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Our effort has been continued for developing and analyzing high order methods for the interface problems. Specifically, We have developed an adaptive mesh refinement for the Immersed Boundary and the Immersed Interface Method using the level set method for elliptic interface problems, the sensitivity calculation with respect to the parameter and shape for flow past obstacle and multi-moment immersed interface method for wave equation and Hamilton-Jacobi equation. Our approach uses the physical augmented variables and the domain embedding technique and allows us to develop fast and robust implementation of the immersed interface method for general interface problems.

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Applications of Sharp Interface Method for Flow Dynamics, Scattering and Control Problems

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July 30, 2012

Abstract Our effort has been continued for developing and analyzing high order methods for the interface problems. Specifically, We have developed an adaptive mesh refinement (AMR) for the Immersed Boundary (IB) and the Immersed Interface Method (IIM) using the level set method for elliptic interface problems, the sensitivity calculation with respect to the parameter and shape for flow past obstacle and multi-moment immersed interface method for wave equation and Hamilton-Jacobi equation. Our approach uses the physical augmented variables and the domain embedding technique and allows us to develop fast and robust implementation of the immersed interface method for general interface problems. In particular, the sensitivity calculation with respect to the Reynolds number for the vortex shading in the flow past obstacle problem near the critical Reynolds number (Re=47) shows the enhanced and detailed sensitivity toward the instability. The sensitivity calculation provides more precise and detailed sensitivity of the solution and describes the dynamical change due to the variation in the Reynolds number. The immersed interface method accurately computes the velocities and the stress on the interface.

This is the first time that an AMR has been applied to the immersed interface method. The new algorithm is based on the level set representation of the interface. This unique feature enables us to refine the mesh only in the neighborhood of the interface. The results for elliptic interface problems with arbitrary shapes are tested. An accurate sensitivity analysis with respect to the change in the interface condition and geometry has been computed efficiently via the immersed interface method as the forward and the sensitivity equation solver. We develop and analyze a new multi-moment method for one-dimensional hyperbolic equations with discontinuous coefficients. The method is based on the backward characteristic method and uses the solution and its derivative as unknowns and cubic Hermite interpolation for each computational cell. An exact update formula for solution and its derivative for variable wave speed is derived and used for efficient time integration. At points of discontinuity we develop a piecewise cubic Hermite interpolation based on interface conditions.

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The method is extended to the one-dimensional Maxwell equation with variable material properties. The extension of methods to multi-dimensional case is under progress, in which we use Poisson formula for wave equation as a building block.

1 Project Summary

Our effort has been continued for developing and analyzing high order methods for the interface problems. We have developed an adaptive mesh refinement (AMR) for the Immersed Boundary (IB) and the Immersed Interface Method (IIM) using the level set method for elliptic interface problems. We have continued the sensitivity calculation with respect to the Reynolds number for the vortex shading in the flow past obstacle problems and a accurate sensitivity analysis with respect to the change in the interface condition and geometry has been computed efficiently via the immersed interface method as the forward and the sensitivity equation solver. We have developed and analyzed a new multi-moment method for one-dimensional hyperbolic equations with discontinuous coefficients. The method is based on the backward characteristic method and uses the solution and its derivative as unknowns and cubic Hermite interpolation for each computational cell. An exact update formula for solution and its derivative for variable wave speed is derived and used for efficient time integration. At points of discontinuity we develop a piecewise cubic Hermite interpolation based on interface conditions. The method has been applied to the Hamolton-Jacobi equation.

2 Status/Progress

Elliptic interface problems are fundamental problems for incompressible flow solvers for Stokes flow and Navier-Stokes equations. This is the first time that an AMR has been applied to the immersed interface method. The new algorithm is based on the level set representation of the interface. This unique feature enables us to refine the mesh only in the neighborhood of the interface. The results for elliptic interface problems with arbitrary shapes are excellent. Let $\varphi(x)$ be a levelset function whose zero level set ($\varphi = 0$) defines the interface. If we have a uniform Cartesian grid with mesh size h_1. For the grid points (x_i, y_j) in the tube $|\varphi| \leq \delta$, we generate a finer grid with smaller mesh size h_2 ($h_2 < h_1$). Generate the finite difference equations using the standard 5-point or 9-point stencil. The procedure can be repeated hierarchically. Whenever the finite difference scheme at a grid point in the new mesh needs values at points that are not grid points in any level of the mesh (hanging nodes), an interpolation scheme is used. The resulting linear system of finite difference equations is solved by an algebraic multi-grid solver.

The AMR features include:

- Better accuracy near the interface

Table 2.1: Comparison of results using a uniform mesh and that of the AMR-IIM with the same finest resolution h. The runtimes in second by algebraic multigrid solver are also provided.

	Uniform				AMR(+1)[4]					
m	#	Error	Ratio	CPU	m	λ	#	Error	Ratio	CPU
40	1521	8.44e-4		.03	20	1	805	9.07e-4		.02
80	6241	2.45e-4	3.4	.08	40	2	2745	2.86e-4	3.2	.06
160	25281	6.69e-5	3.7	.31	80	4	10201	7.80e-5	3.7	.19
320	101761	1.57e-5	4.3	1.3	160	8	39225	1.89e-5	4.1	.67
640	408321	3.65e-6	4.3	5.4	320	16	153805	5.04e-6	3.8	2.5

- Less degree of the freedom, i.e. smaller system to solve

- Combined with the level set method so it is flexible in dealing with complicated domain

- Use an algebraic multi-grid solver which is fast.

We have applied to an elliptic interface problem:

$$u_{xx} + u_{yy} = 0, \quad \text{on} \quad \Omega \backslash \Gamma, \qquad [u]_\Gamma = 0, \qquad \left[\frac{\partial u}{\partial n}\right]_\Gamma = 2,$$

Figure 2.1 shows the three level refinement for a eight star shape and Table 2.1 shows the AMR performance for the corresponding solutions.
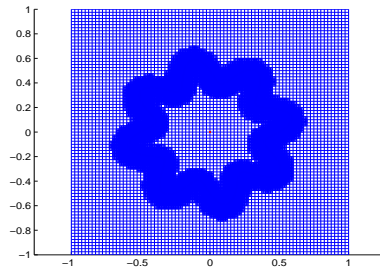

Figure 2.1: A two level AMR for a eight star shape

2.1 The sensitivity analysis and flow control

We haveinvestgated the sensitivity analysis with respect to the Reynolds number for the flow past obstacle problem. To carry out such analysis, at each time step, we need to solve

the incompressible Navier-Stokes equations on irregular domains twice, one for the primary variables; the other is for the sensitivity variables with homogeneous boundary conditions. The Navier-Stokes solver is the augmented immersed interface method for Navier-Stokes equations on irregular domains which is developed under the current grant. Our sensitivity analysis can predict the critical Reynolds number at which the vortex shading begins to develop in the wake of the obstacle. Numerical experiments are shown to illustrate how the critical Reynolds number varies with different geometric settings.

The sensitivity equation with respect to the constant viscosity μ for the incompressible Navier Stokes equation is given by; $\mathbf{v} = \frac{d}{d\mu}\mathbf{u}$, $q = \frac{d}{d\mu}p$ satisfy

$$\rho\,(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{u}) + \nabla q = \nabla \cdot \frac{\mu(\nabla \mathbf{v} + \nabla \mathbf{v}^T)}{2} + \nabla \cdot \frac{(\nabla \mathbf{u} + \nabla \mathbf{u}^T)}{2}$$

$$\nabla \cdot \mathbf{v} = 0.$$

We have carried out our sensitivity analysis to examine the effect of the shape and orientation of the obstacle on the vortex shading. The obstacle that we considered is an ellipse whose level set function is

$$\varphi(x,y) = \sqrt{(x+7)^2/0.6^2 + y^2/0.4^2} - 1$$

with different orientations. We see that critical Reynolds number of $Re^* = 1/\mu^*$ is quite different for different orientations. The sensitivity analysis does give a good estimate of the critical number at which the vortex shading is about to develop. In Figure 2.2 (left), we show the sensitivity analysis of an ellipse at different orientations. The plots on the left are the plots of the vorticity (top) and sensitivity vorticity (bottom) strengths versus the parameter $1/\mu \sim Re$. From the plot, we can predict that the critical Reynolds number is significant different with the orientation. The results are also agree with intuition. With the orientation is parallel to the flow direction, the flow in the wake of the obstacle is more stable than that against the flow direction. As we can expected, we can predict that the critical Reynolds number is the largest for the horizontal case (around 70), decreases for the oblique ellipse (around 45), and the smallest for the vertical case (around 40). The plots on the right are vorticity (top) and sensitivity vorticity (bottom) at some numbers which supports our discussions above.

We have also carried out the sensitivity analysis for a flow past two separated cylinders. The level set function is defined as follows

$$\varphi_1(x,y) = \sqrt{(x+7)^2 + y^2} - 1, \quad \varphi_2(x,y) = \sqrt{(x-x_2)^2 + y^2} - 1,$$

$$\varphi_1(x,y) = \min\{\varphi_1(x,y), \varphi_2(x,y)\}.$$

In Figure 2.2 (right), we show the sensitivity analysis for the two stationary cylinders. We show three different cases. In the first case (a), the two cylinders are centered at $(-7, 0)$
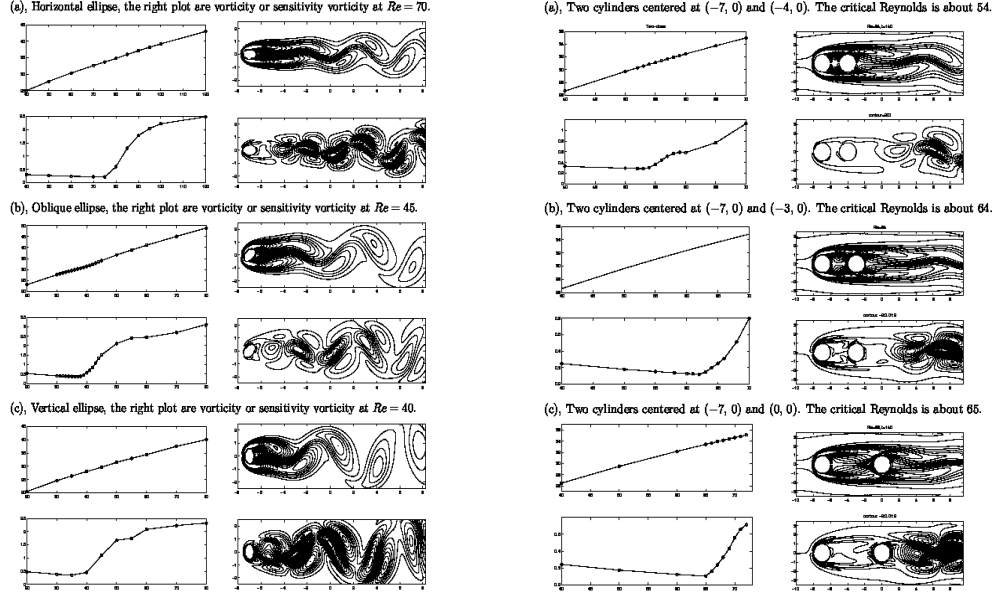
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(a), Horizontal ellipse, the right plot are vorticity or sensitivity vorticity at $Re = 70$.

(a), Two cylinders centered at $(-7, 0)$ and $(-4, 0)$. The critical Reynolds is about 54.

(b), Oblique ellipse, the right plot are vorticity or sensitivity vorticity at $Re = 45$.

(b), Two cylinders centered at $(-7, 0)$ and $(-3, 0)$. The critical Reynolds is about 64.

(c), Vertical ellipse, the right plot are vorticity or sensitivity vorticity at $Re = 40$.

(c), Two cylinders centered at $(-7, 0)$ and $(0, 0)$. The critical Reynolds is about 65.

Figure 2.2: Vorticity strength versus the Reynolds: The top row is for the primitive and the bottom row is for the sensitivity equations.

and $(-4, 0)$, respectively. They are considered as relatively close. We see that the critical Reynolds number is about $Re^* = 54$ which is larger than that of a single cylinder. As we increase the distance between two cylinders by one unit, we see that the critical Reynolds number increases by 10, see Fig. **??** (b) for which the two cylinders are centered at $(-7, 0)$ and $(-3, 0)$ respectively. If we further increase the distance between two cylinders by another 3 units, the critical Reynolds number remains almost the same. In Fig. **??**, the left plots are the strength of the vorticity and sensitivity versus the Reynolds number defined as $1/\mu$. The right plots are the vorticity and sensitivity plot at $t = 100$ and a Reynolds number that is close to the critical Reynolds number 54, 64, and 65 respectively.

We have investigated with rotating objects for the single obstacle and two separated cylinders. Our sensitivity analysis can provide a good estimate of the critical number at which the vortex shading is about to develop even for dynamical flow cases. We will continue to investigated for the different flow regime (the both backward and forward steps and cavity flows) under various applied flow controls and develop the sensitivity analysis tool for the actuator and sensor dynamics.

2.2 Multi-moment method for hyperbolic equations

We have developed a multi-moment method for the wave propagation in discontinuous media. We considered and transport as well as advection equation as a test model:

$$u_t + c(x)u_x = 0, \quad u_t + (c(x)u)_x = 0, \quad x \in R. \tag{2.1}$$

where the wave speed $c = c(x) > 0$ is variable and discontinuous. Our method is motivated and closely related to CIP methods [?]. CIP is one of the numerical methods that provides an accurate, less-dispersive and less-dissipative numerical solution. Our method uses the exact integration in time by the characteristic method and uses the cubic Hermite interpolation in each cell $[x_{j-1}, x_j]$ based on solution values and its derivatives at two endpoints x_{j-1}, x_j. That is, we develop the exact solution formula for solution and its derivative for (2.1). The Hermite cubic interpolation is then used to evaluate the formula locally. Our development of CIP is entirely based on the characteristic method and results in a different (improved and simpler) scheme than the conventional one for equations with variable wave speed. For example the new method allows us to take an arbitrary time step (no CFL limitation) without losing the stability and accuracy. We analyzed the von-Neumann stability of the proposed method for a constant speed. We develop an immersed interface method [?] for the discontinuous wave speed case, i.e., we construct a piecewise cubic Hermite interpolation at a cell $[x_{j-1}, x_j]$ which contains a point of discontinuity of $c(x)$ using proper interface conditions and solution values and its derivatives at two endpoints x_{j-1}, x_j. This interface treatment is applied for the one-dimensional Maxwell equation with variable material properties. We first approximate the variable coefficients by a piecewise constant (discontinuous) coefficient. The d'Alembert's based method for the Maxwell equation that extends our characteristic based method to Maxwell system is developed for the piecewise constant media and then applied to Maxwell system with piecewise constant coefficients.

In the case of the transport equation we have

$$u(t + \Delta t, x) = u(t, y), \quad u_x(t + \Delta t, x) = \frac{c(y)}{c(x)} u_x(t, , y)$$

where y is the backward characteristic of $x \in R$ determined by

$$\frac{d}{dt}x(t) = c(x(t)), \quad x(t + \delta t) = x \text{ and } y = x(t).$$

Suppose $c(x)$ is piecewise constant and discontinuous at $x^* \in (x_{j-1}, x_j)$, We construct the piecewise cubic interpolation $F^{\pm}(x)$ at each time step t_n:
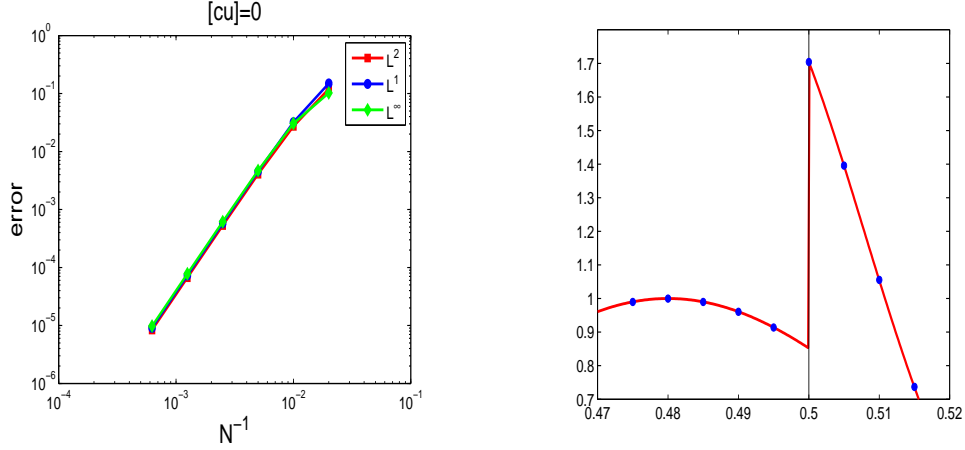
$$F^{\pm}(x) = \sum_{k=0}^{3} a_k^{\pm}(x - x^*)^k$$

6

Figure 2.3: Discontinuous wave speed: advection case

in each interval $[x_{j-1}, x^*]$ and $[x^*, x_i]$, respectively. We determine the eight unknowns via the interface relations $[u] = [cu_x] = [c^2 u_{xx}] = [c^3 u_{xxx}] = 0$ and and the interpolation conditions $u(x_{j-1}) = u_{j-1}^n$, , $u_x(x_{j-1}) = v_{j-1}^n$ at x_{j-1} and: $u(x_j) = u_j^n$, , $u_x(x_j) = v_j^n$ at x_j. Thus, the update formula becomes

$$u_j^{n+1} = F^+(x_n - c^+ \Delta t), \quad v_j^{n+1} = \frac{c^-}{c^+} F_x^+(x_n - c^+ \Delta t).$$

In Figure 2.3 solutions in discontinuous wave speed media for the advection equation.

We are currently extending our treatment to the multi-dimensional wave equations. The building block of the extension is the Poisson formula for solutions to the wave equation, i.e. one can derive an exact time integration method in time for the homogeneous media case. Also, we uses the (cell-wise) bi-cubic interpolation based on moments u, u_x, u_y, u_{xy}. We analyze the von-Neumann stability of the proposed method and we establish CFL number is one for the proposed method. Despite its complexity, the method offers a highly efficient method to attain the desired accuracy due to the relaxed CFL number limitation and accurate space resolution by the bi-cubic profile. The method preserves discontinuous profiles very accurately without any smearing and distortion, e.g. see Figure 2.4. Also, the method computes directly the physical quantities, e.g., current and electric field gradient, very accurately. The immersed interface treatment will be integrated to develop a higher order fully time-space method for heterogeneous media.

2.3 Multi-moment method for Hamilton-Jacobi equation

We develop a multi-moment approximation method for Hamilton Jacobi (HJ) equations, which arise in many applications such as geometrical optics, crystal growth, etching, com-
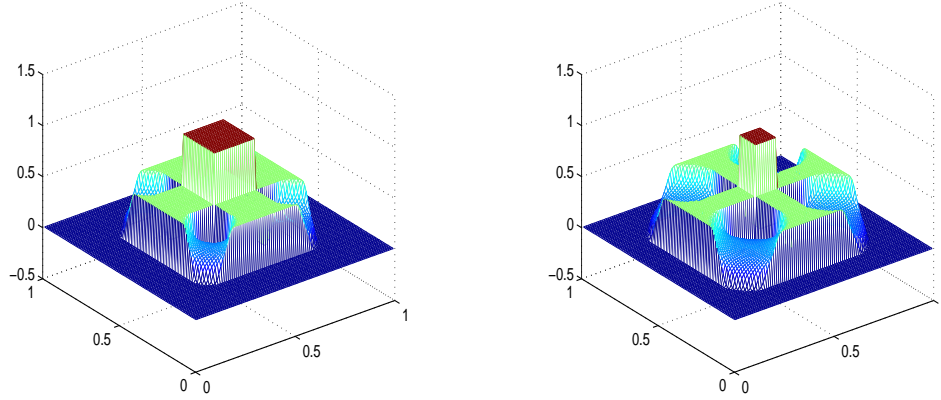
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Figure 2.4: Scattering from a square (CFL=1)

puter vision, obstacle navigation, path planning, photolithography, and seismology and control problems. In general, the solutions usually develop singularities in their gradient even with smooth initial conditions. In many application of HJ it is very essential to compute the gradient of the solution along with solution accurately. Especially, for control problems the optimal feedback solution is determined using the gradient of the value function (solution).

We develop approximation methods based on the Lax-Hoph formula for the convex Hamiltonian and the maximum principle for the control problem as the (exact) time integration. The formula calculates the unique viscosity solution of HJ equations. In order to update the solution to the next time step we use a locally defined quadratic and cubic interpolation of the solution based on the solution value and its gradient at each square local cell. To complete the multi-moment method we develop the update formula for the gradient.

For Hamilton-Jacobi equation

$$V_t + H(V_x) = 0, \quad x \in \Omega,$$

where H is a convex Hamiltonian, for example $H(P) = \frac{1}{2}|p|^2$, $|p|$. Let H^* be the convex conjugate of H:

$$H^*(v) = \sup_p \left((v,p) - H(p) \right).$$

We have the solution update formula:

$$V(z, t + \Delta t) = \min_{\zeta \in \Omega} \{ V(\zeta, t) + \Delta t H^*(\tfrac{z-\zeta}{\Delta t}) \}.$$

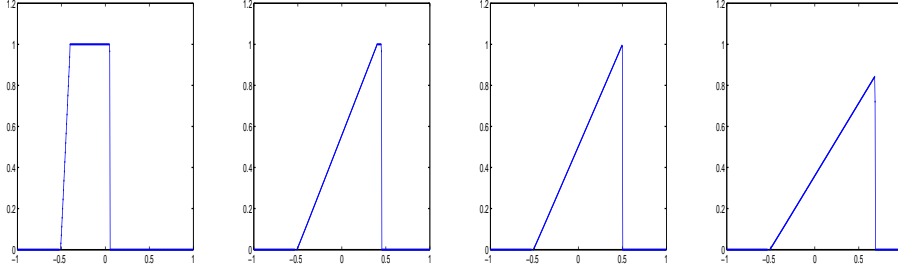$$\nabla V(z, t + \Delta t) \in \partial H^*(\tfrac{z-\zeta_z}{t}).$$

8

Figure 2.5: 1D Hamilton-Jacobi equation: $u = V_x$

where ζ_x is a minimizer.

The following is our proposed algorithm:

- Initialize (v_i^0, u_i^0)

- Solve the two-sided quadratic minimizations

$$y_i^- = \arg\min \{F_{i-1,i}(t_n, y) + \Delta t H^*(\tfrac{x_i - y}{\Delta t})\} \text{ over } [x_{i-1}, x_i]$$

and

$$y_i^+ = \arg\min \{F_{i,i+1}(t_n, y) + \Delta t H^*(\tfrac{x_i - y}{\Delta t})\} \text{ over } [x_i, x_{i+1}]$$

- Find y_i^n (either y_i^+ or y_i^+) that achieves the minimum of the two-sided minimization;

$$v_i^{n+1} = \min(F_{i-1,i}(t_n, y_i^-), F_{i,i+1}(t_n, y_i^+))$$

$$u_i^{n+1} = \frac{x_i^n - y_i^n}{\Delta t}$$

Here $F_{i-1,i}$ and $F_{i,i+1}$ are the interpolation function on each interval $[x_{i-1}, x_i]$ and $[x_i, x_{i+1}]$, respectively. The building block of our integration method for Hamilton-Jacobi equation is the Hoph-Lax formula and variant of methods can be developed based on various proper interpolation method for the local solution profile. We have also successfully applied the method for Hamilton-Jacobi equation associated with optimal control problems. In Figure 2.5 solutions are shown for the case $H(p) = \frac{|p|^2}{2}$ for square wave $u = V_x$ (Burgers equation).

Personnel Supported During Duration of Grant

K. Ito, Prof., North Carolina State University.

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Sarah King, Cary Humber, Graduate Student at North Carolina State University.

Publications

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