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Final Report

Distributed Control for Networked Systems with Non-Traditional Communication Constraints

Lossy Links, Power and Usage Limitations, and Induced Cooperation

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0. Introduction

This is the final report for the AFOSR Grant“Distributed control for networked systems with non-traditional communication constraints: Lossy links, power and usage limitations, and induced cooperation,” covering the period March 1, 2009 - November 30, 2011. The report is comprised of 5 sections. Section 1 provides an overview of the relevant research area and the issues addressed by the PI during the period of the Grant. Section 2 describes the research conducted and the results obtained, broken down to 7 subsections, each one covering a specific subarea. After a brief conclusions section (Section 3), the report contains a list of plenary talks and special colloquia given by the PI on topics covered by the Grant (Section 4), and a list of publications and conference presentations resulting from this research, along with additional bibliography cited in the report (Section 5).

1. Overview of the Research Area and Issues Addressed

As wireless sensing and control become increasingly applicable in fields ranging from real-time alarm systems and vehicle systems to aeronautical guidance and formation control, the need for establishing a theoretical foundation for what is known as *networked systems* [R1] has grown likewise. Such systems have sensors and controllers distributed generally in an ad-hoc manner, but have to be connected virtually either through communication and information transmission or because of the need to achieve some level of performance driven by individual or common goals, or *both.* Any effective effort to develop a theoretical foundation for this relatively new paradigm necessitates pooling together of tools (both conceptual and algorithmic) from multiple seemingly disparate disciplines, such as control theory, information theory, coding, communication, computing, and game theory. Some salient aspects of this paradigm, and the challenging issues that arise in this context, which we have addressed in the research supported by this AFOSR grant, are as follows, where the generic term *agents* is used for entities that are responsible for decision making that leads to actions, be they sensors or controllers or even dynamical systems.

• **Incomplete information.** In most networked systems, the agents do not have access to the complete *map* of the network on which they operate, and know only the presence of their neighbors with whom they interact. Incompleteness of information (which is different from receiving noisy information or measurements) necessitates that each agent develop a subjective probabilistic model of the network and update its model (through learning) as more information becomes available. One of the questions that arise in this connection is whether such iterations converge and if they do whether they converge to the same limiting model (*consensus*). Another question is the dependence of the various issues listed below, and particularly the sensitivity of the performance attainable for the network, on inaccuracies in the modeling by different agents. In other words, what is the *cost of incompleteness of information*?

• **Decentralization.** Even if all agents share the same (probabilistic) model of the network on which they operate, the on-line measurements they make and the information they receive are not broadcast and hence are not centrally available. Decentralized optimal decision making and control problems, particularly in stochastic environments, are known to be notoriously difficult (not only to solve numerically, but even conceptually) because of the tradeoff between two generally conflicting roles of a control/decision policy in such systems with *nonclassical information*: contributing toward performance improvement at the present versus carrying useful information into future stages which could potentially lead to better overall performance [R2]. This *information-carrying capability* of control laws lies at the heart of the difficulty in constructing optimal policies when different agents operating on the same network do not share the same information or the same measurement channels. One may argue that the current sensor technology partially alleviates this problem by making it possible for sensors linked to individual control channels or agents to communicate with each other and transfer the necessary information so that all needed data are eventually shared by all relevant parties. However, what emerges is still not a centralized control problem because of the power and bandwidth limitations inherent to sensor communication and the presence of time delay(s) in the sharing of information in a distributed system. There have been some success stories in using some indirect methods, particularly information theory based bounds, in solving optimal decision problems with nonclassical information [R3], but the field is still in its infancy. Even though to obtain a general theory for optimal control problems with general nonclassical information could be out of reach, still for some special structures the problem may be tractable, as in [R3] and [R2], using however some nontraditional tools. Carving out such special structures rooted in real applications, and generating new approaches for these problems has been one of the goals of our research.

• **Bandwidth limitations.** Most networked systems have digital (wired or wireless) links between the plant, the controllers, and the sensors, which are necessarily bandwidth limited. To match the input to a channel's characteristics one has to pass the signal through a sampler (if it is continuous time) followed by a quantizer and an encoder. At the output of the channel the message will have to be decoded, de-quantized, and passed through a sample-hold to lift it back to a continuos-time and/or continuous-alphabet signal or message. The questions that arise here at the front end are how to sample (time invariant or time varying) and at what rate (variable or constant); how to quantize (uniform, logarithmic, etc) under given channel capacity constraints; and how to encode (time invariant, time varying, memoryless, with memory, etc), so that certain performance guarantees (including stability) can be assured. The presence of the feedback loop and the natural requirements of real-time implementability and causality on the designs make standard coding theory techniques inapplicable in these scenarios. Even though we have not addressed these problems in our research under this grant, it is worth noting that some strides have been made here by many (including us; [R4]-[R6]) during the last decade, which have however only scratched the surface, leaving still several open problems, such as when channels are noisy [R7] (not quantization noise) or when controls are distributed and information is decentralized [R8]-[R10] in which case signaling through control plays an important role in enhancing information flow in the network.

• **Usage limitations and sharing of resources.** Another type of a communication constraint arises when there are restrictions on channel usage and/or on the frequency of interactions between neighboring agents. The former arises when, for example, communication or information transmission is carried out using wireless devices which are naturally power limited due to battery lifespan. This requirement imposes severe restrictions on the duration of time the wireless device can be on/awake and the number of transmissions it can make. This is because the radio frequency (RF) communication consumes a significant portion of the battery power when the wireless unit is awake. Therefore, life of the wireless device can be lengthened by optimizing the duty cycle (or reporting frequency) of the unit as well as by transmitting data only when it is necessary. This type of limitation brings in a constraint on the decision making function of sensors and controllers, which is not of the standard type. Some of the questions that arise in this context are [R11]-[R14]: How does one optimally schedule transmission of data/information from one agent (such as a sensor) to another (say a controller) on a network under a constraint on the frequency of transmissions? How can one quantify the degradation in performance due to such constraints on the number of interactions between different decision units? If a communication link is used by multiple sensors and multiple controllers, how should transmissions be scheduled (or bandwidth be time-shared) so that optimum performance is achieved? A related question is [R15]: If both sensing and control are to be executed using the same bandwidth-limited channel, how should the resource be allocated between these two uses? Should a higher fraction (of the resource) be allocated to control/action signal transmission or to receiving more data to improve the performance of less frequent control actions? Toward the beginning of the grant period, we had initiated research on these problems with non-traditional constraints, and since then we have made significant inroads into the underlying challenges, such as obtaining recursively computable threshold-type optimal policies for some specific models. Details are given in the next section.

- **Lossy, leaky, and unreliable links, and adversarial interference.** In a network, link failures cause the information flow between the controller and the plant (or in general between two or more agents) to be disrupted, which results in control and/or measurement packets being lost. This disruption of communication has a deteriorating effect on the networked control system performance. Therefore, it is important to develop an understanding of how much loss the control system, or the network in general can tolerate before the system becomes unstable, or in the case of estimation before the estimation error becomes unbounded, or in the case of a general network before overall performance falls below an unacceptable level. Further, if the statistical description of the link failure process is given a priori, a problem of interest is to determine the optimal control and estimation policies under the link failure constraints. In a remote control setting, packet losses may occur both from the sensor to the controller, and from the controller to the actuator. In the first case, the measurement packets are lost and therefore the controller has access to perfect or imperfect information on the state only intermittently. In the latter case, the control or actuation packets are lost, and this causes the actuator to have access to the controls intermittently. Optimal control of stochastic systems in a networking environment under such unreliability constraints is a problem of great relevance and importance to networked systems, and we had initiated research on this class of problems just prior to the start of the *Grant*, and we have expanded on this line of enquiry during the course of this *Grant*, as discussed in the next section. We note that unreliability could be the result of some malfunction or fading, which admits a statistical description [R16], [R17], or could be the result of some adversarial malicious attack, as in [R18], or *both* [R19]. Further, one has to distinguish between the case where the sender agent (such as the controller) receives an acknowledgment for each control packet it sends to the receiving agent (such as the actuator), and the case where he does not. The former resembles the Internet, with the acknowledgement mechanism built into TCP (Transmission Control Protocol), whereas the latter resembles a best-effort, or UDP (User Datagram Protocol) type network. In our fairly recent work prior to the start of this *Grant*, we had shown that one can formulate reasonable and tractable stochastic optimization formulations which capture the salient aspects underlying these problems, and this had opened up promising avenues for research toward developing a comprehensive theory for such systems. One of these, which we have initiated under this *Grant*, was looking at stochastic control problems under usage limitations as in the earlier bullet, where now the communication links connecting different agents are not reliable, with *unreliability* given a precise mathematical description in terms of a Bernoulli process. Another one on which we have obtained substantial new results during the *Grant* period is the class of problems where there is adversarial interference on the interactions of different agents, which could be in the form of jamming the communication links, controlling link failure rates, and limiting usage of the channels. These are further discussed in the next section.

- **Distributed local-performance driven decision making.** In a networked system, or equivalently in a network of distributed agents, even though there may be a central goal, it is unrealistic to assume that this overall goal is communicated to and even shared by all individual agents. Instead, agents will have their individual *local* performance-driven objectives, and will carry out their actions computed in accordance with these objectives and under the informational and communication constraints of the types introduced and discussed above. Further, they will be acting non-cooperatively (as in Nash equilibrium), even though the overall mission will necessitate team coordination. A question of interest here is whether, in the absence of a higher-level coordinator, the interactions of the agents under the given informational constraints and driven by their local objectives, will converge to a Nash equilibrium, and how far this equilibrium would be from an *efficient* solution which requires collaboration at some cost of additional communication. That is, what is the *price of non-cooperative behavior* that is *due to localization of objectives*? Another question is: If there is a common underlying goal or mission (such as detection of a target), is consensus reached by the agents, as in [R20]? These questions have been addressed during the course of this *Grant* within the context of the various scenarios introduced above (as discussed in the next section).

• **Coordination through multi-level interaction.** The discussion above has assumed that all agents on a network are same-level decision makers or computing units, and that there is no higher level coordinator. The presence of a higher level coordinator (or one agent acting as such), with the coordinator sending appropriate signals to lower-level units based on information received from the other agents, could facilitate coordination. This could in fact induce all agents to cooperate as though they were acting as members of a team working toward a common goal, even though each is optimizing its own objective function, as shown in the context of a network services problem [R21]. But signaling is costly, and effective signaling requires a two-way communication, with also information fed from the agents to the coordinator. How should this information be properly aggregated and quantized, is a challenging question that arises here. Another related one is whether the signaling can be restricted to one- or two-bit messaging, as in [R22], without significant degradation in performance. These and related issues have also been addressed in our research.

2. Description of Research Conducted and Selected Results

Forty-six publications and presentations based on research supported by the *Grant* are listed in Section 5, in reverse chronological order. Here we discuss some salient aspects of the results obtained, organized into seven subsections.

2.1. Decision making in adversarial environments with limits on the frequency of actions

We have introduced in the previous section the rationale for placing limits on the number of times different decision units in a distributed network would interact with their environment, and particularly with other decision units. These problems can be formulated as dynamic stochastic optimization problems (in discrete time), with a total usage constraint (over a finite horizon) on the number of actions by each decision unit. One specific problem that we have looked at is that of transmission of a discrete-time random process (a finite string) over a channel with the goal of generating an accurate estimate at the other end (under minimum mean squared error (MMSE) or minimum probability of error (MPE) criteria) [11]. If the sequence is of length N, and the channel can be used only $M < N$ times, then the question is determination of the optimum memoryless transmission (encoding) policy and the corresponding optimum estimator (decoder) policy under these constraints. We have shown that the optimum transmission policy is to carry out a comparison at each point in (discrete) time the observed random variable in the sequence against a time-varying, pre-computable threshold, and decide whether to *transmit* or *not transmit* accordingly. A salient characteristic of this optimal solution is that even when transmission does not take place, this still has some information content regarding the true value of the random variable at that point in time, which can be used at the receiving end to improve the estimate of the random sequence. The optimal solution can also be seen leading to an event driven system, with the events being associated with transmissions, each one being triggered by the norm of the random variable's realized value exceeding a given threshold. The difference from a more standard event-driven system though is that here the trigger mechanism is optimally controlled and hence is part of the overall design process, rather than being exogenous to the system.

We have subsequently introduced an adversarial component into the problem, not in an estimation but in a control context: in a standard stochastic networked control system setting there is an additional entity, *strategic adversary*, who has the capability to jam the channel that connects the controller to the plant and prevent the control signal from reaching the plant [32], but under some budget constraints. The jammer acts only intermittently, limited to a given finite number (say M) of actions over a horizon of $N(N > M)$ time steps. Such a constraint is introduced to capture the fact that, since jamming is a power intensive activity and available power on-board a jammer is typically limited, continuous action throughout the entire decision horizon is not possible. Such a formulation leads to a dynamic zero-sum game between two players, in this case the controller and the jammer. We show in [32] that for this game a saddle-point equilibrium exists under full state, total recall information structure for both players, and obtain the corresponding control and jamming strategies. The nature of the solution is such that the jammer acts according to a *threshold policy*, which means that at every time step, the jammer jams if and only if a particular norm of the plant's state is larger than an off-line computable and time-varying threshold. We derive in [32] various properties of the threshold functions, and complement the study with numerical simulations.

2.2. Communication jamming and allocation of resources, with application in formation of UAVs

In a series of papers [1, 9, 14, 16, 23, 28, 29, 30, 37, 40, 42, 44], we have brought in additional elements into the jamming scenario above such as mobility, continuous dynamics, and multiplicity of agents. In [1] and [37], we have investigated a jamming attack on the communication network of a team of unmanned aerial vehicles (UAVs) flying in a formation under a communication and a motion model for the UAVs, where communication is essential for a team of UAVs to sustain formation. We formulated the problem as a zero-sum pursuit-evasion (P-E) game (not between two players as in standard P-E games, but between two teams) where the cost function is the termination time of the game, with termination defined as breakdown of communication among the team of UAVs. Using the framework of Isaacs, we have obtained motion strategies for the UAVs to evade the jamming attack, and have also provided motion strategies for aerial intruders to jam the communication between the UAVs. [37] was restricted to the case of 2 UAVs and a jammer, and [1] provided an extension to multiple UAVs and jammers. And in [9], we extended the results to a scenarion involving also AGVs. We have addressed the connectivity maintenance problem also in [14], but now when two mobile autonomous agents (holonomic agents) navigate in an environment containing polygonal obstacles. One of the agents, the pursuer, is assumed to follow the other agent so as to maintain a constant line-of-sight, which is the path of the dominant signal. The other agent is modeled as an adversary that tries to break the line-of-sight with the pursuer. Therefore, the problem of maintaining a healthy communication link has been modeled as a visibility based pursuit-evasion game, where we have adopted the Rician fading model for the communication channel. We have investigated a specific kind of singular surface that appears in the solution to the underlying pursuit-evasion game, namely the *dispersal surface*. In the paper, we have presented construction of the projection of several dispersal surfaces for various obstacle geometries by fixing the initial position of the evader. Further, we have worked several numerical simulations for specific environments containing obstacles.

We have considered in [44] again the problem of maintaining connectivity in a network of mobile agents in formation in the presence of a jammer, but now from a control-theoretic point of view. For the underlying differential game, we have obtained a complete set of necessary conditions for optimal controls for each agent. One novelty is the introduction of a model that constructs a state-dependent graph based on the state-space of the agents. We use tools from algebraic-graph theory on the state-dependent graph in order to provide locally optimal control laws for the agents in the formation. Simulations validate the control scheme introduced in the paper.

In another paper, [17], we again investigate, but from a different perspective, a jamming attack on the communication network of a multi-agent system in a formation and formulate the problem as a zero-sum pursuit-evasion game. In the models of [1] and [37], we had used Isaacs' framework (as discussed above) to obtain motion strategies for a network of agents to evade the jamming attack. In this work, however, we imagine a scenario in which each agent has prior knowledge about the underlying value function under perfect state information. Due to lack of information about all the agents in the team, each agent is forced to make a local decision based only on the information about his neighbors. We develop online algorithms under two different decentralized information patterns which converge for each player to local strategies that use estimators designed based on state equations and the value function. A further work in this direction is [30], where we address the problem where each UAV determines its control strategy based on limited information available from its neighbors in the network graph. The limitations are posed in a way such that each UAV receives the state information about other UAVs in the formation that are at most n hops away in the network graph. Under this information structure, we study the performance of the entire formation when each UAV runs an estimator based on the underlying information pattern in order to compute its actions. The performance measure considered in this game is the maximum time for which the network remains connected in the presence of an aerial adversary. We again use tools and conceptual framework of differential game theory to obtain the saddle-point strategies of the underlying P-E game under the constrained information structure. We present results on the attainable performance when $1 \leq n \leq \mathrm{diam}(G)$, where $\mathrm{diam}(G)$ represents the diameter of the graph of the underlying communication network.

Two other recent papers on jamming games between two teams are [28] and [29], where we study the problem of power allocation and adaptive modulation in teams of decision makers. We restrict the study initially to the case of two teams with each team consisting of two mobile agents. Agents belonging to the same team communicate over wireless ad hoc networks, and they try to split their available power between the tasks of communication and jamming the nodes of the other team. The agents have constraints on their total energy and instantaneous power usage. The cost function adopted is the difference between the rates of erroneously transmitted bits of each team. We model the adaptive modulation problem as a zero-sum matrix game which in turn gives rise to a continuous kernel game to handle power control. Based on the communications model, we present sufficient conditions on the physical parameters of the agents for the existence of a pure strategy saddle-point equilibrium (PSSPE). We present simulation results for the case when the agents are holonomic.

In a recent paper [16], we have considered a variation of the problem above where now each player has an omnidirectional antenna for jamming the communication between the members of the other team. Again we consider the case of two teams with each team consisting of two mobile agents. Agents belonging to the same team communicate over wireless ad hoc networks, and they try to split their available power between the tasks of communication and jamming the nodes of the other team. The agents again have constraints on their total energy and instantaneous power usage, and the cost function adopted is the difference between the rates of erroneously transmitted bits of each team. Formulating the problem as a zero-sum differential game between two teams, we prove the existence of a PSSPE and obtain a characterization of optimal strategies. What we observe is a switching behavior in the optimal communication strategy within a team, over the time horizon of the entire game.

Our final work on this general topic is [15], where we study efficient transmission of information in a wireless medium with stationary nodes and relays, but again in an adversarial environment. We study the complex decision making processes between such a network of wireless users that perform uplink transmission via relay stations and an active malicious node, that is able to act as an eavesdropper and as a jammer. We formulate a noncooperative game in which the wireless users and the malicious node are the players. On the one hand, the users seek to choose the relay station that maximizes their utilities which reflect their potential mutual interference as well as the security of the chosen data transmission path. On the other hand, the objective of the malicious node is to choose whether to eavesdrop, jam, or use a combination of both strategies, in a way to reduce the total capacity at all the hops of the network. To solve the game, we introduce a fictitious play-based algorithm using which the users and the malicious node reach a mixed-strategy Nash equilibrium. Simulation results show that the proposed approach improves the average expected utility per user by up to 49.4neighbor algorithm. The results also show how the malicious node can strategically decide on whether to jam or eavesdrop depending on its capabilities and objectives.

2.3. Quantization and transmission over noisy channels

In the publication [7], we have considered the problem of remotely controlling a continuous-time linear time-invariant system driven by Brownian motion process, when communication takes place over noisy memoryless discrete- or continuous-alphabet channels. What makes this class of remote control problems different from all the previously studied models is the presence of noise in both the forward channel (connecting sensors to the controller) and the reverse channel (connecting the controller to the plant). For stability of the closed-loop system, we look for the existence of an invariant distribution for the state, for which we show that it is necessary that the entire control space and the state space be encoded, and that the reverse channel be at least as reliable as the forward channel. We obtain necessary conditions and sufficient conditions on the channels and the controllers for stabilizability. Using properties of the underlying sampled Markov chain, we show that under variable-length coding and some realistic channel conditions, stability can be achieved over discrete-alphabet channels even if the entire state and control spaces are to be encoded and the number of bits that can be transmitted per unit time is strictly bounded. For control over continuous-alphabet channels, however, a variable rate scheme is not necessary. We also show that memoryless policies are rate-efficient for Gaussian channels.

Quantization is also the centerpiece of research reported in [5], [18], [25] and [36], which study its effect on the performance of $\mathcal{L}_1$ adaptive controllers. In [5] and [18], we address the problem of tracking for a general class of uncertain nonlinear MIMO systems with input quantization, without requiring any matching conditions. We consider the $\mathcal{L}_1$ adaptive controller and analyze its performance bounds in the presence of input quantization of two types: uniform and logarithmic. In both cases we provide the transient performance bounds, which are decoupled into two positive terms. One of these terms can be made arbitrarily small by increasing the rate of adaptation, while the other term can be made small by increasing the quantization density. The performance bounds imply that with sufficiently dense quantization and fast adaptation, the output of an uncertain MIMO nonlinear system can follow the desired reference input sufficiently closely. We notice that with $\mathcal{L}_1$ adaptive control architecture fast adaptation does not lead to high-gain control and retains guaranteed time-delay margin, which is bounded away from zero.

An earlier work [36] has looked at the case of linear uncertain systems again with input quantization. We show that the performance bounds of the $\mathcal{L}_1$ adaptive controller (in the presence of input quantization) have an additional term, dependent upon the quality of quantization. The signals of the closed-loop $\mathcal{L}_1$ adaptive systems can be rendered arbitrarily close to the corresponding signals of a bounded reference system by increasing the adaptation rate and improving the quantizer.

2.4. Inefficiency of Nash equilibrium, and different notions of price

It is well known that the non-cooperative Nash equilibrium in nonzero-sum games is generally inefficient, which means that it would be possible for all players to do better in terms of attaining higher utilities or lower costs (than they would attain under Nash equilibria, even if the equilibrium is unique) through cooperation. This is true for static deterministic games, and naturally also for stochastic games as well as dynamic and differential games. In these latter of classes of games, one could bring up additional issues with regard to Nash equilibria beyond efficiency or lack thereof, such as whether an increase in information to one player (or all or a subset of the players) would be advantageous to that player (or groups of players), in terms of attaining higher utilities or lower costs, or whether acquiring more information would be undesirable for a player. In the special class of games where all players have the same utility function or cost function (that is, team problems) and what is sought is the global maximum or global minimum of these functions, the answer to such a query is clean, which is that additional information (defined as expansion of sigma fields) can never hurt. The same is true for the special class of zero-sum games. In stochastic games, or dynamic and differential games which are not team problems or zero-sum games, however, the answer is not that clean, and one could encounter quite surprising and at the outset counter-intuitive results. Perhaps the first demonstration of this was reported by the PI some 40 years ago, when he studied two classes of two-player stochastic static games, one a linear-quadratic-Gaussian (LQG) model and the other one a stochastic Cournot duopoly model, both of which admit unique Nash equilibria. It was shown that for the LQG model better information (on some stochastic variables) for *only one* player leads to lower average Nash equilibrium costs for *both* players, but in the duopoly model only the player whose information is improved benefits while the other one hurts (in the sense that his average Nash equilibrium cost increases). Another way of comparison would be in terms of the relative values of the average Nash equilibrium costs attained by the players, when one player has informational advantage over the other. It was again shown by the PI that, in an otherwise completely symmetric game, the player who has better information attains higher cost than the other player in the LQG model (the counter-intuitive result), whereas he attains lower cost in the duopoly model (the intuitive result). Several manifestations of these conclusions can be seen also in dynamic and differential games; for example time-consistent open-loop Nash equilibrium is not necessarily inferior to the strongly time-consistent closed-loop feedback Nash equilibrium.

Now coming back to inefficiency of Nash equilibrium in a fixed nonzero-sum game, one question of interest is exploration of the extent of this inefficiency, that is how far off is a Nash equilibrium from the socially optimal solution, which is obtained as the maximum of the sum of the utilities of the players, or some convex combination of the utilities (or minimum in the case of cost functions). The notion of the *price of anarchy* (PoA)was introduced earlier as a quantification of this offset, as a utility ratio between the worst possible Nash solution (among multiple Nash equilibria) and the social optimum. In a way, this index serves to quantify the loss of efficiency due to competition. It has been shown that in routing games and resource allocation games, PoA is bounded by a constant, allowing agents to achieve some level of efficiency despite being suboptimal.

The idea of quantifying the gap between social optimality and game equilibrium solutions sparked many follow-up work in that same vein. Also *price of simplicity* has been introduced for a pricing game in communication networks as the ratio between the revenue collected from a flat pricing rule and the maximum possible revenue. Further, *price of uncertainty* has been introduced to measure the relative payoff of an expert user of a security game under complete information to the one under incomplete information. In another earlier work, *price of leadership* has been proposed as a measure of comparison of utilities in a power control game between Nash equilibria and Stackelberg solutions. In all of these works, primarily communication networks have been used as a backdrop application domain, be it routing, resource allocation, power control, or security. Game-theoretical methods along with Nash equilibrium have found many applications in communication networks.

In our recent work [8, 38], we introduce and discuss several indices which quantify variations or offsets in the payoff values or costs attained under Nash equilibria in the context of differential games (DGs). We first extend the notion of *PoA* to DGs, which heretofore has been primarily limited to static continuous kernel games. We provide a characterization of *PoA* for a class of scalar linear-quadratic (LQ) DGs, and quantify the efficiency loss in the long run when the players behave non-cooperatively under the Nash equilibrium concept. We consider both open-loop (OL) and closed-loop (CL) information structures (ISs). We show that for the class of scalar LQ DGs with CL IS using the strongly time-consistent CL feedback Nash equilibrium, the *PoA* has some appealing computable upper bounds, which can further be approximated when the number of players is sufficiently large (that is, the large population regime), whereas, under the OL IS, it is possible to obtain an expression for the *PoA* in closed form.

As mentioned, going from static to dynamic (differential) games brings in the possibility of various ISs which add richness to the (Nash equilibrium) solution of a game. Different ISs (generally) yield different equilibrium solutions, and hence as already pointed out, IS is a crucial factor in the investigation of *PoA* in DGs. Motivated by this, we introduce another index, the *price of information* (PoI), which is a result of the comparison of the equilibrium utilities or costs under different ISs. For the class of scalar LQ DGs above, we have shown that the *PoI* between the feedback and open-loop ISs is bounded from below by $\sqrt{2}/2$ and from above by $\sqrt{2}$, again in the large population regime. Finally, motivated by some recent results on the level of cooperation between players in a routing game, captured by the degree of willingness of a player to place partial weight on other players' utilities in his utility function, we introduce the *price of cooperation* (PoC) as a measure of benefit or loss to a player on his base Nash equilibrium payoff due to cooperation.

Another set of new results on nonzero-sum games, but with decision hierarchy, has been reported in papers [10, 41], which introduce the notion of *mixed leadership* in non-zero-sum differential games, where there is no fixed hierarchy in decision making with respect to the players. Whether a particular player is leader or follower depends on the instrument variable s/he is controlling, and it is possible for a player to be both leader and follower, depending on the control variable. We have studied two-player open-loop differential games in this framework, and obtained a complete set of equations (differential and algebraic) which yield the controls in the mixed-leadership Stackelberg solution. The underlying differential equations are coupled and have mixed boundary conditions. Our work also discusses the special case of linear-quadratic differential games, in which case solutions to the coupled differential equations can be expressed in terms of solutions to coupled Riccati differential equations which are independent of the state trajectory.

Finally, in the recent paper [2], we formulate an evolutionary multiple access control game with continuous variable actions and coupled constraints. We characterize Nash equilibria of the game and show that the pure equilibria are efficient (Pareto optimal) and also resilient to deviations by coalitions of any size, i.e., they are strong equilibria. We use the concepts of price of anarchy and strong price of anarchy to study the collective performance of the players in the game. We also address the question of how to select one specific equilibrium solution using the concepts of normalized equilibrium and evolutionarily stable strategies. We examine the long-run behavior of these strategies under several classes of evolutionary game dynamics, such as Brown-von Neumann-Nash dynamics, Smith dynamics, and replicator dynamics. In addition, we examine correlated equilibrium for the single-receiver model. Correlated strategies are based on signaling structures before making decisions on rates. We then focus on evolutionary games for hybrid additive white Gaussian noise multiple access channel with multiple users and multiple receivers, where each user chooses a rate and splits it over the receivers. Users have coupled constraints determined by the capacity regions. Building upon the static game formulation and results, we formulate a system of hybrid evolutionary game dynamics using G-function dynamics and Smith dynamics on rate control and channel selection, respectively. We show that the evolving game has an equilibrium and illustrate these dynamics with numerical examples.

2.5. Learning and iterative computation under minimal exchange of information

An important issue in multi-agent systems is the convergence of iterative schemes adopted by agents (players) based on various behavioral patterns and using minimal information on the actions of others, to an equilibrium which may be computable offline should all information be available and centralized (*which it is not*). In a series of papers [3, 4, 20, 26, 33, 45, 39], we have addressed this problem within the context of static Nash games. We have introduced a non-model based approach for locally stable convergence to Nash equilibria in static, noncooperative games with a finite number of (say, N) players. In classical game theory algorithms, each player employs the knowledge of the functional form of his payoff and the knowledge of the other players' actions, whereas in our approach the players need to measure only their own payoff values (and hence online information on other players' actions is provided only to the extend they affect an individual player's payoff). The response strategies of the players in our work, and our analysis, are based on the extremum seeking approach, which has previously been developed for standard optimization problems and employs sinusoidal perturbations to estimate the gradient. We first consider static games with quadratic payoff functions before generalizing our results to games with non-quadratic payoff functions that are the output of a dynamic system. Specifically, we consider general nonlinear differential equations with N inputs and N outputs, where in the steady state, the output signals represent the payoff functions of a noncooperative game played by the steady-state values of the input signals. We employ *local averaging theory* and obtain local convergence results both for quadratic payoffs, where the actual convergence is semi-global, and for non-quadratic payoffs, where the potential existence of multiple Nash equilibria precludes semi-global convergence. Our convergence conditions coincide with conditions that arise in model-based Nash equilibrium seeking. However, in our framework the user is not meant to check these conditions because the payoff functions are presumed to be unknown. For non-quadratic payoffs, convergence to a Nash equilibrium is not perfect, but is biased in proportion to the perturbation amplitudes and the third derivatives of the payoff functions. We quantify the size of these residual biases and confirm their existence numerically in an example noncooperative game. In this example, we present the first application of extremum seeking with projection to ensure that the players actions remain in a given closed and bounded action set. We also consider extensions of these results to countably and uncountably infinite number of players.

Another line of research on iterative schemes for games and a new set of algorithms have been introduced in [34], where the framework is that of a class of two-person zero-sum stochastic games with an arbitrary number of states and a finite number of actions for each player, with possibly probabilistic (mixed) strategies. When each player has a complete knowledge of its payoff function and has past access to past actions of the others, then there is an arsenal of tools such as fictitious play algorithms, best response dynamics, and gradient-based algorithms, that can be used to arrive at the equilibrium of the game. However, it is well known that these algorithms may fail to converge even under the perfect observation of actions and payoffs. A new *learning challenge* hence arises when a player does not know its own payoff function and/or has no information about the past actions of the other players (as also discussed above). In this case, the player needs to interact with the environment to find out its expected payoff and its optimal strategy. In practical applications, we are often in search of distributed learning algorithms that require a minimal amount of information and a minimal amount of resources. It is then natural to ask whether there exists a learning scheme that demands less information and less memory within a dynamically evolving environment, and leads to an efficient, stable and fair outcome. We address this challenge by proposing a class of heterogeneous learning algorithms in a scenario where the players do not know their own payoff functions. At each time t, each player chooses an action and receives a numerical value for its payoff or perceived payoff as an outcome of the instantaneous game. In contrast to fictitious play and best response dynamics which require the knowledge of the history of actions played by the other players, our learning algorithm relaxes this assumption. Indeed, it is often implausible and impractical in applications to assume the capability of observations of the actions of the other players. Furthermore, we assume that the state space of the game and its transition law between the states are unknown to the players. In addition, the players also do not have the knowledge of the action spaces of the others. The question we have addressed is how much the players can expect to learn under such circumstances. We have introduced in [34] different coupled (or combined) and fully distributed learning schemes that enable learning optimal strategies and concurrently estimating the optimal payoffs. In contrast to the standard reinforcement learning algorithms which focus only on either strategy or payoff reinforcement for the equilibrium learning, the algorithm that couples the payoff-reinforcement learning together with strategy reinforcement learning enables an immediate prediction and updates the strategies by updated estimations based on recent experiences. Our learning algorithms also offer the degrees of freedom to model different levels of rationality and learning rates of the players. The ordinary differential equations (ODEs) associated with the stochastic learning algorithms differ from the standard replicator dynamics, best response dynamics and fictitious play dynamics. We also establish particular connections to logit dynamics and imitative logit dynamics. Using stochastic approximation techniques, and under suitable assumptions on the learning rates, we show the convergence of different learning algorithms to a new class of game dynamics and establish their asymptotic properties within a class of zero-sum stochastic games.

2.6. Large population games, and equilibrium analysis

An important research direction in multi-agent systems is the analysis of collective behavior that arises when the population is large. In the terminology of game theory this entails the analysis of different equilibria under different ISs when the number of players is arbitrarily large. We have conducted such a study in [21, 31], within the context of risk-sensitive stochastic differential games. Risk-sensitivity is captured by exponentiating the integral cost (over the duration of the game) of a player, before taking the expectation, and this brings in additional robustness to the resulting equilibrium strategies (or optimal control laws, in the case of risk-sensitive stochastic control)–robustness to unmodeled inputs to the system by say an adversary. We first introduce in [21] a mean-field stochastic differential game model where the players are coupled not only via their risk-sensitive cost functionals but also via their states. The main coupling term is the mean-field process, also called *occupancy process* or *population profile process.* Then, using a particular structure of state dynamics, we derive the mean-field limit of the individual state dynamics, leading to a nonlinear controlled macroscopic McKean-Vlasov equation. Combining together with the convergence of the risk-sensitive cost functional, we provide the mean-field optimality principle, and obtain compatibility with the density distribution using the Fokker-Planck-Kolmogorov forward equation. The mean-field value of the exponentiated cost functional coincides with the value function of a Hamilton-Jacobi-Bellman (HJB) equation with an additional quadratic term. We provide an explicit solution of the mean-field best response when the instantaneous cost functions are log-quadratic and the state dynamics are affine in the control. We formulate an equivalent mean-field risk-neutral problem and characterize the corresponding mean-field equilibria in terms of backward-forward macroscopic McKean-Vlasov equations, Fokker- Planck-Kolmogorov equations and HJB equations.

In [27], we have looked at the consensus problem within a differential game-theoretic mean-field framework. In networked systems, agents typically seek to achieve a task with some knowledge of their neighbors or immediate friends. Consensus is one of the fundamental and pivotal problems in decision making involving a large number of distributed agents reaching consensus in their opinions, resources, security, and the like. in [27], we work in the framework of linear-quadratic nonzero-sum differential games defined on an infinite horizon with discounted cost, which we study under different information structures and with a view to consensus. We characterize the open-loop (OL) and strongly time-consistent closed-loop (STC CL) Nash equilibrium (NE) strategies for finite population and large population regimes. For the finite population game, the STC CL NE strategy of each agent is affine in the states of its neighbors and consensus is achieved depending on the initial states of the agents. For a large homogeneous population, the STC CL NE requires the solution of a nonlinear PDE that describes the state evolution of the population, which is coupled with a set of coupled algebraic Riccati equations. We also study the relationship between OL and STC CL as the population or the neighborhoods grow.

2.7. Goal-oriented coalition formation in networks of agents

In one piece of work [6, 45] we have looked at formation of coalitions among multiple agents (players) in a distributed network, with the process driven by goals of individual agents. The specific model (which has application involving UAVs) is as follows: A number of agents are required to collect data from several arbitrarily located tasks. In a wireless network framework, each task represents a queue of packets that require collection and subsequent wireless transmission by the agents to a central receiver. The problem is modeled as a hedonic coalition formation game between the agents and the tasks that interact in order to form disjoint coalitions. Each formed coalition is modeled as a polling system consisting of a number of agents, designated as collectors, which move between the different tasks present in the coalition, collect and transmit the packets. Within each coalition, some agents might also take the role of a relay for improving the packet success rate of the transmission. The hedonic coalition formation algorithm developed allows the tasks and the agents to take distributed decisions to join or leave a coalition, based on the achieved benefit in terms of effective throughput, and the cost in terms of polling system delay. As a result of these decisions, the agents and tasks structure themselves into independent disjoint coalitions which constitute a Nash-stable network partition. Moreover, the proposed coalition formation algorithm allows the agents and tasks to adapt the topology to environmental changes such as the arrival of new tasks, the removal of existing tasks, or the mobility of the tasks. Simulation results show how the proposed algorithm allows the agents and tasks to self-organize into independent coalitions, while improving the performance, in terms of average player (agent or task) payoff, of at least 30:26 % (for a network of 5 agents with up to 25 tasks) relative to a scheme that allocates nearby tasks equally among agents.

3. Recap

This report has discussed a number of topics and issues that are of paramount importance to networked systems, and has described the broad field of study we have undertaken in our research and the results we have obtained during the close to 3-year course of this *AFOSR Grant*. Our research has led to a deeper understanding of the various trade-offs that exist in design and decision making in networked systems, which involve incompleteness of information, decentralization, communication constraints, resource allocation, distributed sensing and control, and coordination and consensus formation. During the course of this research we have also started working on a book [46] on stochastic networked control systems, which is now closer to completion, which will contain an in-depth analysis of some of the issues discussed above, particularly those that involve quantization, uncertainty, performance, stabilization, decentralization, and learning.

4. Related Plenary Talks and Special Colloquia given by the PI

Below is a list of plenary talks and special colloquia given by Tamer Başar during the 3-year course of the *Grant*:

- Chinese Automation Congress (CAC 2011), Beijing, China, November 27, 2011 (Plenary) (Title: *Multi-Agent Networked Systems: Efficiency Through Coordination and Control*)

- Chinese Control Conference (CCC 2011), Yantai, Shangdong Province, China, July 22-24, 2011 (Plenary) (Title: *Sensing, Coordination and Control in Adversarial Environments with Limited Actions*)
- 5th International ICST Conference on Performance Evaluation Methodologies and Tools (ValueTools 2011), ENS, Canches, France, May 17-19, 2011 (Plenary) (Title: *Dynamic Teams and Games with Non-standard Information*)
- Turkish National Committee on Automatic Control, National Symp. on Automatic Control (TOK'10), Gebze Institute of Technology, Gebze, Turkey, September 21-23, 2010 (Plenary) (Title: *Variations on the LQG Paradigm and the Emerging Subtleties*)
- Amorph International Workshop – Amorphous Computing & Complex Biological Networks, The Halifax, Endcliffe Village, Sheffield, UK, August 17-20, 2010 (Plenary) (Title: *Games, Networks, and Distributed Computation)*
- 6th Spain, Italy and Netherlands Meeting on Game Theory (SING 6), Palermo, Italy, July 7-9, 2010 (Plenary) (Title: *Dynamic Teams, Games, and Non-Classical Information*)
- 10th International Conference on Automation Technology (Automation 2009), Tainan, Taiwan, June 27-28, 2009 (Plenary) (Title: *Networked Sensing and Control with Limits on Transmission*)
- 2009 (21st) Chinese Control and Decision Conference (CCDC 2009), Guilin (Guangxi), China, June 17-19, 2009 (Plenary) (Title: *Games, Decisions, Control and Communications: Common Threads and Coping with Non-Neutrality*)
- 2009 (5th) Northeast Control Workshop (NECW 2009), Pittsburgh, Pennsylvania, April 24-26, 2009 (Plenary) (Title: *Non-Neutral Decision Making in Control and Dynamic Games*)
- Workshop on "Control Systems Security: Challenges and Directions," CDC/ECC 2011, Orlando, FL, December 11, 2011 (Title: *Game Theoretic Approaches to Security*)
- Workshop on "Game Theory for Finance, Social and Biological Sciences (GAM)," the University of Warwick, Coventry, England, April 14-17, 2010 (Title: *Non-Neutral Decision Making in Stochastic Teams and Games*)
- ISR Distinguished Lecture, University of Maryland, College Park, Maryland, October 11, 2010 (Title: *Sensing, Control, and Decision Making with Limited Actions*)

5. Publications and Conference Presentations supported by the Grant

[1] S. Bhattacharya and T. Başar, "Differential game-theoretic approach to a spatial jamming problem," *Advances in Dynamic Game Theory and Applications*, Annals of Dynamic Games, vol. 11, Birkhäuser, 2012 (to appear)

[2] Q. Zhu, H. Tembine, and T. Başar, "Evolutionary games for multiple access control," *Advances in Dynamic Game Theory and Applications*, Annals of Dynamic Games, vol. 11, Birkhäuser, 2012 (to appear)

[3] P. Frihauf, M. Krstic, and T. Başar, "Nash equilibrium seeking for dynamic systems with non-quadratic payoffs," *Advances in Dynamic Game Theory and Applications*, Annals of Dynamic Games, vol. 11, Birkhäuser, 2012 (to appear)

[4] P. Frihauf, M. Krstic, and T. Başar, "Nash equilibrium seeking in noncooperative games," *IEEE Trans Automatic Control*, 57(5): 1192-1207, May 2012.

[5] H. Sun, N. Hovakimyan, and T. Başar, "L_1 adaptive controller for uncertain nonlinear multi-input multi-output systems with input quantization.," *IEEE Trans Automatic Control*, 57(3):565-578, March 2012.

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