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# COMPUTATIONAL DESIGN OPTIMIZATION UNDER UNCERTAINTY OF SYSTEMS WITH NONLINEAR AEROELASTIC CONSTRAINTS

## FINAL REPORT GRANT FA9550-10-1-0353

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University of Arizona, Tucson

### **1 Abstract**

The work funded through this grant has enabled the development of a new methodology to substantially facilitate the design of systems with aeroelastic constraints. Techniques focusing on the optimal design and the propagation of uncertainties have been developed. The fundamental paradigm shift in this work stems from the use of a classification-based approach to construct explicit “decision” boundaries such as stability boundaries. This enables one to manage problems with discontinuities (e.g., problems with subcritical limit cycle oscillations) or problems with a large number of constraints. This approach has been used as the foundation for reliability-based design optimization and multifidelity algorithms for the reduction of computational time. The proposed techniques have been successfully applied to several design examples with aeroelastic constraints.

### **2 Objectives**

The main objective of this grant was to introduce a drastically novel methodology for the optimal design under uncertainty of systems with aeroelastic constraints. The need for a new methodology stems from the presence of major hurdles such as the large computational times associated with aeroelasticity simulations, response discontinuities, high sensitivities to uncertainties, and a large number of failure modes.

During the two year grant, the objective of the first year was to demonstrate the applicability and feasibility of the new solution schemes with a focus on the construction of stability boundaries using Support Vector Machines (SVMs). A particular focus was on problems with discontinuous behaviors encountered in the case

of sub-critical limit-cycle oscillations (LCOs). In particular, the objective was to introduce uncertainty and perform reliability-based design optimization (RBDO). The objective of the second year was to use realistic simulations from a commercial code (ZAERO) and develop new strategies to reduce the computational burden required for the construction of stability boundaries (multifidelity approach) and to perform design optimization.

### 3 Methodology and accomplishments

The methodology is based on the construction of explicit decision boundaries (boundaries of the feasible or failure region) using a Support Vector Machine (SVM) [1]. Because it is a classification technique, as opposed to an approximation technique, it has the advantage of handling problems with discontinuous and binary responses. In addition, the construction of explicit boundaries provides a flexible framework to propagate uncertainties with multiple failure modes, thus greatly enhancing the potential for optimal and reliable designs.

**This work is the result of a fruitful and intense collaboration with Dr. Phil Beran at AFRL/RBSD, Wright Patterson Air-Force Base, OH.**

The research during the two years has been characterized by the following milestones:

#### 3.1 Construction of explicit boundaries. Nonlinear aeroelasticity.

A methodology for the construction of explicit stability (flutter and/or divergence) boundary has been developed. The boundary is constructed using a design of experiments and an SVM which separates stable and unstable configurations. Figure 3.1 provides an example of flutter boundary for a two degree-of-freedom airfoil:

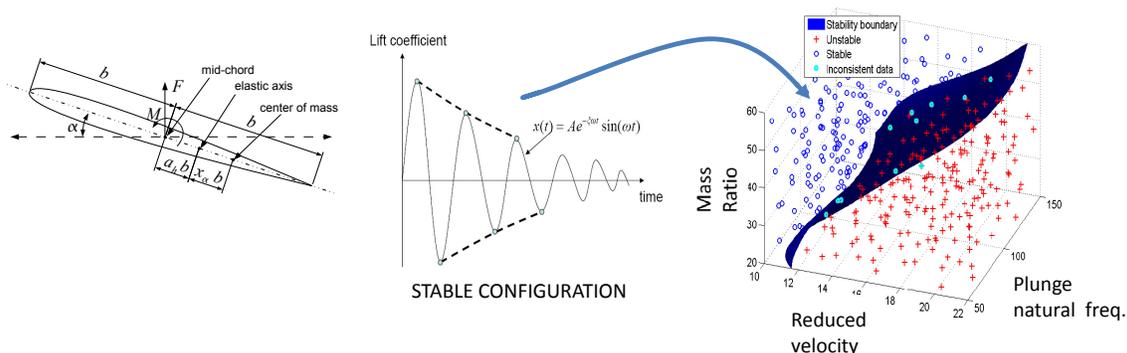


Figure 1: **Example of construction of explicit flutter boundary for a two degree-of-freedom airfoil.**

This approach is particularly useful for problems with discontinuities. This is the case for systems that exhibit sub-critical limit cycle oscillations (LCO). Consider the

case of the two-degrees of freedom airfoil with structural nonlinearities (i.e., nonlinear stiffness). The airfoil can exhibit sub-critical LCO which happen at lower speed than the predicted linear flutter velocity (Figure 3.1). By using clustering techniques, the discontinuities can be detected, thus providing the classification to construct the explicit sub-critical LCO boundary [2].

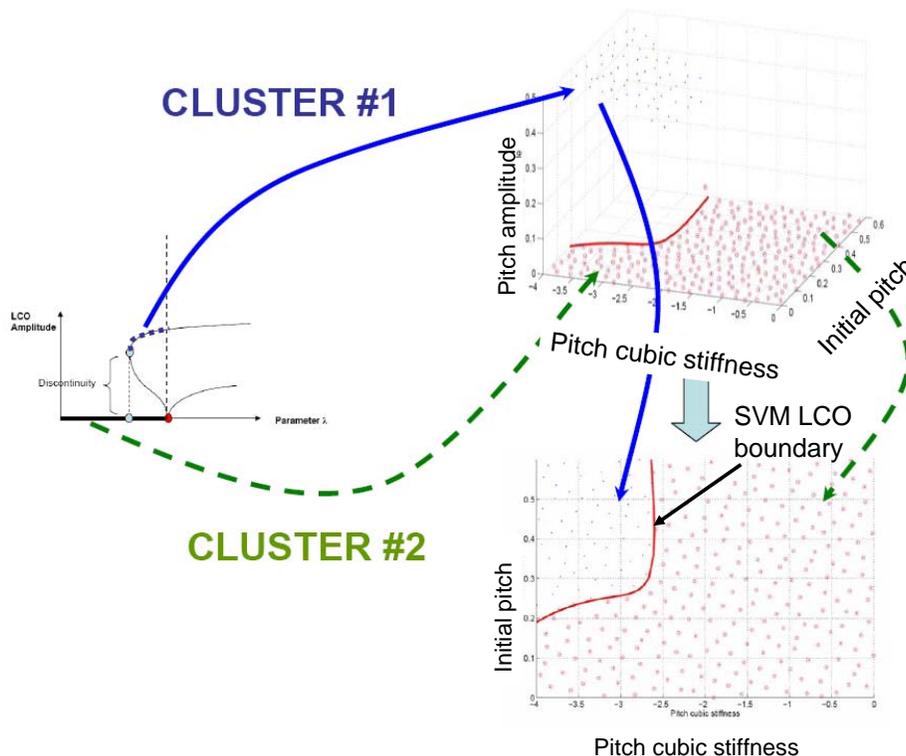


Figure 2: Construction of explicit boundaries for sub-critical LCO. Detection of the discontinuities, classification and construction of SVM.

### 3.2 Probability of failure and Reliability-based design optimization.

Once the boundary is created, one can easily propagate uncertainties, calculate probabilities of failure and perform reliability based design optimization. For instance, we have solved a problem where one wants to minimize weight while making sure that subcritical LCOs do not appear with a given probability [2]. A new approach has also been developed to obtain a relatively conservative probability of failure using probabilistic support vector machines [3].

### 3.3 Multifidelity models

An important milestone in this research has been the development of a scheme to handle multiple levels of fidelity (e.g., low and high) in order to limit the problems due to large computational times. This is a major step towards the design of full

scale reliable optimal aerospace systems. In this research, the construction of explicit stability boundaries can be enhanced by using lower and “cheaper” models as well as experimental data. A sampling scheme has been developed with the basic idea of selecting the region where high fidelity data need to be used. An example is the construction of a nonlinear flutter boundary based on the linear flutter boundary. Figure 3 provides an example for the two degree-of-freedom airfoil [4].

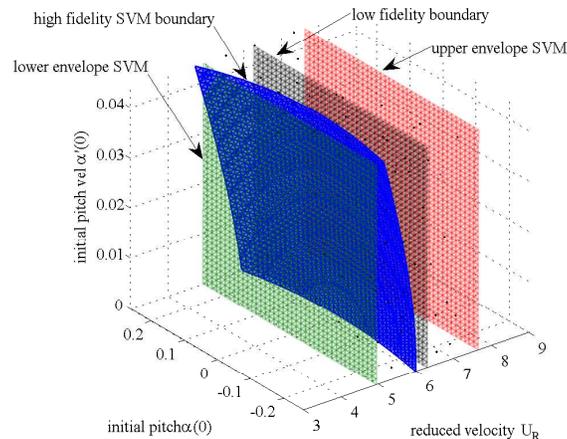


Figure 3: **Example of construction of “high fidelity” (nonlinear) explicit flutter boundary based on a lower fidelity (linear) flutter boundary.**

Another example is the construction of boundaries of regions of the design space where the deflections due to a gust are smaller than a certain value. This was applied to a General Transport Aircraft model available from ZONA Technology. The lower fidelity model is a simple intuitive model whereas the higher fidelity model is the finite element model subjected to a gust (Figure 4).

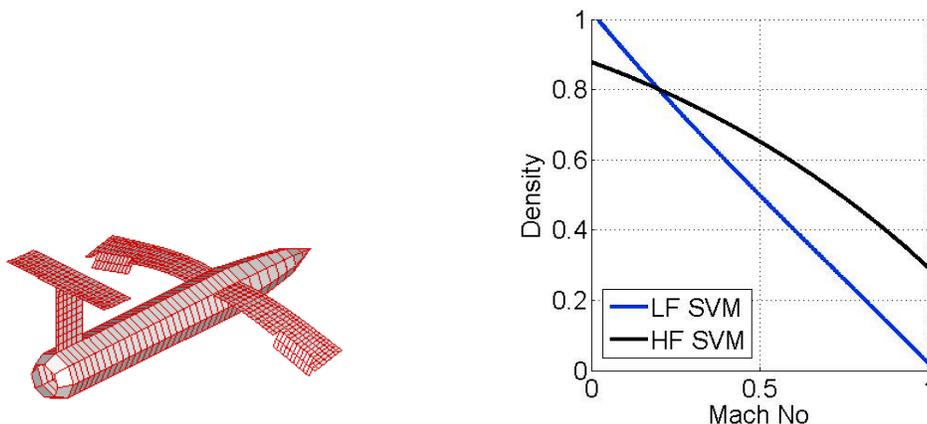


Figure 4: **Lower and higher fidelity boundary for wing tip deflection for a GTA subjected to gust. GTA model from ZONA Technology**

In the second year of this research, the multifidelity scheme was significantly

improves to reduce the number of calls to the high fidelity models. The new algorithm is described in detail in [5]. The approach was applied to the following analytical example:

Consider the  $n$ dimensional problem. The low-fidelity boundary is given as:

$$0.16 - \sum_{i=1}^n (x_i - 0.5)^2 = 0 \quad (1)$$

And the high-fidelity boundary is:

$$0.16 - \sum_{i=1}^n (x_i - 0.5 - 0.02 \times (-1)^i)^2 = 0 \quad (2)$$

The results are depicted in Figure 3.3. The plot gives the error as a function of the number of high-fidelity function calls for the proposed multifidelity scheme, a traditional design of experiments (CVT), and the adaptive sampling scheme based on high-fidelity calls only (EDSD). The plot shows a clear advantage of the approach compared to the other techniques.

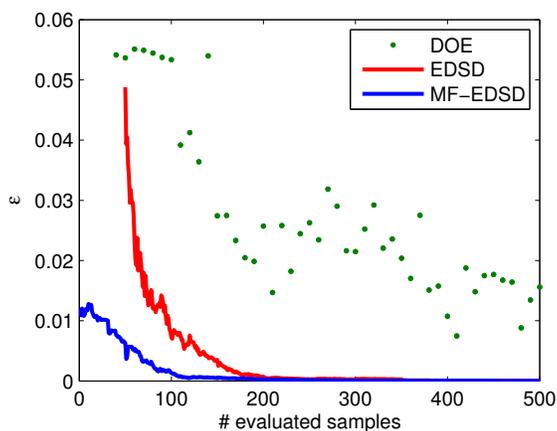


Figure 5: Comparison of the multifidelity approach, traditional design of experiments (CVT), and adaptive sampling based on high fidelity only (EDSD).

In addition, the multifidelity scheme was applied to an aeroelasticity problem solved using ZAERO [[6]]. The objective was to construct the the stability boundary which includes both flutter and divergence. the wing planform and the parameters are depicted in Figure 3.3. The low and high fidelity scheme are segregated based on the number of structural finite elements and the number of aero-boxes (see Table 3.3 and Figure 3.3).

The stability boundary has been constructed in the three dimensional space with the chord ratio ( $\lambda$ ), the sweep angle, and the half. The low fidelity and high fidelity boundaries are depicted in Figure 3.3.

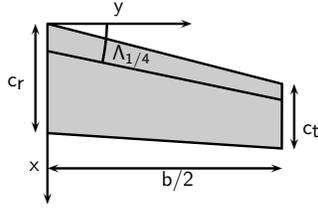


Figure 6: Parameters of wing planform for construction of stability boundary using the new multifidelity scheme.

Parameter	LF	HF
elements (chord)	10	15
elements (semi-span)	24	36
modes	10	15
aero-boxes (chord)	10	15
aero-boxes (semi-span)	12	18
CPU time	1 min	5 min

Table 1: Low (LF) and high (HF) fidelity configurations for wing model.

For the 3D case, the convergence of the multifidelity scheme is depicted in Figure 3.3. As for the analytical example, it is compared to EDS and a DOE.

### 3.4 Constrained Efficient Global Optimization

In order to perform an effective and efficient optimization with an SVM-based stability constraint, the PI has developed an approach based on Efficient Global optimization (EGO) for the objective function [7]. EGO allows one to select samples that have the highest “expected” improvement for a given objective function approximated as a Gaussian process.

A sampling scheme has been developed to maximize the expected improvement,  $EI$ , while refining a constraint on the probability of feasibility  $P(+1|\mathbf{x})$ . The probability of feasibility  $P(+1|\mathbf{x})$  is calculated using a novel probabilistic SVM model, which accounts for the probability of misclassification [8].

The overall scheme is a two-level optimization (details are available in [8]). A first stage minimizes globally the objective function by maximizing the expected improvement ( $EI$ ) and constraining the probability of feasibility  $P(+1|\mathbf{x})$ :



Figure 7: Low (LF) and high (HF) fidelity configurations for wing model.

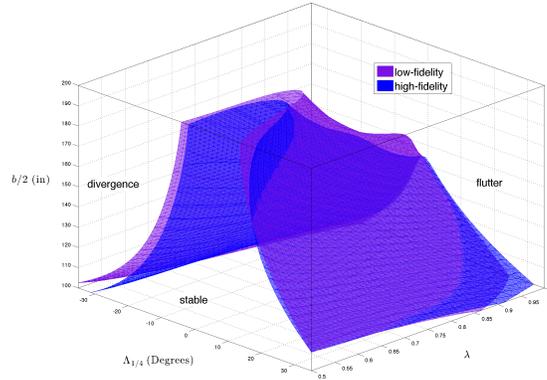


Figure 8: High and low fidelity stability boundaries. Two failures modes: divergence and flutter.

### 3D Case

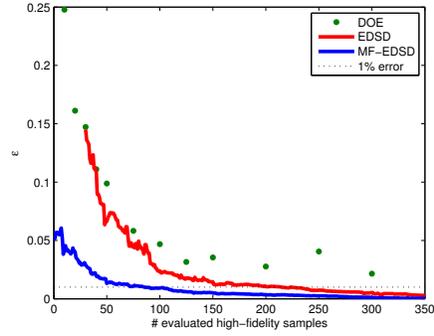


Figure 9: Comparison of HF function evaluations for the multifidelity scheme (MF-EDSD), HF-based EDSD, and Central Voronoi Tessellation designs of experiments (DOE).

$$\begin{aligned} \max_{\mathbf{x}} \quad & EI(\mathbf{x}) \\ \text{s.t.} \quad & P(+1|\mathbf{x}) \geq 0.5 \end{aligned}$$

The second level refines the SVM boundary locally within a hypersphere and is based on the probability of misclassification  $P_m$ :

$$\begin{aligned} \max_{\mathbf{x}} \quad & \|\mathbf{x} - \mathbf{x}_{nearest}\| \\ \text{s.t.} \quad & P_m(\mathbf{x}) \geq 0.5 \\ & \|\mathbf{x} - \mathbf{x}_c\| \leq R_u^{(k)} \end{aligned}$$

Details on the update of the radius  $R_u$  at each iteration is given in [8].

**Application example.** Consider a wing for which one wants to find the thickness distribution (modeled with an exponential) along with the sweep angle, taper ratio, and span (a total of five variables) so as to minimize weight while maintaining a stable configuration (accounting for flutter and divergence). Figure 10 provides a depiction of the wing and its optimal thickness distribution. Figure 11 provides the evolution of the objective function (weight) and an example of representation of the stability boundary.

The optimization problem to find the optimal planform and optimal thickness distributions are:

$$\begin{aligned} \min_{\mathbf{x}} \quad & Weight = \int_0^{\frac{b}{2}} \rho t(y) c(y) dy \\ \text{s.t.} \quad & s(\mathbf{x}) \leq 0 \quad (\text{i.e., } \mathbf{x} \in \Omega_{stable}) \\ & \mathbf{x}_{min} \leq \mathbf{x} \leq \mathbf{x}_{max} \end{aligned}$$

Where the vector of design variables  $\mathbf{x}$  is  $\mathbf{x} = \{\theta, t_1, t_2, b, c_r\}^T$  and the exponential thickness distribution is defined as:  $t(y) = t_1 e^{-\frac{2t_2 y}{b}}$ . Convergence results and optimal results are provided in the following figures.

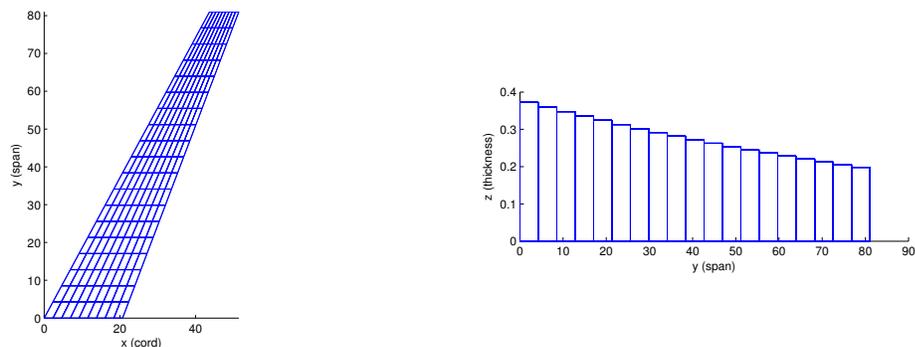


Figure 10: Wing planform to optimize for weight with stability constraint. Optimal thickness distribution of the wing.

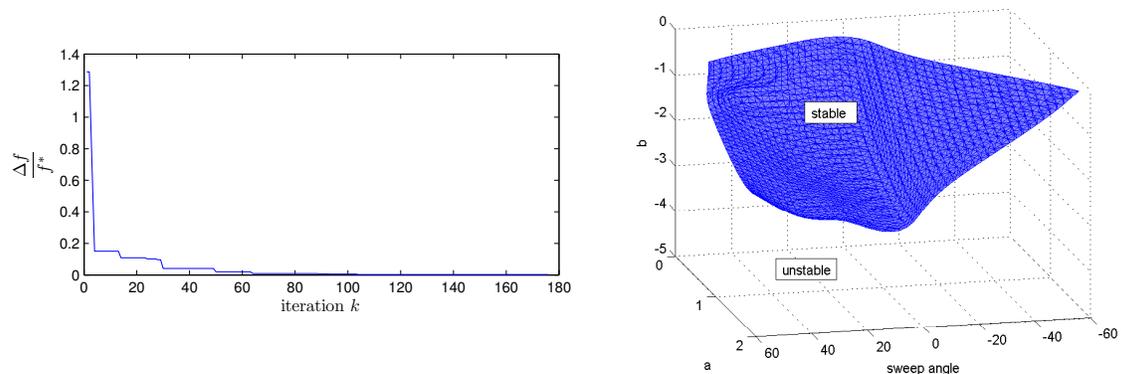


Figure 11: Evolution of objective function. Example of stability boundary with 3 variables.

## References

- [1] Cristianini, N. and Scholkopf, B. Support vector machines and kernel methods: The new generation of learning machines. *Artificial Intelligence Magazine*, 23(3):31–41, 2002.
- [2] S. Missoum, C. Dribusch, and P. Beran. Reliability-based design optimization of nonlinear aeroelasticity problems. *Journal of Aircraft*, 47(3):992–998, 2010.
- [3] Basudhar, A. and Missoum, S. Reliability assessment using probabilistic support vector machines (psvms). In *51<sup>st</sup> AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference*, 2010. Orlando, FL.
- [4] C. Dribusch, S. Missoum, and P. Beran. A multifidelity approach for the construction of explicit decision boundaries: application to aeroelasticity. *Structural and Multidisciplinary Optimization*, pages 693–705, 2010.
- [5] C. Dribusch and S. Missoum. Construction of aeroelastic stability boundaries using a multi-fidelity approach. In *In Proceedings of the 53<sup>rd</sup> AIAA/ASME/ASCE/ASC Structures Dynamics and Materials Conference*, volume AIAA 2012-1803, April 2012.
- [6] Zona Technology, Inc, 9489 E. Ironwood Square Drive, Scottsdale, AZ 85258-4578. *Zaero Version 8.3 Theoretical Manual*, 19 edition, October 2008.
- [7] BJ Bichon, S. Mahadevan, and MS Eldred. Reliability-based design optimization using efficient global reliability analysis. In *11th AIAA Non-Deterministic Approaches Conference, Palm Springs, CA*, 2009.
- [8] A. Basudhar, C. Dribusch, Lacaze S., and S. Missoum. Constrained efficient global optimization with support vector machines. *Structural and Multidisciplinary Optimization*, 46:210–221, 2012.

## 4 Personnel Supported During Duration of Grant

Christoph Dribusch, Graduate Student, University of Arizona, Tucson

Samy Missoum, Associate Professor, University of Arizona, Tucson

## 5 Publications resulting from this grant

“Reliability-based Design Optimization of Nonlinear Aeroelasticity Problems”. S. Missoum, C. Dribusch, P. Beran. *AIAA Journal of Aircraft*. Vol. 473 No. 3, 2010, pp. 992–998.

“A Multifidelity Approach for the Construction of Explicit Decision Boundaries: application to aeroelasticity. C. Dribusch, S. Missoum, P. Beran. *Structural and Multidisciplinary Optimization*. Vol. 42. No. 5, 2010, pp. 693-70.

“Constrained Efficient Global Optimization using Support Vector Machines”. A. Basudhar, C. Dribusch, S. Lacaze, S. Missoum. Structural and Multidisciplinary Optimization. Vol. 46, No. 2, 2012, pp. 201–221.

”Construction of Aeroelastic Stability Boundaries Using a Multi-Fidelity Approach”. C. Dribusch, S. Lacaze, S. Missoum. Proceedings of the 53<sup>rd</sup> AIAA/ASME/ASCE/ASC Structures, Structural Dynamics, and Materials Conference. Honolulu, HI, 2012

## **6 Interactions**

A large part of this research has been carried out as a collaboration with Philip S. Beran, AFRL/RBSD, WPAFB, OH.