

# Using an Ordinal Outranking Method Supporting the Acquisition of Military Equipment

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## **ABSTRACT**

*In some acquisition procedures of military equipment there are no crisp data available and/or no willingness to provide quantitative data. This might often occur in an early stage of the procedure, but might also be a characteristic of the qualitative nature of some or all the criteria involved in the assessment of the alternative equipments to be selected or ranked. In such a case we propose the use of an ordinal outranking method of the “ORESTE” type, which requires only ordinal data and rankings of criteria according to their importance.*

*We discuss how to use such a method in the context of an acquisition procedure of military equipment, and how to conduct the incomparability analysis. A visualization of the incomparabilities and of the total and partial rankings is presented.*

*Finally we discuss the combined use of an ordinal method with other outranking methods and the interpretation of the results in the context of the acquisition procedure of military equipment.*

## **INTRODUCTION**

In the SAS-080 (NATO SAS-080 Specialist Meeting – Brussels 22-23 October 2009) contributions on “Using an outranking method supporting the acquisition of military equipment” (SAS-080 14) and on “Assessing and Visualizing Incomparabilities by using an outranking method supporting the acquisition of military equipment” (SAS-080 15), we reminded that outranking methods for multicriteria decision aid belong typically to the so-called European School of Multicriteria Decision Making (MCDM), which came into existence with the stimulating work of B. Roy ([13],[14],[15],[16]). We will repeat here some of the points addressed in these papers. The outranking approach is based on a fundamental partial comparability axiom where incomparability is a key concept ([5], p.80). In contrast with this approach there is the so-called American School in which Th. Saaty plays an important role with his “Analytical Hierarchy Process” (AHP Method) in which there is no place for incomparabilities [17]. In the European School we think that incomparabilities between alternatives to be ranked or to be selected, are a natural aspect of any MCDM problem, in which criteria evaluating the performance of these alternatives are conflicting – meaning that for instance two different criteria can have inversed preferences between couples of the same alternatives. If this happens on a large set of couples of criteria, then we claim that neglecting these conflicts, is leading to decisions which are often far from the original data of the MCDM problem. Although the final objective in practice is to decide about a ranking or about a selection of a subset of the alternatives, we claim that the decision maker should be supported by methods which are warning about the presence of incomparabilities. We even claim that it should be possible to assess the importance (the intensity) of these incomparabilities in order to fully inform the decision maker about it, before the final decision is made.

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Many different methods belong to the outranking class. For overviews we refer to [5], [18] and [19]. For a detailed description of industrial applications with the oldest member (ELECTRE) of this class we refer to [11]. In this paper we will concentrate on the well-known ORESTE method ([10],[12]) which is complementary to the PROMETHEE methods. There are other methods belonging to the European School like MACBETH [1] which in SAS-080 is the subject of a keynote address by C. Bana e Costa.

The ORESTE method is sufficiently well-known by System Analysis specialists, to skip in this paper all mathematical aspects. For details we refer to [10] and [12] .

The ORESTE method has sometimes been used in the eighties and nineties of last century by small teams of Belgian MoD equipment acquisition services as a complement to the PROMETHEE methods. This MCDM method is taught in the curriculum of the High Staff College for Military Administrators of the Belgian MoD. Currently personnel involved with equipment acquisition can use these methods on an individual basis.

*In this paper we concentrate on practical features of the ORESTE method especially related to the incomparability analysis, the typical use for military equipment acquisition, and we illustrate the discussion primarily by an implementation we called MCDMTool [7].*

## **INPUT DATA**

We start with the same data as in the paper SAS-080 on “Using an outranking method supporting the acquisition of military equipment”, but now we intend to apply the ORESTE method for the sub-problem related to the criterion assessing “Human Resources” issues.

This is illustrated in Figure 1 with MCDMTool in which we will now only look at the sub-tree “Human Resources”.

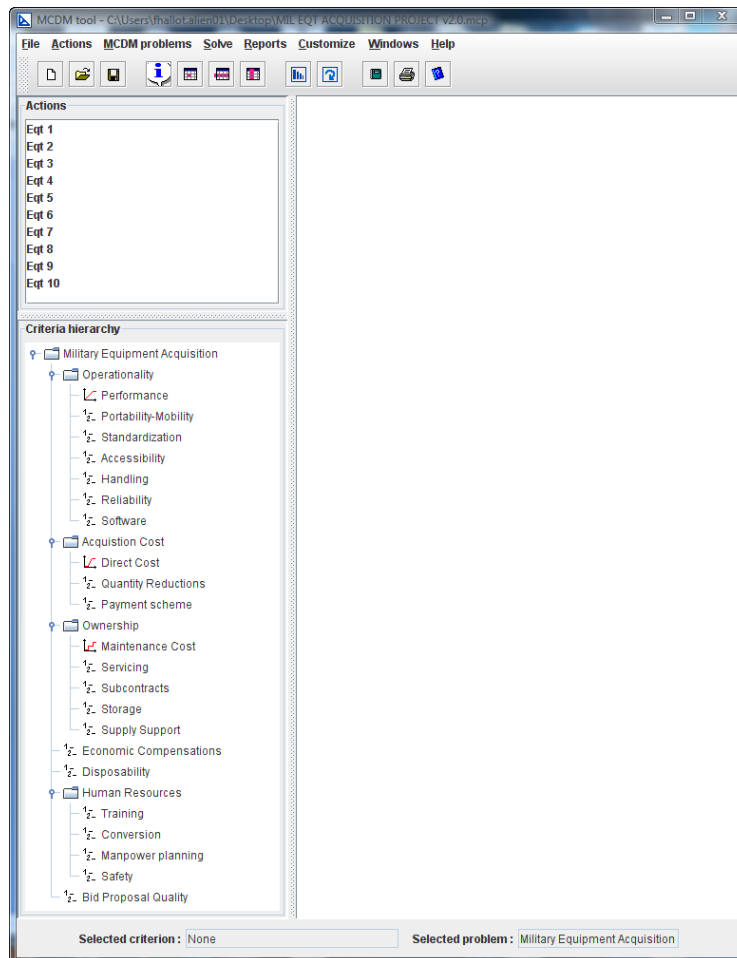


Figure 1: Input structure

All assessments of alternatives for all criteria are ordinal (represented by ranks). The relative importance of the criteria are also given by ranks.

This is illustrated in Figure 2 with MCDMTool.

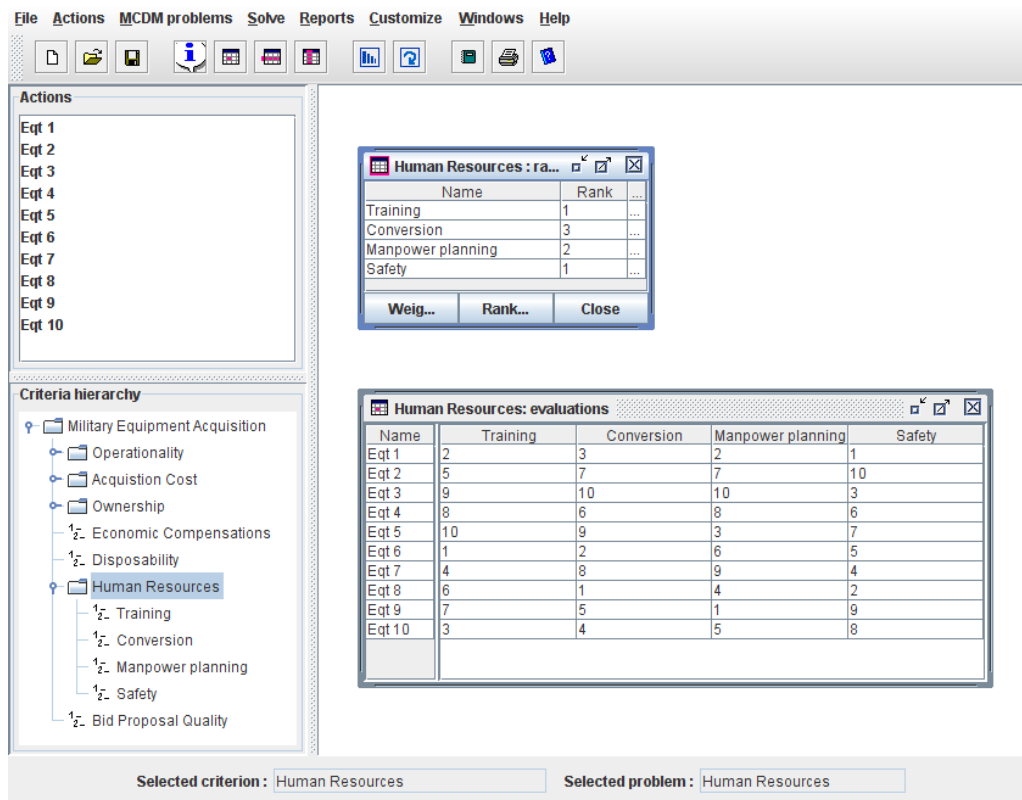


Figure 2: Input ordinal data

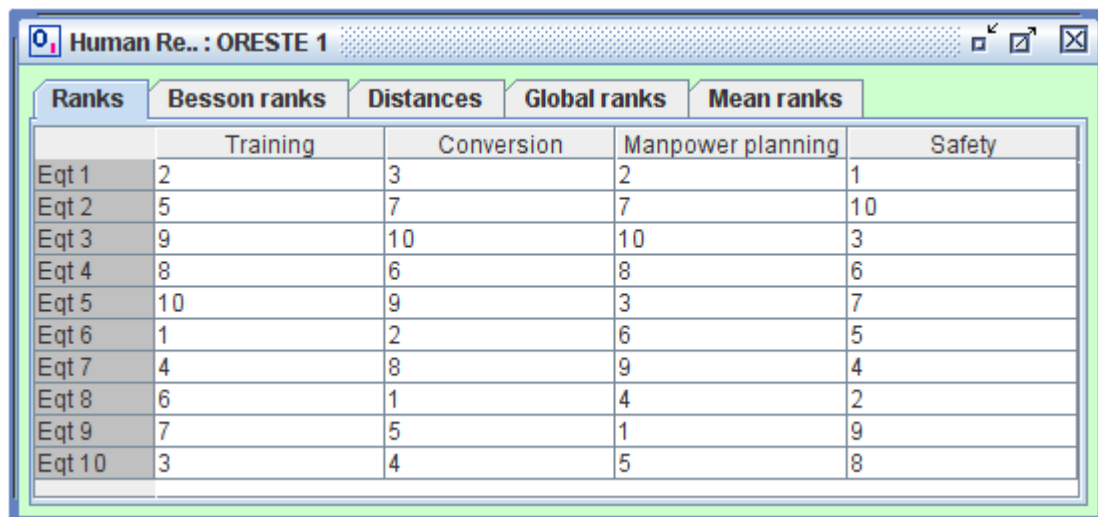
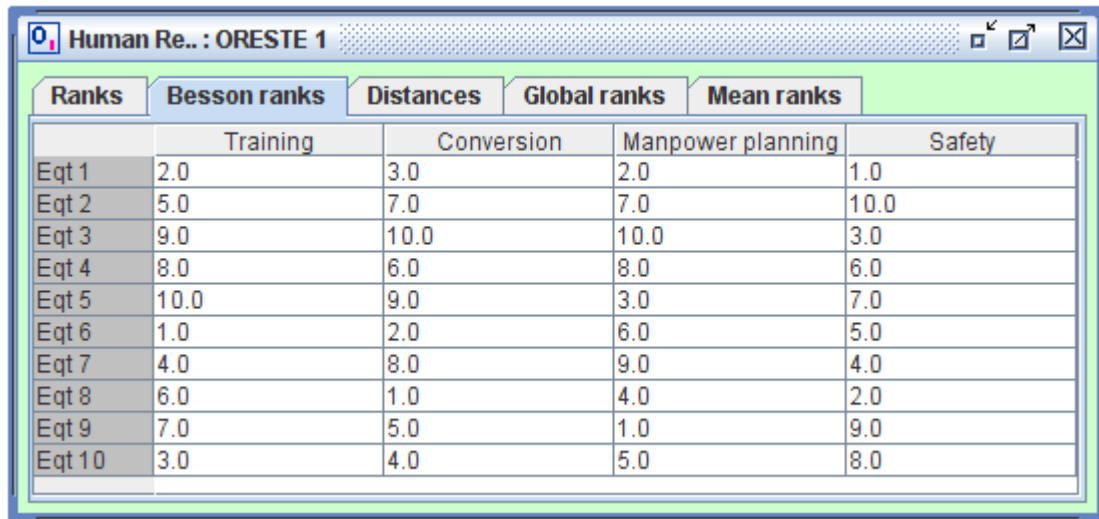


Figure 3: Ranks of Alternatives for criteria

## ORESTE I COMPUTATIONS

First all ranks are converted into Besson ranks [10] to take into account possible ties (in this example there are none). The results are in Figure 4.



Ranks	Besson ranks	Distances	Global ranks	Mean ranks
	Training	Conversion	Manpower planning	Safety
Eq1 1	2.0	3.0	2.0	1.0
Eq1 2	5.0	7.0	7.0	10.0
Eq1 3	9.0	10.0	10.0	3.0
Eq1 4	8.0	6.0	8.0	6.0
Eq1 5	10.0	9.0	3.0	7.0
Eq1 6	1.0	2.0	6.0	5.0
Eq1 7	4.0	8.0	9.0	4.0
Eq1 8	6.0	1.0	4.0	2.0
Eq1 9	7.0	5.0	1.0	9.0
Eq1 10	3.0	4.0	5.0	8.0

Figure 4: Besson ranks

Then a first aggregation of the two-dimensional alternatives-criteria data (Figure 2) is performed by computing for each couple alternative-criteria as “distance” score to and ideal position occupied by the best alternative for the most important criterion. This score is the mean value of the Besson-rank  $r_{c_j}$  of criterion  $c_j$ , and of the Besson-rank  $r_{c_j}(a)$  of the alternative  $a$  in criterion  $c_j$ . This mean value can be

computed by  $d(a, c_j) = \left[ \frac{1}{2} r_{c_j}^R + \frac{1}{2} r_{c_j}(a)^R \right]^{1/R}$  with  $R \in ]0, \infty[$ . The parameter  $R$  can be adapted for two

different parts of the input data table. Most users will take the default value  $R=1$  (see Figure 5). Also the coefficient  $\frac{1}{2}$  can take values  $A$  and  $(1-A)$  for two different parts of the input data table. Most users will however take the default value  $A=1/2$ .

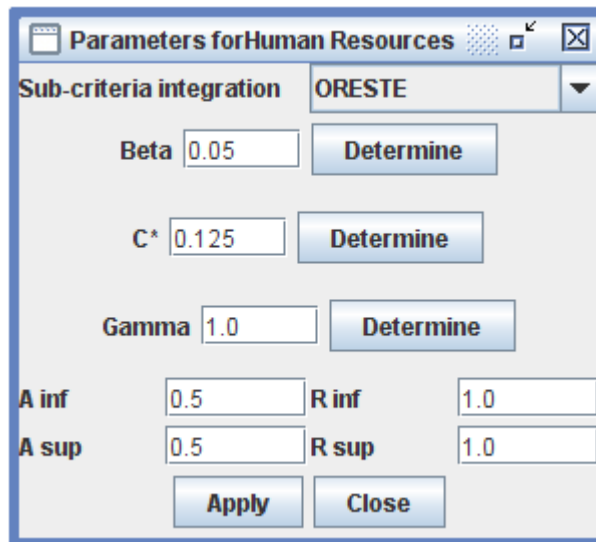
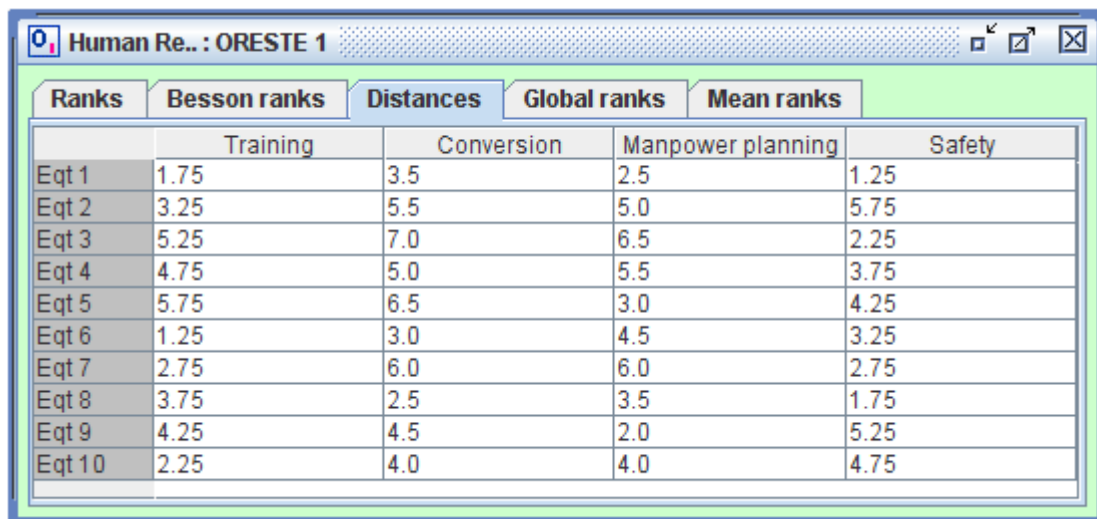


Figure 5: Parameter setting

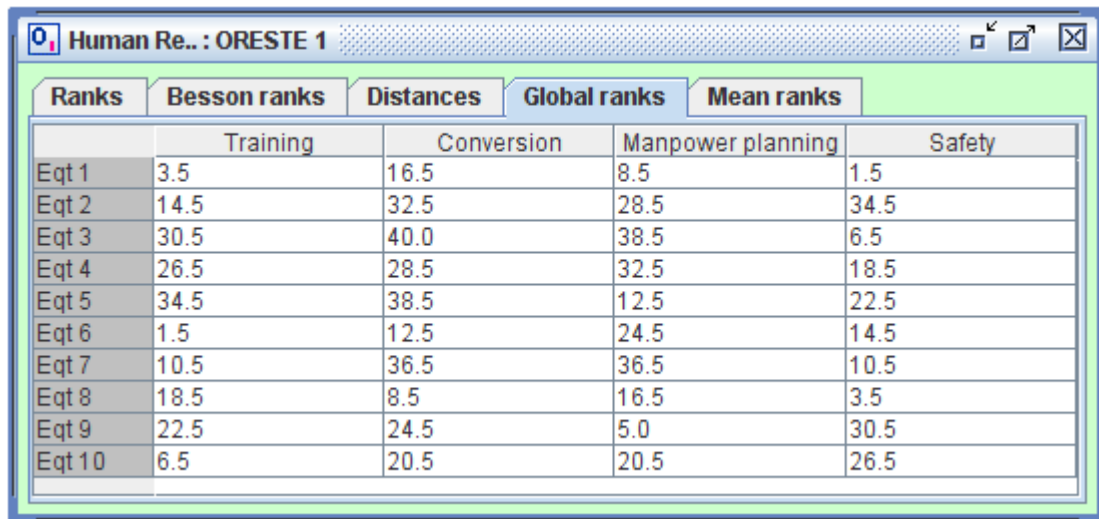


Ranks	Besson ranks	Distances	Global ranks	Mean ranks
	Training	Conversion	Manpower planning	Safety
Eq1	1.75	3.5	2.5	1.25
Eq2	3.25	5.5	5.0	5.75
Eq3	5.25	7.0	6.5	2.25
Eq4	4.75	5.0	5.5	3.75
Eq5	5.75	6.5	3.0	4.25
Eq6	1.25	3.0	4.5	3.25
Eq7	2.75	6.0	6.0	2.75
Eq8	3.75	2.5	3.5	1.75
Eq9	4.25	4.5	2.0	5.25
Eq10	2.25	4.0	4.0	4.75

Figure 6: “distance” scores

Then all these “distance” scores are converted into Besson-ranks in order to keep the method fully ordinal. We call these Besson-ranks global ranks  $\rho_{c_j}(a)$  which are upper-bounded by the value  $mk$  with  $m$  the number of alternatives (here 10) and  $k$  the number of criteria (here 4).

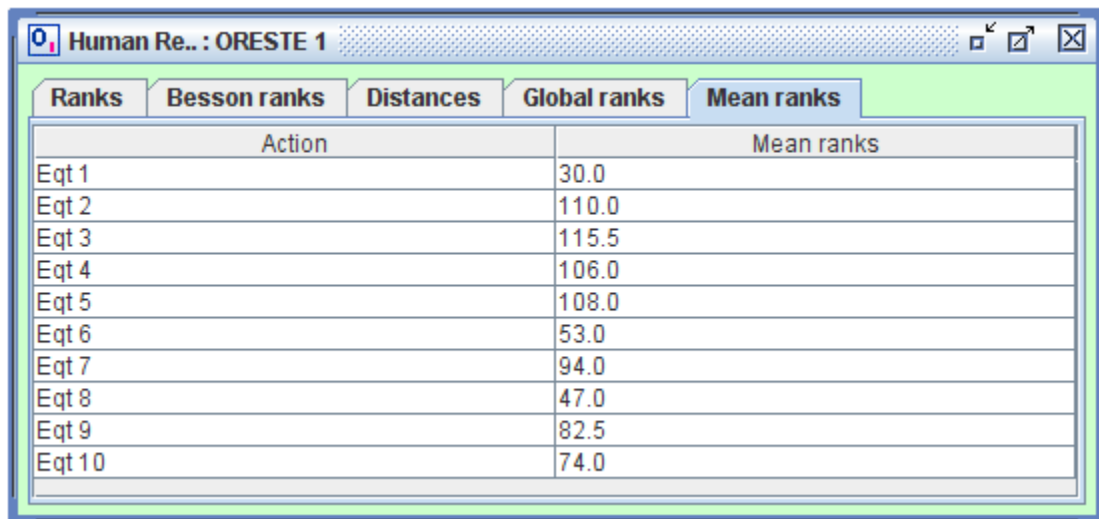
These global ranks are illustrated in Figure 7.



Ranks	Besson ranks	Distances	Global ranks	Mean ranks
	Training	Conversion	Manpower planning	Safety
Eq1	3.5	16.5	8.5	1.5
Eq2	14.5	32.5	28.5	34.5
Eq3	30.5	40.0	38.5	6.5
Eq4	26.5	28.5	32.5	18.5
Eq5	34.5	38.5	12.5	22.5
Eq6	1.5	12.5	24.5	14.5
Eq7	10.5	36.5	36.5	10.5
Eq8	18.5	8.5	16.5	3.5
Eq9	22.5	24.5	5.0	30.5
Eq10	6.5	20.5	20.5	26.5

Figure 7: Global ranks

Then we compute the “mean” rank for each alternative by  $r(a) = \sum_j \rho_{c_j}(a)$  which is actually only the numerator of the mean of all the global ranks the alternative  $a$  got in the previous table. The values of these  $r(a)$  are in Figure 8.



Ranks	Besson ranks	Distances	Global ranks	Mean ranks
Action			Mean ranks	
Eq1			30.0	
Eq2			110.0	
Eq3			115.5	
Eq4			106.0	
Eq5			108.0	
Eq6			53.0	
Eq7			94.0	
Eq8			47.0	
Eq9			82.5	
Eq10			74.0	

Figure 8: Mean ranks

Finally the alternatives are ranked in increasing order of the mean ranks. This yield for our example the representation in Figure 9 together with the profiles of the alternatives in Figure 10.



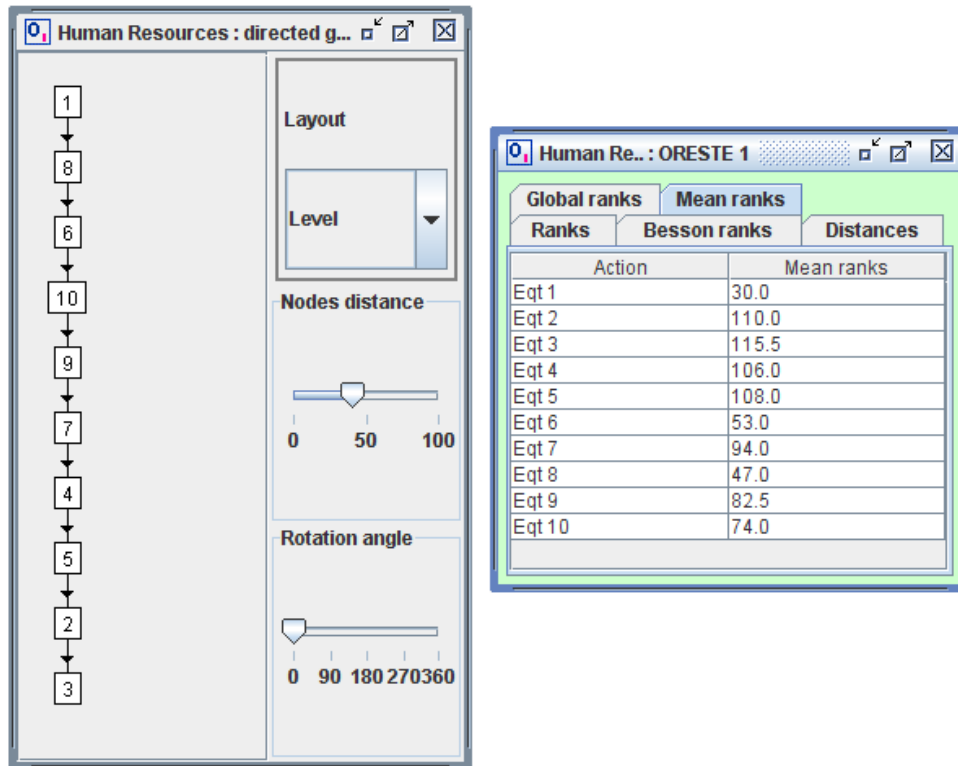


Figure 9: Final ranking

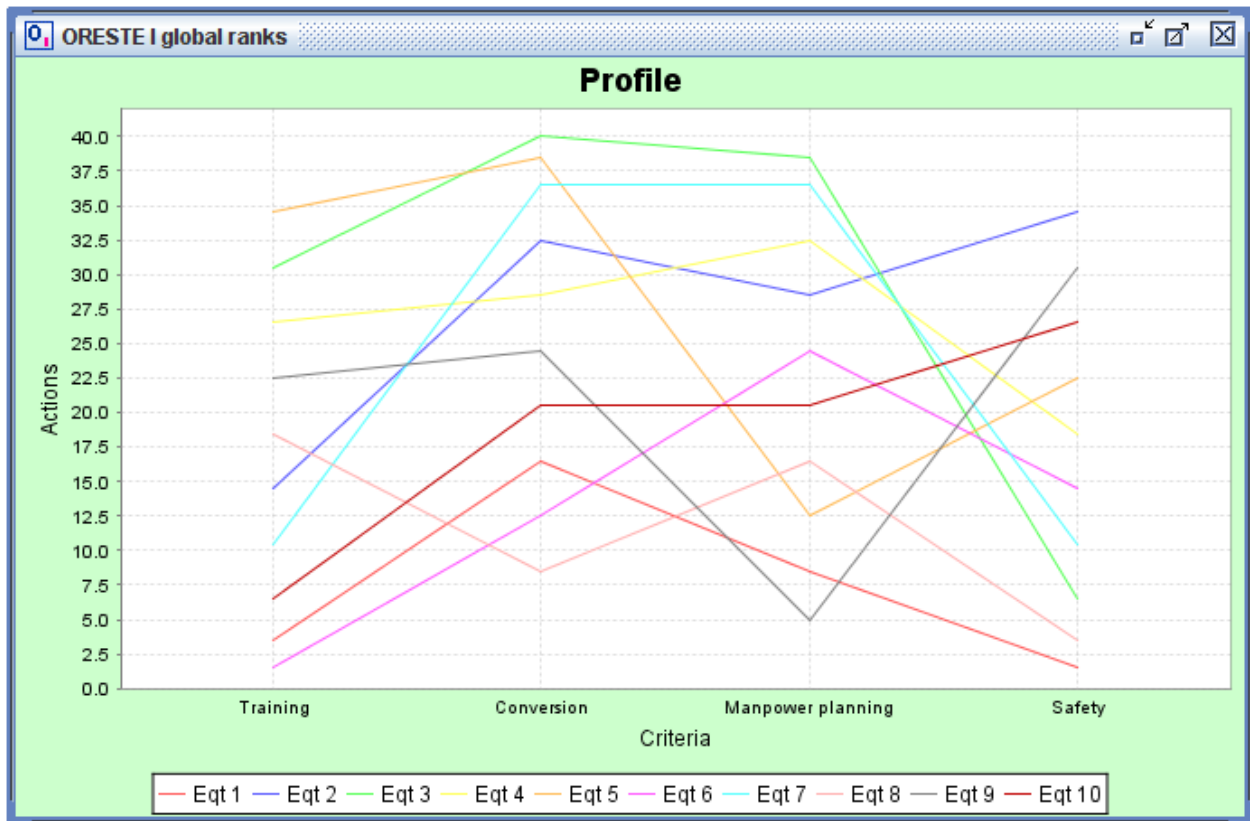


Figure 10: Profiles

## ORESTE II COMPUTATIONS

Now we compute for each couple of alternatives  $a$  and  $b$  two preference intensities  $C'(a,b)$  and  $C'(b,a)$

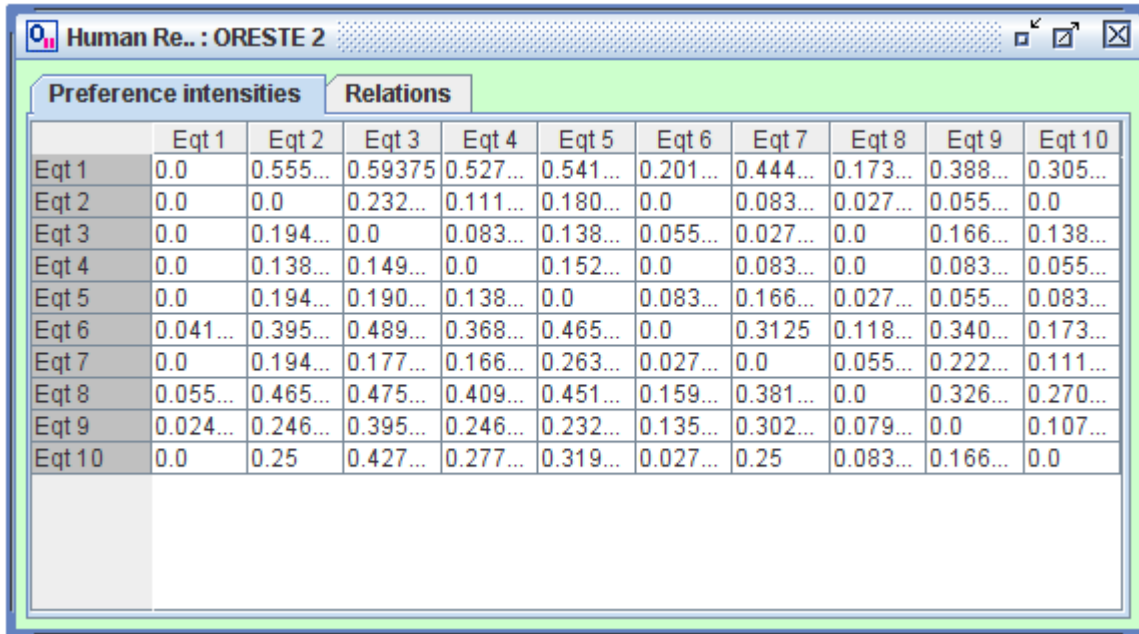
$$C'(a,b) = \sum_{j:\rho_j(b) \geq \rho_j(a)} \rho_j(b) - \rho_j(a)$$

with

$$C'(b,a) = \sum_{j:\rho_j(a) \geq \rho_j(b)} \rho_j(a) - \rho_j(b)$$

These preference intensities are upper-bounded by  $k^2(m-1)$ . (see [10])

When we divide the preference intensities by this upper bound, we obtain normalized preference intensities  $C(a,b)$  and  $C(b,a)$  similar to the preference indicators  $\pi(a,b)$  and  $\pi(b,a)$  of the PROMETHEE methods (see [4] and SAS-080 14 and SAS-080 15). These preference intensities are in Figure 11.



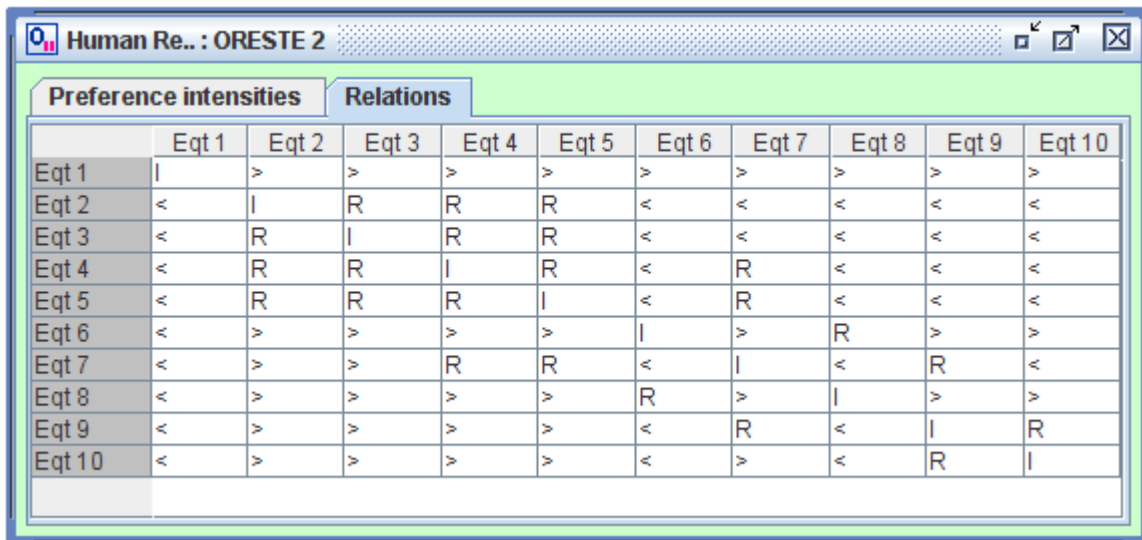
	Eq1	Eq2	Eq3	Eq4	Eq5	Eq6	Eq7	Eq8	Eq9	Eq10
Eq1	0.0	0.555...	0.59375	0.527...	0.541...	0.201...	0.444...	0.173...	0.388...	0.305...
Eq2	0.0	0.0	0.232...	0.111...	0.180...	0.0	0.083...	0.027...	0.055...	0.0
Eq3	0.0	0.194...	0.0	0.083...	0.138...	0.055...	0.027...	0.0	0.166...	0.138...
Eq4	0.0	0.138...	0.149...	0.0	0.152...	0.0	0.083...	0.0	0.083...	0.055...
Eq5	0.0	0.194...	0.190...	0.138...	0.0	0.083...	0.166...	0.027...	0.055...	0.083...
Eq6	0.041...	0.395...	0.489...	0.368...	0.465...	0.0	0.3125	0.118...	0.340...	0.173...
Eq7	0.0	0.194...	0.177...	0.166...	0.263...	0.027...	0.0	0.055...	0.222...	0.111...
Eq8	0.055...	0.465...	0.475...	0.409...	0.451...	0.159...	0.381...	0.0	0.326...	0.270...
Eq9	0.024...	0.246...	0.395...	0.246...	0.232...	0.135...	0.302...	0.079...	0.0	0.107...
Eq10	0.0	0.25	0.427...	0.277...	0.319...	0.027...	0.25	0.083...	0.166...	0.0

Figure 11: Preference intensities

These normalized preference intensities are then used as coordinates of points representing couples of alternatives in an IPR-diagram. For a parameter set defining the I, P and R regions (see [10]) this yields then a set of couples for which a warning is triggered, announcing an incomparability if the point is located in the (red) R-zone of the IPR-diagram.

The result for a very risk-averse parameter setting (meaning that incomparability is announced even for low intensities of the incomparability) is given in Figure 12.

The intensity of the incomparability can be measured by the distance of the point representing the couple in the IPR-diagram, to the bisector of quadrant of the IPR-diagram. This distance can on its turn be measured by the parameter Gamma ( $\gamma$ ) for which MCDMTool provides an interactive analysis



	Eq1	Eq2	Eq3	Eq4	Eq5	Eq6	Eq7	Eq8	Eq9	Eq10
Eq1		>	>	>	>	>	>	>	>	>
Eq2	<		R	R	R	<	<	<	<	<
Eq3	<	R		R	R	<	<	<	<	<
Eq4	<	R	R		R	<	R	<	<	<
Eq5	<	R	R	R		<	R	<	<	<
Eq6	<	>	>	>	>		>	R	>	>
Eq7	<	>	>	R	R	<		<	R	<
Eq8	<	>	>	>	>	R	>		>	>
Eq9	<	>	>	>	>	<	R	<		R
Eq10	<	>	>	>	>	<	>	<	R	

Figure 12: IPR analysis

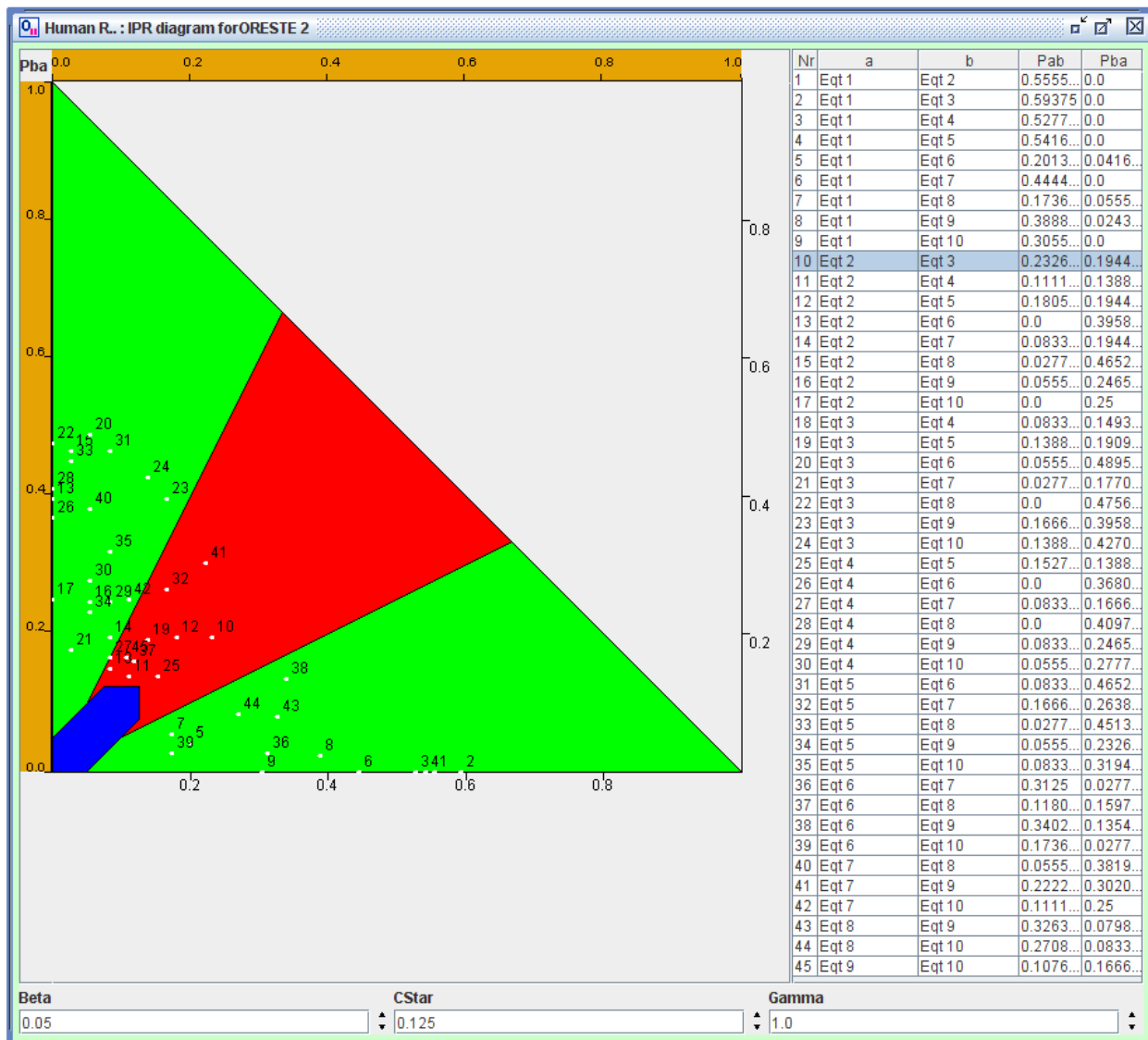


Figure 13: IPR diagram

Notice in Figure 13 that for instance the point 41 is in the incomparability region, and indeed the couple Eq7 and Eq9 is announced by the IPR analysis as incomparable.

In Figure 14 we illustrate the use of reference (standard) profiles to determine the parameter gamma. This gamma determines the incomparability zone and indirectly gives a measure of the distance of the point representing a couple of alternatives to the bisector (axis of symmetry of the R-region) of the quadrant of the IPR-diagram.

This interactive analysis for two different values of gamma is illustrated in Figure 15.

It shows for instance that the couple Eq6 and Eq8 (represented by the point 37 in the IPR-diagram) has roughly the same incomparability (conflict) intensity than the couple Eq2 and Eq3 (represented by the point 10 in the IPR-diagram) because these points are almost on the same “gamma”-distance from the bisector of the quadrant. However it is much more critical and more important to trigger the warning of incomparability for Eq6 and Eq8 than for the other couple, because the couple is among the top-ranked in ORESTE I. This means for instance that it is very relevant to have this analysis tool in the case when

this MCDM problem consists of the selection of two alternatives among ten. In that case the choice of the second alternative to be selected (after the overall winner Eq1) is a hard decision to be taken, which might require some further investigation (re-assessments, additional criteria,...).

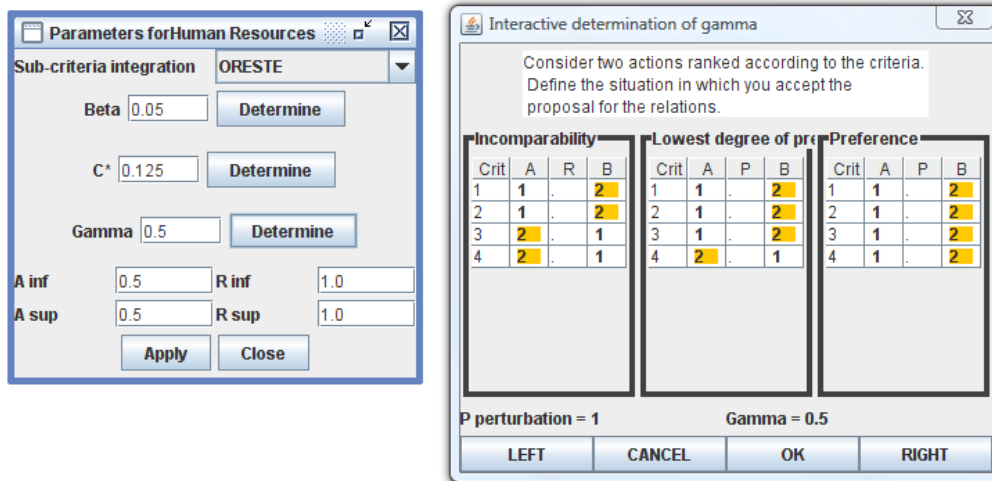


Figure 14: Interactive determination of gamma for reference (standard) profiles

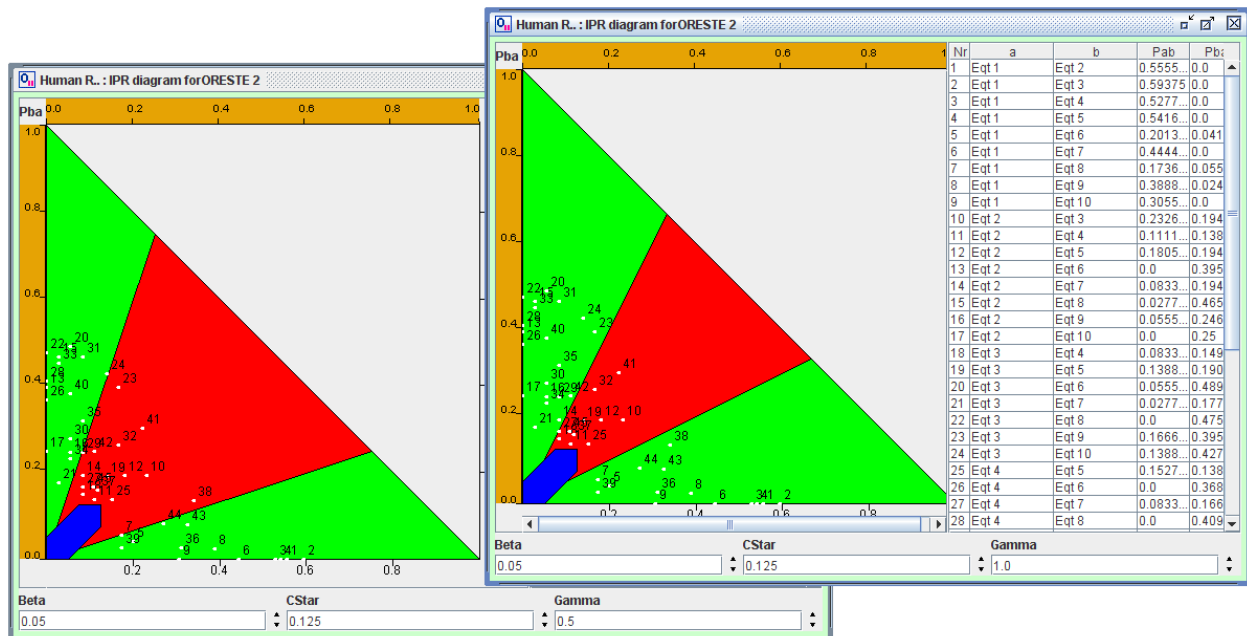


Figure 15: Interactive assessment of the incomparability intensity

## CONCLUSION

This ordinal outranking method ORESTE, implemented with MCDMTool, is a powerful and relevant analysis tool, providing the decision maker with the necessary information in the case of hard decisions about expensive military equipment to be taken (or advise to be given) in the context of conflicting criteria leading to incomparable alternatives (especially on the top of the ranking). This is the main reason why we integrated it into the MCDMTool software.

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