Report Documentation Page

Form Approved
OMB No. 0704-0188

1. REPORT DATE **FEB 2006**	2. REPORT TYPE	3. DATES COVERED **00-00-2006 to 00-00-2006**

4. TITLE AND SUBTITLE **Accelerating Around an Unbanked Curve**	5a. CONTRACT NUMBER
	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)	5d. PROJECT NUMBER
	5e. TASK NUMBER
	5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **U.S. Naval Academy,Physics Department,Annapolis,MD,21402-5002**	8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSOR/MONITOR'S ACRONYM(S)
	11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited

13. SUPPLEMENTARY NOTES

14. ABSTRACT

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT **unclassified**	b. ABSTRACT **unclassified**	c. THIS PAGE **unclassified**	**Same as Report (SAR)**	**2**	

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std Z39-18

Accelerating Around an Unbanked Curve

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The December 2004 issue of *TPT* presented a problem concerning how a car should accelerate around an unbanked curve of constant radius r starting from rest if it is to avoid skidding.[1] Interestingly enough, two solutions were proffered by readers.[2] The purpose of this note is to compare and contrast the two approaches. Further experimental investigation of various turning strategies using a remote-controlled car and overhead video analysis could make for an interesting student project.

One approach, exemplified by Scott Wiley's solution, assumes that the tangential acceleration a_t of the car is constant throughout the turn, while the centripetal acceleration $a_c = v^2/r$ continuously increases as the car's speed v builds up. But the magnitude of the total acceleration $a = \sqrt{a_t^2 + a_c^2}$ must never exceed $\mu_s g$, where μ_s is the coefficient of static friction between the tires and road. To minimize the travel time t, the car's acceleration just attains this slipping value as it completes the turn. Using the kinematic relations $v^2 = 2a_t r\phi$ and $r\phi = \frac{1}{2}a_t t^2$ (where ϕ is the angle through which the car has turned), straightforward algebraic manipulations lead to

$$\frac{v}{V} = \sqrt{\frac{2\phi}{k}}, \quad \frac{t}{T} = \sqrt{2k\phi}, \quad \frac{a_c}{A} = \frac{2\phi}{k}, \quad \text{and} \quad \frac{a_t}{A} = \frac{1}{k}, \tag{1}$$

where $k \equiv \sqrt{1 + 4\phi_{max}^2}$ and the variables have been normalized by $V \equiv \sqrt{\mu_s g r}$, $T \equiv \sqrt{r/\mu_s g}$, and $A \equiv \mu_s g$. Equations (1) are plotted in red in Fig. 1 for the case of a 45° turn (i.e., $\phi_{max} = \pi/4$ rad).

The other approach further reduces the driving time by making a equal to $\mu_s g$ during the *entire* turn, rather than merely as the car *completes* it. Following the solution of Eugene Mosca, one can equate $a_t = \sqrt{a^2 - a_c^2} = \sqrt{V^4 - v^4}\big/r$ to $a_t = dv/dt = (ds/dt) \times (dv/ds) = vdv/rd\phi$ (where $s = r\phi$ is the distance traveled around the curve) to obtain

$$\phi = \int \frac{v\,dv}{\sqrt{V^4 - v^4}} = \frac{1}{2}\sin^{-1}\left(\frac{v}{V}\right)^2. \tag{2}$$

Inverting this result gives

$$\frac{v}{V} = \sqrt{\sin 2\phi} \Rightarrow \frac{a_c}{\mathrm{A}} = \sin 2\phi \text{ and } \frac{a_t}{A} = \cos 2\phi. \tag{3a}$$

(If ϕ_{max} is greater than 45°, then v remains constant with value V, so that $a_c = A$ and $a_t = 0$, at all angles beyond $\pi/4$.) The time is found by substituting $v = rd\phi/dt$ into the first equality in Eq. (3a) to get

$$\frac{t}{T} = \int_0^{\phi} \frac{d\phi'}{\sqrt{\sin 2\phi'}} \approx \frac{\phi}{N}\sum_{n=1}^{N} \csc^{1/2}\left[\frac{2\phi}{N}\left(n - \frac{1}{2}\right)\right], \tag{3b}$$

where N is any large integer.[3] This summation can be easily performed in a spreadsheet program.[4] The results are plotted (using $N = 1000$), along with Eq. (3a), in blue in Fig. 1.

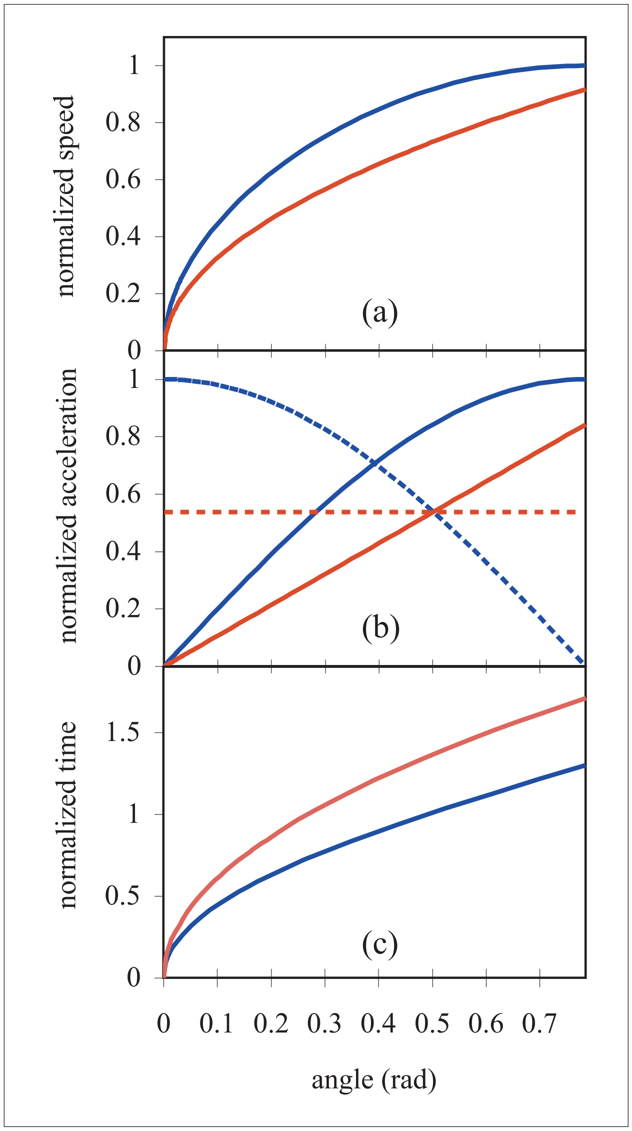

Fig. 1. In each graph, Eq. (1), which assumes constant tangential acceleration, is plotted in red, and Eq. (3), which assumes constant total acceleration, is in blue. (a) The normalized speed *v*/*V* of the car. (b) The normalized tangential acceleration a_t/A (dashed curves) and centripetal acceleration a_c/A (solid curves). (c) The normalized driving time *t*/*T*.

The shapes of the a_t curves in Fig. 1(b) give an indication of how the driver should depress the gas pedal. In the first approach (red dashed curve), he needs to maintain a constant but intermediate pressure on the pedal. In contrast, in the second scheme (blue dashed curve) he begins by "flooring it," such that the tires are on the verge of slipping, to get the maximum possible initial increase in speed. As he proceeds around the curve, he then eases off the pedal at an increasing rate in such a manner that his foot comes fully off it after having turned through 45° (assuming air drag and rolling friction are negligible).

As a final remark, one can use the preceding equations to determine how the car travels around the curve even if it starts with a nonzero speed $v_0 < V$. (If the car's initial speed is greater than *V*, it needs to first slow down to *V* and then execute the turn at that constant speed, as discussed in Ref. 5.) Simply solve Eq. (1) or (2) as appropriate to find the angle, call it ϕ_0, that corresponds to the initial speed v_0. Now, rather than performing the subsequent calculations for angles 0 to ϕ_{max}, instead consider the automobile to be traveling from angles ϕ_0 to $\phi_0 + \phi_{max}$.

References

1. "Physics Challenges for Teachers and Students: A home stretch," *Phys. Teach.* **42**, 555 (Dec. 2004).
2. "Solutions to December 2004 Challenges: A home stretch," *Phys. Teach*. **43**, 19–20 (2005) online at http://www.aapt.org/tpt.
3. The factor of ½ in the argument of the cosecant causes the integrand to be evaluated at the midpoint of each summation interval of width ϕ/N. This gives slightly more accurate results than evaluating the integrand at the endpoint, because cosecant is a monotonically decreasing function over the range of integration.
4. Note that the integral in Eq. (3b) for $\phi = \pi/4$ can be expressed exactly in terms of the gamma function $\Gamma(5/4)$, whose value can be looked up in standard mathematical tables. The result is $t/T = [2\Gamma(1.25)]^2/\sqrt{2\pi} \approx 1.31$, in excellent agreement with the value 1.30 obtained from the spreadsheet summation. The gamma function is a generalization of factorials to non-integers. See M.L. Boas, *Mathematical Methods in the Physical Sciences,* 2nd ed. (Wiley, New York, 1983), Chap. 11.
5. R.A.D. Hewko, "The racing car turn," *Phys. Teach.* **26**, 436–437 (Oct. 1988).

PACS codes: 45.40.Aa, **45.20.da,** 02.60.-x

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