A MIXED INTEGER PROGRAMMING MODEL FOR IMPROVING THEATER DISTRIBUTION FORCE FLOW ANALYSIS

THESIS

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Abstract

Obtaining insight into potential vehicle mixtures that will support theater distribution, the final leg of military distribution, can be a challenging and time-consuming process for United States Transportation Command (USTRANSCOM) force flow analysts. The current process of testing numerous different vehicle mixtures until separate simulation tools demonstrate feasibility is iterative and overly burdensome.

Improving on existing research, a mixed integer programming model was developed to allocate specific vehicle types to delivery items, or requirements, in a manner that would minimize both operational costs and late deliveries. This gives insight into the types and amounts of vehicles necessary for feasible delivery and identifies possible bottlenecks in the physical network. Further solution post-processing yields potential vehicle beddowns which can then be used as approximate baselines for further distribution analysis.

A multimodal, heterogeneous set of vehicles is used to model the pickup and delivery of requirements within given time windows. To ensure large-scale problems do not become intractable, precise set notation is utilized within the mixed integer program to ensure only necessary variables and constraints are generated.

To my wife, who has supported me from the beginning and was a phenomenal helpmate throughout the research process.

To my parents, who taught me the value of hard work.

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Micah J. Hafich

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A MIXED INTEGER PROGRAMMING MODEL FOR IMPROVING THEATER DISTRIBUTION FORCE FLOW ANALYSIS

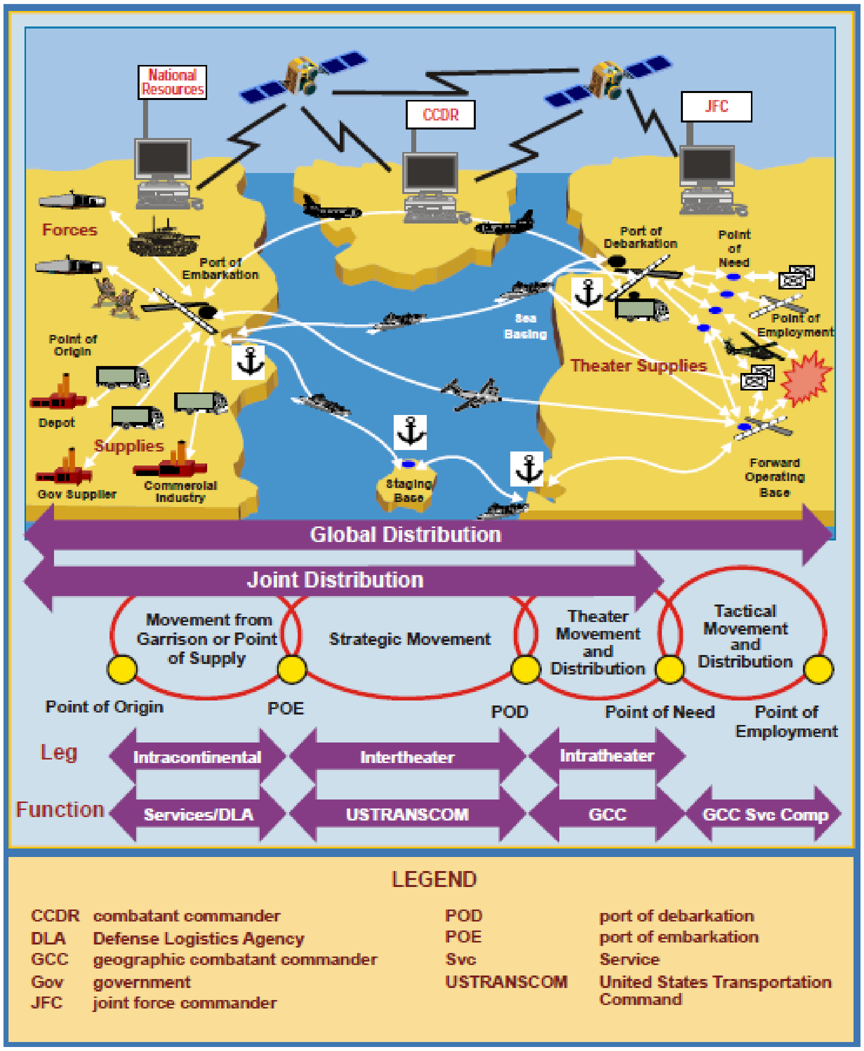
I. Introduction

Background

Although varying facets of warfare have changed considerably throughout the history of combat operations, theater distribution has remained an important concept. In fact, Alexander the Great successfully conquered much of the known world in the 4th century B.C. largely because of his proficiency in supplying his army (Engels, 1978). Theater distribution, a principal component of military logistics, is defined as the flow of personnel, equipment, and materiel within a given theater as necessitated by the geographic combatant commander to support theater missions (Joint Chiefs of Staff, 2010). A military force cannot operate in-theater as intended if the war-fighters and their required provisions are not in the appropriate place at the necessary time. Therefore, effective theater distribution must be achieved in any military contingency.

The United States (US) military places great emphasis on the superior distribution of troops and materiel. As such, the core logistic capability of Deployment and Distribution is an underpinning of the US military's doctrine on joint logistics. This doctrinal capability focuses on moving forces, along with their equipment and materiel, around the globe while maintaining time deadlines dictated by combatant commanders (Joint Chiefs of Staff, 2008). United States Transportation Command (USTRANSCOM), the unified command responsible for the deployment and distribution of troops and equipment, supports this logistic capability with sound planning and execution.

Joint military distribution is typically carried out in three specific phases, known as legs. The first leg, or intracontinental movement, is the movement of forces and cargo from their initial point of origin to a Port of Embarkation (POE). The first leg typically remains within the United States, with troops and cargo departing from unit bases to a POE for further movement. The second leg, intertheater movement, involves movement from a POE to an in-theater Port of Debarkation (POD). This leg usually entails the movement of forces and goods from the United States to a specific theater of operations. The final leg, known as intratheater movement or theater distribution, occurs when personnel and materiel are moved from an in-theater POD to their final delivery destination, or Point of Need, within the operating area (Joint Chiefs of Staff, 2010). This final leg occurs entirely within the operational theater. Throughout the distribution process, ports (both PODs and POEs) may be either aerial ports or sea ports. An example of how the three legs of distribution work together to deliver goods from origin to theater is shown below in Figure 1.



(Joint Chiefs of Staff, 2010, p. I2)

Figure 1. The Three Legs of Joint Military Distribution

Military operations are typically planned with an operation plan (OPLAN). For operations requiring the movement of forces, Time Phased Force Deployment Data (TPFDD) accompanies the OPLAN. The TPFDD document details the required personnel, equipment, and materiel that must be delivered to support the OPLAN. Each individual item to be distributed is known as a requirement, and TPFDDs list considerable information for each individual requirement. Among other things, requirements in a TPFDD will have their planned origin, POE, POD, final delivery destination, and weight all listed. Additionally, timing information such as different pickup and delivery windows are included. A properly employed TPFDD will ensure that all necessary items arrive to the theater in a sequential, phased manner, allowing geographic combatant commanders to successfully conduct missions as capabilities arrive within the area of operations.

In a TPFDD, time constraints are planned for all legs of the movement. However, a few specific time-related attributes are of great importance to theater distribution planning. The Earliest Arrival Date (EAD) and Latest Arrival Date (LAD) describe the earliest and latest dates in which the stated POD for a requirement can accept the delivery of a specific requirement from its POE. This creates an EAD-LAD delivery window. Therefore, each requirement is to arrive at its POD within this window. Once an item has arrived at the POD, it may then begin the final leg of its journey to the final delivery destination. The Required Delivery Date (RDD) is the date in which a requirement departing its POD must arrive at its final delivery destination. Table 1 below illustrates what some requirement attributes and information in a TPFDD might look like.

Table 1. Partial Data from Sample TPFDD

Requirement	POE	EAD	LAD	POD	RDD	Destination	Total Short Tons
1	FGSL	5	8	TWTH	10	GHOS	300
2	TWBI	7	10	HSNP	12	BHEL	100

Another important time constraint is the Commander's Required Delivery Date (CRD). While not listed in a TPFDD, the CRD is a date beyond the RDD, decided upon by the geographic combatant commander, in which a requirement must have arrived at the final delivery destination. Therefore, while undesirable, delivery after the RDD but on or before the CRD can be allowed in modeling to assess late impacts. (Joint Chiefs of Staff, 2011a).

As part of distribution planning, and in order to ensure successful future military movements, USTRANSCOM holds recurring force flow conferences. At these conferences, proposed OPLANs and accompanying TPFDDs are tested against logistical capabilities to determine the feasibility of planned actions. Analysts and planners must determine whether or not requirements listed in an OPLAN's TPFDD can be realistically delivered based upon the planned delivery network, assigned transportation vehicles, and the timelines for movements. If analysis shows that the transportation of the required equipment and materiel needed to begin and sustain operations cannot be conducted in a feasible manner, an iterative process of refining the OPLAN and TPFDD is conducted until a satisfactory and feasible operation plan is established (Joint Chiefs of Staff, 2010).

While USTRANSCOM force flow conferences may examine all three legs of military distribution during their analysis, particular attention must be given to theater distribution, the intratheater movement between PODs and final destinations. Firstly, theater distribution normally requires a beddown of vehicles within the theater in order to sustain delivery to the final destination. Thus, determining how to allocate requirements to vehicles and deciding which vehicles to position at theater locations to support theater distribution can be a challenging task. Secondly, the theater distribution phase is crucial to ensuring war-fighters receive their goods and materiel on time. Timeliness is imperative in this last leg as late deliveries could negatively impact military operations and potentially harm US forces. Movement requirements shipped on-time to the POD are useless to troops in combat if they do not also arrive on-time to the theater locations. Thus, it is imperative that appropriate analysis is conducted on theater distribution.

At USTRANSCOM force flow conferences various mobility simulation tools are used to find feasible delivery options by examining the transportation networks and assets under consideration. An internal research paper authored by Longhorn & Kovich (2012) of USTRANSCOM points out that while these simulation models are helpful in conducting theater distribution analysis, they only describe limitations to theater distribution without prescribing any potential fixes. In other words, the simulation tools report only on the feasibility or infeasibility of specific transportation plans based upon the constraints of the specific network under consideration and the transportation assets selected to be utilized within the simulation. Once limitations or infeasibilities are found, no current tool exists to describe an appropriate vehicle mixture that will allow the operation to then become feasible. In fact, it may take many time-consuming “trial and error” runs with differing transportation vehicle mixtures until one that supports feasible movement is found.

To address this, Longhorn & Kovich (2012) propose an integer programming optimization formulation, known throughout this thesis as the Theater Distribution Model (TDM). The TDM, discussed thoroughly in Chapter II, would prescribe, before simulation of the theater distribution phase, a specific multimodal vehicle mixture that is needed to successfully deliver the materiel for a specific operation. Once determined, the specific vehicle mixtures would be used as input in the simulation tools as analysts continue with distribution analysis. Because the vehicle mixture solutions drawn from the TDM would demonstrate sufficient transportation assets for the requirements, they should yield feasible transportation plans. Thus, analysts can avoid the iterative, timely process of checking for feasibility and adapting as necessary. Furthermore, by making cost changes in the optimization models, analysts can also compare how different policy changes would impact theater distribution efforts. (Interested readers should contact Dr. Jeff Weir, AFIT/ENS, at jeffery.weir.2@us.af.mil for information on obtaining the Longhorn & Kovich internal research paper).

Research Purpose and Objectives

The purpose of this research is to improve contingency planning capabilities at USTRANSCOM, specifically for force flow analysis of theater distribution. At present, analysts at USTRANSCOM have no functioning optimization models that dictate, for a given operation, a feasible number of vehicles needed to conduct theater distribution in an on-time, least-cost method. Currently, planners initially select a vehicle mixture that may or may not yield feasible transportation after analysis. Next, simulation tools are run to examine whether or not that particular predetermined vehicle mixture will allow for feasible flow within the network. If the analysis shows infeasibility, another vehicle mixture is tested.

Because the simulations are descriptive in nature, they do not give insight into what types of vehicle mixtures would provide for feasible transportation and because of this, potential vehicle mixtures are often selected via "trial and error". However, even if a particular vehicle mixture is found to yield feasible transportation within the network, there is certainly no guarantee that the vehicle mixture is even remotely optimal in terms of costs. This iterative technique of finding vehicle mixtures can be extremely time consuming, requiring hours of simulation every time a new vehicle mixture is tested for feasible transportation. The objective of the proposed TDM is to find on-time, least-cost delivery options for all requirements within the TPFDD, detailing on what days different types of vehicles should be available for transportation. However, the TDM has yet to be thoroughly tested.

The first objective of this research is to test the proposed TDM and determine if it is capable of finding solutions to large-scale problems, such as those engendered with TPFDDs for US military contingencies. A typical TPFDD may easily contain thousands of movement requirements. Thus, it is important to ensure that any proposed model is computationally efficient as problems can grow rapidly in size.

The second objective of this research is to determine if the TDM optimization model adequately matches reality. That is, the validity of the model must be inspected to ensure that it appropriately finds the vehicle mixture necessary for requirements in an on-time, least-cost method.

Thirdly, this research will examine possible changes to the formulation of the model. In particular, the process by which vehicles are allocated to requirements will be investigated.

Lastly, the research will attempt to construct approximate vehicle beddowns that would be necessary at each POD based upon the model solutions. Beddowns may be helpful to analysts as they attempt to model the theater distribution portion of movements with simulation tools.

With these objectives in mind, this research intends to save USTRANSCOM countless hours of analysis and planning at their force flow conferences. A functional optimization model for force flow analysis will allow operational planners to quickly find feasible vehicle mixtures for intratheater transportation needs rather than going through multiple stages of guesswork, followed by hours of simulation, when selecting a vehicle mixture for successful theater distribution in a planned contingency operation.

Furthermore, in addition to reducing the man-hours required to conduct the planning, testing, and analysis of OPLANs/TPFDDs, an optimization model would allow analysts to explore different feasible vehicle mixtures by changing model inputs as a demonstration of different policy decisions or other driving forces. Through this research, improved efficiency in planning of theater distribution will help ensure war-fighters are given the materiel and equipment they need in an on-time and least-cost manner.

Organization

The remainder of this thesis contains four additional chapters. Chapter II provides a literature review of airlift optimization modeling, the Pickup and Delivery Problem with Time Windows, and other relevant models focused on distribution. Additionally, the proposed TDM is introduced and explained in detail. In Chapter III, the methodology utilized in this research is discussed. In particular, a reduced-size, mixed integer programming solution method is developed. Chapter IV shows the implementation of the methodology and demonstrates improvements over the TDM. Chapter V offers concluding remarks and discusses how this research might be extended with further work.

II. Literature Review

This chapter will provide a review of relevant literature, focusing mainly on distribution-related models. The research mentioned herein is not entirely exhaustive, but gives the reader a general understanding of past efforts in areas such as airlift optimization modeling, the Pickup and Delivery Problem with Time Windows, and specific Tabu Search approaches to theater distribution. Additionally, great detail is given on the Theater Distribution Model, or TDM. This model, developed for the purpose of force flow analysis, was the basis of this thesis research.

Background

The US Military utilizes a number of simulation tools to assist in mobility planning. Interested readers are directed to McKinzie & Barnes (2004) for a review of some of these models. However, as discussed by Longhorn & Kovich (2012), these models tend to describe rather than prescribe various aspects of theater distribution. While various optimization techniques have been applied to military transportation problems throughout the years, many of them are aimed at the specific routing and scheduling of individual vehicles. However, force flow analysts are not concerned with creating individual routes for vehicles.

Force flow analysis is strictly for planning purposes, in which analysts attempt to judge the feasibility of future transportation plans and adapt plans when necessary. Furthermore, combat is a dynamic environment in which many aspects cannot be planned for exactly because scenarios often can, and do, change instantly. For example, physical factors such as terrain, weather, and the impacts of friendly and enemy forces greatly affect operations and sustainment (Joint Chiefs of Staff, 2011b). For these reasons, the creation of individual vehicle routes and schedules is neither necessary nor desired for force flow analysis. Instead, analysts simply desire a baseline vehicle mixture that will successfully support distribution operations. This chapter will offer a review of distribution modeling efforts as well as specific mathematical approaches to closely related problems such as the Pickup and Delivery Problem with Time Windows.

Airlift Optimization Modeling

Early optimization efforts on military distribution often focused on airlift capabilities**.** Rappoport, Levy, Toussaint, & Golden (1994) developed a transportation problem formulation to be utilized in airlift planning for Military Airlift Command, the predecessor to the US Air Force's Air Mobility Command (AMC). The model was utilized to assign differing airlift vehicle types, such as bulk or outsize, and shipment days to specific requirements. Then, once these matches were made, the results were preprocessed and then placed into a heuristic routing and scheduling procedure known as the Airlift Planning Algorithm (APA). The model, set up as a linear programming transportation problem, minimized the costs of assigning capacity to different requirements. While the model matches vehicle types to movements as a preprocessor to further modeling, the transportation model does not dictate the number of vehicles needed to sustain flow within the network.

Shortest path techniques have also been applied to AMC aircraft routing. Rink, Rodin, Sundarapandian, & Redfern (1999) applied a double-sweep algorithm to find the k - shortest paths between each onload location and offload location given in a TPFDD. However, the time factor (i.e. avoiding lateness) is not considered in this model. Shortest path methods can be a hindrance to successful analysis. Due to certain policy decisions regarding concerns like safety or enemy in the area, a shortest path may not be the best path. Additionally, shortest paths are not guaranteed to have enough outloading and unloading resources to support distribution.

Rosenthal et al. (1997) discuss the use of THRUPUT II, a model developed at the Naval Post Graduate School, in order to model the entire transportation network. Linear programming is used to yield on-time throughput of both cargo and passengers. That is, given the inputs of units to be moved, airfields available, aircraft available, and routes available, the model provides routes and mission start times for aircraft within the model.

All airlift models have an inherent drawback for use in theater distribution analysis because they fail to consider movement amongst other modes of transportation, such as rail or road. Thus, the effects and tradeoffs between different modes cannot be properly assessed. In theater distribution, multiple modes are usually available and thus multimodal modeling is important.

Pickup and Delivery Problem with Time Windows

In theater distribution, requirements are to be picked up at their respective POD and then delivered to their in-theater destination. A TPFDD will dictate what the time windows for both the pick-up at the POD and delivery at the Destination are. Because of the time windows on both the pickup and delivery, this problem is related to an optimization problem known as the Pickup and Delivery Problem with Time Windows (PDPTW). The PDPTW involves transportation requests that have both a pickup and delivery location along with time windows in which the pickup and delivery must occur. Solutions to the PDPTW yield optimal routes for vehicles in which demand is met within the appropriate time windows while meeting capacity and precedence constraints (Dumas, Desrosiers, & Soumis, 1991).

Dumas et al. (1991) offer a PDPTW mathematical formulation that utilizes a homogenous fleet of vehicles and is solved utilizing column generation with a shortest path subproblem. Many other solution attempts to the PDPTW have been developed, such as the Reactive Tabu Search method employed by Nanry & Barnes (2000)**.** Furthermore**,** Baldacci, Bartolini, & Mingozzi (2011) utilize a set partitioning formulation to solve the PDPTW. Readers interested in exploring the different formulations and applications of the PDPTW may review Cordeau, Laporte, Potvin, & Savelsbergh (2007).

Because the US military has numerous vehicle types in their inventory, the PDPTW with a homogenous fleet is not a particularly useful model. However, pickup and delivery models utilizing multiple vehicle types have been studied. Lu & Dessouky (2004) developed an exact algorithm for solving the multiple vehicle pickup and delivery problem (MVPDP), which may include time windows. Their integer programming formulation allows for multiple heterogeneous vehicles. Many heuristic solution methods to the MVPDP have also been developed and interested readers may reference Savelsbergh & Sol (1995). Xu, Chen, Rajagopal, & Arunapuram (2003) developed a Practical Pickup and Delivery Problem (PPDP) that extends the PDPTW to include, not only multiple vehicle types, but many additional considerations such as multiple time windows, travel time restrictions, and compatibility constraints.

It is important to point out that the PDPTW typically involves an assumption that a set number of vehicles are located at depots from which vehicles begin their routes.

However, in theater distribution, vehicles are typically not centrally located at some depot where they are then scheduled and routed for missions. Instead, transportation assets are typically delivered into the theater of operations. In fact, a goal of force flow analysis is to determine how many vehicles of each type need to be located at different PODs to begin supporting transportation requirements.

Tabu Search Approaches to Theater Distribution

Tabu Search approaches have recently been applied specifically to theater distribution problems. Crino, Moore, Barnes, & Nanry (2004) utilized Group Theoretic Tabu Search in order to solve the Theater Distribution Vehicle Routing and Scheduling Problem. This is a powerful approach which prescribes the routing and scheduling of multimodal theater transportation assets at the individual vehicle level in order to provide time-definite delivery of cargo. Likewise, Burks, Moore, Barnes, & Bell (2010) utilized Adaptive Tabu Search in an attempt to solve the theater distribution problem. This model focuses on solving two separate problems simultaneously. It solves both the Location Routing Problem and the Pickup and Delivery Problem with Time Windows to optimally choose locations of depots and supply points as well as the specific routes of vehicles while satisfying all demand requirements. As with many other models discussed in this chapter, these models prescribe individual vehicle routes and schedules.

While these Tabu Search approaches optimize time-definite delivery and allow multiple modes to be utilized within the transportation network, the models are of such high-fidelity that they are of little use in force flow analysis. Because too many factors could change an individual vehicle's route under combat scenarios, a general approximating solution approach, at the aggregate vehicle level, is preferred for force flow analysis. Thus, while a model employing Tabu Search may provide practical results for a day-to-day outlook on theater distribution operations, these models are not particularly insightful for force flow analysis, where a generalized solution that provides baseline estimates for necessary vehicles is more favorable (Longhorn & Kovich, 2012).

Time-Space Network Approaches

In order to model disaster relief operations Haghani & Oh (1996) developed a multicommodity, multimodal network flow model that finds the optimal use of different modes in a network to meet commodity and time requirements. To do this, a time-space network is utilized, which means that nodes in the network represent not only the physical locations of supply and demand, but also moments in time. Thus, time can be captured as flow occurs through the network. A time-space network technique is also utilized by Clark, Barnhart, & Kolitz (2004) to model the distribution of US Army Munitions, where ammunition and ship movements are scheduled within the distribution system.

Theater Distribution Model (TDM)

TDM Overview.

To determine an appropriate mixture of vehicles necessary to conduct theater distribution for specific contingencies, Longhorn & Kovich (2012) proposed a pure integer programming model. The Theater Distribution Model (TDM) attempts to find an optimum allocation of requirements to vehicles such that time-definite delivery occurs in a least-cost manner. Unlike other distribution models, the TDM does not specify routes and schedules for individual vehicles. As previously discussed, those sorts of high-fidelity models are impractical for force flow analysis. Instead, the TDM answers questions such as when, where, what type, and how many when discussing vehicles needed to conduct theater distribution subject to physical network constraints.

In the TDM, users must select which modes of transportation and vehicle types they wish to enter into the model. Selected modes form the set M. The individual Modes $m \in M$ will typically contain all or some elements of the set {Air, Road, Rail}. Vehicle Types are selected by the user to form a set of vehicle Types K. Each vehicle Type $k \in K$ is a specific vehicle (e.g. C-17) of a single Mode m, and has two input parameters associated with it. The first parameter is the daily cost of utilizing vehicle Type k, b_k. This cost could be financial in nature, but it may also be utilized as an arbitrary cost in order to analyze the impact certain policy decisions have upon solutions. The second parameter is p_k, the average payload (measured in short tons) of a vehicle of Type k.

The TDM draws much data for use in analysis from the TPFDD that is associated with the theater distribution plan under analysis. The TPFDD under consideration will list n_{max} separate movement requirements. Thus, the set $N = \{1, \ldots n_{max}\}$ contains a unique identifier for all movement requirements in the TPFDD. Each movement Requirement $n \in N$ has associated data with it such as the specific requirement's POD, Destination, EAD, RDD, and total weight. The set I contains all PODs i included in the TPFDD requirements while the set J contains all Destinations j. Each movement Requirement n, to be delivered from POD i to Destination j, has a requirement weight r_{nij} which is measured in short tons. Within the model, it is assumed that all requirements are standard cargo requirements. Passenger requirements and any potential restrictions on outsize or oversize cargo are ignored.

The variable ad_n describes the day in which Requirement n arrives at its stated POD. The TDM assumes that a requirement may not ship from its POD until the day immediately following its arrival at the POD. In other words, the first day in which Requirement n can deliver from its POD to its Destination would be the day $ad_n + 1$. The variable rd_n indicates the Required Delivery Date, or RDD, at the Destination for each Requirement n. Any requirement arriving after the RDD is considered late. Analysts and commanders may work together to determine how late a requirement may be for analysis. Each Requirement n may be given qd_n extension days in order to be delivered. Delivering on an extension day is allowable, but the movement will be denoted as late and a penalty, g, will be assessed per vehicle for each day late. The value of g is user-defined.

The TDM does not allow requirements to be delivered beyond their RDD plus any input extension days. Mathematically, this means that each requirement n must be picked up and delivered within the time window beginning at day $ad_n + 1$ and ending at $rd_n + qd_n$. Thus, the Days utilized within the model range from $\min_{n \in N} ad_n + 1$ to $\max_{n \in N} rd_n + qd_n$. The set V describes this set of Days v for delivery, spanning the absolute earliest possible day of requirement delivery and the absolute latest possible delivery day based upon information located in the TPFDD.

Physical limitations of the distribution network are captured in the TDM with restrictions on the number of vehicles which may be outloaded at PODs and unloaded at Destinations within a given day. Characteristics such as space and manning may impact the amount of vehicles that may pass through a POD or Destination daily. The TDM assumes that these outloading and unloading limits are not based upon specific vehicle Types, only vehicle Modes. In the model, o_{imv} describes the maximum number of Mode m vehicles that can be outloaded at POD i on Day v of the operation. Likewise, u_{jmv} describes the maximum number of Mode m vehicles that can be unloaded at Destination j on Day v. If a certain POD or Destination does not support the movement of a certain Mode, then the associated parameters o_{imv}, or u_{jmv} respectively, would have a value of zero. Typically, subject matter experts can provide these parameters.

The TDM assumes that a vehicle type assigned to a requirement will transport directly from the POD to Destination, and back and forth as necessary, until the entire requirement has completely been delivered. Thus, the model requires data on how many direct trips may be completed in a single day. The parameter w_{nijmk} details the approximate number of daily cycles that can be completed by a Mode m, Type k vehicle delivering Requirement n from POD i to Destination j. These approximate cycle values must be calculated before being input into the model and should take into account outloading and unloading times as well as distance between locations and vehicle speeds. Interested readers are encouraged to reference Longhorn & Kovich (2012) to see their cycle calculations.

The decision variable of the TDM is x_{nijmkv}, which describes the number of vehicles of Mode m, Type k that are required on Day v to deliver Requirement n from POD i to Destination j. Thus, the decision variables provide much pertinent information when assessing a vehicle mixture solution output. Table 2 - Table 4 below summarize the sets, parameters, and decision variables utilized in the TDM's pure integer programming formulation.

Table 2. TDM Sets

Set	Description
N	Set of all Movement Requirements n
I	Set of all PODs i
J	Set of all Destinations j
M	Set of all vehicle Modes m
K	Set of all vehicle Types k
V	Set of all possible delivery Days v

Table 3. TDM Parameters

Parameter	Description
b_k	Daily operating cost for a Type k vehicle
p_k	Average payload of a Type k vehicle
r_{nij}	Total weight (in short tons) of Requirement n that must be delivered from POD i to Destination j
ad_n	Day in which Requirement n arrives at its given POD
rd_n	Day describing the Required Delivery Date (RDD) at the given Destination for Requirement n
qd_n	Maximum allowable extension days beyond RDD in which Requirement n can be delivered late to given destination (with penalty)
g	Late penalty per vehicle per day
o_{imv}	Maximum number of Mode m vehicles that can be outloaded at POD i on Day v
u_{jmv}	Maximum number of Mode m vehicles that can be unloaded at Destination j on Day v
w_{nijmk}	Number of possible cycles in a day between POD i and Destination j via Mode m, Type k vehicles transporting Requirement n

Table 4. TDM Decision Variables

Variable	Description
x_{nijmkv}	Number of vehicles of Mode m, Type k that are required on Day v to deliver Requirement n from POD i to Destination j

TDM Formulation.

The TDM, a pure integer linear program formulated by Longhorn & Kovich, is shown below in Model 1.

$$\text{Minimize} \sum_{N}\sum_{I}\sum_{J}\sum_{M}\sum_{K}\sum_{v=ad_n+1}^{rd_n+q_n} b_k x_{nijmkv} + \sum_{N}\sum_{I}\sum_{J}\sum_{M}\sum_{K}\sum_{v=rd_n+1}^{rd_n+q_n} g(v-rd_n)x_{nijmkv} \qquad (1)$$

Subject to

$$\sum_{M}\sum_{K}\sum_{v=ad_n+1}^{rd_n+q_n} w_{nijmk}\, p_k x_{nijmkv} \ge r_{nij} \qquad \forall n \forall i \forall j \qquad (2)$$

$$\sum_{N}\sum_{J}\sum_{K} w_{nijmk} x_{nijmkv} \le o_{imv} \qquad \forall i \forall m \forall v \qquad (3)$$

$$\sum_{N}\sum_{I}\sum_{K} w_{nijmk} x_{nijmkv} \le u_{jmv} \qquad \forall j \forall m \forall v \qquad (4)$$

$$x_{nijmkv} \in \{0\} \cup \mathbb{Z}^{+} \qquad \forall n \forall i \forall j \forall m \forall k \forall v \qquad (5)$$

Model 1. Theater Distribution Model (TDM)

The TMD has two objectives, both of which are captured in a single objective function seen in (1). The objective minimizes the cost of vehicles allocated to execute the deliveries and minimizes the number of late vehicles. Recall a late vehicle is one that delivers a requirement on an extension day, after its stated RDD. Though the penalty value g in the objective is user-defined, it should be scaled large enough to ensure that it is less-preferred to any potential costs associated with on-time movement. Because the penalty factor is multiplied by the number of days past the RDD that the delivery is made, increased lateness causes higher penalties. Thus, this objective will seek minimum cost vehicle mixtures that will meet all delivery requirements while also minimizing lateness.

The two objectives are combined into a single objective function through the use of the weighted sum method, albeit with the weight on each objective set to 1. In other words, the objectives are simply added together. Readers interested in the weighted sum method are directed to Ehrgott (2010). While both objectives are weighted equally, an appropriately high penalty value in the latter objective steers solutions away from late requirement deliveries, which would incur penalties and yield high objective values.

There are three general sets of constraints in the model, including demand, outloading, and unloading constraints. The demand constraints at (2) ensure that enough vehicles, and thus capacity, are selected to deliver each requirement's weight. This constraint specifically allows for delivery to be accomplished through a combination of different vehicle types. Constraints at (3) ensure that the vehicles departing each POD do not exceed the specific outloading capacity of each specific POD, Mode, and Day combination. Likewise, (4) ensures that unloading capacities at Destinations are not violated. Lastly, (5) dictates that vehicle decision variable values may only take on either zero or nonnegative integer values.

Because the decision variables are indexed across so many different sets, much information is conveyed by the decision variables once the TDM is solved. For example, one decision variable and value taken from an arbitrary solution might be $x_{6,VTFP,WMAL,Air,C130,5} = 4$. This means that Requirement 6, being delivered from POD VTFP to Destination WMAL would require 4 C-130 aircraft on Day 5 to complete delivery. Thus, appropriate post-processing can inform analysts greatly.

TDM Conclusion.

The TDM was developed specifically for force flow analysis with the purpose of analyzing the movement of requirements in a multimodal network with differing vehicle types while seeking optimal vehicle allocations for requirements. Thus, the goal of the TDM is to provide feasible vehicle mixtures that would sustain movement operations based upon TPFDD requirements and outload and unload capabilities at PODs and Destinations. This would be an improvement over current force flow analysis processes in which vehicle mixtures are found essentially through trial and error.

Conclusion

Much of the previous research on theater distribution has involved the precise routing and scheduling of individual vehicles within a network. However, these types of models are simply too high-fidelity for use at USTRANSCOM force flow conferences. Additionally, many related optimization problems such as the PDPTW are also routing-focused at the individual vehicle level. However, when assessing theater distribution from a force flow analysis standpoint, approximate vehicle mixtures are preferred. For this reason, the TDM does not develop routes and instead assumes allocated vehicles will travel directly between its requirement's stated POD and Destination.

Another key difference between the TDM and other previous models is that most approaches, such as the PDPTW and Tabu Search, assume that a predetermined set of vehicles are available for the model to route and schedule. For example, one might say that 20 vehicles are available in a PDPTW. Thus, the overall capacity of transportation assets within the network is defined up front and the model attempts to route and schedule those 20 vehicles. However, in the TDM, no such overall transportation capability is input. In fact, the transportation capability is exactly what the model outputs as decision variables. That is, the TDM gives the minimum-cost set of vehicles that will sufficiently support requirement delivery. This is a better approach than limiting vehicles up front, as any output vehicle mixture deemed unsatisfactory by decision makers can be modified by either redesigning operations or implementing policy changes, such as including other vehicle types, or by adding more port capabilities.

While the proposed TDM detailed in this chapter can offer some insight into theater distribution, it has great room for improvement. The solution methodologies outlined in this thesis are aimed at improving the pure integer programming TDM in both ease of solving and also in goodness of solutions, providing for better theater distribution force flow analysis. Chapter III details the methodology which results in an improved model.

III. Methodology

Introduction

This research is carried out in three distinct steps. Firstly, work is conducted to drastically reduce the problem size of the TDM. The TDM includes a number of extraneous decision variables, causing the associated constraint matrix to be extremely sparse. Additionally, numerous unnecessary constraints are included. To reduce computational difficulties by ridding the problem of unnecessary variables and constraints, the Reduced Theater Distribution Model (RTDM) is developed. Next, once model reduction is complete, the mixed integer programming Improved Theater Distribution Model (ITDM) is developed which maintains model reduction principles but changes the modeling process by introducing a set of continuous decision variables. Lastly, analysis is conducted on the models.

Assumptions

Many assumptions are drawn directly from Longhorn & Kovich (2012). Allocated vehicles are assumed to travel only between their stated POD and Destination. That is, vehicles may not pick up at multiple PODs nor deliver to multiple Destinations. Furthermore, a vehicle allocated at a POD can never accomplish the delivery of requirements leaving from another POD. Additionally, it is assumed that for all transportation modes, there is only one (if any) path between two locations. It is also assumed that requirements may not leave their POD until the day following their arrival at the POD. Thus, a requirement's delivery window goes from the day after its arrival at the POD to the RDD plus any extension days. For post-processing, it is assumed that vehicles allocated at a POD for the distribution of requirements are eligible to be utilized in subsequent days as well. Lastly, it is assumed that any requirement may be placed on any vehicle, and that requirements may be split in any possible way and any number of times. Again, as this model only approximates vehicle mixtures, precise modeling of the exact shape and type of each requirement and/or vehicle is not conducted. Lastly, outload and unload constraints are applied to modes only, not specific vehicle types.

Reduced Theater Distribution Model (RTDM)

RTDM Motivation.

As detailed thoroughly in Chapter II, the TDM prescribes the number and type of vehicles, along with timing information, needed to successfully conduct a theater distribution operation. However, as formulated, the model can be incredibly burdensome to generate. This is because the formulation leads to a large number of decision variables and numerous unnecessary constraints.

For example, recall the TDM objective function, (1) which contains summations which go across the entire sets $N, I, J, M, K,$ as well as portions of V. Because of this, decision variables x_{nijmkv} are created for every possible combination of indices n, i, j, m, k along with some values of v. However, many of the 6-tuples (n, i, j, m, k, v) correspond with unrealistic, and even impossible, decisions. For example, consider the sample sets below in Figure 2.

$N =$	{1,2,3}
$I =$	{A,B}
$J =$	{C,D}
$M =$	{Air, Road}
$K =$	{C-130, M1083}
$V =$	{3,4,5,6}

Figure 2. Arbitrary Example Sets

Assuming Day 4 is within the deliver window for Requirement 2, that is that $ad_2 + 1 \le 4 \le rd_2 + qd_2$, one possible 6-tuple (n,i,j,m,k,v) from the given sets is $(2, A, C, Road, C-130, 4)$. This 6-tuple corresponds with decision variable $x_{2,A,C,Road,C-130,4}$ which would be generated within the integer program's objective. However, this decision variable is illogical, for the C-130 is an aircraft platform, and is not a vehicle of Mode Road.

Mathematically, $x_{2,A,C,Road,C-130,4}$, and other decision variables with similar circumstances, will always be zero upon solving the model. Because the C-130 is not of the Mode Road, there can be no daily cycles between POD A and Destination C for Mode Road, Type C-130 vehicles, regardless of Requirement number. Thus, in parameter input, a user would define the daily cycles parameter $w_{2,A,C,Road,C-130} = 0$, to demonstrate no movement via this Mode/Type combination is possible. With $w_{2,A,C,Road,C-130} = 0$, $w_{2,A,C,Road,C-130}x_{2,A,C,Road,C-130,4} = 0$. Therefore, giving $x_{2,A,C,Road,C-130,4}$ any nonzero value adds to the objective but fails to impact constraints (2) through (4) in the model. In particular, the requirement's demand constraint, where delivery is enforced, would not be met at all by giving such a decision variable nonzero value.

Therefore, the TDM does not give variables such as $x_{2,A,C,Road,C-130,4}$ a nonzero value as that would absolutely increase the objective while failing to impact any of the constraints. Thus, because this decision variable, and others like it, will always be zero and have no impact on the solution, they should not be generated and included in the model. The same can be said for extraneous decision variables unnecessarily generated by TDM constraints.

In addition to extraneous variables being generated by the model, the TDM also creates numerous unnecessary constraints with a right-hand side (RHS) of 0. For example, recall our sample sets in Figure 2. Again, assume that Requirement 2 is to be delivered from A to C and has weight of 100 short tons. Then $r_{2,A,C} = 100$, by definition of parameter r_{nij}. Furthermore, $r_{2,A,D} = r_{2,B,C} = r_{2,B,D} = 0$ because Requirement 2 is not delivered along any of those POD i, Destination j pairs. Then when implementing Constraints (2) for all combinations of i and j with $n = 2$, the following four constraints are obtained:

$$\sum_M \sum_K \sum_{v=ad_n+1}^{rd_n+q_n} w_{nijmk} p_k x_{nijmkv} \geq 100 \qquad n = 2, i = A, j = C$$

$$\sum_M \sum_K \sum_{v=ad_n+1}^{rd_n+q_n} w_{nijmk} p_k x_{nijmkv} \geq 0 \qquad n = 2, i = A, j = D$$

$$\sum_M \sum_K \sum_{v=ad_n+1}^{rd_n+q_n} w_{nijmk} p_k x_{nijmkv} \geq 0 \qquad n = 2, i = B, j = C$$

$$\sum_M \sum_K \sum_{v=ad_n+1}^{rd_n+q_n} w_{nijmk} p_k x_{nijmkv} \geq 0 \qquad n = 2, i = B, j = D$$

Note that the latter three constraints are completely unnecessary. As the TDM assumes that each of $x_{nijmkv}, w_{nijmk}, p_k \geq 0$, it is clear that the latter three constraints above will always be trivially greater than or equal to 0 and thus satisfied. Therefore, their inclusion in the model is unwarranted because the constraints will always be satisfied regardless of decision variable or parameter values. A similar happening occurs with the TDM's outloading and unloading constraints in that extra unneeded constraints may also be created.

While including superfluous decision variables with a value of zero and unnecessary constraints in the model will not dictate different solutions, it may have drastic impacts on memory allocation and problem size. Recall that a large scale TPFDD may have thousands of requirements, hundreds of Days, and numerous PODs, Destinations, Modes, and Vehicles. Thus, as the problem increases in size, many more 6-tuples (n,i,j,m,k,v) are possible and thus many more decision variables must be generated even though many may, by default, have value of 0 as discussed above. This causes the constraint matrix to become increasingly sparse, possibly causing problems to become intractable if enough computer memory is not available to generate or solve the problem. Even if the problem is tractable, the extraneous variables and unnecessary constraints increase the problem size and thus slow solution time.

To avoid this dilemma, a Reduced Theater Distribution Model (RTDM) is designed which sensibly reduces the problem while keeping all necessary variables and constraints intact. This is done in two ways. Firstly, decision variables are generated by the model only when there exists a chance for a decision variable to become nonzero, which implies that a vehicle allocation is theoretically possible. Secondly, constraints that do not affect the feasible space are not entered into the model. These problem reducing concepts are implemented with a series of decomposing sets and binary functions which are used to determine which portions of a set to sum through, as well as which constraints are valid and necessary constraints to include in the model.

RTDM Overview.

While the parameters and decision variables from the TDM remained unchanged in the RTDM, new sets are introduced with the purpose of reducing model sparsity and ridding the problem of unnecessary variables and constraints. This assists in quicker model generation. Some of the sets are simple decomposing sets and some sets require the use of binary functions to determine inclusion. These sets, paired with an adjusted formulation, greatly reduce the problem size while keeping the concepts and intent of the TDM fully intact. This subsection will detail changes to the sets that are utilized in the RTDM.

Firstly, new decomposing sets are introduced. These sets simply decompose the original TDM sets of M, K and N. The set M_{ij} is introduced to describe the eligible modes that may be selected between any POD i and Destination j. For example, if Air and Road are possible transportation modes between i and j, but Rail is not, then $M = \{Air, Road, Rail\}$ yet $M_{ij} = \{Air, Road\}$. The RTDM also introduces the set K_m which describes the set of vehicles Types $k \in K$ which are of Mode m. For example, K_{Air} may contain the air platforms C-130, C-5, and C-17. The set N_i is introduced to include only requirements $n \in N$ such that Requirement n departs POD i. Likewise, the set N_j is introduced to include only requirements $n \in N$ such that Requirement n

arrives at Destination j. These decomposing sets are easily determined with preprocessing and are of great value in reducing problem size by eliminating extraneous decision variable creation within constraints.

In addition to the decomposing sets, the RTDM also utilizes five Function Derived Tuple Sets: $VOTM$, VLM, VR, VO, and VU. Binary functions are used to evaluate the inclusion of tuples within these sets. Thus, these sets can be utilized to determine which tuples' corresponding variables should be included within the objective and constraints. Functions (6) to (11) below describe the binary functions used to create the new sets.

$$A(n,v) = \begin{cases} 1, \text{ if Requirement } n \text{ delivered on Day } v \text{ would be on-time} \\ 0, \text{ otherwise} \end{cases} \tag{6}$$

$$B(n,v) = \begin{cases} 1, \text{ if Requirement } n \text{ delivered on Day } v \text{ would be late} \\ 0, \text{ otherwise} \end{cases} \tag{7}$$

$$C(m,k) = \begin{cases} 1, \text{ if vehicle of Type } k \text{ is also a Mode } m \text{ vehicle} \\ 0, \text{ otherwise} \end{cases} \tag{8}$$

$$D(n,i,j) = \begin{cases} 1, \text{ if Requirement } n \text{ is to be delivered from POD } i \text{ to Destination } j \\ 0, \text{ otherwise} \end{cases} \tag{9}$$

$$E(i,m,v) = \begin{cases} 1, \text{ if } \exists \text{ some Requirement } n \text{ that may outload at POD } i \text{ onto a Mode } m \text{ vehicle on day } v \\ 0, \text{ otherwise} \end{cases} \tag{10}$$

$$F(j,m,v) = \begin{cases} 1, \text{ if } \exists \text{ some Requirement } n \text{ that may unload at Destination } j \text{ off of a Mode } m \text{ vehicle on day } v \\ 0, \text{ otherwise} \end{cases} \tag{11}$$

The first set, $VOTM$, describes 6-tuples which are utilized in the decision variables x_{nijmkv}. The set $VOTM$, or Valid On Time Movements, yields tuples which correspond to decision variables indicating valid, on-time movements. Mathematically, $VOTM = \{(n,i,j,m,k,v) \mid A(n,v) \cdot C(m,k) \cdot D(n,i,j) = 1\}$. This implies that Requirement n is eligible to deliver from POD i to Destination j via a Mode m, Type k vehicle on Day v where $v \le rd_n$. Proper decision variable tuples in $VOTM$ may not have Mode/Type mismatches, delivery Days after the RDD, or POD/Destination pairs that are not the proper, designated POD and Destination for specific requirements. Thus, for decision variables x_{nijmkv}, Functions (6), (8), and (9) work together to determine if the corresponding 6-tuple (n,i,j,m,k,v) warrants inclusion in the set $VOTM$. Function (6) determines if Requirement n would be on-time if shipped on Day v. Function (8) determines if a Type k vehicle is of Mode m and Function (9) checks to ensure that Requirement n ships from i to j. Only if all functions return a value of 1, and thus the product of the functions is also 1, will the 6-tuple be included in the set $VOTM$ and the corresponding decision variable be generated and placed in the objective.

The second set, VLM, also describes 6-tuples which are utilized in the decision variables. The set VLM corresponds to decision variables for Requirement n shipping from POD i to Destination j via a Mode m, Type k vehicle on Day v such that $rd_n < v \le rd_n + qd_n$. This set is dissimilar to $VOTM$ in that it describes 6-tuples (n,i,j,m,k,v) whose corresponding decision variable would indicate a requirement being delivered past the RDD. Inclusion in VLM requires that a 6-tuple's associated decision variable not imply a Mode/Type mismatch, POD/Destination mismatch, or delivery prior to or on the RDD. Therefore, $VLM = \{(n,i,j,m,k,v) \mid B(n,v)\cdot C(m,k)\cdot D(n,i,j) = 1\}$. Functions (8), and (9) work as described in $VOTM$ and Function (7) determines if the decision variable would indicate Requirement n being delivered late after the RDD. If such conditions are met, a 6-tuple's corresponding decision variable will be generated and included in the objective function.

The final three Function Derived Tuple Sets are utilized for ridding the formulation of unnecessary constraints. The set of Valid Routes is defined by Function (9). That is, $VR = \{(n,i,j) \mid D(n,i,j) = 1\}$. As each Requirement n has only a single POD i and Destination j, there is only a single 3-tuple for each Requirement n that describes its one and only Valid Route. Function (10) checks whether or not for a given 3-tuple (i,m,v), some Requirement $n \in N$ may outload at POD i onto a Mode m vehicle on Day v. This is used to construct the set of Valid Outload tuples, VO. Mathematically, $VO = \{(i,m,v) \mid E(i,m,v) = 1\}$. Likewise, Function (11) utilizes the same methodology to construct Valid Unload tuples, VU. The set VU is defined mathematically by $VU = \{(j,m,v) \mid F(j,m,v) = 1\}$. All of the new sets discussed lead to the reduced formulation of the RTDM by eliminating extraneous decision variables and unnecessary constraints from the problem. Table 5 - Table 8 below summarize the sets, parameters, and decision variables utilized in the pure integer programming RTDM.

Table 5. RTDM Basic Sets

Set	Description
N	Set of all Movement Requirements n
I	Set of all PODs i
J	Set of all Destinations j
M	Set of all vehicle Modes m
K	Set of all vehicle Types k
V	Set of all possible delivery Days v
M_{ij}	Set of all Modes m with direct paths between POD i and Destination j
K_m	Set of all vehicle Types k which are of Mode m
N_i	Set of movement Requirements n that depart from POD i
N_j	Set of movement Requirements n that arrive at Destination j

Table 6. RTDM Function Derived Tuple Sets

Set	Description	Mathematical Notation
$VOTM$	Valid On-Time Movements	$\{(n,i,j,m,k,v) \mid A(n,v)\cdot C(m,k)\cdot D(n,i,j)=1\}$
VLM	Valid Late Movements	$\{(n,i,j,m,k,v) \mid B(n,v)\cdot C(m,k)\cdot D(n,i,j)=1\}$
VR	Valid Routes	$\{(n,i,j) \mid D(n,i,j)=1\}$
VO	Valid Outloading	$\{(i,m,v) \mid E(i,m,v)=1\}$
VU	Valid Unloading	$\{(j,m,v) \mid F(j,m,v)=1\}$

Table 7. RTDM Parameters

Parameter	Description
b_k	Daily operating cost for a Type k vehicle
p_k	Average payload of a Type k vehicle
r_{nij}	Total weight (in short tons) of Requirement n that must be delivered from POD i to Destination j
ad_n	Day in which Requirement n arrives at its given POD
rd_n	Day describing the Required Delivery Date (RDD) at the given Destination for Requirement n
qd_n	Maximum allowable extention days beyond RDD in which Requirement n can be delivered to given destination (with penalty)
g	Late penalty per vehicle per day
o_{imv}	Maximum number of Mode m vehicles that can be outloaded at POD i on Day v
u_{jmv}	Maximum number of Mode m vehicles that can be unloaded at Destination j on Day v
w_{nijmk}	Number of possible cycles in a day between POD i and Destination j via Mode m, Type k vehicles transporting Requirement n

Table 8. RTDM Decision Variables

Variables	Description
x_{nijmkv}	Number of vehicles of Mode m, Type k that are required on Day v to deliver Requirement n from POD i to Destination j

RTDM Formulation.

The RTDM, which greatly reduces problem size, is shown below in Model 2.

$$\text{Minimize} \sum_{(n,i,j,m,k,v)\in VOTM\cup VLM} b_k x_{nijmkv} + \sum_{(n,i,j,m,k,v)\in VLM} g(v-rd_n)x_{nijmkv} \tag{12}$$

Subject to

$$\sum_{M_{ij}}\sum_{K_m}\sum_{v=ad_n+1}^{rd_n+q_n} w_{nijmk}\, p_k x_{nijmkv} \ge r_{nij} \qquad \forall (n,i,j)\in VR \tag{13}$$

$$\sum_{N_i}\sum_{J}\sum_{K_m} w_{nijmk} x_{nijmkv} \le o_{imv} \qquad \forall (i,m,v)\in VO \tag{14}$$

$$\sum_{N_j}\sum_{I}\sum_{K_m} w_{nijmk} x_{nijmkv} \le u_{jmv} \qquad \forall (j,m,v)\in VU \tag{15}$$

$$x_{nijmkv} \in \{0\}\cup \mathbb{Z}^+ \qquad \forall (n,i,j,m,k,v)\in VOTM\cup VLM \tag{16}$$

Model 2. Reduced Theater Distribution Model (RTDM)

The purpose of the RTDM objective and constraints remain the same as discussed in the original TDM (page 20)—to find on-time, least-cost vehicle allocations to accomplish delivery. However, the model is reduced significantly by taking advantage of the new sets introduced in the RTDM Overview subsection. With the RTDM, the objective (12) retains the purpose of minimizing both vehicle utilization costs and penalties for utilizing vehicles for late deliveries.

By summing across all 6-tuples in $VOTM \cup VLM$, the first part of the objective function multiplies vehicle operating cost b_k and decision variable x_{nijmkv} for each and every theoretically possible decision variable. However, no extraneous decision variables, those with 6-tuples $(n,i,j,m,k,v) \notin VOTM \cup VLM$, are generated. Likewise, only the logical decision variables whose 6-tuples correspond to late movements, that is those where $(n,i,j,m,k,v) \in VLM$, are multiplied by the penalty factor. Thus, the objective function includes the all theoretically possible decision variables and associated costs and penalties.

The RTDM constraints shown in (13) to (16) are the demand, outloading, unloading, and integrality constraints for the model. These are similar to (2) through (5) of the TDM. However, the left-hand side (LHS) summations in the RTDM constraints do not simply go across entire sets. Instead, some decomposing sets are utilized, which keeps extraneous variables from being created. Additionally, the “for all” statements for each general constraint that dictate which combinations of variables are used to generate a constraint are restricted in the RTDM. Recall that the TDM generated constraints for each and every combination of indices for the requirement, outloading, and unloading constraints. However, this is not necessary and thus the RTDM ensures a totally reduced format.

In the demand constraint at (13), the LHS summation is across sets M_{ij}, K_m, and appropriate values of v. Thus, the decomposed sets ensure extraneous variables are not included in the model. Likewise, only necessary demand constraints are included in the model because a constraint is only generated for $(n,i,j) \in VR$. Thus, the use of the Function Derived Tuple Set VR ensures that unnecessary constraints are not generated when the 3-tuple (n,i,j) is illogical.

In the outload and unload constraints at (14) and (15), the sets N_i and N_j, respectively, are utilized in the LHS summations in place of the set N as was done in the TDM. Additionally, the set K_m is utilized rather than K. Again, the use of these decomposing sets ensures that extraneous variables are not generated in the RTDM. Furthermore, 3-tuples are checked for inclusion in the Function Derived Tuple Sets to check if a constraint should be made. A 3-tuple in VO will generate a necessary outload constraint and a 3-tuple in VU will generate an unloading constraint. Constraints are not constructed for 3-tuples not included in VO or VU as they would have no impact on the feasible space.

RTDM Conclusion.

By restricting the objective function to consider only theoretically possible variables, and using decomposing sets on summations on the LHS of the constraints, the RTDM ensures that no extraneous decision variables are created. Only those decision variables that may theoretically take on nonzero value are included. Properly conducted preprocessing and the use of binary functions to determine set inclusion guarantees that no decision variable is taken out that could potentially take on a nonzero value. Furthermore, limiting the tuples for which constraints are generated reduces the total number of constraints in the model. Because only extraneous decision variables are removed and no constraints that affect the feasible space are removed, solving the same arbitrary problem with both the TDM and RTDM should yield the same objective value and solution. The difference will be in number of decision variables, number of constraints, and problem size. Thus, a reduced formulation yielding the same vehicle allocations given by the TDM can be successfully, and more easily, generated and attained with the RTDM.

Improved Theater Distribution Model (ITDM)

ITDM Motivation.

While the RTDM greatly reduces problem size by removing extraneous decision variables and unnecessary constraints, the core of the modeling formulation remains unchanged from the TDM. The RTDM's pure integer programming formulation includes an objective for on-time least cost vehicle mixtures along with three general constraints which are demand, outloading, and unloading. However, research into RTDM solutions indicate changes are needed to the formulation, particularly with respect to new decision variables and constraints. Thus, a mixed integer linear program is developed, known as the Improved Theater Distribution Model (ITDM) which improves upon the pure integer program RTDM. In making these new additions, the ITDM also requires some new sets to make certain that, like the RTDM, the ITDM is minimally formulated to ensure no extraneous decision variables or unnecessary constraints are generated. The ITDM is the main contribution of this research, encompassing both model reduction and a new mixed integer programming approach to force flow analysis.

Recall that the decision variable of the TDM and RTDM was x_{nijmkv}, representing the number of vehicles of Mode m, Type k that are required on Day v to deliver Requirement n from POD i to Destination j. That is, each requirement is associated with a specific mixture of vehicles and accompanying delivery dates, indicated by those decision variables assuming nonzero value. However, there is an inherent flaw in this choice of decision variable as it requires that each Requirement n

be allocated at least one vehicle specifically for that requirement. This construct does not necessary match reality. For example, consider two requirements, each with the exact same attributes of POD, Destination, Arrival Date at POD (ad_n) and Required Delivery Date at Destination (rd_n). If both requirements each weigh only 10 short tons, it should be clear that the two requirements could possibly be allocated to a single 20 short ton transport vehicle. However, modeling with the RTDM (or TDM) would not allow this, as each vehicle is specifically matched with a requirement due to decision variables having an index of Requirement n. The mixed integer formulation presented in the ITDM overcomes this shortfall.

The RTDM also models lateness poorly and the ITDM addresses this. This improvement is important as a model that inappropriately models lateness may give solutions that are not truly representative of the best on-time, least-cost solution. Recall that in the TDM/RTDM formulations, lateness was penalized per vehicle per day late. However, it is clear that two vehicles arriving equally late would not necessarily deserve to be penalized equally. Arbitrarily, assume that a truck delivers only two short tons late while an aircraft delivers 50 short tons late. Logically, the aircraft holding the larger cargo shipment should be penalized more severely for lateness. However, the RTDM does not consider this. The RTDM only measures the truck and the aircraft as a single, late vehicle. However, the ITDM penalizes lateness not by measuring the number of vehicles that arrive late per day, but rather, how many short tons arrive late per day. The next subsection will explain concepts developed in the ITDM before the model formulation is given.

ITDM Overview.

The ITDM has two different types of decision variables. Rather than examining vehicle decisions only as with the RTDM, the ITDM has one set of decision variables to model the flow of requirement short tonnage throughout the network and another to represent the vehicles necessary to support these flows. Continuous decision variables y_{nijmkv} are utilized to represent the number of short tons of Requirement n being delivered by a Mode m, Type k vehicle from POD i to Destination j on Day v. Additionally, the integer decision variable x_{ijmkv} is used, representing the number of vehicles of Mode m, Type k that are needed on Day v to deliver any requirements from POD i to Destination j. Note that the integer vehicle variables are not tied to any particular requirement number, n. Thus, the vehicle allocations dictated by decision variables x_{ijmkv} may embody the movement of one, or many different requirements. Furthermore, both on-time and late cargo may be delivered on the same vehicle. The use of both continuous and integer decision variables allows for a much more accurate representation vehicle use.

In regards to sets utilized in the ITDM, many are carried over from the RTDM (page 34), namely the sets $N, I, J, M, K, V, K_m, M_{ij}, VR, VO,$ and VU. Like the RTDM, the ITDM addresses model reduction. However, the ITDM's differing decision variables causes some different decomposing sets and Function Derived Tuple Sets to be implemented in the ITDM. The RTDM sets $VOTM, N_{i,}$ and N_j are not utilized in the ITDM. Four new sets are introduced with the ITDM, three of which are derived from functions as well as a single new decomposing set. Together, these new sets work to ensure a reduced model that removes all extraneous flow and vehicle decision variables from both the objective and constraints, as well as ridding the problem of unnecessary constraints.

While much model reduction in the ITDM is carried out using the same binary functions developed in (6) to (11) from the RTDM, one new binary function is introduced with the ITDM. The new binary set defining function $G(i, j, v)$ is introduced which determines whether or not there exists any Requirement $n \in N$, from POD i to Destination j, that may be delivered, either on-time or late, on Day v. This binary function appears below:

$$G(i, j, v) = \begin{cases} 1, \text{ if } \exists \text{ some Requirement } n, \text{ from POD } i \text{ to Destination } j \text{ s.t. } ad_n + 1 \le v \le rd_n + qd_n \\ 0, \text{ otherwise} \end{cases} \tag{17}$$

This binary function is crucial in the creation of vehicle decision variables within the ITDM, which populate the set VV, or Valid Vehicles. The set of tuples in VV, where $VV = \{(i, j, m, k, v) \mid G(i, j, v) \cdot C(m, k) = 1\}$, includes those tuples which correspond to valid vehicle variables that may take on value within the mixed integer program. Thus, a vehicle variable is created only when the 5-tuple (i, j, m, k, v) corresponds to a theoretically possible vehicle assignment.

Additionally, the ITDM also reduces the tuples utilized for flow decision variables. Two sets are utilized. The first set, Valid Flows, yields 6-tuples (n, i, j, m, k, v) which correspond to a valid decision variable on flow within the network, both on-time and late. This set is defined mathematically as

$VF = \{(n,i,j,m,k,v) \mid A(n,v)\cdot C(m,k)\cdot D(n,i,j) + B(n,v)\cdot C(m,k)\cdot D(n,i,j) = 1\}$.

Additionally, the set Late Flows describes valid 6-tuples which correspond to a valid flow decision variable which indicate late movement. Mathematically, this set is defined as $LF = \{(n,i,j,m,k,v) \mid B(n,v)\cdot C(m,k)\cdot D(n,i,j) = 1\}$. Note that LF is mathematically equivalent to the RTDM set VLM . However, the tuples in this case correspond to flow variables, not vehicle variables.

The decomposing set N_{ijv} is also introduced which includes all requirements $n \in N$ which are to be delivered from POD i to Destination j and are eligible to deliver on Day v . This set is crucial to one of the main constraints of the problem, the vehicle linking constraint, which ensures that enough vehicles are allocated to move the necessary requirements.

The parameters of the ITDM remain mostly the same, save for two slight, yet important, adjustments. Firstly, the penalty parameter, g , no longer represents the penalty per vehicle per day late. This is because in the ITDM, lateness is measured by short tons delivered late rather than vehicles delivering late. Thus, in the ITDM, the penalty variable g actually represents the late penalty per short ton per day delivered late. The cycle parameter also changes within the ITDM. While its purpose remains the same, the index of $n \in N$ is removed from the cycle parameter. Thus, in the ITDM, cycles are given by the parameter w_{ijmk} . Recall that in the TDM /RTDM formulations, cycle values w_{nijmk} were defined by their 5-tuples (n,i,j,m,k) . However, because a cycle is simply a time and distance calculation for a Mode m , Type k vehicle along the path from i to j, the requirement number is irrelevant. Thus, a cycle value is just as insightful when only defined across the 4-tuple (i, j, m, k).

The seven set defining binary functions utilized in the ITDM are listed below in Functions (6)-(11), and (17). Following the functions, the sets, parameters, and decision variables of the ITDM are listed in Table 9 to Table 12.

$$A(n,v) = \begin{cases} 1, \text{ if Requirement } n \text{ delivered on Day } v \text{ would be on-time} \\ 0, \text{ otherwise} \end{cases} \tag{6}$$

$$B(n,v) = \begin{cases} 1, \text{ if Requirement } n \text{ delivered on Day } v \text{ would be late} \\ 0, \text{ otherwise} \end{cases} \tag{7}$$

$$C(m,k) = \begin{cases} 1, \text{ if vehicle of Type } k \text{ is also a Mode } m \text{ vehicle} \\ 0, \text{ otherwise} \end{cases} \tag{8}$$

$$D(n,i,j) = \begin{cases} 1, \text{ if Requirement } n \text{ is to be delivered from POD } i \text{ to Destination } j \\ 0, \text{ otherwise} \end{cases} \tag{9}$$

$$E(i,m,v) = \begin{cases} 1, \text{ if } \exists \text{ some Requirement } n \text{ that may outload at POD } i \text{ onto a Mode } m \text{ vehicle on day } v \\ 0, \text{ otherwise} \end{cases} \tag{10}$$

$$F(j,m,v) = \begin{cases} 1, \text{ if } \exists \text{ some Requirement } n \text{ that may unload at Destination } j \text{ off of a Mode } m \text{ vehicle on day } v \\ 0, \text{ otherwise} \end{cases} \tag{11}$$

$$G(i,j,v) = \begin{cases} 1, \text{ if } \exists \text{ some Requirement } n, \text{ from POD } i \text{ to Destination } j \text{ s.t. } ad_n + 1 \le v \le rd_n + qd_n \\ 0, \text{ otherwise} \end{cases} \tag{17}$$

Table 9. ITDM Basic Sets

Set	Description
N	Set of all Movement Requirements n
I	Set of all PODs i
J	Set of all Destinations j
M	Set of all vehicle Modes m
K	Set of all vehicle Types k
V	Set of all possible delivery Days v
M_{ij}	Set of all Modes m with direct paths between POD i and Destination j
K_m	Set of all vehicle Types k which are of Mode m
N_{ijv}	Set of Requirements n that are eligible to deliver from POD i to Destination j on Day v

Table 10. ITDM Function Derived Tuple Sets

Set	Description	Mathematical Notation
VV	Valid Vehicle	$\{(i,j,m,k,v) \mid G(i,j,v)\cdot C(m,k)=1\}$
VF	Valid Flows	$\{(n,i,j,m,k,v) \mid A(n,v)\cdot C(m,k)\cdot D(n,i,j)+B(n,v)\cdot C(m,k)\cdot D(n,i,j)=1\}$
LF	Late Flows	$\{(n,i,j,m,k,v) \mid B(n,v)\cdot C(m,k)\cdot D(n,i,j)=1\}$
VR	Valid Routes	$\{(n,i,j) \mid D(n,i,j)=1\}$
VO	Valid Outloading	$\{(i,m,v) \mid E(i,m,v)=1\}$
VU	Valid Unloading	$\{(j,m,v) \mid F(j,m,v)=1\}$

Table 11. ITDM Parameters

Parameter	Description
b_k	Daily operating cost for a Type k vehicle
p_k	Average payload of a Type k vehicle
r_{nij}	Total weight (in short tons) of Requirement n that must be delivered from POD i to Destination j
ad_n	Day in which Requirement n arrives at its given POD
rd_n	Day describing the Required Delivery Date (RDD) at the given Destination for Requirement n
qd_n	Maximum allowable extension days beyond RDD in which Requirement n can be delivered to given destination (with penalty)
g	Late penalty per short ton late per day
o_{imv}	Maximum number of Mode m vehicles that can be outloaded at POD i on Day v
u_{jmv}	Maximum number of Mode m vehicles that can be unloaded at Destination j on Day v
w_{ijmk}	Number of possible cycles in a day between POD i and Destination j for Mode m, Type k vehicles

Table 12. ITDM Decision Variables

Variables	Description
x_{ijmkv}	Number of vehicles of Mode m, Type k that are required on Day v to deliver any requirement(s) from POD i to Destination j
y_{nijmkv}	Short tons of Requirement n delivered from POD i to Destination j on Mode m, Type k vehicle(s) on Day v

ITDM Formulation.

The Improved Theater Distribution Model, the main thesis contribution, is formulated below in Model 3.

$$\text{Minimize} \sum_{(i,j,m,k,v)\in VV} b_k x_{ijmkv} + \sum_{(n,i,j,m,k,v)\in LF} g(v-rd_n) y_{nijmkv} \tag{18}$$

Subject to

$$\sum_{M_{ij}} \sum_{K_m} \sum_{v=ad_n+1}^{rd_n+q_n} y_{nijmkv} = r_{nij} \qquad \forall\ (n,i,j) \in VR \tag{19}$$

$$\sum_{J} \sum_{K_m} w_{ijmk} x_{ijmkv} \le o_{imv} \qquad \forall (i,m,v) \in VO \tag{20}$$

$$\sum_{I} \sum_{K_m} w_{ijmk} x_{ijmkv} \le u_{jmv} \qquad \forall (j,m,v) \in VU \tag{21}$$

$$\sum_{N_{ijv}} y_{nijmkv} \le x_{ijmkv} w_{ijmk} p_k \qquad \forall (i,j,m,k,v) \in VV \tag{22}$$

$$y_{nijmkv} \ge 0 \qquad \forall (n,i,j,m,k,v) \in VF \tag{23}$$

$$x_{ijmkv} \in \{0\} \cup \mathbb{Z}^{+} \qquad \forall (i,j,m,k,v) \in VV \tag{24}$$

Model 3. Improved Theater Distribution Model (ITDM)

The ITDM presents significant improvements over both the RTDM and TDM. Firstly, the introduction of flow decision variables allow for a better modeling process. Model 3 above demonstrates how both flow, y_{nijmkv}, and vehicle, x_{ijmkv}, variables are implemented where appropriate. Because decisions are made on both flow and vehicles, vehicles are no longer allocated to single requirements. Thus, in this formulation a specific mixture of vehicles may be matched with portions (measured in short tons) of requirements. Each vehicle mixture could support partial requirements or one or multiple requirements together. Because vehicles are no longer tied to specific requirements, a much more accurate modeling process is achieved.

The ITDM objective, given in (18), attempts to minimize vehicle costs while also looking to minimize penalties associated with the short tons being delivered late. Thus, optimal solutions to the ITDM will not necessarily include a minimal number of late vehicles, but rather a minimal number of late short tons. This presents a more realistic objective in terms of real-world considerations.

Constraints at (19) ensure that the sum of valid flow variables are large enough to equal the demand associated with each requirement. That is, each requirement must have associated flows that will meet the requirement's weight. Constraints at (20) and (21) ensure that the number of vehicles selected by the model do not exceed the outloading and unloading capacities, respectively. The TDM and RTDM had similar constraints, however, formulation is different because vehicles are no longer tied to specific requirements. Thus, no information on the requirement number is needed within these two outloading and unloading constraints as they are concerned only with vehicles.

The vehicle linking constraint at (22) is what ties together the continuous flow variables and the integer vehicle variables. It ensures that, for flow decisions corresponding to matching (i, j, m, k, v) values, enough vehicles are allocated to provide transportation capacity for appropriate requirements included as part of those flows. This allows vehicles to hold cargo from a number of different requirements. The constraint also allows late cargo from some number of requirements to be delivered with on-time cargo from other requirements. In a real-world scenario, there is nothing that would prevent this from happening. Lastly, (23) ensures that all flow variables are nonnegative and (24) ensures vehicle variables are nonnegative and integer.

ITDM Conclusion.

The ITDM is the main contribution of this thesis. The formulation, with use of two separate types of decision variables, adjusted constraints, and the addition of an important linking constraint, allows the ITDM to better model flow across a network and allocate vehicles to requirements while ensuring a minimum cost vehicle solution that also minimizes lateness can be adequately found. Furthermore, decomposing sets and Function Derived Tuple Sets are utilized to maintain a minimum size problem formulation, promoting tractability. Thus, the mixed integer formulation provides a useful, powerful tool that can aid in force flow analysis.

Measuring Vehicle Capacity Utilization

The Approximate Capacity Utilization (ACU) is defined as the total short tonnage included in the TPFDD divided by the approximate amount of cargo-space obtained by the model's vehicle allocations. The measure is approximate because any noninteger cycle values can make it difficult to estimate exactly how many vehicle allocations were possible. To calculate allocated cargo-space, each vehicle variable is multiplied by its payload and cycle value. This value is then summed for all vehicles (i.e. nonzero vehicle decision variables). By letting S represent the sum of all requirements' short tonnage listed in a TPFDD and letting X represent all nonzero vehicle decision variables, mathematically we may define ACU as

$$\frac{S}{\sum_{X} x_{nijmkv} p_k w_{nijmk}} \tag{25}$$

for the TDM/RTDM and

$$\frac{S}{\sum_{X} x_{ijmkv} p_k w_{ijmk}} \tag{26}$$

for the ITDM. Note that the summation goes across all nonzero vehicle decision variables, regardless of whether or not the model ties those vehicles to specific requirements. Thus, the ACU measures the same quantity in both formulae above.

The ACU is used to measure how well the model is allocating different vehicle resources to requirements. A low ACU implies that much of the capacity provided by the allocated vehicles is going unused. Conversely, an ACU near 100% implies vehicles are being used near their full capacity. In reality, it is highly unlikely that a vehicle is filled to 100% of its capacity every time. However, using the model, one can modify the average payload value, p_k, such that an ACU of 100% actually implies a smaller amount of "filling" is conducted.

For example, if a vehicle has a true payload of 20 short tons, analysts may determine that if the vehicle is filled to 15 short tons it would be a "good" load. Therefore, by setting $p_k = 15$, the model is actually assessing capacity based on a typical fullness amount rather than a vehicle's actual capacity. Thus, while the ACU may be near 100%, analysts can be sure that they are not making unrealistic allocations to vehicles.

Approximating Beddowns

After running the ITDM, it is possible to post-process solutions to develop a possible beddown at each POD for each vehicle type selected by the model. This information may be very useful to analysts who require information on the number of vehicles needed to conduct distribution. The process of approximating beddowns involves taking the vehicle allocation outputs from the model and deriving a possible vehicle beddown. Mathematically, the beddown of vehicles of Mode m, Type k needed at POD i can be approximated by

$$Beddown_{imk} = \max_{v \in V} \left(\sum_{J} x_{ijmkv} \right) \tag{27}$$

This measure will first find how many vehicle to requirement allocations are made within each day at every POD for every vehicle Type. The maximum allocation value across all days for each POD and vehicle Type will yield the approximate number of Mode m, Type k vehicles needed to be beddown at POD i. Thus, this measure converts the allocation decisions determined by the model into beddown information. For example, assuming integer cycle values, if the model states that two trains are needed at POD Alpha on Days 3, 4, and 5 to deliver requirements, then in actuality, the beddown is simply that two trains are needed at POD Alpha. This measure is applicable under the assumption that vehicles utilized at a POD on Day v will also be available again on all Days subsequent to v. With integer cycle values, vehicles will complete full cycles and be available at the POD again the very next day within the model. To achieve only integer cycle values, it may be best to simply take the floor of any noninteger calculated cycle value to ensure an overestimated solution rather than an underestimated solution, which would perhaps not allow for successful delivery.

Although the TDM/RTDM decision variables are indexed over requirements n, an equivalent beddown measure, shown below, can be utilized where variables tied to requirements only departing the POD under consideration are summed into the measure.

$$Beddown_{imk} = \max_{v \in V} \left(\sum_{N_i} \sum_{J} x_{nijmkv} \right) \tag{28}$$

Aggregation of Requirements

With each of the three aforementioned models, it is possible to aggregate some requirements as a pre-processing step, before the optimization model is run. If the EAD is used as a requirement's arrival date at the POD, then aggregation of requirements would entail combining requirements with the same POD, Destination, EAD, and RDD. If multiple requirements have the exact same values for these attributes, their short tonnage is combined and the requirements are represented by a new, single requirement**.** Figure 3 below demonstrates how 21 separate requirements drawn from a TPFDD are combined into a single requirement.

Requirement	POD	Destination	Short Tons	EAD	RDD
237	KUHE	KUHA	2.1	52	61
240	KUHE	KUHA	13	52	61
242	KUHE	KUHA	13	52	61
251	KUHE	KUHA	41.6	52	61
619	KUHE	KUHA	41.6	52	61
622	KUHE	KUHA	2.1	52	61
624	KUHE	KUHA	13	52	61
626	KUHE	KUHA	13	52	61
628	KUHE	KUHA	41.6	52	61
631	KUHE	KUHA	2.1	52	61
633	KUHE	KUHA	13	52	61
635	KUHE	KUHA	13	52	61
637	KUHE	KUHA	41.6	52	61
640	KUHE	KUHA	2.1	52	61
642	KUHE	KUHA	13	52	61
644	KUHE	KUHA	13	52	61
646	KUHE	KUHA	41.6	52	61
649	KUHE	KUHA	2.1	52	61
651	KUHE	KUHA	13	52	61
653	KUHE	KUHA	13	52	61
655	KUHE	KUHA	41.6	52	61

$\longrightarrow$

Requirement	POD	Destination	Short Tons	EAD	RDD
28	KUHE	KUHA	390.1	52	61

Figure 3. Aggregation of Like Requirements

Attaining fewer requirements will lead to fewer decision variables and may impact the number of vehicles necessary for delivery. However, because extension days are not listed on a TPFDD, but rather determined by commanders, aggregation may lead to incorrectly assigned extension days. Furthermore, it is not possible to disaggregate once aggregation has been conducted. Because aggregation may be implemented as a pre-processing step in the TDM, RTDM, or ITDM, aggregation only affects the input, specifically of requirements, into the model. The formulation and mathematics of each model remain unchanged.

Conclusion

This chapter has extensively detailed the models developed in this research, namely the RTDM and ITDM. The concept of aggregating requirements as a preprocessing step for inputs was also discussed. Additionally, an approximating measure for vehicle beddowns is introduced. The next chapter of this thesis will detail the implementation of these models on a handful of different test cases.

IV. Implementation and Results

Implementation

The RTDM and ITDM presented in Chapter III, along with the TDM, were implemented using both Microsoft Office Excel 2007 and the optimization software LINGO 13 (Lindo Systems Inc, 2012). A Decision Support System was built in the Excel environment where the user uploads a TPFDD and enters all other input parameters for the model. Once all data has been entered, Visual Basic for Applications (VBA) code is used within Excel to construct and write the math programming models to LINGO files. Once built, these files are solved in LINGO 13. Upon completion, the raw solution data is converted into information and then reported back within the Excel environment. All testing was conducted on a Dell Precision T7500 computer running Windows Vista (Service Pack 2) with two Intel Xeon W5590 processors and 48 GB of RAM.

To encourage fast solutions for larger models, a relative optimality tolerance setting was utilized. The solver was set to search for the true optimal solution for the first two minutes of solving. If, after those two minutes, the true optimal solution was not found, feasible solutions found within at least 0.2% of the Linear Program Relaxation lower bound were reported as globally optimal. Other LINGO 13 settings used in this analysis are available in Appendix A.

Recall from Chapter III that the TDM and RTDM unnecessarily index cycle parameters across requirement number n, as requirement numbers have no impact on the cycle value itself. Because the ITDM addresses this, ITDM cycle inputs are not indexed over the requirement number. Thus, two different cycle inputs are used in testing depending upon the model being implemented. However, cycle parameters for the ITDM were matched to align with the cycle parameters used in the TDM and RTDM. Therefore, comparisons between models remain sound.

Model Testing

In this analysis, each model (TDM, RTDM, ITDM) was tested on three different test cases. The first two test cases were entirely notional, while the third test case was a large-scale problem with data typical of an actual TPFDD. In all cases, solutions were found in less than 3 minutes, and small test cases (i.e. Test Cases 1 and 2) solved in less than a second.

For each test case, solution information regarding the number of air, road, and rail vehicle allocations made to requirements was collected. This information is drawn directly from the nonzero vehicle decision variables. For example, $x_{KUHE,KUHA,Air,C-130,4} = 3$, implying that 3 C-130s are needed on day 4 to deliver requirements from KUHE to KUHA, means that three vehicle allocations are made. Note that if the cycle value with matching tuple to this decision variable is greater than 1, more than one pickup and delivery is conducted with this allocation. For example, if $w_{KUHE,KUHA,Air,C-130} = 2$, then the three vehicle allocations actually imply six pickup and deliveries were made. Approximate Capacity Utilization values were also collected during testing.

Problem size information such as number of variables (integer and continuous) and number of constraints were also recorded. Potential vehicle beddowns derived from vehicle allocations are developed for the large scale solutions found in Test Case 3. Also, an example of how different inputs can be utilized to model policy decisions is shown using the ITDM. Lastly, model solutions are compared when the aggregation of like requirements is conducted before optimization.

In all tests, it was assumed that requirements arrived at the POD on the EAD stated in the TPFDD. Thus, for each requirement, ad_n is set to the requirement's EAD. Additionally, every requirement was given a single extension day within all test cases. That is, $qd_n = 1$ for all requirements.

Test Case 1.

The first test case utilized the exact TPFDD and data used as an example in the internal research paper by Longhorn & Kovich (2012). The TPFDD for this case listed 16 movement requirements, two PODs, and two Destinations. The TPFDD is shown below in Table 13. Note that the Short Tons column gives the r_{nij} values, the EAD column gives the ad_n values, and the RDD column gives the rd_n values. Note also that the possible delivery Days, including extension days, (i.e. the set V) ranged between Day 3 and Day 10. Three modes (Air, Rail, Road) and three vehicle Types (C-130, M1083, and DODX) were utilized. The penalty per day per late vehicle (TDM/RTDM) and penalty per day per short ton (ITDM) was set to $g = 1,000,000$. The payload, cost, outloading , unloading, and cycle parameters used are shown in Appendix B.

Table 13. TPFDD for Test Case 1

Requirement	POD	Destination	Short Tons	EAD	RDD
1	i1	j1	500	2	4
2	i1	j1	250	3	5
3	i1	j1	750	4	6
4	i1	j1	200	5	7
5	i1	j1	100	6	8
6	i1	j2	600	2	5
7	i1	j2	400	3	6
8	i1	j2	200	4	7
9	i1	j2	300	5	8
10	i1	j2	500	6	9
11	i2	j1	500	4	5
12	i2	j1	400	5	6
13	i2	j1	300	6	7
14	i2	j2	1000	3	5
15	i2	j2	200	5	7
16	i2	j2	500	7	9

After the TDM, RTDM, and ITDM were all tested on this case, the model outputs and statistics were collected, which appear below in Table 14 and Table 15. The solution information parsed from nonzero decision variables for Test Case 1 is shown in Figure 4 and Figure 5 with late vehicles/requirements highlighted.

Table 14. Test Case 1 Model Results

Model	Total Vehicles Allocated	Air Vehicles Allocated	Road Vehicles Allocated	Rail Vehicles Allocated	ACU[1]	Late Vehicles[2]	Late Short Tons[3]
TDM	166	41	94	31	97.0%	6	N/A
RTDM	166	41	94	31	97.0%	6	N/A
ITDM	161	41	90	30	99.7%	N/A	205

[1]Approximate Capacity Utilization
[2]TDM and RTDM only
[3]ITDM only

Table 15. Test Case 1 Model Statistics

Model	Objective Value	Constraints	Total Variables	Integer Variables	Continuous Variables
TDM	6,419,431	160	4608	4608	-
RTDM	6,419,431	106	100	100	-
ITDM	205,419,030	158	152	52	100

Note that the TDM and RTDM yield the same solution and model outputs. This is to be expected, as the models are conceptually the same. Meanwhile, the ITDM showed that five fewer vehicle allocations are actually necessary to move the TPFDD requirements. Thus the ITDM has a higher ACU than the TDM and RTDM. Although the two models output the same solution, the RTDM has much fewer constraints and variables than the TDM, yielding 33.5% and 97.8% reductions respectively. Meanwhile, the mixed integer ITDM offers a 1.3% decrease in constraints and a 96.7% decrease in variables.

Regarding the physical network, the model solutions indicate that some requirements simply cannot be entirely delivered on time. The TDM/RTDM report 6 vehicle allocations will arrive late, delivering Requirements 3 and 12. The ITDM reports that 205 short tons, comprised of parts of Requirements 3 and 12, will arrive beyond the RDD. Late requirements and vehicles are easily identified by comparing a requirement's rd_n to the v index of corresponding x_{nijmkv} for the TDM/RTDM and y_{nijmkv} for the ITDM.

Requirement	1	leaving POD	I1	for	destination	J1	requires	4	C130(s)	(AIR)	on day	3
Requirement	1	leaving POD	I1	for	destination	J1	requires	12	M1083(s)	(ROAD)	on day	3
Requirement	1	leaving POD	I1	for	destination	J1	requires	9	M1083(s)	(ROAD)	on day	4
Requirement	2	leaving POD	I1	for	destination	J1	requires	4	C130(s)	(AIR)	on day	4
Requirement	2	leaving POD	I1	for	destination	J1	requires	4	M1083(s)	(ROAD)	on day	4
Requirement	3	leaving POD	I1	for	destination	J1	requires	4	C130(s)	(AIR)	on day	5
Requirement	3	leaving POD	I1	for	destination	J1	requires	5	C130(s)	(AIR)	on day	6
Requirement	3	leaving POD	I1	for	destination	J1	requires	3	C130(s)	(AIR)	on day	7
Requirement	3	leaving POD	I1	for	destination	J1	requires	2	M1083(s)	(ROAD)	on day	5
Requirement	3	leaving POD	I1	for	destination	J1	requires	10	M1083(s)	(ROAD)	on day	6
Requirement	4	leaving POD	I1	for	destination	J1	requires	2	C130(s)	(AIR)	on day	7
Requirement	4	leaving POD	I1	for	destination	J1	requires	7	M1083(s)	(ROAD)	on day	7
Requirement	5	leaving POD	I1	for	destination	J1	requires	7	M1083(s)	(ROAD)	on day	8
Requirement	6	leaving POD	I1	for	destination	J2	requires	3	DODX(s)	(RAIL)	on day	3
Requirement	6	leaving POD	I1	for	destination	J2	requires	1	DODX(s)	(RAIL)	on day	4
Requirement	6	leaving POD	I1	for	destination	J2	requires	1	DODX(s)	(RAIL)	on day	5
Requirement	7	leaving POD	I1	for	destination	J2	requires	3	DODX(s)	(RAIL)	on day	6
Requirement	8	leaving POD	I1	for	destination	J2	requires	2	DODX(s)	(RAIL)	on day	7
Requirement	9	leaving POD	I1	for	destination	J2	requires	3	M1083(s)	(ROAD)	on day	6
Requirement	9	leaving POD	I1	for	destination	J2	requires	1	DODX(s)	(RAIL)	on day	7
Requirement	9	leaving POD	I1	for	destination	J2	requires	1	DODX(s)	(RAIL)	on day	8
Requirement	10	leaving POD	I1	for	destination	J2	requires	2	DODX(s)	(RAIL)	on day	8
Requirement	10	leaving POD	I1	for	destination	J2	requires	2	DODX(s)	(RAIL)	on day	9
Requirement	11	leaving POD	I2	for	destination	J1	requires	7	C130(s)	(AIR)	on day	5
Requirement	11	leaving POD	I2	for	destination	J1	requires	11	M1083(s)	(ROAD)	on day	5
Requirement	12	leaving POD	I2	for	destination	J1	requires	6	C130(s)	(AIR)	on day	6
Requirement	12	leaving POD	I2	for	destination	J1	requires	1	C130(s)	(AIR)	on day	7
Requirement	12	leaving POD	I2	for	destination	J1	requires	3	M1083(s)	(ROAD)	on day	6
Requirement	12	leaving POD	I2	for	destination	J1	requires	2	M1083(s)	(ROAD)	on day	7
Requirement	13	leaving POD	I2	for	destination	J1	requires	5	C130(s)	(AIR)	on day	7
Requirement	13	leaving POD	I2	for	destination	J1	requires	4	M1083(s)	(ROAD)	on day	7
Requirement	14	leaving POD	I2	for	destination	J2	requires	12	M1083(s)	(ROAD)	on day	4
Requirement	14	leaving POD	I2	for	destination	J2	requires	8	M1083(s)	(ROAD)	on day	5
Requirement	14	leaving POD	I2	for	destination	J2	requires	4	DODX(s)	(RAIL)	on day	4
Requirement	14	leaving POD	I2	for	destination	J2	requires	4	DODX(s)	(RAIL)	on day	5
Requirement	15	leaving POD	I2	for	destination	J2	requires	2	DODX(s)	(RAIL)	on day	6
Requirement	16	leaving POD	I2	for	destination	J2	requires	2	DODX(s)	(RAIL)	on day	8
Requirement	16	leaving POD	I2	for	destination	J2	requires	3	DODX(s)	(RAIL)	on day	9

Figure 4. TDM/RTDM Case 1 Solution

3 C130(s)	leaving POD I1 for destination J1 on day 3 (AIR)		
		144.00 Short Tons of Requirement	1
5 C130(s)	leaving POD I1 for destination J1 on day 4 (AIR)		
		181.40 Short Tons of Requirement	1
		58.60 Short Tons of Requirement	2
4 C130(s)	leaving POD I1 for destination J1 on day 5 (AIR)		
		192.00 Short Tons of Requirement	3
4 C130(s)	leaving POD I1 for destination J1 on day 6 (AIR)		
		192.00 Short Tons of Requirement	3
4 C130(s)	leaving POD I1 for destination J1 on day 7 (AIR)		
		176.00 Short Tons of Requirement	4
		10.00 Short Tons of Requirement	5
7 C130(s)	leaving POD I2 for destination J1 on day 5 (AIR)		
		336.00 Short Tons of Requirement	11
7 C130(s)	leaving POD I2 for destination J1 on day 6 (AIR)		
		336.00 Short Tons of Requirement	12
7 C130(s)	leaving POD I2 for destination J1 on day 7 (AIR)		
		64.00 Short Tons of Requirement	12
		270.00 Short Tons of Requirement	13
3 DODX(s)	leaving POD I1 for destination J2 on day 3 (RAIL)		
		400.00 Short Tons of Requirement	6
1 DODX(s)	leaving POD I1 for destination J2 on day 4 (RAIL)		
		133.33 Short Tons of Requirement	6
1 DODX(s)	leaving POD I1 for destination J2 on day 5 (RAIL)		
		66.67 Short Tons of Requirement	6
		66.67 Short Tons of Requirement	8
3 DODX(s)	leaving POD I1 for destination J2 on day 6 (RAIL)		
		400.00 Short Tons of Requirement	7
2 DODX(s)	leaving POD I1 for destination J2 on day 7 (RAIL)		
		133.33 Short Tons of Requirement	8
		133.33 Short Tons of Requirement	9
2 DODX(s)	leaving POD I1 for destination J2 on day 8 (RAIL)		
		166.67 Short Tons of Requirement	9
		100.00 Short Tons of Requirement	10
3 DODX(s)	leaving POD I1 for destination J2 on day 9 (RAIL)		
		400.00 Short Tons of Requirement	10
4 DODX(s)	leaving POD I2 for destination J2 on day 4 (RAIL)		
		400.00 Short Tons of Requirement	14
4 DODX(s)	leaving POD I2 for destination J2 on day 5 (RAIL)		
		400.00 Short Tons of Requirement	14
2 DODX(s)	leaving POD I2 for destination J2 on day 6 (RAIL)		
		200.00 Short Tons of Requirement	15
3 DODX(s)	leaving POD I2 for destination J2 on day 8 (RAIL)		
		300.00 Short Tons of Requirement	16
2 DODX(s)	leaving POD I2 for destination J2 on day 9 (RAIL)		
		200.00 Short Tons of Requirement	16
12 M1083(s)	leaving POD I1 for destination J1 on day 3 (ROAD)		
		180.00 Short Tons of Requirement	1
13 M1083(s)	leaving POD I1 for destination J1 on day 4 (ROAD)		
		3.60 Short Tons of Requirement	1
		191.40 Short Tons of Requirement	2
2 M1083(s)	leaving POD I1 for destination J1 on day 5 (ROAD)		
		30.00 Short Tons of Requirement	3
13 M1083(s)	leaving POD I1 for destination J1 on day 6 (ROAD)		
		195.00 Short Tons of Requirement	3
11 M1083(s)	leaving POD I1 for destination J1 on day 7 (ROAD)		
		141.00 Short Tons of Requirement	3
		24.00 Short Tons of Requirement	4
6 M1083(s)	leaving POD I1 for destination J1 on day 8 (ROAD)		
		90.00 Short Tons of Requirement	5
20 M1083(s)	leaving POD I2 for destination J2 on day 4 (ROAD)		
		200.00 Short Tons of Requirement	14
11 M1083(s)	leaving POD I2 for destination J1 on day 5 (ROAD)		
		165.00 Short Tons of Requirement	11
2 M1083(s)	leaving POD I2 for destination J1 on day 7 (ROAD)		
		30.00 Short Tons of Requirement	13

Figure 5. ITDM Case 1 Solution

Figure 4 shows the vehicle allocations determined by the model by parsing through the nonzero decision variables and their indices. In the RTDM solution, each requirement is allocated a number of different vehicles. In the ITDM solution, vehicle allocations are made to POD Destination pairs for specific days which support the delivery of different requirements. Figure 5 demonstrates this principal with the vehicle allocations on the left-hand side and the different requirements the model has allocated to flow on those vehicles on the right-hand side.

Test Case 2.

The second test case utilized a modified version of the TPFFD utilized in Test Case 1. The Case 2 TPFDD is shown below in Table 16. The difference between the two TPFDDs is a result of two distinct changes. Firstly, the short tonnage for each requirement has been set to 1 short ton. Secondly, delivery windows were constructed such that an intersection of at least one day exists for requirements within each POD/Destination pair. For example, notice that Requirements 1 through 5 all are to be delivered from $i1$ to $j1$ and each requirement could be delivered on-time on Day 6.

Table 16. TPFDD for Test Case 2

Requirement	POD	Destination	Short Tons	EAD	RDD
1	i1	j1	1	2	6
2	i1	j1	1	3	6
3	i1	j1	1	4	6
4	i1	j1	1	5	7
5	i1	j1	1	5	8
6	i1	j2	1	2	7
7	i1	j2	1	3	7
8	i1	j2	1	4	7
9	i1	j2	1	5	8
10	i1	j2	1	6	9
11	i2	j1	1	4	8
12	i2	j1	1	5	8
13	i2	j1	1	6	8
14	i2	j2	1	3	9
15	i2	j2	1	5	8
16	i2	j2	1	7	9

All input parameter values remain unchanged from Test Case 1 and are available in Appendix B. Using these inputs, the TDM, RTDM, and ITDM were all tested. The outputs and statistics from each model are depicted in Table 17 and Table 18. The solution information parsed from nonzero decision variables for Test Case 1 is shown in Figure 6 and Figure 7.

Table 17. Test Case 2 Model Results

Model	Total Vehicles Allocated	Air Vehicles Allocated	Road Vehicles Allocated	Rail Vehicles Allocated	ACU[1]	Late Vehicles[2]	Late Short Tons[3]
TDM	16	0	8	8	1.5%	-	N/A
RTDM	16	0	8	8	1.5%	-	N/A
ITDM	4	0	2	2	6.1%	N/A	-

[1]Approximate Capacity Utilization

[2]TDM and RTDM only

[3]ITDM only

Table 18. Test Case 2 Model Statistics

Model	Objective Value	Constraints	Total Variables	Integer Variables	Continuous Variables
TDM	808	160	4608	4608	-
RTDM	808	106	136	136	-
ITDM	202	160	190	54	136

Note that in this test case, the inherent modeling differences between the TDM/RTDM and ITDM yield drastically different outputs. The TDM/RTDM find that 16 vehicle allocations are needed to deliver the requirements. However, the ITDM finds that the same requirements could be delivered with only four vehicle allocations. Note that four vehicles is the minimum allocation possible, as there are four separate POD/Destination pairs in the TPFDD. The ITDM has the higher ACU. Regarding model statistics, as seen in Test Case 1, both the ITDM and RTDM have greatly reduced variables when compared to the TDM.

Requirement	1	leaving POD	I1	for	destination	J1	requires	1 M1083(s)	(ROAD)	on	day	6
Requirement	2	leaving POD	I1	for	destination	J1	requires	1 M1083(s)	(ROAD)	on	day	6
Requirement	3	leaving POD	I1	for	destination	J1	requires	1 M1083(s)	(ROAD)	on	day	6
Requirement	4	leaving POD	I1	for	destination	J1	requires	1 M1083(s)	(ROAD)	on	day	7
Requirement	5	leaving POD	I1	for	destination	J1	requires	1 M1083(s)	(ROAD)	on	day	8
Requirement	6	leaving POD	I1	for	destination	J2	requires	1 DODX(s)	(RAIL)	on	day	5
Requirement	7	leaving POD	I1	for	destination	J2	requires	1 DODX(s)	(RAIL)	on	day	5
Requirement	8	leaving POD	I1	for	destination	J2	requires	1 DODX(s)	(RAIL)	on	day	5
Requirement	9	leaving POD	I1	for	destination	J2	requires	1 DODX(s)	(RAIL)	on	day	8
Requirement	10	leaving POD	I1	for	destination	J2	requires	1 DODX(s)	(RAIL)	on	day	9
Requirement	11	leaving POD	I2	for	destination	J1	requires	1 M1083(s)	(ROAD)	on	day	8
Requirement	12	leaving POD	I2	for	destination	J1	requires	1 M1083(s)	(ROAD)	on	day	8
Requirement	13	leaving POD	I2	for	destination	J1	requires	1 M1083(s)	(ROAD)	on	day	8
Requirement	14	leaving POD	I2	for	destination	J2	requires	1 DODX(s)	(RAIL)	on	day	6
Requirement	15	leaving POD	I2	for	destination	J2	requires	1 DODX(s)	(RAIL)	on	day	6
Requirement	16	leaving POD	I2	for	destination	J2	requires	1 DODX(s)	(RAIL)	on	day	9

Figure 6. TDM/RTDM Case 2 Solution

1 DODX(s)	leaving POD	I1	for	destination	J2	on	day	7 (RAIL)			
									1.00 Short Tons	of Requirement	6
									1.00 Short Tons	of Requirement	7
									1.00 Short Tons	of Requirement	8
									1.00 Short Tons	of Requirement	9
									1.00 Short Tons	of Requirement	10
1 DODX(s)	leaving POD	I2	for	destination	J2	on	day	8 (RAIL)			
									1.00 Short Tons	of Requirement	14
									1.00 Short Tons	of Requirement	15
									1.00 Short Tons	of Requirement	16
1 M1083(s)	leaving POD	I1	for	destination	J1	on	day	6 (ROAD)			
									1.00 Short Tons	of Requirement	1
									1.00 Short Tons	of Requirement	2
									1.00 Short Tons	of Requirement	3
									1.00 Short Tons	of Requirement	4
									1.00 Short Tons	of Requirement	5
1 M1083(s)	leaving POD	I2	for	destination	J1	on	day	7 (ROAD)			
									1.00 Short Tons	of Requirement	11
									1.00 Short Tons	of Requirement	12
									1.00 Short Tons	of Requirement	13

Figure 7. ITDM Case 2 Solution

Test Case 3.

The third test case utilized a sample portion of a large-scale TPFDD acquired from USTRANSCOM. The TPFDD was inspected and certain requirements, such as passenger-only requirements, were removed or modified to ensure a complete data set where all requirements had positive short tons values and had a POD that was different from the Destination. After adjustment, the TPFDD contained 4,426 requirements, which totaled 872,667.2 short tons. See Appendix C for information on obtaining this TPFDD.

The TPFDD listed 10 different PODs and 13 Destinations. Three Modes were utilized (Air, Rail, Road), with 3 different vehicle Types for each Mode resulting in 9 total vehicle Types. Possible delivery Days ranged from Day 1 to Day 296. The payload and cost parameters are located below in Table 19. Note that the DODX train is both the cheapest and largest capacity vehicle.

Table 19. Vehicle Parameters for Test Case 3

Type	Average Payload (Short Tons) p_k	Daily Cost b_k
C130	12	100
C17	35	101
C5	60	102
HEMTT	7	11
M1083	5	10
M35	8	12
DODX	200	1
FTTX	150	2
ITTX	180	3

POD and Destination outloading and unloading capacities were made arbitrarily high, with $o_{imv} = u_{jmv} = 250$ for all appropriate tuples. Likewise, all cycle values were set arbitrarily high to $w_{nijmk} = 3$ (TDM/RTDM) and $w_{ijmk} = 3$ (ITDM). This implies that any vehicle can make three trips between its POD and Destination daily. The penalty per day per late vehicle (TDM/RTDM) and penalty per day per short ton (ITDM) was set to $g = 10,000$. The results of Test Case 3 are located below in Table 20 and Table 21. For complete solution outputs, please reference Appendix C.

Table 20. Test Case 3 Model Results

Model	Total Vehicles Allocated	Air Vehicles Allocated	Road Vehicles Allocated	Rail Vehicles Allocated	ACU[1]	Late Vehicles[2]	Late Short Tons[3]
TDM							
RTDM	5,159	0	0	5,159	28.2%	2	N/A
ITDM	1,476	0	0	1,476	98.5%	N/A	13.5

[1]Approximate Capacity Utilization

[2]TDM and RTDM only

[3]ITDM only

Table 21. Test Case 3 Model Statistics

Model	Objective Value	Constraints	Total Variables	Integer Variables	Continuous Variables
TDM			4,390,644,960	4,390,644,960	-
RTDM	25,159	7,673	714,321	714,321	-
ITDM	136,476	15,104	721,863	7,542	714,321

Note that the TDM could not be successfully tested in this case, though the number of decision variables may be manually calculated. The TDM integer program was successfully developed utilizing VBA, however, LINGO 13 could not compile and/or solve the integer program with its more than 4.3 billion decision variables. However, as the RTDM is the same model conceptually, and produces the same solutions as demonstrated in Test Cases 1 and 2, it is assumed that the TDM, with enough computing power, would eventually arrive at the same solution as the RTDM.

An all DODX train solution is obtained by both models. As was seen with the first two test cases, the ITDM finds that fewer vehicle allocations are necessary to move the requirements than are reported by the RTDM. Additionally, over 70% more capacity is utilized by ITDM vehicle allocations compared to the RTDM.

Both the RTDM and ITDM report that vehicles carrying requirements 223 and 231 arrive at the destination late. In terms of model statistics, both the RTDM and ITDM offer a reduction in decision variables greater than 99.9%. Due to linking constraints, the ITDM actually has more constraints than the RTDM. However, the problem remains very tractable.

Determining a Vehicle Beddown

As discussed in Chapter III, it is possible to determine potential vehicle beddowns at the PODs by analyzing the model outputs on vehicle allocations. To demonstrate the use of the formulae in (27) and (28), the vehicle beddowns from the large scale Test Case 3 for both the RTDM and ITDM are given below in Table 22. Note that in Test Case 3, an integer cycle value was utilized, and therefore the measure is applicable as it is assumed any vehicle utilized on any day will be available in following days.

Table 22. Beddowns of Mode Rail, Type DODX vehicles by POD for Test Case 3

Model	ARKJ	AZTG	FUQN	HNTK	HNTS	KUHE	TMKH	TYFR	VKNP	YVGQ	TOTAL
RTDM	3	83	13	2	47	83	3	14	55	83	386
ITDM	1	82	2	1	30	74	1	12	54	29	286

Note that the beddowns at each POD for Mode Rail, Type DODX vehicles are inherently larger with the RTDM than the ITDM. This follows from the fact that the RTDM dictates more vehicle allocations than does the ITDM. Therefore, as the ITDM projects less vehicle allocations, the beddowns are also smaller.

Limitations exist with this beddown methodology. For example, if a requirement needs ten allocations of a certain vehicle, but has a two day window to accomplish it, different beddowns may be derived by the model at the same cost. If all ten allocations are made on the first day, a ten vehicle beddown would be reported. If five were made on the first day, and five on the second day, only a five vehicle beddown would be reported as the same five vehicles used on the first day could be used again on the second. Both beddowns, having ten allocations, would both impact the objective function equally. In Chapter V, ideas for further research on investigating beddowns are discussed.

Policy-Driven Solutions

To demonstrate how the models are responsive to policy changes encoded into model parameter inputs, a slight modification was made to Test Case 3. As an example, consider a scenario in which decision makers decide that road and rail travel will place unnecessary harm on ground troops and air solutions are to be encouraged. Thus, operational costs of each aircraft type could be changed to reflect this policy. In this test, the TPFDD from Case 3 is utilized, however, some parameter inputs are modified. The vehicle attributes were changed as shown in Table 23. Furthermore, outloading and unloading settings were changed to $o_{imv} = u_{jmv} = 4300$ for all appropriate tuples. Lastly, all cycle values were updated to $w_{nijmk} = 43$ (RTDM) and $w_{ijmk} = 43$ (ITDM). Note that in this test example, the C-130, an air platform, was made to have the lowest cost.

The higher outloading, unloading, and cycle values are arbitrary, and likely illogical, but allow for a demonstration of an all aircraft solution.

Table 23. Vehicle Parameters for Policy-Driven Solutions Example

Type	Average Payload (Short Tons) p_k	Daily Cost b_k
C130	12	1
C17	20	2
C5	30	3
HEMTT	7	11
M1083	5	10
M35	8	12
DODX	200	100
FTTX	150	101
ITTX	180	103

As a result of this setup, an all C-130 solution was obtained by both the RTDM and ITDM (like Test Case 3, the problem was too large to compile with the TDM). This is a direct result of the fact that C-130s were the cheapest vehicle to select for delivery. The RTDM reported an objective of 25,241 with 5,017 C-130 allocations. The ITDM reported an objective of 136,710 with 1,710 C-130 allocations. These results, paired with Test Case 3, demonstrate that users can drive the model towards solutions that are consistent with policy directives or other impacting considerations. Readers interested in complete solutions should reference Appendix C.

Although not modeled in this research, one could easy redefine the costs such that they are based upon attributes other than vehicle Type. For example, replacing the cost

b_k with cost b_{ik} would imply a separate operational cost for each POD i and vehicle Type k combination. With this formulation, similar vehicles departing from different PODs may be given different costs. As seen above, the model would steer towards solutions which meet user defined objectives. If road travel out of a certain POD is dangerous, perhaps due to Improvised Explosive Devices, road vehicles leaving this POD could be given a higher cost than other road vehicles leaving from safer PODs. This would require more user input. However, it is one of the many ways in which the costs utilized in the objective value could be adapted to meet policy, guidance, or doctrine relevant to the situation.

Aggregation

Recall from Chapter III that aggregation may be used as a precursor to optimization. That is, aggregation of like requirements with both the same POD/Destination pair and EAD/RDD pair may be performed. Test Cases 1 and 2 had no such requirements. However, the aggregation of the TPFDD from Test Case 3 is possible. To see the impact aggregation has on model outputs and statistics, the same setup from Test Case 3 was implemented, only with the TPFDD aggregated. This brought the number of requirements from 4,426 down to 148, although the total short tonnage of the TPFDD remained unchanged. Table 24 and Table 25 below show the result of the runs. For full solutions, reference Appendix C.

Table 24. Model Results with Aggregated TPFDD

Model	Total Vehicles Allocated	Air Vehicles Allocated	Road Vehicles Allocated	Rail Vehicles Allocated	ACU[1]	Late Vehicles[2]	Late Short Tons[3]
TDM							
RTDM	1,547	0	0	1,547	94.0%	1	N/A
ITDM	1,476	0	0	1,476	98.5%	N/A	14

[1]Approximate Capacity Utilization
[2]TDM and RTDM only
[3]ITDM only

Table 25. Model Statistics with Aggregated TPFDD

Model	Objective Value	Constraints	Total Variables	Integer Variables	Continuous Variables
TDM			153,766,080	153,766,080	-
RTDM	11,547	3,395	24,471	24,471	-
ITDM	136,476	10,862	32,013	7,542	24,471

Note that with aggregation, the ITDM reports 1,476 train allocations (all of which were DODX). This is the exact same number of DODX train allocations given by the ITDM in the non-aggregated test run in Case 3. However, the RTDM reports a reduction in the number of allocations, from 5,159 without aggregation to 1,547 with aggregation even though the total short tonnage of the TPFDD remains unchanged. While aggregation does lessen the gap between the number of allocations required between the RTDM and ITDM, the fact that the number of allocations changes based upon aggregation of like requirements in the RTDM is problematic. Aggregation does lead to smaller problem size, as there are less requirements. Even so, the TDM could not generate such a large model.

In both the aggregated and nonaggregated cases, the ITDM achieved an ACU of 98.5%. However, even aggregation before optimization with the RTDM does not achieve such an ACU. Recall that in regards to time windows, requirements are aggregated only when there are exact matches with time windows. However, the ITDM can allocate requirements with intersecting time windows regardless of whether aggregation has been performed or not.

The result that the ITDM produces the same objective value for any dataset, regardless of whether or not aggregation is done, is actually fairly straightforward. For with aggregation, only requirements with the exact same attributes are aggregated, and their short tonnage values are summed. Note within the ITDM, the outloading and unloading constraints are not affected by aggregation, because nothing is indexed over the requirements n. However, changes do occur in the objective, linking constraint, and requirement constraint. In the objective (18), rather than summing multiple continuous disaggregated requirement flow variables, a continuous flow variable representing aggregated flow is in their place. Likewise, in (22) the LHS summation includes continuous aggregated variables rather than multiple individual variables. For (19), there are now different (n,i,j) tuples generating constraints. The aggregated constraints have the aggregated sum on the RHS, and on the LHS, the aggregated continuous flow variables replace individual disaggregated variables.

Verification and Validation

As with any model, proper verification and validation must be performed as part of the analysis. Verification seeks to ensure that one is building the model right.

Validation focuses on whether or not one is building the right model. These checks are enacted upon the three models discussed in this thesis.

Verification.

Verification is conducted on each of the three models by examining the results of Test Case 1. Firstly, as the Test Case 1inputs are drawn directly from Longhorn & Kovich (2012), the TDM is easily verified in seeing that, within this research, the TDM obtained the same objective value as Longhorn & Kovich did in their research. Although alternate optimal solutions may exist, their solution was replicated. Furthermore, as the RTDM produced the exact same output, the RTDM is verified as well. The ITDM is verified in seeing that similar vehicle allocations were needed, although slight lower numbers were seen due to increased ACU values. Another indication of successful verification of the ITDM is that both the TDM/RTDM and ITDM indicated that Requirements 3 and 12 would be delivered late, indicating the same bottleneck present in the network.

Validation.

Test Case 2 offers one reason why the TDM and RTDM cannot be "the right model." Far too many vehicles are allocated to move the 16 requirements under both the TDM and RTDM. This is what led this research to pursue an alternate modeling technique. With the ITDM, the model avoids the issue of not being allowed to allocate vehicles to multiple different requirements. Additionally, when examining the solutions from all Test Cases, lower utilization rates are seen in the TDM and RTDM. This is because the model forces at least one vehicle to be allocated to every requirement, no matter how small. This is a poor modeling construct, and the ITDM averts this dilemma.

Further verification of the ITDM is not entirely possible, as results of force flow analysis is typically classified. However, as stated above, the ITDM resolves much of the issues seen with the TDM and RTDM.

The fact that the RTDM solutions are so sensitive to aggregation also demonstrates it is not a useful model. Though the exact same amount of cargo needs to be delivered with aggregation, drastic changes in solutions were seen in the aggregation testing of the large-scale TPFDD. Conversely, the ITDM gives the same solution whether aggregated or not because its solutions are not sensitive to the actual number of requirements, but rather the amount of short tons in the TPFDD.

V. Conclusions and Future Research

Conclusions

The ITDM is the best model to use in approximating vehicle mixtures for theater distribution. As the TDM, the baseline model, was tested and analyzed, it became quite clear that problem sizes would be too large for real world problems. Thus, the RTDM was developed, which solved the problem the same way but only considered relevant decision variables and constraints. However, this reduced model still had deficiencies in how requirements were allocated to vehicles, and thus the ITDM was developed to address this.

The ITDM can give force flow analysts great insight into vehicles needed for a contingency. In terms of solution quality, the ITDM is better than the TDM/RTDM as it more accurately allocates vehicles to requirements. The RTDM forces every requirement to have at least one vehicle allocated to it. Even if aggregation is attempted in order to reduce the number of requirements, the RTDM still fails to achieve the ACU that is accomplished with the ITDM. This is because aggregation only combines requirements with exact time window matches whereas the ITDM can allocate different requirements on a single vehicle whenever there is an intersection in the delivery windows of the requirements.

The ITDM is also far smaller in terms of variables and constraints when compared to the TDM. In fact, it was seen that the TDM failed to even generate for larger problems. Because the ITDM has both continuous and integer variables, and an additional linking constraint, it is actually a larger problem than the RTDM. However, this is a tradeoff of no consequence that results in far better solutions.

Although the ITDM gives the same solutions whether the TPFDD is aggregated or not, keeping the TPFDD disaggregated is preferred as it keeps requirements in their initial, disaggregated state allowing for better analysis and allowing differing extension day values to be input by commanders during analysis. However, if computing resources are scarce, using aggregation before solving the ITDM may be an option.

Lastly, with appropriate cycle selection, solutions may be post-processed to determine approximate vehicle beddowns required at each POD. These approximate vehicle beddowns can provide important answers for force flow analysts. Thus, rather than arbitrarily selecting a vehicle beddown to test in theater distribution simulations, the ITDM can help drive feasible solutions.

The ITDM is able to find feasible vehicle mixtures that minimize operational cost and minimize late deliveries. Because costs are user-defined, solutions may be steered towards vehicle mixtures that align with current policy or direction. By post-processing solutions, insights into limitations of the physical network and potential vehicle beddowns may be gained. While the beddown measures may be sensitive to alternate optimal solutions, finding beddowns after analysis with the ITDM can provide strong starting points as analysts test different vehicle mixtures as part of force flow analysis. Through the use of the ITDM and associated Decision Support System tool, force flow analysts should be able to provide data input, model generation, solution analysis, and solution transfer to simulation tools much faster than current guess and check methods in place. Force flow analysts will be able to receive insight into required vehicle mixtures and beddowns as they plan contingencies. This use of ITDM to model theater distribution has the potential to save many man-hours amongst USTRANSCOM analysts and planners.

Future Research

There are many possible adjustments to the ITDM formulation which would allow further modeling of operational realities. The greatest potential for bettering force flow modeling is to investigate the best way to determine vehicle beddowns. Through the research process, it became clear that actual vehicle beddowns at PODs may be more useful outputs for force flow analysts than what the tested models provide which are the minimum cost allocations of vehicles to different requirements. While this thesis develops a methodology for measuring approximate vehicle beddowns with the ITDM, the beddowns appear to be sensitive to alternate optimal solutions. Thus, while analysts may find such beddowns useful as starting points in distribution analysis, better beddown solutions may exist.

An exploration of how different objectives, including minimizing lateness, beddown size, beddown costs, and operational costs, all impact vehicle solutions yielded by the models should prove fruitful. Changing objectives could cause further constraints to be introduced into the model. Tradeoffs exist within these different objectives, and thus solutions may be impacted depending on which objectives are included, as well as any possible weighting assigned to objectives. Investigating this multiobjective problem, and determining which objectives and measures provide proper beddowns as needed by USTRANSCOM is a practical next step for research.

Another way to improve the ITDM would be to introduce a multi-commodity flow approach. The ITDM currently can allocate any vehicle to any requirement's short tons. However, in reality, there are some requirements that cannot go on certain vehicles. For example, an M1 tank cannot fit onto M1083 truck. It could, however, be placed on a C-17 aircraft. Information regarding the type of cargo is easily accessible on a TPFDD and therefore, restricting which vehicles may carry each requirement could produce more realistic solutions. TPFDDs also contain passengers (i.e. troops) that need transport into the theater. These could also be modeled as a commodity to be allocated to passenger vehicles.

Modeling could also be expanded to include all three legs of the distribution process. In other words, a model could show the flow of troops and materiel from home base to POE to POD to Destination. This would greatly increase the number of variables and would likely require the use of heuristics.

Lastly, further research into defining cycles should be conducted. Rather than relying on user-input, a tool could be developed to calculate the greatest circular distance (or other measure) between a POD and Destination and then, taking into consideration vehicle speeds, outload/unload times, and other operational capabilities, report back a particular cycle value. Furthermore, research into how to address noninteger cycle values should be considered. As it stands now, a noninteger cycle value gives an imprecise location of vehicles between days.

Appendix A. LINGO 13 Settings File Contents

The LINGO.CNF file contains settings which have been changed from their default values within LINGO 13. The contents of the LINGO.CNF file as utilized in this thesis appear below.

```
Lingo CNF info:
! LINGO Custom Configuration Data:
MXMEMB=  25000
ABSINT=  0.10000000E-11
IPTOLR=  0.20000000E-02
TIM2RL=  120
LINLEN=  150
DUALCO=  0
PRECIS=  12
```

Appendix B. Additional Model Inputs for Test Case 1 and Test Case 2

Table 26. Vehicle Parameters for Test Cases 1 and 2

Type	Average Payload p_k	Daily Cost b_k
C130	12	10000
M1083	5	100
DODX	200	1

Table 27. Outloading Parameters for Test Cases 1 and 2

POD	Mode	Outload Capacity o_{imv}
i1	Air	20
	Road	50
	Rail	2
i2	Air	28
	Road	50
	Rail	2

*Note, for each POD/Mode pair, the outload capacity is assumed constant for all days v.

Table 28. Unloading Parameters for Test Cases 1 and 2

Destination	Mode	Unload Capacity u_{jmv}
j1	Air	44
	Road	40
	Rail	0
j2	Air	0
	Road	60
	Rail	3

*Note, for each Destination/Mode pair, the outload capacity is assumed constant for all days v.

Table 29. Cycle Values for Test Cases 1 and 2 (TDM/RTDM Only)

Movement	POD	Destination	Mode	Type	Cycles w_{nijmk}
1	i1	j1	AIR	C130	4
1	i1	j1	ROAD	M1083	3
1	i1	j1	RAIL	DODX	0
2	i1	j1	AIR	C130	4
2	i1	j1	ROAD	M1083	3
2	i1	j1	RAIL	DODX	0
3	i1	j1	AIR	C130	4
3	i1	j1	ROAD	M1083	3
3	i1	j1	RAIL	DODX	0
4	i1	j1	AIR	C130	4
4	i1	j1	ROAD	M1083	3
4	i1	j1	RAIL	DODX	0
5	i1	j1	AIR	C130	4
5	i1	j1	ROAD	M1083	3
5	i1	j1	RAIL	DODX	0
6	i1	j2	AIR	C130	0
6	i1	j2	ROAD	M1083	2.66666667
6	i1	j2	RAIL	DODX	0.66666667
7	i1	j2	AIR	C130	0
7	i1	j2	ROAD	M1083	2.66666667
7	i1	j2	RAIL	DODX	0.66666667
8	i1	j2	AIR	C130	0
8	i1	j2	ROAD	M1083	2.66666667
8	i1	j2	RAIL	DODX	0.66666667
9	i1	j2	AIR	C130	0
9	i1	j2	ROAD	M1083	2.66666667
9	i1	j2	RAIL	DODX	0.66666667
10	i1	j2	AIR	C130	0
10	i1	j2	ROAD	M1083	2.66666667
10	i1	j2	RAIL	DODX	0.66666667
11	i2	j1	AIR	C130	4
11	i2	j1	ROAD	M1083	3
11	i2	j1	RAIL	DODX	0
12	i2	j1	AIR	C130	4
12	i2	j1	ROAD	M1083	3
12	i2	j1	RAIL	DODX	0
13	i2	j1	AIR	C130	4
13	i2	j1	ROAD	M1083	3
13	i2	j1	RAIL	DODX	0
14	i2	j2	AIR	C130	0
14	i2	j2	ROAD	M1083	2
14	i2	j2	RAIL	DODX	0.5
15	i2	j2	AIR	C130	0
15	i2	j2	ROAD	M1083	2
15	i2	j2	RAIL	DODX	0.5
16	i2	j2	AIR	C130	0
16	i2	j2	ROAD	M1083	2
16	i2	j2	RAIL	DODX	0.5

*Note, while the RTDM does not consider illogical tuples (e.g. with Mode Rail, Type C-130), the TDM does and all such illogical cycle values (not shown) are set to 0.

Table 30. Cycle Values for Test Cases 1 and 2 (ITDM Only)

POD	Destination	Mode	Type	Cycles w_{ijmk}
i1	j1	AIR	C130	4
i1	j1	ROAD	M1083	3
i1	j1	RAIL	DODX	0
i1	j2	AIR	C130	0
i1	j2	ROAD	M1083	2.66666667
i1	j2	RAIL	DODX	0.66666667
i2	j1	AIR	C130	4
i2	j1	ROAD	M1083	3
i2	j1	RAIL	DODX	0
i2	j2	AIR	C130	0
i2	j2	ROAD	M1083	2
i2	j2	RAIL	DODX	0.5

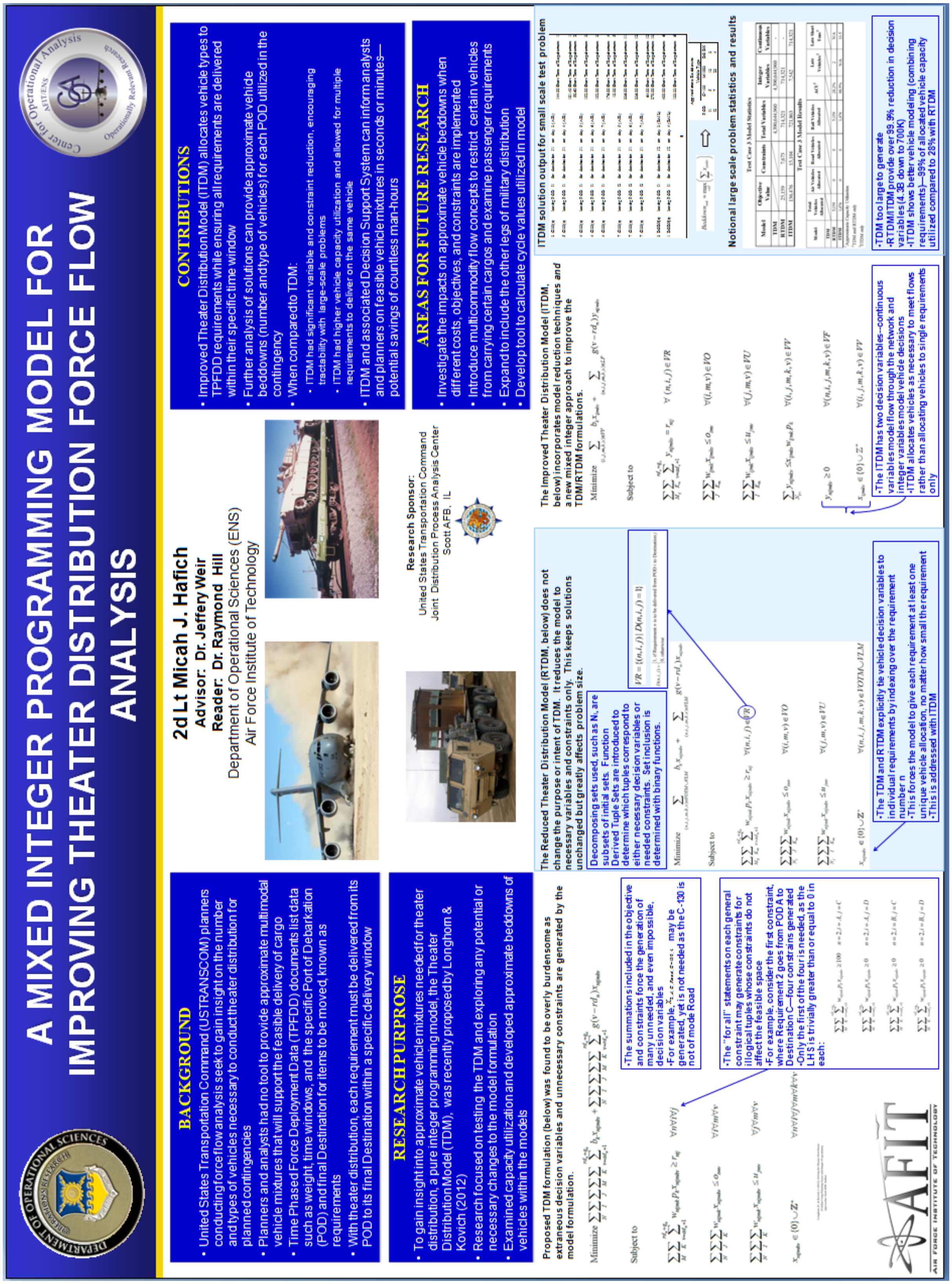
Appendix C. TPFDD and Solutions for Test Case 3

The large scale TPFDD utilized for this research contained 4,426 requirements, resulting in a very lengthy document. Therefore, those interested in this dataset should contact Dr. Jeff Weir, of the Air Force Institute of Technology's Department of Operational Sciences (AFIT/ENS). Dr. Weir can be reached at jeffery.weir.2@us.af.mil or at (937) 255-6565 x4523. Readers interested in seeing the complete solution outputs should do the same.

Appendix D. Model Coding

The VBA coding utilized in this research to generate the RTDM and ITDM is lengthy but available upon request. The coding of the TDM is available as well. Readers interested in obtaining the code should contact Dr. Jeff Weir, of the Air Force Institute of Technology's Department of Operational Sciences (AFIT/ENS). Dr. Weir can be reached at jeffery.weir.2@us.af.mil or at (937) 255-6565 x4523.

Appendix E. Research Summary Chart



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Vita

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14. ABSTRACT
Obtaining insight into potential vehicle mixtures that will support theater distribution, the final leg of military distribution, can be a challenging and time consuming process for United States Transportation Command (USTRANSCOM) force flow analysts. The current process of testing numerous different vehicle mixtures until separate simulation tools demonstrate feasibility is iterative and overly burdensome. Improving on existing research, a mixed integer programming model was developed to allocate specific vehicle types to delivery items, or requirements, in a manner that would minimize both operational costs and late deliveries. This gives insight into the types and amounts of vehicles necessary for feasible delivery and identifies possible bottlenecks in the physical network. Further solution post-processing yields potential vehicle beddowns which can then be used as approximate baselines for further distribution analysis. A multimodal, heterogeneous set of vehicles is used to model the pickup and delivery of requirements within given time windows. To ensure large scale problems do not become intractable, precise set notation is utilized within the mixed integer program to ensure only necessary variables and constraints are generated.

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