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1. REPORT DATE *(DD-MM-YYYY)*	2. REPORT TYPE	3. DATES COVERED *(From - To)*
06-30-2012	Final Report	30-Sep-2008 – 31-Mar-2012

4. TITLE AND SUBTITLE		
Complex Network Information Exchange in Random Wireless Environments	**5a. CONTRACT NUMBER**	FA9550-08-1-0480
	5b. GRANT NUMBER	
	5c. PROGRAM ELEMENT NUMBER	

6. AUTHOR(S)		
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Eldar, Yonina		
	5f. WORK UNIT NUMBER	

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)	8. PERFORMING ORGANIZATION REPORT NUMBER
Stanford University Stanford, CA 94305	

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSOR/MONITOR'S ACRONYM(S)
Air Force Office of Scientific Research Mathematics, Information and Life Sciences AFOSR/RSL 875 North Randolph Street, 4036 Arlington, VA 22203	AFOSR
	11. SPONSOR/MONITOR'S REPORT NUMBER(S) AFRL-OSR-VA-TR-2012-1077

12. DISTRIBUTION/AVAILABILITY STATEMENT

Distribution A: Approved for Public Release

13. SUPPLEMENTARY NOTES

14. ABSTRACT

This document summarizes the research performed under AFOSR contract FA9550-08-1-0480, entitled ``Complex network information exchange in random wireless environments.'' The objective of this project was to develop novel techniques, structures, and algorithms for optimization of complex wireless networks where channels change dynamically and randomly, effecting network performance and reliability. These random dynamics, while challenging for ensuring robust high-performance network operation, also create opportunities that adaptive network control policies can exploit. This is particularly important for advanced military networks operating in rapidly changing, heterogeneous and sometimes hostile environments. A main focus of this research was to develop the new technique of Wireless Network Utility Maximization (WNUM), extending NUM ideas to include network and traffic dynamics. The research also explore optimization of network security protocols that exploit random wireless environments. Finally, compressed sensing and matrix completion ideas were explored to develop low-complexity network control.

15. SUBJECT TERMS

complex wireless networks, wireless network utility maximization, stochastic control, compressed sensing, matrix completion, reduced-dimension network control, sparse approximation.

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			Andrea Goldsmith
U	U	U	U	11	**19b. TELEPHONE NUMBER** *(Include area code)* 650 766 8317

Complex Network Information Exchange
in Random Wireless Environments

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Abstract:

This document summarizes the research performed under AFOSR contract FA9550-08-1-0480, entitled "Complex network information exchange in random wireless environments." The objective of this project was to develop novel techniques, structures, and algorithms for optimization of complex wireless networks where channels change dynamically and randomly, effecting network performance and reliability. These random dynamics, while challenging for ensuring robust high-performance network operation, also create opportunities that adaptive network control policies can exploit. This is particularly important for advanced military networks operating in rapidly changing, heterogeneous and sometimes hostile environments. A main focus of this research was to develop the new technique of Wireless Network Utility Maximization (WNUM). This technique built on previous work in Network Utility Maximization (NUM), but extended those ideas to include network and traffic dynamics, blending techniques from stochastic optimization, stochastic approximation, reinforcement learning and economics to yield optimal network control policies that adapt to randomly changing conditions. WNUM addressed the following questions: what are the important network control variables; what are the critical flows of control information; what are the optimal control policies? The research also explore optimization of network security protocols that exploit random wireless environments for security purposes. Finally, compressed sensing and matrix completion ideas were explored to develop low-complexity network control policies based on sparse approximations of the network state.

Summary of Results:

During the course of the project, we have obtained the following key results: 1) We developed WNUM techniques to optimize the reliability and throughput tradeoffs of networks operating in random wireless environments, in particular with respect to adaptive modulation; 2) We developed multi-period NUM (MPNUM) techniques to find optimal control policies in dynamic environments, taking into account time-sensitivity of traffic; We also developed learning methods for WNUM and MPNUM where the statistics of the environment are unknown, and must be learned; 3) We developed physical-layer security protocols for relay networks that embed security into the transmission strategy; 4) We have developed reduced-complexity control techniques for complex wireless networks based on sparse approximation, whereby the full network state is approximated from a small set of samples. Details of our results in each of these areas are given in the next section.

The work has resulted in 9 conference papers and four journal papers.

Detailed Results:

1. **Reliability and Throughput Tradeoffs with Wireless Network Utility Maximization**

   In this effort, we focused on a distributed algorithm for optimizing the rate-reliability tradeoff in wireless networks where the physical channel has a randomly time-varying characteristic [5]. The idea is to develop an online distributed algorithm to find optimal control policies for network link power, rate and reliability in wireless environments, based on stochastic approximation. Utilizing a stochastic version of dual decomposition, we develop an algorithm that learns the channel characteristics and converges to optimal policies under a broad set of conditions. The proposed algorithm does not need to know the distribution of the channel states in advance and learns it along the way by sampling channel conditions and updating the policy accordingly.

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In particular, consider M logical source/destination pairs and L links in the network. Each source and destination pair is associated with an upper layer protocol stack and the routing of information flows over links is described by the routing matrix A, where $A(l, m) = 1$ if information on flow m traverses link l and is otherwise zero. For the m'th data session, r_m denotes the rate of information sent into the link encoder. The ratio of the total number of useful information bits to the total number of bits exiting the encoder per unit time is termed the code rate $0 \leq \theta_l \leq 1$. Encoded bits are removed from the link buffer and transmitted by the wireless link at rate R_l. Also for the channel condition, consider the channel matrix $G \in \mathbf{R}^{L \times L}$, where G_{ij} is the power gain from the transmitter on link j to the receiver on link i. The vector of transmitter powers is given by $\underline{S} \in \mathbf{R}^L$. Each transmitter, say at link l, has an average power budget $\bar{S}_l$. Also, the error probability of bits flowing over the link is defined as $X(\theta)$. This error probability is a nondecreasing function of the code rate θ for any useful code. We used the following model in our calculations:

$$X(\theta) = \frac{1}{2} 2^{-N(R_0 - \theta)} \tag{1}$$

where N is the code block length used by the encoder and R_0 is the cutoff rate. Then, the reliability of an information flow m is defined by ϕ_m as follows :

$$\underline{\phi} = 1 - A^T \underline{X}(\theta) \tag{2}$$

where $A^T \underline{X}(\theta)$ is the sum of the error rates on the links traversed by the flow.

The performance of upper layer protocols are modeled as utility functions. Each source m has a utility function $U(r_m, \phi_m)$. Utility functions are strictly concave increasing functions of the information rate and information reliability. In this work, we use the following parameterized family of utility functions:

$$U(r, \phi) = \beta \log r + (1 - \beta) \log \phi \tag{3}$$

where $0 \leq \beta \leq 1$ weights the relative importance of information rate and reliability.

The system can adapt to changing channel conditions by estimating G and adapting parameters such as transmit power $S = S(G)$, transmitter link rate $R = R(S(G), G)$, the information rate $r = r(G)$, code rate $\theta(G)$ and information reliability $\phi(G)$. Given the above definitions, the following is the problem formulation to obtain the optimal rate, reliability, and power. The goal here is to find adaptive rate vector $\underline{r}(G)$, reliability vector $\underline{\phi}(G)$, and power vector $\underline{S}(G)$, which maximize the average utility of the network, under constraints on information rates, link rates, reliability, and average power transmitted, in the following sense

$$\text{Maximize :} \quad \mathbf{E}\left[\sum_m U_m(r_m(G), \phi_m(G))\right] \tag{4}$$

$$\text{Subject to :} \quad \mathbf{E}\left[S_l(G)\right] \leq \bar{S}_l \quad l \in \{1, 2, ...L\}, \tag{5}$$

$$\mathbf{E}\left[A\underline{r}\right] \leq \mathbf{E}\left[\text{Diag}(\underline{\theta}(G))\underline{R}(S(G), G)\right], \tag{6}$$

$$\mathbf{E}\left[\underline{\phi}(G)\right] \leq 1 - \mathbf{E}\left[A^T \underline{X}(\theta(G))\right], \tag{7}$$

$$0 < \underline{\theta}(G) < 1, \tag{8}$$

$$0 < \underline{\phi}(G) < 1, \tag{9}$$

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where $\mathbf{E}$ is the expectation operator and optimization variables are $\underline{S}(G), \underline{r}(G), \underline{\theta}(G), \underline{\phi}(G)$.

The main algorithm that solves the aforementioned problem can be presented now.

(Primal-Dual iteratitive solution for W-NUM): For the design parameters specified above, consider the following coupled iterative equations, where t is a non-negative integer counter:

Initialization: Initialize all the parameters with a random feasible point.

Primal step:

$$\hat{S}_l^t = \hat{S}_l^{t-1} + \sigma_t \left(\lambda_{S,l} e^{\hat{S}_l^{t-1}} + \lambda_{\pi,l} \frac{\partial R_l(S,G)}{\partial \hat{S}_l} \right) \tag{10}$$

$$\hat{r}_m^t = \hat{r}_m^{t-1} + \sigma_t \left(\beta - \sum_{l=1}^{L} \lambda_{q,l} \frac{A(l,m) e^{\hat{r}_m}}{A(l,:) \, \underline{r}} \right) \tag{11}$$

$$\hat{\phi}_m^t = \hat{\phi}_m^{t-1} + \sigma_t \left(1 - \beta + \lambda_{\phi,m} \right) \tag{12}$$

$$\hat{\theta}_l^t = \hat{\theta}_l^{t-1} + \sigma_t \left(\frac{\lambda_{q,l}}{\theta_l^{t-1}} + \sum_{F_l} \lambda_{\phi,m} A(l,m) X(\theta_l) \right) \tag{13}$$

$$\hat{\pi}_l^t = \hat{\pi}_l^{t-1} + \sigma_t \left(\frac{\lambda_{q,l}}{\pi_l^{t-1}} - \lambda_{\pi,l} \pi_l^{t-1} \right) \tag{14}$$

Where $\{\sigma^t\}_{k=0}^{\infty}$ is a positive, square summable but not summable sequence and $F_l \overset{def}{=} \{(l,m) : A(l,m) \neq 0\}$ is the set of all the flows traversing from link l.

Price update:

$$\lambda_{S,l}^t = \left[\lambda_{S,l}^{t-1} + \sigma^t \left(e^{\hat{S}_l^{(t-1)}} - \bar{S}_l \right) \right]^+ \tag{15}$$

$$\lambda_{\pi,l}^t = \left[\lambda_{\pi,l}^{t-1} + \sigma^t \left(\pi_l^{(t-1)} - R_l^{(t-1)}(S,G) \right) \right]^+ \tag{16}$$

$$\lambda_{q,l}^t = \left[\lambda_{q,l}^{t-1} + \sigma^t \left(\log(A(l,:) \, \underline{r}^{(t-1)}) - \right. \right.$$
$$\left. \left. \log(\theta_l) - \log(\pi_l^{(t-1)}) \right) \right]^+ \tag{17}$$

$$\lambda_{\phi,m}^t = \left[\lambda_{\phi,m}^t + \sigma^t \left(\hat{\phi}_l - 1 + A(:,m)^T \, \underline{X}(\theta^{(t-1)}) \right) \right]^+ \tag{18}$$

Where for any real s, $[s]^+ \overset{def}{=} \max\{s,0\}$ and $\underline{\lambda} = [\underline{\lambda_S}, \underline{\lambda_\pi}, \underline{\lambda_q}, \underline{\lambda_\pi}]$ is the vector of Lagrange multipliers, associated with the constraints in W-NUM formulation.

We have studied the convergence of this algorithm where we proved that the output is the global optimum of the aforementioned W-NUM problem. We also showed that different steps of this algorithm can be calculated based on local information and hence it can be implemented in a distributed fashion. In the future, we will further modify and apply the above control framework to unreliable networks under attacks.

2. **Multi-Period WNUM (MPNUM)**: MPNUM captures the time sensitivity of wireless traffic by introducing a new class of utility functions and reformulating the problem as an infinite horizon average cost Markov decision problem (MDP). Our MPNUM work thus developed theoretical methods to find optimal control policies in dynamic environments when stochastic properties are well described. We applied these techniques to adaptive modulation and power control and compared its performance against algorithms such as water-filling that do not take traffic statistics into account [6, 8]. We also examine complex utility functions that involve the ratio of stochastic network parameters in [9]. We further extended MPNUM to wireless environments when stochastic properties are unknown using reinforcement learning techniques, such as Least Squares Temporal Dynamic Learning (LSTD-Learning). Periodic NUM(PNUM) extends WNUM to cyclo-stationary wireless or wireline traffic, and like MPNUM yields adaptive on line control policies to control rate power and queuing delay [11].

MPNUM [6,12,13] uses time smoothed utility functions to model the upper layer performance of data flows through a wireless network as functions of the time averaged rate at which packets are injected into the network. The time averaging serves two purposes. First, it captures differences in the characteristics of different data types, and second it reflects the observation that upper layer protocols often operate at longer time scales than those used by the physical layer. The associated MDP is of the form

$$\lim_{N\to\infty} \frac{1}{N} E\left[\sum_{t=0}^{N-1}(\sum_m \mathbf{U}_m(r_m^t) - \nu^T \phi(G^t, r^t))\right], \tag{19}$$

where $\mathbf{U}_m(r_m^t)$ is the time smoothed utility function of the mth data flow and $\nu^T \phi(G^t, r^t))$ are the resources required to support traffic carried by the network. Finding analytical solutions to this equation is very challenging and our work in this first phase of MPNUM focused on characterizing the optimal control policies and exploring different numerical techniques.

In the second phase of MPNUM, we explored ways to find basis functions to drive LSTD-Learning. There does not yet exist a general method to find these basis functions, so our approach used a novel new method based on finding solutions to differential equations derived from (19), as reported in [13]. In addition, our analysis found wireless networks modeled in this way exhibit state space collapse. The notion of state space collapse comes from the heavy traffic theory of stochastic networks. In this context, a reduction in dimension is obtained through a separation of time-scales, much like in singular perturbation analysis in dynamical systems and Markov chains.

3. **Physical-Layer Network Security** In this work we considered the design and analysis of security protocols that exploit the physical properties of the wireless medium for relay networks. We study a half-duplex relay network in which communication between a source node and a destination node is assisted by a relay station, while a passive eavesdropper can overhear radio signals from all legitimate nodes. All communication parties including the eavesdropper can be equipped with multiple antennas.

One of the key discoveries of this body of work is that it is possible to achieve secrecy rates that grow linearly in the SNR in some very challenging scenarios, in which the eavesdropper has more antennas than all the legitimate nodes [4]. The main insight is to let the legitimate nodes suitably cooperate so that they can have a collective advantage over the attacker despite their

individual disadvantages. From a designers viewpoint, the main lesson learnt is to let the source and the destination nodes alternately jam the eavesdropper in the different phases of the relaying protocols.

We also find that to achieve high-rate secure communication, it suffices for the relay station to employ off-the-shelf protocols such as amplify-and-forward and compress-and-forward relaying [14]. Furthermore the operation at the relay station does not require any explicit information about the channel state of the eavesdropper. In practice, this finding implies that low-cost low-complexity relay nodes with no secure encoding and decoding can be used with little loss in performance.

4. **Structure-Based Learning in Wireless Networks via Sparse Approximation** In this work, a novel framework for the online learning of expected cost-to-go functions characterizing wireless networks performance is proposed. The work is motivated by the fact that physical network states can change drastically after a WMD attack and such changes must be quickly assessed for optimizing communication and control schemes. The proposed framework is based on the observation that wireless protocols induce structured and correlated behavior of the Finite State Machine (FSM) modeling the operations of the network. As a result, a significant dimension reduction can be achieved by projecting the cost-to-go function on a graph wavelet basis set capturing typical sub-structures in the graph associated with the FSM. Sparse approximation with random projection is then used to identify a concise set of coefficients representing the cost-to-go function in the wavelet domain. This Compressed Sensing (CS) approach enables a considerable reduction in the number of observations needed to achieve an accurate estimate of the cost-to-go function.

Specifically, the network is modeled as a FSM whose state evolves within the state space $\mathcal{S}$ with $N=|S|$. Define $S(t)\in\mathcal{S}$ as the state of the FSM at time $t=0, 1, 2, ...$ We assume that the sequence $\mathbf{S}=\{S(0), S(1), S(2), \ldots\}$ is a Markov process with transition probabilities

$$p(s, s') = \mathcal{P}(S(t+1)=s'|S(t)=s), \tag{20}$$

where $P(\cdot)$ denotes the probability of an event. The performance of the network is measured by a function $c(s, s')$ that assigns a positive and bounded cost to the transition from state s to state s'. The average cost from state s is

$$c(s)=E_{s'\in\mathcal{S}}[c(s, s')] = \sum_{s'\in\mathcal{S}} p(s, s')c(s, s'). \tag{21}$$

The function

$$\overline{c}(S(t)) = c(S(t))+E\left[\sum_{\tau=1}^{\infty} \gamma^{\tau} c(S(t+\tau))\right], \tag{22}$$

where $E[\cdot]$ denotes expectation and $\gamma\in(0, 1)$ is the discount factor, is the expected discounted long-term cost. This function is also known as the *cost-to-go* function and is central to DP and optimal control. For any fixed $S(t)=s\in\mathcal{S}$, the function $\overline{c}(\cdot)$ is independent of the time index t and can be rewritten as

$$\overline{c}(s) = c(s)+\sum_{s'\in\mathcal{S}}\sum_{\tau=1}^{\infty} \gamma^{\tau}p^{\tau}(s, s')c(s'), \tag{23}$$

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where

$$p^{\tau}(s, s') = \mathcal{P}(S(t+\tau)=s'|S(t)=s) \tag{24}$$

is the τ-step transition from state s to s'. We propose an algorithm for the online learning of cost-to-go functions in wireless networks from the observation of a state-cost trajectory of the associated FSM.

The algorithm is composed of three elements:

- **observation:** the transition probabilities and cost function $c(\cdot)$ are estimated by observing a state-cost sample-path;

- **projection:** $\overline{c}$ is projected onto a graph wavelet basis set capturing typical structures in the graph;

- **sparse estimation of** $\overline{c}$**:** a sparse estimation algorithm is used to identify a concise set of basis functions providing the best fit with the estimated transition probabilities and cost function.

We define the $N \times N$ matrix $\mathbf{P}$ to be the probability transition matrix where $\mathbf{P}[s, s']=p(s, s')$ as in Eq. (20). The long-term cost $\overline{c}$ can be rewritten as

$$\overline{\mathbf{c}} = \mathbf{c} + \sum_{\tau=1}^{\infty} \gamma^{\tau} \mathbf{P}^{\tau} \mathbf{c} = \mathbf{c} + \gamma \mathbf{P}\overline{\mathbf{c}}. \tag{25}$$

Thus, $\overline{c}$ can be computed as the fixed point solution $\overline{c}=\Omega(\overline{c})$ of the operator $\Omega(\overline{c})=c+\gamma P\overline{c}$. The transition matrix $\mathbf{P}$ and cost vector $\mathbf{c}$ are not known *a priori* and need to be estimated from observation. At time T, the sample-path $\mathbf{O}_T$ is used to compute the estimates $\hat{\mathbf{P}}(T)$ and $\hat{\mathbf{c}}(T)$ of $\mathbf{P}$ and $\mathbf{c}$. We use the estimator

$$\left[\hat{\mathbf{P}}(T)\right]_{ij} = \begin{cases} \frac{\sum_{t=1}^{T-1} 1(S(t)=i, S(t+1)=j)}{\sum_{t=1}^{T-1} 1(S(t)=i)} & \text{if } \exists S(t) = i, t = 0, \ldots, T-1 \\ 0 & \text{otherwise,} \end{cases}, \tag{26}$$

$$\left[\hat{\mathbf{c}}(T)\right]_{i} = \begin{cases} \frac{\sum_{t=1}^{T-1} 1(S(t)=i)c(S(t), S(t+1))}{\sum_{t=1}^{T-1} 1(S(t)=i)} & \text{if } \exists S(t) = i, t = 0, \ldots, T-1 \\ 0 & \text{otherwise} \end{cases}, \tag{27}$$

where $1(\cdot)$ is the indicator function.

A fundamental element of the proposed framework is the projection of the cost-to-go function $\overline{c}$ on a set of basis functions capturing the typical substructures of the graph at various time scales. We employ the recently proposed Diffusion Wavelets (DWs) as a basis set for the projection. DWs are a multiresolution geometric construction for the multiscale analysis of operators on graphs. DW functions are computed by sequentially applying a diffusion operator (for instance, the transition matrix $\mathbf{P}$) at the current scale k, compressing the range via a local orthonormalization procedure, representing the operator in the compressed range and computing the $\mathbf{P}^{2k}$ on this range. Functions defined on the support space are analyzed in multiresolution fashion, where dyadic powers of the diffusion operator correspond to dilations, and projections correspond to downsampling. Even if $\mathbf{P}$ is not known *a priori*, we assume that the location of the non-zero

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elements of $\mathbf{P}$, that is, the connectivity structure of $\mathbf{P}$, is known. Define $\mathbf{I}(\mathbf{P})=\mathrm{sgn}(\mathbf{P} + \mathbf{P}^T)$. The basis set W is then computed on $\mathbf{P}_{symm}$ where the i–th row of $\mathbf{P}_{symm}$ is

$$[\mathbf{P}_{symm}]_i=[\mathbf{I}(\mathbf{P})]_i/\sum_j[\mathbf{I}(\mathbf{P})]_{ij}. \tag{28}$$

Define $\mathbf{W}$ as a diffusion wavelet basis set computed on $\mathbf{P}_{symm}$, where the DW functions are the columns of $\mathbf{W}$. We have then $\tilde{c}\approx\mathbf{Wx}$, where $\mathbf{x}$ is the representation vector collecting the coefficients of the wavelet functions in $\mathbf{W}$. Given $\mathbf{P}$ and $\mathbf{c}$, the representation vector $\mathbf{x}^*$ providing the most accurate approximation of $\overline{\mathbf{c}}$ on $\mathbf{W}$ minimizes the Bellman residual $\|\Omega(\mathbf{Wx}) - \mathbf{Wx}\|_2^2$. We have then

$$\mathbf{x}^* = \arg\min_{\mathbf{x}}\|\mathbf{c} - (\mathbf{I} - \gamma\mathbf{P})\mathbf{Wx}\|_2^2. \tag{29}$$

The main idea behind the proposed estimation paradigm is that the DW set of functions is a sparsifying basis for the cost-to-go function $\overline{\mathbf{c}}$. Due to the structured behavior defined by networking protocols, a small number of functions can represent the evolution and, thus, the collected cost, from large groups of states. The Least Angle Selection and Shrinkage Operator (LASSO) algorithm minimizes the residual norm of the residual plus a regularization term. For the considered problem, the LASSO is formulated as

$$\mathbf{x}^*(T) = \arg\min_{\mathbf{x}}\|\mathbf{R}(T)\hat{\mathbf{c}}(T) - \mathbf{R}(T)\hat{\mathbf{B}}(T)\mathbf{Wx}\|_2^2 + \lambda\|\mathbf{x}\|_1, \tag{30}$$

where $\hat{\mathbf{B}}(T) = \mathbf{I}-\gamma\hat{\mathbf{P}}(T)$, $\mathbf{R}$ is a random matrix, and $\mathbf{R}(T)$ is the submatrix formed by retaining the columns of $\mathbf{R}$ indexed by states hit in the observation interval T. We begin with the definition of the properties we wish to show.

Definition 1 (Restricted Isometry Property): The observation matrix $\mathbf{B}$ is said to satisfy the *restricted isometry property* of order $S \in \mathbb{N}$ with parameter $\delta_S \in (0, 1)$, *i.e.* RIP(S, δ_S) if

$$(1 - \delta_S)\|\mathbf{x}\|_2^2 \leq \|\mathbf{Bx}\|_2^2 \leq (1 + \delta_S)\|\mathbf{x}\|_2^2, \tag{31}$$

holds for all $x\in \mathbb{R}^N$ having no more than S non-zero entries. Note that $\mathbf{B}$ is a $K\times N$ matrix. RIP implies that $\mathbf{B}$ is approximately an isometry for S-sparse signals.

We have the following theorem,

Theorem 1 The matrix $\mathbf{R}(T)(\mathbf{I} - \gamma\mathbf{P})$ does not satisfy RIP(S, δ_S) with the following probability bound,

$$\mathcal{P}\left(\mathbf{R}(T)\mathbf{B} \text{ does not satisfy RIP}(\delta_S, S)\right) \ \leq\ \exp\left(-\frac{c_1 K}{S^2}\right)$$

if $K^2 \geq \frac{192\log nS^2}{\delta_S^2-64c_1}$ and $c_1 \geq \frac{\delta_S^2}{64}$.

This result states that if the number of observations K is of order $O\left(S^2\sqrt{n\log n}\right)$ then RIP is satisfied with high probability as the network grows large. We contrast this with the more typical results seen in say channel estimation problems where the order is $O\left(S^2\log n\right)$. We remark
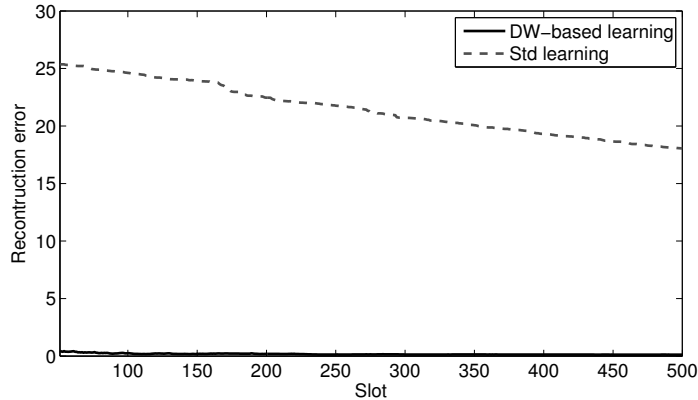
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Figure 1: Comparison between the reconstruction error as a function of the time slot achieved by the proposed algorithm and Q-learning. All states hit by the sample path are used in the estimation.

that our result on the RIP is not limited to LASSO, but leads to the more general conclusion that sparse estimation algorithms can be used to approximate cost-to-go functions of wireless networks.

we present numerical results for an example of a wireless network to demonstrate the potential of the compressed sensing approach. We consider a wireless network where terminals store packets in a finite buffer of size Q and employ Automatic Retransmission reQuest (ARQ) to improve the delivery rate of packets. Time is divided in slots of fixed duration.

The FSM tracking the state of each individual terminal is composed of two sub-chains: a random walk-like sub-chain tracking the number of packets in the buffer (state space $\{0, 1, \ldots, Q\}$) and a forward counter-like sub-chain tracking the retransmission index of the packet being transmitted (state space $\{0, 1, \ldots, F\}$, where F is the maximum number of transmissions of a packet). The FSM tracking the state of the overall network is the composition of the FSMs of the individual terminals. The cost function c measures the normalized cost in terms of throughput loss with respect to the saturation throughput achieved by the terminals in the absence of interference.

For $Q=5$ and $F=4$ and 2 terminals the size of the state space is 1681. In order to keep complexity low, the columns of $\mathbf{W}$ are subsampled. In particular, we select 400 wavelet functions at different time scales. Fig. 1 depicts the reconstruction error achieved by the proposed compressed sensing based framework and that of standard Q-learning as a function of the length of the observed sample-path. The proposed algorithm achieves a considerable accuracy in the estimation of $\overline{c}$ after a very short number of state-cost observations, whereas standard learning converges slowly to $\overline{c}$. Moreover, *the solution is extremely stable* and the compressed sensing based algorithm appears to be very robust to estimation noise. In the future, we will study the design of reduced-dimension network protocols that recover quickly from drastic topology changes.

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