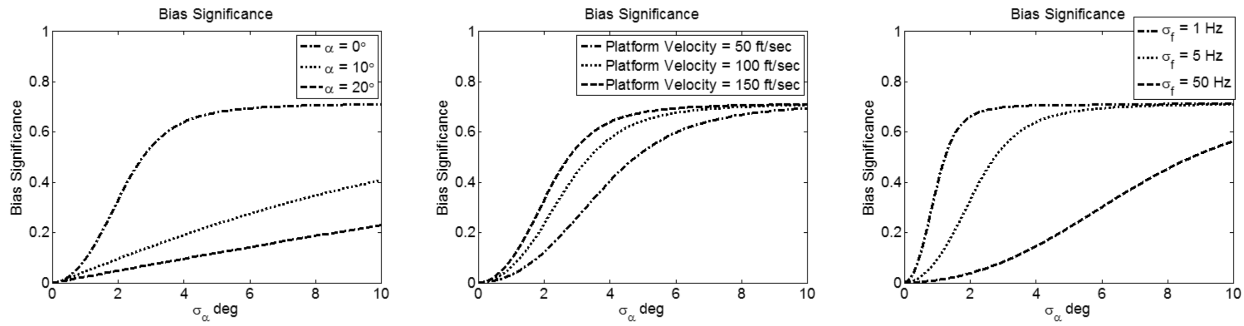

Figure 13. Bias significance vs. σ_α of a 50 kHz sonar for various bearing angles, platform velocities and frequency measurement accuracies. Unless specified otherwise, platform velocity = 150 ft/s, σ_f = 5Hz and $\alpha = 0°$

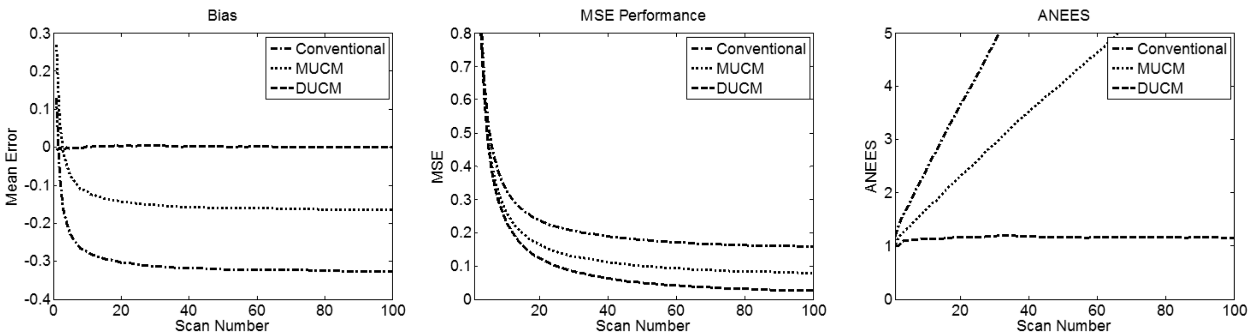

Figure 14. Performance of the conventional, MUCM and DUCM conversion techniques utilized in a range rate tracker.

The MSE approximation from the standard conversion is

$$R_{\text{STANDARD}} = \frac{\sigma_f^2 c^2}{4 f_t^2} + V^2 \sin^2 \alpha_m \tag{38}$$

The true MSE of the unbiased conversion is

$$R = \frac{\sigma_f^2 c^2}{4 f_t^2} + V^2 \left[\frac{1}{2} \left(1 + \cos(2\alpha) e^{-2\sigma_\alpha^2} \right) e^{\sigma_\alpha^2} - \cos^2 \alpha \right] \tag{39}$$

As in the CMKF case, the measurement covariance requires knowledge of the true range. When using the converted measurements in a Kalman filter range rate estimator, or in an extended Kalman filter that uses range rate in the estimation process, the measurement covariance must be approximated. To avoid estimation bias, the DUCM approach of conditioning on the previous value or measurement is chosen.

$$R_{\text{DUCM}} = \frac{\sigma_f^2 c^2}{4 f_t^2} + \frac{1}{2} V^2 \left[\left(1 + \cos(2\alpha_{trk}) e^{-2\sigma_\alpha^2} e^{-2\sigma_{\alpha,trk}^2} \right) e^{\sigma_\alpha^2} - \left(1 + \cos(2\alpha_{trk}) e^{-2\sigma_{\alpha,trk}^2} \right) \right] \tag{40}$$

To evaluate the performance of the conversion techniques, a simple range rate recursive estimator is used. For the case where the seeker and the target are on a collision course, both following constant velocity motion, the bearing rate is zero and the range rate is constant. Using these assumptions, a seeker using a 50 kHz sonar system with a velocity of 150 ft/s, $\sigma_\alpha = 2.5°$ and $\sigma_f = 5$ Hz approaches a target with a true Doppler of 20 ft/s. The true bearing angle, α, is uniformly distributed from -45 to 45 degrees. The estimation bias, MSE performance and consistency, based on the ANEES, of the DUCM approach are superior to the standard conversion for this evaluation, as shown in Figure 14.

The bias of the DUCM approach was consistently less than that of the conventional approach. However, the MSE performance and consistency of the DUCM approach was degraded for very small bearings and large σ_α.

5. CONCLUSION

When using converted measurements in tracking, two sources of bias need to be evaluated and eliminated. The first is measurement conversion bias, which occurs when the conversion process introduces a bias in the mean of the converted measurement. The second source of bias is estimation bias, which occurs when the estimate of the measurement covariance is correlated with the converted measurement noise, leading to a biased Kalman gain. Two tracking problems that exhibit both sources of bias have been examined. The first is the estimation of position and velocity using the Converted Measurement Kalman Filter. The second is the estimation of range rate from a moving platform. It has been shown for both tracking problems that a decorrelated version of the Unbiased Measurement Conversion (DUCM) exhibits improved performance over the previously proposed Modified Unbiased Measurement Conversion, MUCM. Future work may include evaluation of the DUCM technique in additional scenarios, extension from polar to spherical coordinate conversion and evaluation of unbiased range rate measurement conversion in an extended Kalman Filter.

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