

Decentralized Cognitive MAC for Opportunistic Spectrum Access in Ad Hoc Networks: A POMDP Framework

Qing Zhao, Lang Tong, Ananthram Swami, and Yunxia Chen

Abstract— We propose decentralized cognitive MAC protocols that allow secondary users to independently search for spectrum opportunities without a central coordinator or a dedicated communication channel. Recognizing hardware and energy constraints, we assume that a secondary user may not be able to perform full-spectrum sensing or may not be willing to monitor the spectrum when it has no data to transmit. We develop an analytical framework for opportunistic spectrum access based on the theory of Partially Observable Markov Decision Process (POMDP). This decision-theoretic approach integrates the design of spectrum access protocols at the MAC layer with spectrum sensing at the physical layer and traffic statistics determined by the application layer of the primary network. It also allows easy incorporation of spectrum sensing error and constraint on the probability of colliding with the primary users. Under this POMDP framework, we propose cognitive MAC protocols that optimize the performance of secondary users while limiting the interference perceived by primary users. A suboptimal strategy with reduced complexity yet comparable performance is developed. Without additional control message exchange between the secondary transmitter and receiver, the proposed decentralized protocols ensure synchronous hopping in the spectrum between the transmitter and the receiver in the presence of collisions and spectrum sensing errors.

Index Terms— Opportunistic spectrum access. Cognitive MAC. Partially observable Markov decision process (POMDP).

I. INTRODUCTION

A. Opportunistic Spectrum Access

THE EXPONENTIAL growth in wireless services has resulted in an overly crowded spectrum. The current state of spectrum allocation indicates that almost all usable frequencies have already been occupied. This makes one pessimistic about the feasibility of integrating emerging wireless services such as large-scale sensor networks into the existing communication infrastructure.

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In contrast to the apparent spectrum scarcity is the pervasive existence of spectrum opportunity. Extensive measurements indicate that, at any given time and location, a large portion of licensed spectrum lies unused [1]. Even when a channel is actively used, the bursty arrivals of many applications result in abundant spectrum opportunities at the slot level. These observations form the key rationale for opportunistic spectrum access (OSA) envisioned by the DARPA XG program [2]. The idea is to exploit instantaneous spectrum availability by opening licensed spectrum to secondary users (for example, sensor networks). This would allow secondary users to identify available spectrum resources and communicate in a manner that limits the level of interference perceived by the primary users. Even for the unlicensed spectrum, OSA may be of considerable value in improving spectrum efficiency by supporting both subscribers and opportunistic users.

While conceptually simple, OSA presents challenges not present in the conventional wired or wireless networks. We will focus in this paper on two fundamental issues in ad hoc OSA networks where there is no central coordinator or dedicated communication/control channel.

The first issue deals with sensing and access strategies that integrate opportunity identification and exploitation. We do not assume that each secondary user has full knowledge of the availability of all channels; such knowledge implies continuous full-spectrum sensing synchronous among secondary users. While simplifying the design of OSA networks, continuous full-spectrum sensing is energy inefficient and hardware demanding, especially for low-cost battery-powered wireless nodes with bursty traffic. We assume instead that each secondary user can choose to sense a subset of the possible channels (only when it has data to transmit) and must decide whether transmission is possible based on the sensing outcome. When only part of the spectrum can be sensed at a particular time, sensing and access need to be considered jointly. This joint design also allows the handling of spectrum sensing errors at both physical and MAC layers so that interference to primary users is limited below a prescribed level.

The second issue is transmitter-receiver synchronization, which is unique to the medium access control (MAC) in OSA networks. When a secondary user hops in the spectrum, seeking opportunities that are time-varying and location-dependent, its intended receiver needs to hop synchronously in order to carry out the communication. In an ad hoc

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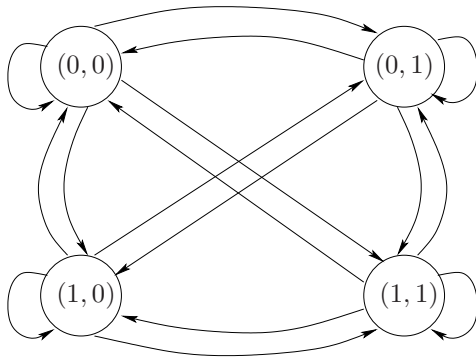


Fig. 1. The underlying Markov process for $N = 2$.

OSA network with collisions and spectrum sensing errors, maintaining transceiver synchronization without introducing extra control message exchange is nontrivial.

B. Summary of Results and Related Work

Summary of Results We present in this paper a cross-layer approach to OSA that integrates spectrum sensing with spectrum access. We adopt a decision-theoretic approach by casting the design of OSA in the framework of Partially Observable Markov Decision Process (POMDP). This formulation leads to optimal policies for spectrum sensing and access and a systematic tradeoff between performance and complexity. Although the POMDP formulation may appear to be natural, specifics of OSA ad hoc networks lead to a number of nontrivial theoretical and practical issues. For example, although the imperfect performance of spectrum sensors has been investigated [3], sensing errors have not been integrated into cognitive MAC design. To our best knowledge, the problem of synchronizing opportunistic users in the presence of collisions and sensing errors has received little attention.

The solution to optimal POMDP has exponential complexity with respect to the number of channels. If the number of channels available for secondary users is relatively small, the optimal policy can be obtained offline, and the implementation cost is acceptable. When the number of channels is large, searching for optimal policy becomes impractical. By exploiting the statistical independencies of primary users, we obtain a sufficient statistic whose dimension grows linearly instead of exponentially with the number of channels. Based on this reduced-dimension sufficient statistics, we propose a suboptimal greedy approach with little complexity yet comparable performance. We recognize that sensing in the presence of noise and fading will lead to errors, which cause opportunity overlook and misidentification. The latter leads to collisions with the primary user, which must be capped below a predetermined design specification. We incorporate the Receiver Operating Characteristics (ROC) of the spectrum sensor into the design of OSA MAC protocols.

Related Work The underutilization of spectrum under the current static spectrum management policy has stimulated a flurry of exciting research activities in searching for dynamic spectrum access strategies. A taxonomy of dynamic spectrum access can be found in [4].

OSA is one of the several approaches to dynamic spectrum access. Differing from this work that mainly addresses the exploitation of temporal spectrum opportunities resulting from the bursty traffic of primary users, a majority of existing work focuses on spatial spectrum opportunities that are static or slowly varying in time. Example applications include the reuse of certain TV-bands that are not used for TV broadcast in a particular region. Due to the slow temporal variation of spectrum occupancy, real-time opportunity identification is not as critical a component in this class of applications, and the prevailing approach to OSA tackles network design in two separate steps: (i) opportunity identification assuming continuous full-spectrum sensing; (ii) opportunity allocation among secondary users assuming full knowledge of spectrum opportunities. Opportunity identification in the presence of fading and noise uncertainty has been studied in [3], [5]–[8]. Spatial opportunity allocation among secondary users can be found in [9]–[12] and references therein. For an overview of challenges and recent development in OSA, readers are referred to [4].

II. THE NETWORK AND PROTOCOL MODEL

The Network Model Consider a spectrum consisting of N channels¹, each with bandwidth B_i ($i = 1, \dots, N$). These N channels are licensed to a primary network whose users communicate according to a synchronous slot structure. The traffic statistics of the primary network are such that the occupancy of these N channels follows a discrete-time Markov process with $M = 2^N$ states. Specifically, the network state in slot t is given by $[S_1(t), \dots, S_N(t)]$ where $S_i(t) \in \{0 \text{ (occupied)}, 1 \text{ (idle)}\}$. The state diagram and a sample path of the state evolution for $N = 2$ are illustrated in Figure 1 and Figure 2, respectively. We assume that the spectrum usage statistics of the primary network remain unchanged for T slots.

We consider a secondary network seeking spectrum opportunities in these N channels (see Figure 2). We focus on an ad hoc network where secondary users join/exit the network and sense/access the spectrum independently without exchanging local information. In each slot, a secondary user chooses a set of channels to sense and a set of channels to access. Limited by its hardware constraints and energy supply, a secondary user can sense no more than L_1 ($L_1 \leq N$) and access no more than L_2 ($L_2 \leq L_1$) channels in each slot.

Our goal is to develop cognitive MAC protocols for the secondary network. For an ad hoc OSA network without a central coordinator or a dedicated communication channel, it is desirable to have a decentralized MAC protocol where each secondary user independently searches for spectrum opportunities, aiming at optimizing its own performance. Such decentralized protocols do not rely on cooperation among secondary users.

The Basic Protocol Structure Without delving into protocol details (which are given in subsequent sections), we present here the basic protocol structure. At the beginning of each slot², a secondary user with data to transmit chooses a set of

¹Here we use the term channel broadly. A channel can be a frequency band with certain bandwidth, a collection of spreading codes in a CDMA network, or a set of tones in an OFDM system.

²The slot information can be broadcasted by the primary network.

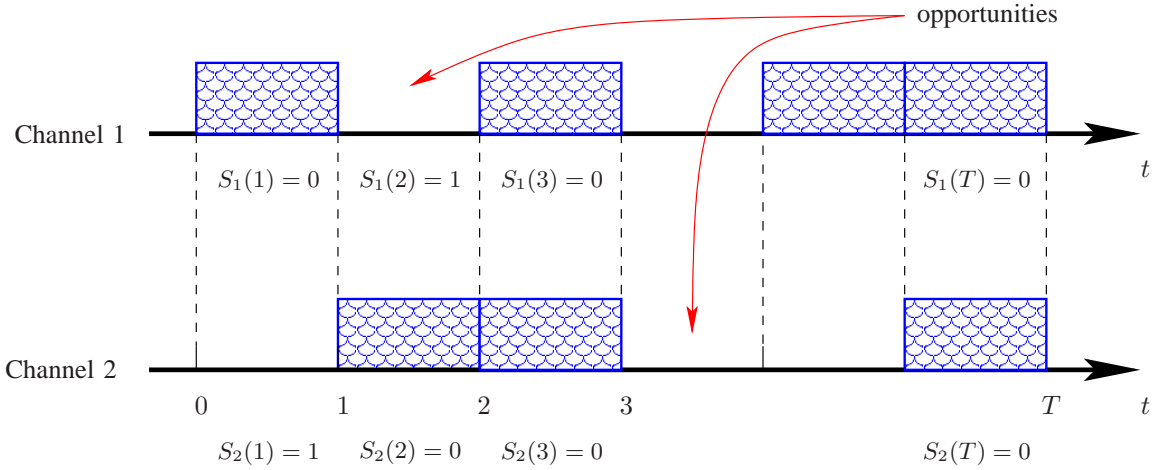


Fig. 2. A sample path of spectrum occupancy.

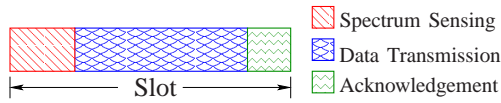


Fig. 3. The slot structure.

channels to sense and a set of channels to access based on the sensing outcome. Such spectrum sensing and access decisions are made to maximize the throughput of the secondary user while limiting the interference to the primary network by fully exploiting the sensing history and the spectrum occupancy statistics. When the secondary user decides to transmit, it generates a random backoff time, and transmits when this timer expires and no other secondary user has already accessed the channel during the backoff time. At the end of the slot, the receiver acknowledges a successful data transmission. The basic slot structure is illustrated in Figure 3.

III. A DECISION-THEORETIC APPROACH BASED ON POMDP

In this section, we develop a decision-theoretic approach to MAC design in OSA networks. We show that the OSA network specified in Section II can be modelled by a POMDP and the spectrum sensing and access component of a MAC protocol corresponds to a policy for this POMDP. Existing techniques and results for POMDP can then be used to develop MAC protocols for OSA networks.

A POMDP Formulation We illustrate in Figure 4 the Markovian dynamics of the OSA network specified in Section II. At the beginning of each slot, a secondary user chooses a set \mathcal{A}_1 ($|\mathcal{A}_1| \leq L_1$) of channels to sense. Given that the current state of the underlying Markov process is j , the user observes $\Theta_{j,\mathcal{A}_1} \in \{0, 1\}^{|\mathcal{A}_1|}$ which indicates the availability of each sensed channel. Based on this observation, the user chooses a set $\mathcal{A}_2 \subseteq \mathcal{A}_1$ ($|\mathcal{A}_2| \leq L_2$) of channels to access. For the chosen action, the user receives a reward $r_{j,\mathcal{A}_1,\mathcal{A}_2}$ at the end of this slot. The sequence of operations in each slot is illustrated in Figure 5.

The objective is to choose the sensing and access action $\{\mathcal{A}_1, \mathcal{A}_2\}$ sequentially in each slot so that the total expected

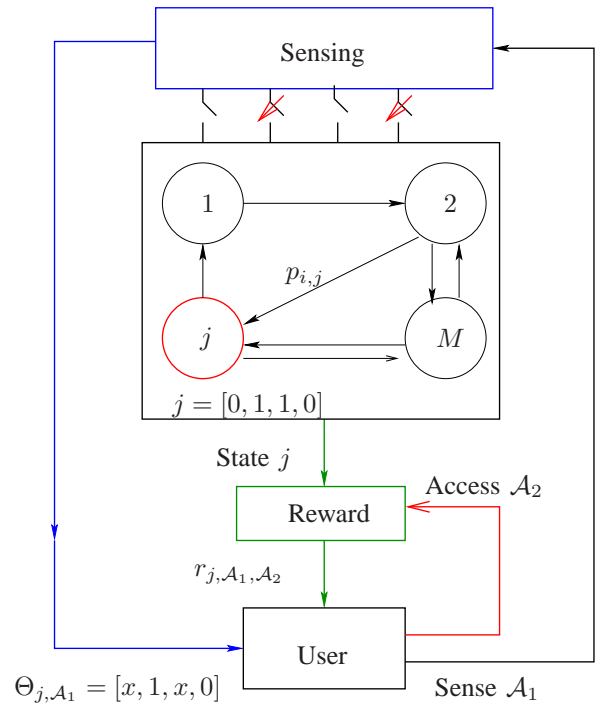


Fig. 4. The Markovian dynamics of the OSA network.

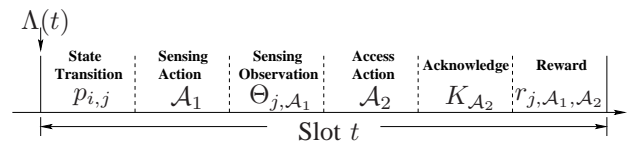


Fig. 5. The sequence of operations in a slot.

reward accumulated over T slots (wherein the spectrum occupancy statistics remain unchanged) is maximized. We now have a POMDP since, in general, the network state cannot be fully observed due to partial spectrum monitoring ($|\mathcal{A}_1| \leq L_1 < N$) or sensing error.

We assume in this paper that the state transition probabilities $\{p_{i,j}\}$ are known. In practice, this may not be available. The problem then becomes one of POMDP with unknown transition probabilities. Such formulations are beyond the scope of this paper. Algorithms for POMDP with an unknown

model exist in the literature [13] and are applicable to the OSA problem.

A Sufficient Statistic For a POMDP, the internal state of the underlying Markov process is unknown. At the beginning of slot t , our knowledge of the internal state of the network based on all past decisions and observations can be summarized by a belief vector $\Lambda(t) = [\lambda_1(t), \dots, \lambda_M(t)]$ where $\lambda_j(t)$ is the conditional probability (given the decision and observation history) that the network state is j at the beginning of slot t prior to the state transition (see Figure 5).

It has been shown in [14] that for any t , the belief vector $\Lambda(t)$ is a sufficient statistic for the design of the optimal action $\{\mathcal{A}_1, \mathcal{A}_2\}$ in slot t . A policy π for a POMDP is thus given by a sequence of functions, each mapping from the current belief vector $\Lambda(t)$ to the sensing and access action $\{\mathcal{A}_1(t), \mathcal{A}_2(t)\}$ to be taken in slot t , *i.e.*, as in (1).

Under this formulation, a spectrum sensing and access strategy is essentially a policy of this POMDP over a finite horizon³.

Reward and Objective Function The reward gained by a secondary user in each slot can be defined in many ways depending on the design objective. For an OSA network, the most obvious way is to define the reward $r_{j, \mathcal{A}_1, \mathcal{A}_2}(t)$ as the number of bits delivered when the user senses channels in \mathcal{A}_1 and transmits using channels in \mathcal{A}_2 given that the network is in state j . Assume that the number of bits delivered over a channel in one slot is proportional to its bandwidth. We define the reward as⁴

$$r_{j, \mathcal{A}_1, \mathcal{A}_2}(t) = \sum_{i \in \mathcal{A}_2} S_i(t) B_i, \quad (2)$$

where $S_i(t) \in \{0, 1\}$ is the state of channel i in slot t . Note that in the presence of sensing error, a secondary user may access an unavailable channel, resulting in a collision with a primary user.

Let ζ denote the maximum probability of collision allowed by the primary network. The design objective for cognitive MAC is to maximize the expected total number of bits transmitted in T slots under the constraint that the probability of collision is bounded below ζ , *i.e.*, the optimal policy π_* is given by (3) where \mathbb{E}_π represents the conditional expectation given that policy π is employed, P_c is the probability of collision, and $\Lambda(1)$ the initial belief vector which can be the stationary distribution of the network state. Note that the probability of collision P_c depends on the sensing and access policy π as well as the operating characteristic of the spectrum sensor.

For ease of presentation, we assume in the rest of the paper that $L_1 = L_2 = 1$. In this case, the action taken in each slot consists of the index $a \in \{1, \dots, N\}$ of the channel to

³The optimal sensing and access strategy can also be formulated as a POMDP over an infinite horizon. Since the statistics of channel occupancy vary with time due to changes in traffic load, it is more appropriate to consider the finite horizon formulation. It is, however, straightforward to extend our formulation to an infinite horizon setup in which stationary policies can be obtained.

⁴In this paper, we focus on decentralized cognitive MAC where secondary users make independent and selfish decisions without coordination. In this case, a secondary user chooses its spectrum sensing and access strategies under the assumption that it will receive a reward when the chosen channel is not used by the primary network.

be sensed and the decision $\Phi_a \in \{0 \text{ (no access)}, 1 \text{ (access)}\}$ on whether to transmit. Results obtained in this paper can be readily extended to general cases.

IV. SPECTRUM SENSING AND ACCESS: OPTIMAL AND SUBOPTIMAL STRATEGIES

In this section, we propose spectrum sensing and access strategies under the POMDP framework developed in Section III. We first assume error-free spectrum sensing to illustrate the structure of the optimal and suboptimal strategies. We then address spectrum sensing and access in the presence of sensing error.

A. An Optimal Channel Sensing and Access Strategy

When the sensing outcome reflects the true channel state, the access decision is straightforward: transmit if and only if the channel is sensed to be available. The constraint on the probability of collision in (3) becomes irrelevant. The design objective is to determine, in each slot, which channel to sense so that the expected total reward obtained in T slots is maximized.

Referred to as the value function, $V_t(\Lambda(t))$ denotes the maximum expected remaining reward that can be accrued starting from slot t when the current belief vector is $\Lambda(t)$. It has two parts: (i) the immediate reward obtained in slot t which is given by $\Theta_{j,a} B_a$ when the network is at state j and the user senses channel a and observes $\Theta_{j,a} \in \{0, 1\}$; (ii) the maximum expected remaining reward $V_{t+1}(\Lambda(t+1))$ starting from slot $t+1$ given a belief vector $\Lambda(t+1) = \mathcal{T}(\Lambda(t)|a, \Theta_{j,a})$ which represents the updated knowledge of the network state after incorporating the action and observation obtained in slot t . Averaging over all possible network states and observations, we arrive at the following Bellman's equation (4) where the updated belief vector $\Lambda(t+1) = \mathcal{T}(\Lambda(t)|a, \Theta_{j,a})$ can be easily obtained via the Bayes rule.

$$\begin{aligned} \Lambda(t+1) &\triangleq \mathcal{T}(\Lambda(t)|a, \theta) \triangleq [\lambda_1(t+1), \dots, \lambda_M(t+1)], \\ \lambda_j(t+1) &= \frac{\sum_{i=1}^M \lambda_i(t) p_{i,j} \Pr[\Theta_{j, \mathcal{A}_1} = \theta]}{\sum_{i=1}^M \sum_{j=1}^M \lambda_i(t) p_{i,j} \Pr[\Theta_{j, \mathcal{A}_1} = \theta]}. \end{aligned} \quad (5)$$

From (4) we can see that an action chosen at a slot affects the total reward in two ways: it acquires an immediate reward θB_a in this slot and transforms the belief vector to $\mathcal{T}(\Lambda|a, \theta)$ which determines the future reward $V_{t+1}(\mathcal{T}(\Lambda(t)|a, \theta))$. The optimal policy strikes a balance between gaining instantaneous reward and gaining information for future use.

Smallwood and Sondik showed in [14] that $V_t(\Lambda(t))$ is convex and piecewise linear as illustrated in Figure 6. Specifically, the domain of $V_t(\Lambda(t))$ can be partitioned into a finite number of convex regions $\{\mathcal{C}_1(t), \dots, \mathcal{C}_L(t)\}$. Associated with each region $\mathcal{C}_i(t)$ is a vector $\Upsilon_i(t)$ such that the value function $V_t(\Lambda(t))$ in this region is given by the inner product of $\Lambda(t)$ ($\Lambda(t) \in \mathcal{C}_i(t)$) and $\Upsilon_i(t)$. Applying this structure of the value function to (4), we obtain (6) where $\langle \cdot, \cdot \rangle$ denotes inner product and $i_{\Lambda(t+1)}$ the index of the region containing the updated belief vector $\Lambda(t+1) = \mathcal{T}(\Lambda(t)|a, \theta)$. Thus, if the convex regions $\{\mathcal{C}_i(t+1)\}$ and the associated Υ -vectors $\{\Upsilon_i(t+1)\}$ have been calculated for slot $t+1$, we can obtain

$$\pi = [\mu_1, \dots, \mu_T], \quad \text{where } \mu_t : \Lambda(t) \in [0, 1]^M \rightarrow \{\mathcal{A}_1(t), \mathcal{A}_2(t)\} \quad (1)$$

$$\pi_* = \arg \max_{\pi} \mathbb{E}_{\pi} \left[\sum_{t=1}^T r_{j(t), \mathcal{A}_1(t), \mathcal{A}_2(t)} \mid \Lambda(1) \right], \quad \text{subject to } P_c \leq \zeta \quad (3)$$

$$V_i(\Lambda(t)) = \max_{a=1, \dots, N} \left\{ \sum_{i=1}^M \lambda_i \sum_{j=1}^M p_{i,j} \sum_{\theta=0}^1 \Pr[\Theta_{j,a} = \theta] (\theta B_a + V_{t+1}(\mathcal{T}(\Lambda(t)|a, \theta))) \right\} \quad (4)$$

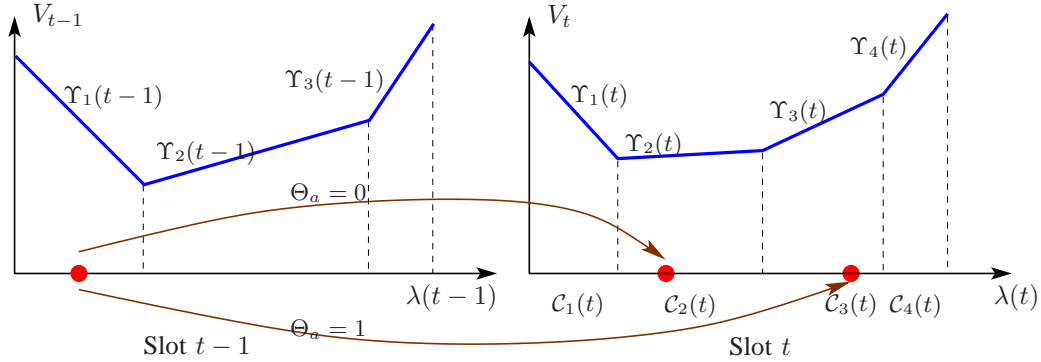


Fig. 6. The structure of $V_i(\Lambda)$. We consider a two-state system ($M = 2$). A belief vector $\Lambda = [\lambda, 1 - \lambda]$ can be represented by a point in $[0, 1]$. As shown above, after we observe θ in slot $t - 1$, the belief vector is transformed into a (different) point in the space of belief vectors for the succeeding slot (see (5)). For the example above, the space of belief vectors for slot t is partitioned into four regions and within each region, $V_i(\Lambda(t))$ is a linear function of $\Lambda(t)$. The corresponding Υ -vectors (the slope of $V_i(\Lambda(t))$ in this example) are denoted by $\{\Upsilon_1(t), \dots, \Upsilon_4(t)\}$.

from (6) the optimal actions and the corresponding Υ -vectors for slot t . A linear programming algorithm is provided in [14] to carry out this computation. The optimal policy is thus given by the convex regions and the associated Υ -vectors and optimal actions for $t = 1, \dots, T$. Note that the computation of the optimal policy can be obtained off-line and the result stored in a table.

B. A Reduced-State Suboptimal Strategy

Finding the optimal policy for a general POMDP can be computationally prohibitive. One reason is that the dimension of the sufficient statistic Λ grows *exponentially* with the number N of channels. Although the optimal policy can be computed off-line and stored before a secondary user starts to access the spectrum, this approach makes it difficult to adapt to changes in the spectrum occupancy statistics. It is thus crucial to exploit the specific structure of the problem and develop suboptimal strategies with reduced complexity. In this section, we show that when channels evolve independently, we can find a sufficient statistic for the optimal policy whose dimension grows *linearly* with N .

Proposition 1: Let $\Omega = [\omega_1, \dots, \omega_N]$ where ω_i is the probability (conditioned on the sensing and decision history) that channel i is available at the beginning of a slot. Then Ω is a sufficient statistic for the optimal OSA protocol under N independent channels.

Proof: See Appendix.

Proposition 1 shows that by exploiting the statistical independency among channels, we can reduce the dimension of the sufficient statistic from 2^N to N . This result points to

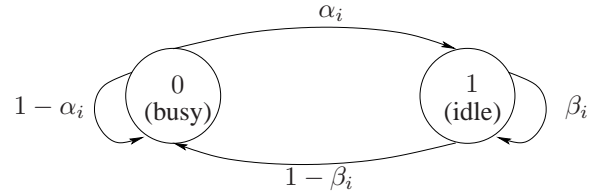


Fig. 7. The Markov channel model

the possibility of significantly reducing the computational and storage complexity of the optimal OSA protocol.

Based on the sufficient statistic Ω , we propose a suboptimal protocol based on a greedy approach that maximizes per-slot throughput⁵. Assume that channels evolve independently. As illustrated in Figure 7, channel i transits from state 0 (unavailable) to state 1 (available) with probability α_i and stays in state 1 with probability β_i . Given that our knowledge of the network state is $\Omega(t)$ at the beginning of slot t prior to the state transition, the expected reward to be gained in slot t if channel a is selected is

$$(\omega_a(t)\beta_a + (1 - \omega_a(t))\alpha_a)B_a, \quad (7)$$

where $(\omega_a\beta_a + (1 - \omega_a)\alpha_a)$ is the probability that channel a will be available in slot t . For the greedy approach, the action in slot t is chosen to maximize the expected immediate reward, *i.e.*, the index $a_*(t)$ of the chosen channel is given by

$$a_*(t) = \arg \max_{a=1, \dots, N} (\omega_a(t)\beta_a + (1 - \omega_a(t))\alpha_a)B_a. \quad (8)$$

⁵When channels are correlated, we can similarly develop a greedy approach based on Λ .

$$V_t(\Lambda(t)) = \max_{a=1, \dots, N} \left\{ \sum_{i=1}^M \lambda_i \sum_{j=1}^M p_{i,j} \sum_{\theta=0}^1 \Pr[\Theta_{j,a} = \theta] (\theta B_a + \langle \Lambda(t+1), \Upsilon_{i_{\Lambda(t+1)}}(t+1) \rangle) \right\} \quad (6)$$

$$\begin{aligned} \Omega(t+1) &= [\omega_1(t+1), \dots, \omega_N(t+1)] \triangleq \mathcal{T}(\Omega(t)|a_*(t), \Theta_{a_*(t)}), \\ \omega_i(t+1) &= \begin{cases} 1 & \text{if } a_*(t) = i, \Theta_{a_*(t)} = 1 \\ 0 & \text{if } a_*(t) = i, \Theta_{a_*(t)} = 0 \\ \omega_i(t)\beta_i + (1 - \omega_i(t))\alpha_i & \text{if } a_*(t) \neq i \end{cases} \end{aligned} \quad (9)$$

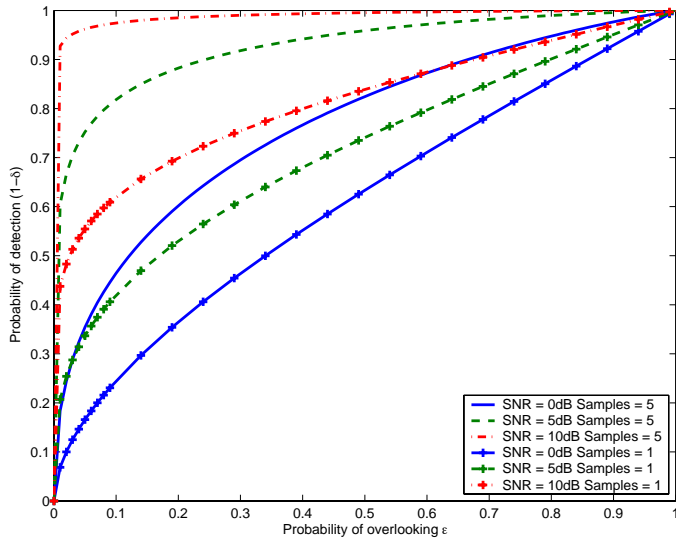


Fig. 8. ROC curves for detecting Gaussian signal in Gaussian noise.

At the end of slot t , the belief vector Ω is updated based on the action $a_*(t)$ and the observation $\Theta_{a_*(t)}$ (indicating the availability of channel a_*) as in (9). Note that when a channel is not sensed, the probability of its availability is updated according to the Markov chain. If the channel is sensed, the state of this channel is the sensing outcome since the belief vector records the channel state prior to the state transition at the beginning of each slot.

Let $W_t(\Omega)$ denote the expected remaining reward starting from slot t achieved by the greedy approach. We obtain a recursive equation for $W_t(\Omega)$ as in (10), where a_* and $\mathcal{T}(\Omega|a, \theta)$ are given by (8) and (9), respectively.

The computational complexity of the greedy approach and a systematic way of trading off performance with complexity are studied in the context of energy-constrained OSA in [15].

C. Spectrum Sensing and Access in the Presence of Sensing Error

We now consider the scenario where sensing errors cannot be ignored. In this case, not only the sensing and access strategy but also the operating characteristics of the spectrum sensor affect the performance of the OSA network and the interference perceived by the primary network. The problem thus includes the design of the spectrum sensor as well as the sensing and access strategy for optimal spectrum utilization under a constraint on the maximum collision probability.

Spectrum sensors perform a binary hypotheses test: \mathcal{H}_0 (null hypothesis indicating that the sensed channel is available) vs. \mathcal{H}_1 (alternative). If the sensor of a secondary user mistakes \mathcal{H}_0 for \mathcal{H}_1 (false alarm), the secondary user may refrain from transmitting, and a spectrum opportunity is overlooked. On the other hand, if the detector mistakes \mathcal{H}_1 for \mathcal{H}_0 (miss detection), a misidentification of spectrum opportunity occurs; the secondary user collides with a primary user if it trusts the sensing outcome. Let ϵ and δ denote, respectively, the overlook (false alarm) and misidentification (miss detection) probabilities. The performance of the sensor is specified by the Receiver Operating Characteristics (ROC) curve which gives $1 - \delta$ as a function of ϵ (examples are given in Figure 8). The objective is to design the optimal spectrum sensing and access policy π_* and the operating point δ_* (on the ROC curve) of the spectrum sensor. Specifically, as in (11).

The above optimization is a constrained POMDP problem which generally requires a randomized optimal policy. To obtain a deterministic strategy with low complexity, we aim at separating the objective function of (11) from the constraint. Specifically, we choose the sensor operating point based on the constraint on the probability of collision: $\delta_* = \zeta$. In this case, the optimal access policy is given by

$$\Phi_a = \begin{cases} 1 & \text{if } \Theta_a = 1 \\ 0 & \text{if } \Theta_a = 0 \end{cases} \quad (12)$$

Since the probability of misidentification of the spectrum sensor is ζ , the probability of colliding with a primary user under this access strategy is ζ , satisfying the design constraint. The problem is then reduced to an unconstrained POMDP where the optimal policy for channel selection is obtained to maximize the throughput of the secondary user.

Both the optimal and suboptimal greedy approaches presented in Section IV.A-B can be extended to incorporate sensing error. We consider here the greedy strategy that maximizes the per-slot throughput. Let U_a denote the number of bits that can be successfully delivered if channel a is chosen. The index a_* of the channel to be selected is then given by (13).

The information gained at the transmitter in slot t includes the decision $\{a_*, \Phi_{a_*}\}$ and the observation $\{\Theta_{a_*}, K_{a_*}\}$ where $K_{a_*} \in \{0, 1\}$ indicates whether an acknowledgement is received at the end of this slot⁶. The information gained at the receiver, however, includes only a_* and K_{a_*} since the receiver does not have the sensing outcome Θ_{a_*} at the transmitter (due to sensing error) and cannot distinguish an unsuccessful transmission from the no-access decision $\Phi_{a_*} = 0$ of the

⁶The transmission of acknowledgement is assumed to be error-free.

$$\begin{aligned}
 W_t(\Omega) &= (\omega_{a_*}\beta_{a_*} + (1 - \omega_{a_*})\alpha_{a_*})B_{a_*} + \sum_{\theta=0}^1 \Pr[\Theta_{a_*} = \theta | \Omega, a_*] W_{t+1}(\mathcal{T}(\Omega | a_*, \theta)) \\
 &= (\omega_{a_*}\beta_{a_*} + (1 - \omega_{a_*})\alpha_{a_*})B_{a_*} + [\omega_{a_*}(1 - \beta_{a_*}) + (1 - \omega_{a_*})(1 - \alpha_{a_*})] W_{t+1}(\mathcal{T}(\Omega | a_*, 0)) \\
 &\quad + [\omega_{a_*}\beta_{a_*} + (1 - \omega_{a_*})\alpha_{a_*}] W_{t+1}(\mathcal{T}(\Omega | a_*, 1))
 \end{aligned} \tag{10}$$

$$\{\pi_*, \delta_*\} = \arg \max_{\{\pi, \delta\}} \mathbb{E}_\pi \left[\sum_{t=1}^T r_{j(t), \mathcal{A}_1(t), \mathcal{A}_2(t)} \mid \Lambda(1) \right], \quad \text{subject to } P_c \leq \zeta \tag{11}$$

$$\begin{aligned}
 a_* &= \arg \max_{a=1, \dots, N} \mathbb{E}[U_a | \Omega] = \arg \max_{a=1, \dots, N} \{B_a \Pr[S_a = 1, \Theta_a = 1 | \Omega]\} \\
 &= \arg \max_{a=1, \dots, N} \{B_a(1 - \epsilon) \{\omega_a \beta_a + (1 - \omega_a) \alpha_a\}\}
 \end{aligned} \tag{13}$$

$$\begin{aligned}
 \Omega(t+1) &\triangleq \mathcal{T}(\Omega(t) | a_*, K_{a_*}) = [\omega_1(t+1), \dots, \omega_N(t+1)] \\
 \omega_i(t+1) &\triangleq \Pr[S_i(t) = 1 | \Omega(t), a_*, K_{a_*}] \\
 &= \begin{cases} \omega_i(t)\beta_i + (1 - \omega_i(t))\alpha_i & \text{if } a_* \neq i \\ 1 & \text{if } a_* = i, K_{a_*} = 1 \\ \frac{\epsilon(\omega_{a_*}\beta_{a_*} + (1 - \omega_{a_*})\alpha_{a_*})}{\epsilon(\omega_{a_*}\beta_{a_*} + (1 - \omega_{a_*})\alpha_{a_*}) + (\omega_{a_*}(1 - \beta_{a_*}) + (1 - \omega_{a_*})(1 - \alpha_{a_*}))} & \text{if } a_* = i, K_{a_*} = 0 \end{cases}
 \end{aligned} \tag{14}$$

transmitter. In order for the transmitter and the receiver to arrive at the same belief vector $\Omega(t+1)$, which ensures that they tune to the same channel in the next slot (see (13)), the belief vector should be updated at both the transmitter and the receiver using only the common information a_* and K_{a_*} . The belief vector $\Omega(t+1)$ is thus given by (14), which is obtained from (15) and (16), where (15) follows from $\Pr[S_{a_*}(t) = 1, \Theta_{a_*} = 1, K_{a_*} = 0 | \Omega(t)] = 0$ (no acknowledgement received when a transmission occurs over an available channel⁷) and (16) from $\Pr[K_{a_*} = 0 | S_{a_*}(t) = 0] = 1$ (no acknowledgement received given that the channel is unavailable). We then obtain (14).

The above specifies the spectrum sensing and access strategy when the spectrum sensor operates at $\delta = \zeta$ (see Figure 8). Implementation details of this protocol are given in Figure 9. The implementation of the optimal sensing strategy can be similarly obtained.

The optimal joint design of spectrum sensor and sensing/access strategy given in (11) is studied in [16], where a separation principle is established that leads to simple, deterministic, yet optimal solutions.

V. PROTOCOL SPECIFICS OF DECENTRALIZED COGNITIVE MAC

In this section, we present protocol specifics of the proposed cognitive MAC strategies. Functions (other than spectrum sensing and access) of cognitive MAC protocols are identified

⁷We assume that a collision with a primary user cannot be distinguished from that with a secondary user and no statistical information on the occurrence of collisions with secondary users is available. The belief vector is thus updated based on the assumption that an unsuccessful transmission results from a collision with a primary user.

and protocol implementations are presented by considering two network settings with increasing complexity.

A. OSA Networks with Spatially Invariant Spectrum Opportunity

We consider first a secondary network where every user is affected by the same set of primary users. In this case, the state of a channel is the same at both the transmitter and the receiver. Detection of channel state can thus be carried out at the transmitter alone. The main issue that needs to be addressed by the MAC protocol is transceiver synchronization: the transmitter and the receiver need to tune to the same channel in order to communicate, and they need to hop synchronously. The synchronization problem can be separated into two phases: the initial handshake between the transmitter and the receiver and the synchronous hopping in the spectrum after the initial establishment of communication.

There are a number of standard implementations to facilitate the initial handshake. Here we borrow the idea of receiver-oriented code assignment in CDMA ad hoc networks [17]. Specifically, each secondary user is assigned a set of channels (not necessarily unique) which it monitors regularly to check whether it is an intended receiver. A user with a message for, say, user A will transmit a handshake signal over one of the channels assigned to user A . Once the initial communication is established, the transmitter and the receiver will implement the same spectrum sensing and access strategy which governs channel selection in each slot. We show below that the sensing and access strategies developed in Section IV ensure synchronous hopping between the transmitter and the receiver in the presence of collisions and sensing errors.

$$\begin{aligned} \Pr[S_{a_*}(t) = 1 | \Omega(t), K_{a_*} = 0] &= \frac{\Pr[S_{a_*}(t) = 1, \Theta_{a_*} = 0 | \Omega(t)] + \Pr[S_{a_*}(t) = 1, \Theta_{a_*} = 1, K_{a_*} = 0 | \Omega(t)]}{\Pr[K_{a_*} = 0 | \Omega(t)]} \\ &= \frac{\Pr[\Theta_{a_*} = 0 | S_{a_*}(t) = 1] \Pr[S_{a_*}(t) = 1 | \Omega(t)]}{\Pr[K_{a_*} = 0 | S_{a_*}(t) = 1] \Pr[S_{a_*}(t) = 1 | \Omega(t)] + \Pr[K_{a_*} = 0 | S_{a_*}(t) = 0] \Pr[S_{a_*}(t) = 0 | \Omega(t)]} \end{aligned} \quad (15)$$

$$= \frac{\Pr[\Theta_{a_*} = 0 | S_{a_*}(t) = 1] \Pr[S_{a_*}(t) = 1 | \Omega(t)]}{\Pr[\Phi_{a_*} = 0 | S_{a_*}(t) = 1] \Pr[S_{a_*}(t) = 1 | \Omega(t)] + \Pr[S_{a_*}(t) = 0 | \Omega(t)]} \quad (16)$$

Decentralized Cognitive MAC

At the beginning of slot t with a belief vector $\Omega(t)$ at both the transmitter and the receiver,

- 1) both the transmitter and the receiver chooses channel a_* given by (13);
- 2) the transmitter senses channel a_* and obtains the sensing outcome Θ_{a_*} ;
- 3) the transmitter chooses the access action Φ_{a_*} according to (12);
- 4) if $\Phi_{a_*} = 1$, the transmitter transmits data over channel a_* using carrier sensing;
- 5) if a data packet is successfully received, the receiver transmits acknowledgement K_{a_*} ;
- 6) both the transmitter and the receiver obtains the new belief vector $\Omega(t+1)$ using $\{a_*, K_{a_*}\}$ according to (14).

Fig. 9. Protocol description for OSA networks with spatially invariant spectrum opportunity.

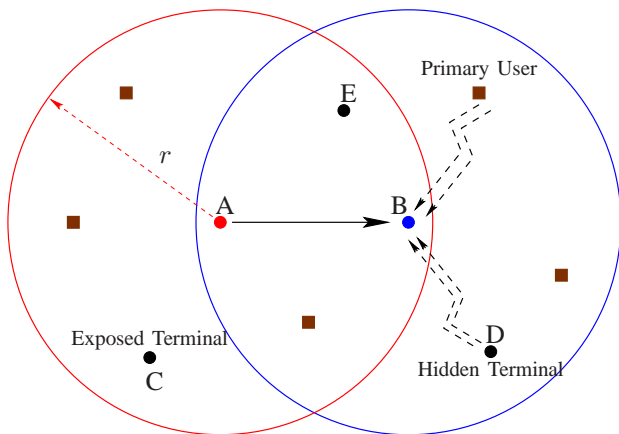


Fig. 10. An OSA network with spatially varying spectrum opportunity.

Proposition 2: In OSA networks with spatially invariant spectrum opportunity, the proposed cognitive MAC protocols ensure transceiver synchronization in the presence of collisions and spectrum sensing errors.

Proof: We focus on the suboptimal greedy approach outlined in Figure 9. The same conclusion can be drawn for the optimal strategy. It is easy to see from the protocol description given in Figure 9 that the transmitter and the receiver maintain the same belief vector independent of collision and sensing error. Since the channel selection is determined by the belief vector, transceiver synchronization is maintained.

B. OSA Networks with Spatially Varying Spectrum Opportunity

When secondary users are affected by different sets of primary users, the state of spectrum occupancy is location dependent; a channel that is idle at a transmitter may not be idle at the corresponding receiver. This spatial variation of spectrum occupancy results in new design challenges as specified below.

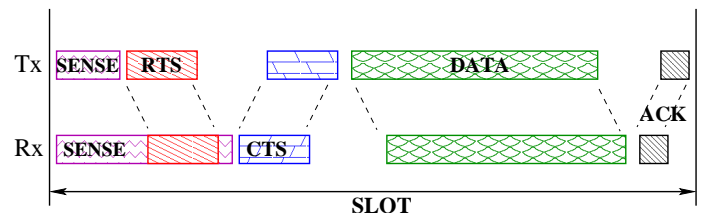


Fig. 11. The protocol structure for OSA networks with spatially varying spectrum opportunity.

Spectrum Opportunity Identification As illustrated in Figure 10, the state of a channel at the transmitter A is determined by the transmission activities of those primary users within A's receiving range r while the state of this channel at the receiver B is determined by primary users within B's receiving range. Since a channel only presents an opportunity to a pair of secondary users if it is available at both the transmitter and the receiver, spectrum opportunities need to be identified jointly by the transmitter and the receiver⁸.

We propose the following modification to the basic protocol structure illustrated in Figure 3 to address the issue of opportunity identification. As shown in Figure 11, at the beginning of a slot, the transmitter monitors the channel for a period of time to ensure the required sensing accuracy. If the channel is sensed to be available, the transmitter generates a random backoff time. If the channel remains idle when its backoff time expires, it transmits a short request-to-send (RTS) message to the receiver, indicating that the channel is available at the transmitter. The receiver, upon receiving the RTS, replies with a clear-to-send (CTS) message if the channel is also available at the receiver. A successful exchange of RTS-CTS completes

⁸In this case, $S_i(t) = 1$ if channel i is available at both the transmitter and the receiver. Otherwise, $S_i(t) = 0$. Strictly speaking, the availability of a channel at the secondary transmitter is determined by primary receivers rather than primary transmitters in its neighborhood [4]. The detection of primary receivers can be transformed to the detection of primary transmitters. A detailed presentation can be found in [4].

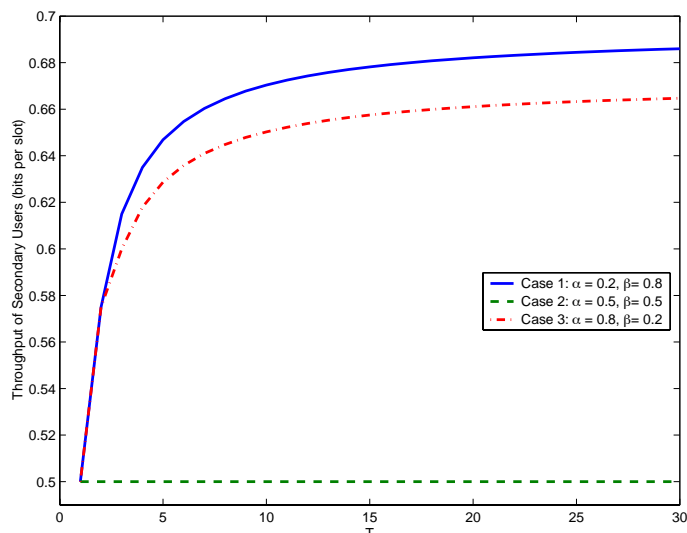


Fig. 12. Performance of the optimal cognitive MAC protocol under different spectrum occupancy statistics ($N = 3$ independent channels with the same bandwidth $B = 1$ and transition probabilities $\{\alpha, \beta\}$).

the identification of a spectrum opportunity.

Hidden and Exposed Terminals The presence of hidden and exposed terminals is a classical problem in MAC design for multihop ad hoc networks. In an OSA network as illustrated in Figure 10, hidden terminals are secondary users within the secondary receiver's range but outside the secondary transmitter's range (for example, D) while exposed terminals are secondary users within the secondary transmitter's range but outside the secondary receiver's range (for example, C). Since hidden terminals can cause collision and exposed terminals may lead to wasted opportunities⁹, the ability to deal with hidden and exposed terminals is crucial to the efficiency of cognitive MAC protocols.

In the proposed protocols, the RTS-CTS exchange has dual functions. Besides facilitating opportunity identification, it also mitigates the hidden and exposed terminal problem as in a conventional communication network [18]. Other collision avoidance schemes such as busy tone and dual busy tone may be incorporated to further reduce the occurrence of collision.

Transceiver Synchronization The issue of initial handshake and transceiver synchronization is similar to that in the first case. The protocol implementation specifics can be similarly defined as in Figure 9. It is easy to show that in OSA networks with spatially varying spectrum opportunity, the proposed protocol maintains the same update of the belief vector at the transmitter and the receiver, thus ensures transceiver synchronization as stated in the proposition below.

Proposition 3: In OSA networks with spatially varying spectrum opportunity, the proposed cognitive MAC protocols ensure transceiver synchronization in the presence of collisions and spectrum sensing errors.

⁹If C chooses to sense the channel used by A, C concludes that the channel is not available thus refrains from transmitting. This leads to a wasted opportunity since C's transmission will not interfere with the communication between A and B.

VI. NUMERICAL AND SIMULATION RESULTS

In this section, we present numerical and simulation results to evaluate the performance of the proposed optimal and suboptimal cognitive MAC protocols.

Optimal Performance and the Impact of Primary Network Traffic Statistics In these examples, we assume sensing errors can be ignored and focus on one secondary user. Shown in Figure 12 is the performance of the optimal protocol under different spectrum occupancy statistics. We consider three independent channels with the same bandwidth $B = 1$ and transition probabilities $\{\alpha, \beta\}$ (see Figure 7). In Case 1, the channel state remains unchanged with a large probability 0.8. This corresponds to bursty traffic arrivals in the primary network: both the message length and the inter-arrival time are large. Case 3 is the opposite of Case 1: the inter-arrival time and the message length of the primary network are relatively small, resulting in more frequent changes in the channel state. In Case 2, the channel is equally likely to change the state or remain at the current state. Note that in all three cases, the channels have the same stationary distribution: with probability 0.5 a channel is available (busy). We see from Figure 12 that although the average traffic load (indicated by the stationary distribution) of the primary network is the same in all cases, different traffic statistics of the primary network lead to different performance of the secondary network. OSA is the most effective when it is overlaid over a primary network with large inter-arrival time and message length. Furthermore, the throughput of the secondary user increases over time in Case 1 and 3, which results from the improved information on the network state drawn from accumulating observations. This demonstrates the cognitive nature of the proposed protocol under the POMDP formulation. Note that in Case 2, the Markovian channel state evolution degenerates to an i.i.d. sequence. No information can be gained from past observations and the optimal strategy reduces to a random selection of channels.

In Figure 13 we study the performance of the suboptimal greedy approach as compared with the optimal protocol. For the channels used in the upper plot, the performance of the greedy approach matches that of the optimal protocol. For the channels in the lower plot, the performance loss of the greedy approach is within 3%. In both cases, significant gain over a random selection of channels is achieved by the proposed cognitive protocols.

Spectrum Efficiency in the Presence of Sensing Error We now study the performance of the proposed OSA MAC protocols based on a specific spectrum sensor at the physical layer. At the beginning of each slot, the sensor takes L measurements $\{Y_i\}_{i=1}^L$ of the chosen channel. We assume that both the channel noise and the signal of primary users can be modelled as white Gaussian processes. We then have the equation on the top of the following page. The SNR is given by $(\sigma_1^2 - \sigma_0^2)/\sigma_0^2$. It can be shown that the Neyman-Pearson detector for this problem is given by the energy detector:

$$\|\mathbf{Y}\|^2 \underset{H_0}{\overset{H_1}{\geq}} \tau. \quad (17)$$

The probability ϵ of false alarm (opportunity overlook) is

$$\begin{cases} \mathcal{H}_0 \text{ (when channel is idle)} & : Y_i \sim \mathcal{N}(0, \sigma_0^2), i = 1, \dots, L \\ \mathcal{H}_1 \text{ (when channel is busy)} & : Y_i \sim \mathcal{N}(0, \sigma_1^2), i = 1, \dots, L \end{cases}$$

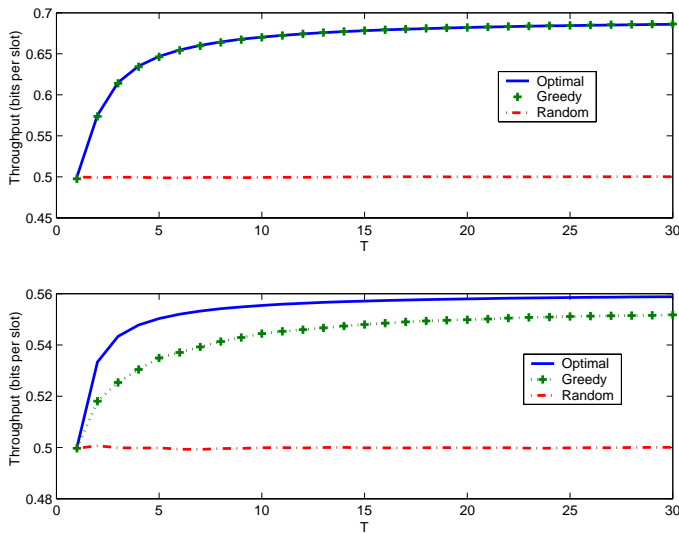


Fig. 13. Performance comparison of the greedy approach and the optimal strategy (in the upper plot, we have $N = 3$ independent channels with the same bandwidth $B = 1$ and transition probabilities $\{\alpha = 0.2, \beta = 0.8\}$; in the lower plot, $N = 3$, $\bar{\alpha} = [0.8, 0.6, 0.4]$, $\bar{\beta} = [0.6, 0.4, 0.2]$, $\bar{B} = [3/4, 1, 3/2]$).

determined by the threshold τ :

$$\epsilon \triangleq \Pr\{|\mathbf{Y}|^2 > \tau | H_0\} = 1 - \Gamma\left(\frac{L}{2}, \frac{\tau}{2\sigma_0^2}\right), \quad (18)$$

where $\Gamma(L, x) = \int_0^x t^{L-1} e^{-t} dt$ is the incomplete gamma function. The ROC for the Neyman-Pearson testing is thus given by

$$1 - \delta \triangleq \Pr\{|\mathbf{Y}|^2 > \tau | H_1\} = 1 - \Gamma\left(\frac{L}{2}, \eta \frac{\sigma_0^2}{\sigma_1^2}\right), \quad (19)$$

where η satisfies $\Gamma\left(\frac{L}{2}, \eta\right) = 1 - \epsilon$. The ROC curves for different SNRs and numbers L of samples are shown in Figure 8.

In Figure 14 we study the performance of the proposed greedy approach (using the above specified spectrum sensors) as a function of the maximum collision probability ζ allowed by the primary network. In the upper plot, we focus on the secondary user. We can see that as ζ increases, the throughput of the secondary user approaches the performance achieved by the optimal protocol in the *absence* of sensing errors. This is because with a large ζ , the probability ϵ of overlook can be very small, leading to improved throughput for the secondary user at a price of more collisions with the primary network. As shown in the lower plot, due to frequent collisions, the overall spectrum efficiency considering both the primary and secondary users decreases when ζ is large. The best spectrum efficiency is obtained when ζ is around 0.15.

Multiple Secondary Users with Random Message Arrivals In this example, we study the performance of the greedy approach when there are multiple secondary users. We assume that sensing errors are negligible. Message arrivals at the

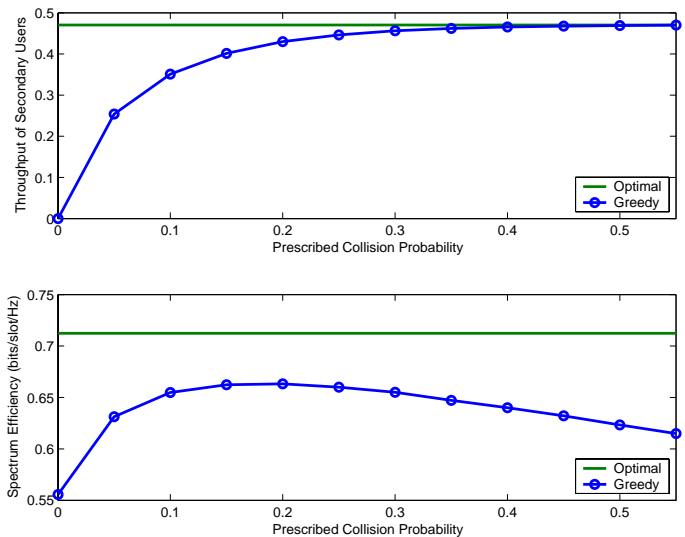


Fig. 14. OSA performance in the presence of sensing error ($N = 3$ independent channels with the same bandwidth $B = 1$ and transition probabilities $\{\alpha = 0.4, \beta = 0.5\}$).

secondary users form a Poisson process with rate λ . The message length is geometrically distributed with an average message length of 50 packets (the transmission time of one packet is assumed to be one slot). If a message arrives, the whole message will be randomly assigned to a secondary user. In each slot, those secondary users who do not have packets to transmit will turn to sleep mode: they do not participate in channel selection and sensing, and their belief states are updated according to the Markovian model of spectrum occupancy. Secondary users with packets to send will choose channels according to the greedy approach¹⁰, and then update their belief states according to the sensing outcomes. If an available channel is chosen by multiple users, we assume that one of these users will succeed. Shown in Figure 15 is the throughput measured in bits/slot of the secondary users as a function of the message arrival rate λ . We can see that the throughput of the secondary users increases with λ .

VII. CONCLUSION

We have presented in this paper an approach to decentralized MAC for ad hoc OSA networks. A novel feature of this work is the exploitation of opportunities at the slot level, allowing low rate applications (such as sensor nodes) coexists with primary users. The framework of POMDP makes the MAC cognitive; an opportunistic user makes optimal decisions for sensing and access based on the belief vector that summarizes the knowledge of the network state based on all past decisions and observations. Our formulation also

¹⁰Note that if the maximum immediate rewards of several channels are the same, the secondary user will randomly choose one of these channels to sense.

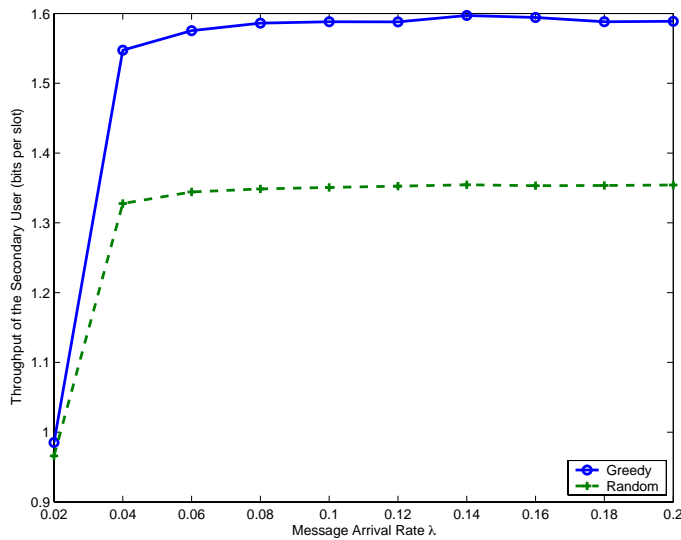


Fig. 15. Multiple secondary users with random message arrivals ($N = 10$ independent channels with the same bandwidth $B = 1$ and transition probabilities $\{\alpha = 0.2, \beta = 0.8\}$; three secondary users).

allows the integration of sensing errors and other practical impairments into the POMDP modeling.

APPENDIX: PROOF OF PROPOSITION 1

We show that when N channels evolve independently, $\Lambda(t)$ can be obtained from $\Omega(t)$. Without loss of generality, we consider $N = 2$. Let $\mathcal{I}(t)$ denote the information obtained up to the beginning of slot t . Let τ_n denote the most recent time instant when channel n is chosen. We can thus write an entry of $\Lambda(t)$ as in (20). Quantities in (20) are entries of $\Omega(t)$. Hence, Ω is a sufficient statistics when channels evolve independently.

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$$\begin{aligned}
& \Pr[S_1(t) = i, S_2(t) = j | \mathcal{I}(t)] = \Pr[S_1(t) = i, S_2(t) = j | S_1(\tau_1) = \theta_1, S_2(\tau_2) = \theta_2] \\
= & \Pr[S_1(t) = i | S_2(t) = j, S_1(\tau_1) = \theta_1, S_2(\tau_2) = \theta_2] \Pr[S_2(t) = j | S_1(\tau_1) = \theta_1, S_2(\tau_2) = \theta_2] \\
= & \Pr[S_1(t) = i | S_1(\tau_1) = \theta_1] \Pr[S_2(t) = j | S_2(\tau_2) = \theta_2]
\end{aligned} \tag{20}$$



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