REPORT DOCUMENTATION PAGE

1. REPORT DATE (DD-MM-YYYY)	2. REPORT TYPE	3. DATES COVERED (From - To)
28-02-2012	Final	15-03-2009 to 30-11-2011

4. TITLE AND SUBTITLE

Hybrid Control for Multi-Agent Systems in Complex Sensing Environments

5a. CONTRACT NUMBER

FA9550-09-1-0221

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)

Geir E. Dullerud

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

University of Illinois
1206 W Green Street
Urbana, IL 61801

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

Air Force Office of Scientific Research
875 N Randolph St
Arlington, VA 22203

10. SPONSOR/MONITOR'S ACRONYM(S)

AFOSR

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

AFRL-OSR-VA-TR-2012-0966

12. DISTRIBUTION/AVAILABILITY STATEMENT

Distribution A: Approved for Public Release

13. SUPPLEMENTARY NOTES

14. ABSTRACT

The overall program provided systematic and computationally responsive research aimed at control of multi-agent systems, and more generally systems that have both discrete and continuous dynamics, using convex optimization and semidefinite programming as the foundation. The program has produced research results in several domains. In the area of distributed control several results were obtained: new stabilization methods were developed for systems in which dynamical agents interact over bandlimited channels; synthesis methods were also developed for metric-based performance optimization of distributed systems over graphs. Systems with simultaneously network latency and finite precision sensing were targeted and algorithms were developed that can guarantee stabilization in these limited information scenarios. For switched systems a separate program accomplishment was the development of equivalence and separation principles for joint synthesis of switching rules and policies. Also, the first known exact convex conditions for receding horizon control of switched systems were found, together with design algorithms for optimizing metric-based performance. New decidability and verification results for general hybrid setting were developed.

15. SUBJECT TERMS

multi-agent networks; distributed control; hybrid systems; switched and Markovian jump systems; receding horizon control; multi-resolution sensing; network latency; decidability; semidefinite programming

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			Geir E. Dullerud
U	U	U	U	26	**19b. TELEPHONE NUMBER** (Include area code) 217-265-5078

HYBRID CONTROL FOR MULTI-AGENT SYSTEMS IN COMPLEX SENSING ENVIRONMENTS: FINAL REPORT

AFOSR FA9550-09-1-0221

Geir E. Dullerud

February 28, 2012

Mechanical Engineering and Coordinated Science Laboratory
University of Illinois at Urbana-Champaign

Contents

1 Executive Summary

The overall program provided systematic and computationally responsive research aimed at control of multi-agent systems, and more generally systems that have both discrete and continuous dynamics, using convex optimization and semidefinite programming as the foundation. The program has produced research results in several domains. In the area of distributed control several results were obtained: new stabilization methods were developed for systems in which dynamical agents interact over bandlimited channels; synthesis methods were also developed for metric-based performance optimization of distributed systems over graphs. Systems with simultaneously network latency and finite precision sensing were targeted and algorithms were developed that can guarantee stabilization in these limited information scenarios. For switched systems a separate program accomplishment was the development of equivalence and separation principles for joint synthesis of switching rules and policies. Also, the first known exact convex conditions for receding horizon control of switched systems were found, together with design algorithms for optimizing metric-based performance. New decidability and verification results for general hybrid setting were developed. A more technical synopsis is provided in the remainder of this executive summary; the sections that follow this summary are devoted to more detailed reports of the results. Information about personnel supported, and honors received during the performance period, can be found at the end of the document.

A major project focus area was switched and Markovian jump systems, a specific subclass of hybrid systems where the parameters of a continuous-state model are switched according to an automaton or Markov chain. One direct application that has been pursued is the use of these systems in modeling latency and packet-loss in communication networks supporting control objectives. The problem of jointly synthesizing a supervisor, a measurement scheduler, and a feedback controller for guaranteed stability and performance levels for discrete-time switched linear systems was considered. It is shown that open-loop supervisory and scheduling laws are nonconservative for uniform exponential stability, and that they can be obtained separately from the feedback controller. The uniformity requirement in exponential stability ensures that both mean-square and worst-case type performance levels are well-defined, and that all the design conditions can be formulated in terms of semidefinite programs. Past work has considered situations in which the automaton is open-loop, and only the control of the continuous states can be controlled via feedback. A recent major result of the program is an exact convex solution to the receding horizon control problem with stochastic performance metrics placed on the performance of the continuous state. This is a problem with a very long history, and to our knowledge our solution is the first exact (i.e., necessary and sufficient conditions) convex solution to this problem. From an operational perspective these results allow for optimal systematic use of advanced or front-running tactical information when controlling physical agents. Future work involves computational algorithm development to enable capability for large-scale application. More technically, considered are nominal systems with state space models that switch in time, whose controllers have access to the precise state space model of the plant for a horizon into the future, but only have fore-knowledge of a set of model possibilities beyond this horizon. The control performance criterion is a bound on the variance over this moving window, and a particular feature of the research that this metric can change depending on the system discrete state; namely, this work provides designers with the direct capability to choose performance objectives that change as tactical situations evolve. In fact, considered is a more general scenario where evolution of the discrete dynamics may be governed by an automaton. Exact convex conditions are provided for the existence of a output feedback policies that can uniformly exponentially stabilize such systems and achieve the desired stochastic performance specification. Each condition is in terms of a nested sequence of semidefinite programs, where (a) feasibility to any element provides an explicit control policy; and (b) infeasibility implies that a controller does not exist for a given performance-stability level.

Another area of accomplishment has been on distributed synthesis. One of the research thrusts has been investigation of spatially distributed heterogeneous systems over infinite lattices. An operator-pencil approach has been employed to develop new analysis and synthesis conditions that are less conservative than existing work, using operator inequalities and convex programming. Also considered has been the related question of decentralized control in the setting of finite wordlength sensing or communication channels. Specifically, considered was decentralized stabilization of a global system by means of multiple agents that receive sensing information through rate-limited channels, while these stations are not capable of communicating with each other directly. The main result is a condition on the data rate of respective channels that guarantees system stabilizability. Provided is an explicit way to construct the associated stabilizing encoder, decoder, and controller. Robustness of the resulting control strategy is also established; specifically, analysis showing that the new control algorithm is structurally robust against model mismatch is provided.

A major area of progress is modeling for general hybrid automata, and in particular classifying models with decidable verification properties. This work is directly aimed at developing tools for verification and validation. In the project is has been shown that for o-minimally definable hybrid automata with a bounded discrete-transition horizon, a finite bisimulation exists and can be constructed given certain decidability assumptions on the underlying o-minimal theory. More importantly, we give specifications for hybrid automata which ensure this boundedness. In addition, we show that these specifications are reasonably tight and that they can model realtime and cyberphysical systems. Another recent line of research that ties together many of the methods used in the project is investigation into the use of pre-orders for analyzing hybrid system properties; pre-orders between processes, like simulation, have played a central role in the verification and analysis of purely discrete-state systems, where verifying correctness is achieved through system abstraction.

The publications associated with the project research are listed in Section 10 of this document. The remainder of this report provides a more detailed account of the project research.

2   Stabilization in Markovian Jump and Switched Systems

Part of the program addressed the problem of jointly synthesizing a switching rule and a feedback controller for automata-switched systems to guarantee stability and performance levels subject to well-posedness-like constraints. Two main results obtained are the equivalence of open-loop and closed-loop switching, and the separation between switching and feedback. That is, open-loop switching rules are non-conservative for stabilization and performance optimization, and in output feedback control they can be obtained separately from the feedback controller. The synthesis conditions for switching and feedback are convex and expressed in terms of linear matrix inequalities. Below this work is overviewed based on the research reported in [6,8,11].

The problem of determining optimal stabilizing switching rules arises in the context of supervisory control and measurement scheduling. In the problem of supervisory control, one is given a set of controllers for a single plant. At each time instant, the supervisor chooses a controller among this set based on the past and present state measurements, and uses the chosen controller to close the feedback loop. Due to the switching among different controllers, the overall closed-loop system is time-varying but can potentially exhibit better stability and performance properties than when the feedback loop is closed using any single controller among the given set. On the other hand, in the problem of measurement scheduling, one is given a set of sensors for a single plant. At each time instant, the scheduler chooses a sensor among this set based on the past state measurements it has made, and samples a new state measurement using the chosen sensor. Again, due to the switching among multiple sensors, the overall system is time-varying and yet, once

feedback-interconnected with a suitable controller, it can potentially yield better stability and performance properties than when feedback-controlled using any single sensor among the given set.

In this research automata-switched systems are considered with the objective of performing joint synthesis of a switching rule and a feedback controller for uniform exponential stability and guaranteed $\mathcal{H}_2$ and $\mathcal{H}_\infty$ type performance levels. A switched linear system under a general switching rule exhibits nonlinear behavior. To ensure such a nonlinear system possesses the kind of robustness that uniformly exponentially stable standard linear systems exhibit against state perturbations, a separation is imposed on the total response of the system into a zero-input response–like term and a zero-state response–like term. Imposing this requirement guarantees that one can speak of "stability under zero input" and "performance under zero initial state" at the same time as in the standard $\mathcal{H}_2$ and $\mathcal{H}_\infty$ control problems. Similarly, to ensure the overall nonlinear system possesses the kind of small gain property and its converse that standard linear systems exhibit against dynamic uncertainty, a separation is imposed on the response of the uncertain system into the dynamics associated with the plant and that with the uncertainty block. This requirement is analogous to the stability requirement: While the stability definition requires robustness against "static" state perturbations, the performance definition requires robustness against "dynamic" feedback uncertainties.

Under these stability and performance requirements the following fundamental results/principles are derived:

- *Equivalence:* Whenever a closed-loop switching rule is stabilizing and guarantees a performance level, there exists an open-loop switching rule that does so and results in a periodic switching sequence.

- *Separation:* The joint synthesis problem for a switching rule and a feedback controller is separated, and one can obtain a switching rule separately from the feedback controller.

The results obtained are in contrast to the fact that there are cases where open-loop switching rules are not sufficient when mere asymptotic stability is required; namely, the additional condition of structural well-posedness is not previously assumed.

Aside from this structural well-posedness requirement on asymptotic stability, it is also required that the feedback controller should be able to recall past switching paths as well as past state measurements. That is, it is required that the feedback controller be not only dynamic (with the information about past measurements stored in its state) but also switching path–dependent (with the information about past switching paths encoded in its coefficients). With this it can be shown that the order of the controller can be no more than that of the plant, and the length of the past switching paths encoded in the controller coefficients can be no more than the period of the open-loop switching sequence. Such a requirement is crucial in guaranteeing nonconservative synthesis of switching rules and feedback controllers. In particular, the derived conditions for joint synthesis of switching and feedback are expressed in terms of linear matrix inequalities. Moreover, these conditions give rise to switching path–dependent Lyapunov functions and cover the common Lyapunov function and multiple Lyapunov functions approaches as special cases of path length zero and one, respectively. An optimal switching sequence can, in principle, be determined by solving an increasing sequence of semidefinite programs indexed by the length of past switching paths that the feedback controller should recall.

The results described are divided below into two parts, one considering only uniform exponential stability, and the other considering the more difficult question of performance.

For $x \in \mathbb{R}^n$ and $\mathbf{A} \in \mathbb{R}^{m \times n}$, denoted by $\|x\|$ and $\|\mathbf{A}\|$ are the Euclidean norm of x and the spectral norm of $\mathbf{A}$, respectively. If $\mathbf{X}, \mathbf{Y} \in \mathbb{R}^{n \times n}$ are symmetric (i.e., $\mathbf{X} = \mathbf{X}^{\mathrm{T}}$ and $\mathbf{Y} = \mathbf{Y}^{\mathrm{T}}$), then we write $\mathbf{X} < \mathbf{Y}$ or

$\mathbf{Y} - \mathbf{X} > \mathbf{0}$ to mean that $\mathbf{Y} - \mathbf{X}$ (resp. $\mathbf{X} - \mathbf{Y}$) is positive definite (resp. negative definite). The identity matrices, with their dimensions understood, are denoted $\mathbf{I}$; similarly, the zero matrices are denoted $\mathbf{0}$.

2.1 Switching and feedback for uniform exponential stability

Let

$$\mathcal{S} = \{(\mathbf{A}_i, \mathbf{C}_i) : i = 1, \dots, N\} \tag{1}$$

be an indexed set of N matrix pairs, where $\mathbf{A}_i \in \mathbb{R}^{n \times n}$ and $\mathbf{C}_i \in \mathbb{R}^{l \times n}$ are given matrices for all $i = 1, \dots, N$. The set $\mathcal{S}$ defines a switched-automata system given by

$$x(t + 1) = \mathbf{A}_{\theta(t)} x(t), \quad t \in \mathbb{N}_0; \tag{2a}$$

$$y(t) = \mathbf{C}_{\theta(t)} x(t), \quad t \in \mathbb{N}_0, \tag{2b}$$

over all *switching sequences*

$$\theta = (\theta(0), \theta(1), \dots) \in \{1, \dots, N\}^\infty.$$

We do not distinguish between the set $\mathcal{S}$ and the difference equation it defines, they are both referred to as switched-automata systems. If $\theta(t) = i$ for some $t \in \mathbb{N}_0$ and $i \in \{1, \dots, N\}$, then the switched-automata system is said to be in *mode i* at time t. Once a switching sequence $\theta \in \{1, \dots, N\}^\infty$ and an initial state $x(0) \in \mathbb{R}^n$ are specified, the state-space model (2) generates a state sequence $x = (x(0), x(1), \dots)$ and a measurement sequence $y = (y(0), y(1), \dots)$.

Presented below is an exact condition for the existence of a stabilizing switching rule for $\mathcal{S}$. This condition will turn out to be equivalent to an exact condition for the existence of a single stabilizing switching sequence θ, which works independently of the state measurements. Furthermore, we will extend the condition to joint synthesis of switching and feedback for stabilizing $\mathcal{S}$.

2.1.1 Definitions

Let $\mathcal{S}$ be as in (1). Let

$$g = \{g_t : t \in \mathbb{N}_0\} \tag{3a}$$

be an indexed family of functions such that

$$\theta(t) = g_t(y^t) \in \{1, \dots, N\}, \quad t \in \mathbb{N}_0, \tag{3b}$$

where $y^t = (y(0), \dots, y(t))$. Then the family g defines a *switching rule* that generates a switching sequence θ for the switched linear system $\mathcal{S}$ based on the measurement sequence y. As in the measurement scheduling case, one may also consider switching rules that generate $\theta(t)$ based on past measurements y^{t-1} only. The results described in this section and subsequent sections carry over to such switching rules as well.

A switching rule g is said to be asymptotically stabilizing for $\mathcal{S}$ if the state equation (2a) satisfies $x(t) \to 0$ as $t \to \infty$ under g. Similarly, a switching sequence θ is said to be asymptotically stabilizing for $\mathcal{S}$ if $x(t) \to 0$ as $t \to \infty$ over all initial states $x(0)$. The stability notions of this research are stronger than just described and require that the state $x(t) \to 0$ with a uniform exponential decay rate over all initial state and over all state perturbations, in the same vein as the MJLS systems described in an earlier research section above. Consider what we call the *internal automata-switched linear system*:

$$x(t + 1) = \mathbf{A}_{\theta(t)} x(t) + r(t), \quad t \in \mathbb{N}_0. \tag{4}$$

Definition 1. *A switching rule g as in* (3) *is said to be* uniformly exponentially stabilizing *for the automata-switched system $\mathcal{S}$ if there exist $c > 0$ and $\lambda \in (0, 1)$ such that the internal switched linear system* (4)*, with $\theta(t) = g_t(y^t)$ for all $t \in \mathbb{N}_0$, satisfies*

$$\|x(t)\| \leq c\lambda^t \|x(0)\| \tag{5a}$$

for all $t \in \mathbb{N}_0$ and $x(0) \in \mathbb{R}^n$ whenever $r(0) = r(1) = \cdots = 0$, and

$$\|x(t)\| \leq c\lambda^{t-t_0} \|r(t_0)\| \tag{5b}$$

for all t_0, $t \in \mathbb{N}_0$ with $t \geq t_0$ and $r(t_0) \in \mathbb{R}^n$ whenever $x(0) = r(0) = \cdots = r(t_0 - 1) = r(t_0 + 1) = \cdots = 0$.

Definition 2. *A switching sequence $\theta \in \{1, \ldots, N\}^\infty$ is said to be* uniformly exponentially stabilizing *for the switched-automata system $\mathcal{S}$ if there exist $c > 0$ and $\lambda \in (0, 1)$ such that*

$$\|\boldsymbol{\Phi}_\theta(t, t_0)\| \leq c\lambda^{t-t_0}$$

for all t_0, $t \in \mathbb{N}_0$ with $t \geq t_0$, where

$$\boldsymbol{\Phi}_\theta(t, t_0) = \begin{cases} \mathbf{A}_{\theta(t-1)} \cdots \mathbf{A}_{\theta(t_0)}, & t > t_0; \\ \mathbf{I}, & t = t_0. \end{cases}$$

It is readily seen that the existence of a uniformly exponentially stabilizing switching sequence implies that of a uniformly exponentially stabilizing switching rule. The converse also holds true. For each $M \in \mathbb{N}_0$, the elements of $\{1, \ldots, N\}^{M+1}$ will be called *switching paths* of length M. A nonempty set $\mathcal{N}$ of switching paths of length M is said to be *admissible* if, for each $(i_0, \ldots, i_M) \in \mathcal{N}$, there exist a $K \in \mathbb{N}$, with $K > M$, and a switching path $(i_{M+1}, \ldots, i_K)$ such that $(i_{K-M}, \ldots, i_K) = (i_0, \ldots, i_M)$ and $(i_t, \ldots, i_{t+M}) \in \mathcal{N}$ for all $t \in \{0, \ldots, K - M\}$. An admissible set $\mathcal{N}$ of switching paths is called *minimal* if none of the proper subsets of $\mathcal{N}$ is admissible. It is readily seen that, if $\mathcal{N}$ is a minimal set of switching paths of length M, then there exists a switching sequence θ such that

$$\mathcal{N} = \{(i_0, \ldots, i_M) : (i_0, \ldots, i_M) = (\theta(t), \ldots, \theta(t + M)), \ t \in \mathbb{N}_0\}. \tag{6}$$

It can be shown that a switching sequence is periodic and is unique up to a time shift.

2.1.2 Synthesis of switching rules

The following theorem characterizes the existence of uniformly exponentially stabilizing switching laws.

Theorem 1. *Let $\mathcal{S}$ be as in* (1)*. The following are equivalent:*

(a) *There is a uniformly exponentially stabilizing switching rule for $\mathcal{S}$.*

(b) *There is a uniformly exponentially stabilizing switching sequence for $\mathcal{S}$.*

(c) *There exist a path length $M \in \mathbb{N}_0$, a minimal set $\mathcal{N}$ of switching paths of length M, and matrices $\mathbf{Y}_{(i_0, \ldots, i_{M-1})} \in \mathbb{R}^{n \times n}$ such that*

$$\mathbf{Y}_{(i_0, \ldots, i_{M-1})} \succ \mathbf{0},$$
$$\mathbf{A}_{i_M} \mathbf{Y}_{(i_0, \ldots, i_{M-1})} \mathbf{A}_{i_M}^{\mathrm{T}} - \mathbf{Y}_{(i_1, \ldots, i_M)} \prec \mathbf{0}$$

for all $(i_0, \ldots, i_M) \in \mathcal{N}$.

Moreover, if condition (c) holds, then any periodic switching sequence θ satisfying (6) is uniformly exponentially stabilizing for $\mathcal{S}$.

Generation of all minimal sets of switching paths of length M amounts to identifying all elementary cycles in a directed graph whose nodes are switching paths of length $M - 1$. There are well-known algorithms for finding all elementary cycles in a directed graph is presented. There are counter examples which show that the existence of a switching rule that guarantees asymptotic stability does not necessarily imply the existence of an asymptotically stabilizing switching sequence. However, Theorem 1 says that adding a uniformity/robustness requirement to the stability notion makes the situation quite different.

Example 1. *Suppose $N = 2$ and $\mathcal{S}$ has*

$$\mathbf{A}_1 = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 2 \end{bmatrix}, \quad \mathbf{A}_2 = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix},$$

$$\mathbf{C}_1 = \mathbf{C}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

It has been shown previously that there exist asymptotically stabilizing switching rules for this $\mathcal{S}$; one such rule $g = \{g_t\}$ is given by

$$\theta(t) = g_t(x^t)$$
$$= \begin{cases} 1 & \text{if } x(t) = [x_1 \ x_2]^{\mathrm{T}} \text{ with } |x_1| > 2|x_2|; \\ 2 & \text{otherwise.} \end{cases}$$

However, it is easy to see that there is no asymptotically stabilizing switching sequence for this case since the determinants of both matrices, and hence any product of them, is one. Therefore, Theorem 1 suggests that no switching rule is uniformly exponentially stabilizing for $\mathcal{S}$. Indeed, if

$$x(0) = \begin{bmatrix} 2 \\ 2 \end{bmatrix}, \quad r(0) = \begin{bmatrix} 0 \\ 1 - \sqrt{3} \end{bmatrix},$$

and

$$r(t) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \text{for } t \geq 1,$$

then the internal system (4) under g generates the switching sequence $\theta = (2, 1, 1, \dots)$; letting $x_0(t)$ (resp. $x_r(t)$) be the response of the internal system to $x(0)$ (resp. $r(0)$), we have that both $x_0(t)$ and $x_r(t)$ diverge even though $x(t) = x_0(t) + x_r(t) \to 0$.

Example 2. *Let us replace $\mathbf{A}_1$ in Example 1 with*

$$\mathbf{A}_1 = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix}.$$

Then the spectral radius of $\mathbf{A}_2 \mathbf{A}_1$ is less than one, and hence the periodic switching sequence

$$\theta = (1, 2, 1, 2, \dots),$$

where modes 1 and 2 alternate, is uniformly exponentially stabilizing. This switching sequence is generated by the following open-loop switching rule:

$$\theta(t) = g_t(x^t) = \begin{cases} 1 & \text{if } t \text{ is even;} \\ 2 & \text{if } t \text{ is odd.} \end{cases}$$

2.1.3 Joint synthesis of switching and feedback

Let $\mathbf{A}_1, \ldots, \mathbf{A}_N \in \mathbb{R}^{n \times n}$, $\mathbf{B}_1, \ldots, \mathbf{B}_N \in \mathbb{R}^{n \times m}$, and $\mathbf{C}_1, \ldots, \mathbf{C}_N \in \mathbb{R}^{l \times n}$ be given. Consider the controlled plant model of the form

$$x(t+1) = \mathbf{A}_{\theta(t)} x(t) + \mathbf{B}_{\theta(t)} u(t), \quad t \in \mathbb{N}_0; \tag{8a}$$

$$y(t) = \mathbf{C}_{\theta(t)} x(t), \quad t \in \mathbb{N}_0. \tag{8b}$$

Assuming that the mode $\theta(t)$ is perfectly observed by the feedback controller at each time instant $t \in \mathbb{N}_0$, and that the controller is able to recall L most recent past modes, consider dynamic output feedback controllers of the form

$$x_K(t+1) = \mathbf{A}_{K,(\theta(t-L),\ldots,\theta(t))} x_K(t) + \mathbf{B}_{K,(\theta(t-L),\ldots,\theta(t))} y(t), \quad t \in \mathbb{N}_0;$$

$$u(t) = \mathbf{C}_{K,(\theta(t-L),\ldots,\theta(t))} x_K(t) + \mathbf{D}_{K,(\theta(t-L),\ldots,\theta(t))} y(t), \quad t \in \mathbb{N}_0.$$

Closing the feedback loop via such a controller yields

$$\tilde{x}(t+1) = \left(\begin{bmatrix} \mathbf{A}_{\theta(t)} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{B}_{\theta(t)} \\ \mathbf{I} & \mathbf{0} \end{bmatrix} \mathbf{K}_{(\theta(t-L),\ldots,\theta(t))} \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{C}_{\theta(t)} & \mathbf{0} \end{bmatrix} \right) \tilde{x}(t), \tag{9}$$

where $\tilde{x}(t) = [x(t)^{\mathrm{T}} \; x_K(t)]^{\mathrm{T}}$ and

$$\mathbf{K}_{(\theta(t-L),\ldots,\theta(t))} = \begin{bmatrix} \mathbf{A}_{K,(\theta(t-L),\ldots,\theta(t))} & \mathbf{B}_{K,(\theta(t-L),\ldots,\theta(t))} \\ \mathbf{C}_{K,(\theta(t-L),\ldots,\theta(t))} & \mathbf{D}_{K,(\theta(t-L),\ldots,\theta(t))} \end{bmatrix} \tag{10}$$

for $t \in \mathbb{N}_0$.

The following result is immediate from Theorem 1 and our previous work, and enables us to determine a switching rule and a feedback controller such that the closed-loop system (9) is uniformly exponentially stable.

Corollary 1. *Let the controlled plant be given by* (8). *The following are equivalent:*

(a) *There exist a switching rule g and a set of controller coefficients* (10), *where $\theta(t) = g_t(y^t)$, such that the closed-loop system* (9) *is uniformly exponentially stable.*

(b) *There exist a switching sequence θ and a set of controller coefficients* (10) *such that the closed-loop system* (9) *is uniformly exponentially stable.*

(c) *There exist a path length $M \in \mathbb{N}_0$, a minimal set $\mathcal{N}$ of switching paths of length M, and matrices $\mathbf{R}_{(i_0,\ldots,i_{M-1})}, \mathbf{S}_{(i_0,\ldots,i_{M-1})} \in \mathbb{R}^{n \times n}$ such that*

$$N(\mathbf{B}_{i_M}^{\mathrm{T}})^{\mathrm{T}} \big(\mathbf{A}_{i_M} \mathbf{R}_{(i_0,\ldots,i_{M-1})} \mathbf{A}_{i_M}^{\mathrm{T}} \\ - \mathbf{R}_{(i_1,\ldots,i_M)} \big) N(\mathbf{B}_{i_M}^{\mathrm{T}}) \prec \mathbf{0}, \tag{11a}$$

$$N(\mathbf{C}_{j_M})^{\mathrm{T}} \big(\mathbf{A}_{i_M}^{\mathrm{T}} \mathbf{S}_{(i_1,\ldots,i_M)} \mathbf{A}_{i_M} \\ - \mathbf{S}_{(i_0,\ldots,i_{M-1})} \big) N(\mathbf{C}_{j_M}) \prec \mathbf{0}, \tag{11b}$$

$$\begin{bmatrix} \mathbf{R}_{(i_1,\ldots,i_M)} & \mathbf{I} \\ \mathbf{I} & \mathbf{S}_{(i_1,\ldots,i_M)} \end{bmatrix} \succeq \mathbf{0}. \tag{11c}$$

for all $(i_0,\ldots,i_M) \in \mathcal{N}$, where $N(\mathbf{M})$ denotes any full-column-rank matrix whose columns span the null space of $\mathbf{M}$.

Moreover, if condition (c) holds, then the closed-loop system (9) is uniformly exponentially stable under a periodic switching sequence θ satisfying (6) and a set of feedback controller coefficients (10) with $L = M$.

Once the semidefinite inequalities (11) have been solved for some path length M and minimal set $\mathcal{N}$, a stabilizing set of controller coefficients (10) can be obtained based on the well-known linear matrix inequality embedding technique.

2.2 Switching and feedback for guaranteed performance

Results on performance, not just stability, are outlined in this section.

Let

$$\mathcal{T} = \{(\mathbf{A}_i, \mathbf{B}_i, \mathbf{C}_{1,i}, \mathbf{D}_{1,i}, \mathbf{C}_{2,i}, \mathbf{D}_{2,i}) \colon i = 1, \ldots, N\} \tag{12}$$

be an indexed set of matrix tuples, where $\mathbf{A}_i \in \mathbb{R}^{n \times n}$, $\mathbf{B}_i \in \mathbb{R}^{n \times m}$, $\mathbf{C}_{1,i} \in \mathbb{R}^{l_1 \times n}$, $\mathbf{D}_{1,i} \in \mathbb{R}^{l_1 \times m}$, $\mathbf{C}_{2,i} \in \mathbb{R}^{l_2 \times n}$, and $\mathbf{D}_{2,i} \in \mathbb{R}^{l_2 \times m}$, $i = 1, \ldots, N$, are given matrices. The set $\mathcal{T}$ defines the family of linear time-varying state-space equations

$$x(t + 1) = \mathbf{A}_{\theta(t)} x(t) + \mathbf{B}_{\theta(t)} w(t), \quad t \in \mathbb{N}_0; \tag{13a}$$

$$z(t) = \mathbf{C}_{1,\theta(t)} x(t) + \mathbf{D}_{1,\theta(t)} w(t), \quad t \in \mathbb{N}_0; \tag{13b}$$

$$y(t) = \mathbf{C}_{2,\theta(t)} x(t) + \mathbf{D}_{2,\theta(t)} w(t), \quad t \in \mathbb{N}_0, \tag{13c}$$

over all switching sequences $\theta \in \{1, \ldots, N\}^\infty$. Given a switching sequence θ, a disturbance input sequence $w = (w(0), w(1), \ldots)$, and an initial state $x(0) \in \mathbb{R}^n$, the linear time-varying system (13) generates an output sequence $z = (z(0), z(1), \ldots)$ in addition to the state sequence x and measurement sequence y.

The $\mathcal{H}_2$-type performance measure considered in this research gives the square root of the average output variance per unit time of the state-space model (13) under white Gaussian disturbance input sequence w, and it indicates how well the system output is regulated under random disturbances. On the other hand, the $\mathcal{H}_\infty$-type performance measure considered in the research requires that the switching rule g satisfy the following small gain property: Whenever a disturbance sequence w generates a switching sequence θ for (12), the state-space model (13) is robustly well-connected with all sufficiently small dynamic uncertainties.

Definition 3. *A switching rule g as in (3) is said to achieve* output regulation level $\gamma > 0$ *for the automata-switched system $\mathcal{T}$ if it is uniformly exponentially stabilizing for $\mathcal{T}$ whenever $w = 0$ and if there exists a $\tilde{\gamma} \in (0, \gamma)$ such that, whenever $x(0) = 0$, the state-space equations (13) satisfy*

$$\lim_{T \to \infty} \frac{1}{T + 1} \sum_{t=0}^{T} \mathbb{E}\|z(t)\|^2 \leq \tilde{\gamma}^2,$$

where w is unit variance white noise.

Definition 4. *A switching sequence $\theta \in \{1, \ldots, N\}^\infty$ is said to achieve output regulation level $\gamma > 0$ for the automata-switched system $\mathcal{T}$ if it is uniformly exponentially stabilizing for $\mathcal{T}$ whenever $w = 0$ and if*

$$\lim_{t \to \infty} \frac{1}{T + 1} \sum_{t=0}^{T} tr\big(\mathbf{C}_{1,\theta(t)} \mathbf{Y}_\theta(t_0, t) \mathbf{C}_{1,\theta(t)}^{\mathrm{T}} + \mathbf{D}_{1,\theta(t)} \mathbf{D}_{1,\theta(t)}^{\mathrm{T}}\big) \leq \tilde{\gamma},$$

where

$$\mathbf{Y}_\theta(t_0, t) = \begin{cases} \sum_{s=t_0}^{t-1} \mathbf{\Phi}_\theta(t, s+1)\mathbf{B}_{\theta(s)}\mathbf{B}_{\theta(s)}^{\mathrm{T}}\mathbf{\Phi}_\theta(t, s+1)^{\mathrm{T}}, \\ \\ \qquad\qquad\qquad\qquad\qquad\qquad t > t_0; \\ \\ \mathbf{0}, \qquad\qquad\qquad\qquad\qquad\quad t = t_0. \end{cases} \tag{14}$$

Definition 5. *A switching rule g as in (3) is said to achieve* disturbance attenuation level $\gamma > 0$ *for the automata-switched system $\mathcal{T}$ if it is uniformly exponentially stabilizing for $\mathcal{T}$ whenever $w = 0$ and if there exists a $\tilde{\gamma} \in (0, \gamma)$ such that, whenever $x(0) = 0$, the state-space model (13) feedback-interconnected with $w = r + \Delta z$ has $z = 0$ as the unique response to $w = 0$ over all linear operators Δ satisfying*

$$\sum_{t=0}^{\infty} \|w(t)\|^2 < \tilde{\gamma}^{-2} \sum_{t=0}^{\infty} \|z(t)\|^2 \tag{15}$$

for all z with $\sum_{t=0}^{\infty} \|z(t)\|^2 < \infty$.

Definition 6. *A switching sequence $\theta \in \{1, \dots, N\}^\infty$ is said to achieve disturbance attenuation level $\gamma > 0$ for the switched linear system $\mathcal{T}$ if it is uniformly exponentially stabilizing for $\mathcal{T}$ whenever $w = 0$ and if there exists a $\tilde{\gamma} \in (0, \gamma)$ such that*

$$\|\mathbf{M}_\theta(t, t_0)\| \leq \tilde{\gamma}$$

for all t_0, $t \in \mathbb{N}_0$ with $t_0 \leq t$, where

$$\mathbf{M}_\theta(t, t_0) = \begin{bmatrix} \mathbf{D}_{1,\theta(t_0)} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{C}_{1,\theta(t_0+1)}\mathbf{B}_{\theta(t_0)} & \mathbf{D}_{1,\theta(t_0+1)} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{C}_{1,\theta(t)}\mathbf{\Phi}_\theta(t, t_0+1)\mathbf{B}_{\theta(t_0)} & \mathbf{C}_{1,\theta(t)}\mathbf{\Phi}_\theta(t, t_0+2)\mathbf{B}_{\theta(t_0+1)} & \cdots & \mathbf{D}_{1,\theta(t)} \end{bmatrix}.$$

If θ is a switching sequence, and if $x(0) = 0$, then $z(0) = \mathbf{D}_{1,\theta(0)}w(0)$ and

$$z(t) = \sum_{s=0}^{t-1} \mathbf{C}_{1,\theta(t)}\mathbf{\Phi}_\theta(t, s+1)\mathbf{B}_{\theta(s)}w(s) + \mathbf{D}_{1,\theta(t)}w(t)$$

for all $t \in \mathbb{N}$. Thus it is readily seen that, if a switching sequence θ achieves an output regulation level, or a disturbance attenuation level, then there exists a switching rule g that achieves the same performance level.

In the research it was shown that the converse also holds, and exact convex synthesis conditions for such a switching rule is given below. An extension to joint synthesis of switching and feedback has also been obtained and will be presented below.

2.2.1 Synthesis of switching rules

The following theorems characterize the existence of switching rules that achieve guaranteed performance levels. They also give convex synthesis conditions for obtaining these rules.

Theorem 2. *Let $\mathcal{T}$ be as in (12); let $\gamma > 0$. The following are equivalent:*

(a) There exists a switching rule that achieves output regulation level γ for $\mathcal{T}$.

11

(b) There exists a switching sequence that achieves output regulation level γ for $\mathcal{T}$.

(c) There exist a path length $M \in \mathbb{N}_0$, a minimal set $\mathcal{N}$ of switching paths of length M, and matrices $\mathbf{Y}_{(i_0,\ldots,i_{M-1})} \in \mathbb{R}^{n \times n}$ such that

$$\mathbf{Y}_{(i_0,\ldots,i_{M-1})} \succ \mathbf{0}, \tag{16a}$$

$$\mathbf{A}_{i_M}\mathbf{Y}_{(i_0,\ldots,i_{M-1})}\mathbf{A}_{i_M}^{\mathrm{T}} - \mathbf{Y}_{(i_1,\ldots,i_M)} \prec \mathbf{B}_{i_M}\mathbf{B}_{i_M}^{\mathrm{T}} \tag{16b}$$

for all $(i_0,\ldots,i_M) \in \mathcal{N}$, and such that

$$\frac{1}{|\mathcal{N}|} \sum_{(i_0,\ldots,i_M)\in\mathcal{N}} tr\left(\mathbf{C}_{1,i_M}\mathbf{Y}_{(i_0,\ldots,i_{M-1})}\mathbf{C}_{1,i_M}^{\mathrm{T}} + \mathbf{D}_{1,i_M}\mathbf{D}_{1,i_M}^{\mathrm{T}}\right) < \gamma^2, \tag{16c}$$

where $|\mathcal{N}|$ denotes the cardinality of $\mathcal{N}$.

Moreover, if condition (c) holds, then any periodic switching sequence θ satisfying (6) achieves output regulation level γ for $\mathcal{T}$.

Theorem 3. *Let $\mathcal{T}$ be as in (12); let $\gamma > 0$. The following are equivalent:*

(a) There exists a switching rule that achieves disturbance attenuation level γ for $\mathcal{T}$.

(b) There exists a switching sequence that achieves disturbance attenuation level γ for $\mathcal{T}$.

(c) There exist a path length $M \in \mathbb{N}_0$, a minimal set $\mathcal{N}$ of switching paths of length M, and matrices $\mathbf{Y}_{(i_0,\ldots,i_{M-1})} \in \mathbb{R}^{n \times n}$ such that

$$\mathbf{Y}_{(i_0,\ldots,i_{M-1})} \succ \mathbf{0} \tag{17a}$$

and

$$\begin{bmatrix} \mathbf{A}_{i_M} & \mathbf{B}_{i_M} \\ \mathbf{C}_{1,i_M} & \mathbf{D}_{1,i_M} \end{bmatrix} \begin{bmatrix} \mathbf{Y}_{(i_0,\ldots,i_{M-1})} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \times \begin{bmatrix} \mathbf{A}_{i_M} & \mathbf{B}_{i_M} \\ \mathbf{C}_{1,i_M} & \mathbf{D}_{1,i_M} \end{bmatrix}^{\mathrm{T}} - \begin{bmatrix} \mathbf{Y}_{(i_1,\ldots,i_M)} & \mathbf{0} \\ \mathbf{0} & \gamma^2\mathbf{I} \end{bmatrix} \prec \mathbf{0} \tag{17b}$$

for all $(i_0,\ldots,i_M) \in \mathcal{N}$.

Moreover, if condition (c) holds, then any periodic switching sequence θ satisfying (6) achieves disturbance attenuation level γ for $\mathcal{T}$.

Example 3. *Suppose $N = 2$ and $\mathcal{T}$ has*

$$\mathbf{A}_1 = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{A}_2 = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}, \quad \mathbf{B}_1 = \mathbf{B}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$\mathbf{C}_{1,1} = \mathbf{C}_{1,2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{C}_{2,1} = \mathbf{C}_{2,2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$\mathbf{D}_{1,1} = \mathbf{D}_{1,2} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \mathbf{D}_{2,1} = \mathbf{D}_{2,2} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

For illustration the condition (c) of Theorem 3 is now used to obtain best switching sequences in terms of their disturbance attenuation performance. There are two minimal sets of switching paths of length zero; namely, $\{1\}$ and $\{2\}$. Since the spectral radii of $\mathbf{A}_1$ and $\mathbf{A}_2$ are not less than 1, inequalities (17) are

infeasible for any $\gamma > 0$ if $M = 0$. However, among three minimal sets of switching paths of length one (namely, $\{(1,1)\}$, $\{(1,2),(2,1)\}$, and $\{(2,2)\}$),

$$\mathcal{N} = \{(1,2),(2,1)\}$$

renders (17) feasible if and only if $M = 1$ and $\gamma \geq 6.845$ (up to four significant digits). Thus the periodic switching sequence

$$\theta = (1,2,1,2,\dots)$$

achieves all disturbance attenuation levels $\gamma \geq 6.845$ but does not achieve any disturbance attenuation level $\gamma \leq 6.844$. Incrementing the path length to $M = 2$, we obtain six minimal sets of switching paths of length two. Among them,

$$\mathcal{N} = \{(1,1,2),(1,2,2),(2,2,1),(2,1,1)\}$$

renders (17) feasible if and only if $M = 2$ and $\gamma \geq 6.484$. Thus the periodic switching sequence

$$\theta = (1,1,2,2,1,1,2,2,\dots)$$

achieves all disturbance attenuation levels $\gamma \geq 6.484$ but does not achieve any disturbance attenuation level $\gamma \leq 6.483$. One can find a better switching sequence by increasing the path length further. For path length $M = 3$, the minimal set

$$\mathcal{N} = \{(1,1,1,2),(1,1,2,2),(1,2,2,2),(2,2,2,1),(2,2,1,1),(2,1,1,1)\}$$

is the best one, and one concludes that the periodic switching sequence

$$\theta = (1,1,1,2,2,2,1,1,1,2,2,2,\dots),$$

where the switching sequence $(1,1,1,2,2,2)$ repeats, achieves all disturbance attenuation levels $\gamma \geq 6.466$ and does not achieve any disturbance attenuation level $\gamma \leq 6.465$. For this particular example, incrementing the path length even further to $M = 4$ does not improve the performance level.

2.2.2 Joint synthesis of switching and feedback

Joint synthesis of a switching rule and a feedback controller is also possible for achieving guaranteed disturbance attenuation performance in view of Theorem 2 and prior work, and for achieving guaranteed output regulation performance in view of Theorem 3. The technical development is similar to that of § 2.1.3.

3 Multi-resolution Sensing, Control and Switched Systems

The program work described in this section was motivated by the need for analytical tools for assessing stability of control systems acting over networks. In particular the goal was to develop tools that could simultaneously handle latency over the network and finite information in the feedback measurement; the latter condition represented either finite-precision in measurement, or finite-bandwidth limitations of transmission channels. Explicit tools and testing conditions were developed for this in a switched-automata system framework, and the results of this effort are reported in [3,10] and the submitted paper [15]. More details about the specifics of this research now follow. This research agenda was pursued by addressing almost sure uniform exponential stabilization and second-order stabilization of Markovian jump linear systems (MJLSs) with

logarithmically quantized state feedbacks. Markovian jump linear systems can be regarded as a special case of switched systems. Introduced were the concepts and explicit constructions of stabilizing mode-dependent and finite path-dependent logarithmic quantizers together with associated controllers. Also developed was a semiconvex way to determine the optimal (coarsest) stabilizing quantization density.

The work investigates quantized control problem of Markovian jump systems (MJSs), where stabilization is considered, either using a mean-square criterion, or an almost sure uniform stability criterion. For MJSs, second-order stability implies almost sure non-uniform stability; however, almost sure uniform exponential stability (UES) requires sample paths to quadratically decrease almost surely uniformly. This guarantees the worst case scenario performance and is therefore a much stronger requirement than the statistical sense second-order stability. In the project research, we consider the situation where measurements are logarithmically quantized before sending to controllers, as an effort to model the finite wordlength constraint of a communication channel, or multi-resolution sensor. MJSs can be used to model the packet loss or delay issue also. Tools developed in the program are extended and then combined with the new concept of finite path-dependent logarithmic quantizer and controller to solve the almost sure uniform exponential stabilization problem of MJSs subject to logarithmically quantized state feedbacks. Explicit constructions of quantizers and controllers are provided along with the semi-convex approach to find the coarsest stabilizing quantization density. The framework we developed provides a systematic approach to model multiple channel constraints in problems of control over communication networks. Most past literature in this area focuses either entirely on the finite bandwidth issues, for example or exclusively on the unreliable transmission problem, for example. Concurrent treatment of these intrinsically related constraints only appears in a few recent works. An application of the framework developed here is uniform stabilization problem of a discrete-time LTI system with logarithmically quantized state feedbacks over a lossy channel, where the packet gets dropped according to a Markovian process. This models the situation where measurements of the system are distorted not only As a special case, one can show that if the underlying Markovian process is Bernoulli, then the system is not stabilizable in the almost sure uniform sense; but it is second-order stabilizable. For the same system, also studied cases when it is almost surely uniformly stabilizable. Main contributions of this work are developing a general framework for stabilizing MJSs with quantized feedbacks, presenting a semi-convex algorithm to approach the coarsest stabilizing quantization density, and providing a systematic paradigm to simultaneously treat multiple feedback channel constraints.

4 Metric-Based Receding Horizon Control

In this research automata-switched systems synthesis was considered with the goal of guaranteeing finite-horizon performance when different operating conditions required different path-dependent performance objectives. Details of this work are reported in [13,16,20].

The focus of the work is a certain class of Pareto optimal controller synthesis. More specifically, a moving finite horizon performance measure over a forward window of length T is considered, where the objective is to design a controller that assures internal stability of the closed-loop system. It is allowed to have a finite memory of the most recent length-M switching path of modes (where M is computed as part of the solution, similar to the previous section) as well as perfect observation of the present mode. The control objective is to minimize the worst-case T-step receding-horizon cost over all start time and over all admissible switching sequences. Novel conditions that guarantee Pareto-optimal path-by-path output regulation are obtained. The novel features of the work are that (a) by focusing on path-by-path Pareto-optimality, one can give different weights to different switching paths; (b) Closed-loop uniform exponential stability is guaranteed and does not require a sufficiently long horizon length T ; and (c) Dynamic output feedback

controllers are allowed to use a (finite) memory of past modes. In the research exact, convex synthesis conditions for dynamic output feedback controllers are expressed in terms of a sequence of semidefinite programs. Provided is an exact solution to the linear receding-horizon control problem in discrete time, a problem of long-standing interest. Considered are nominal systems with state space models that vary in time, whose controllers have access to the precise statespace model of the plant for a fixed number of steps into the future, but only have foreknowledge of the set of model values beyond this horizon. In fact, considered is a more general scenario where evolution within the latter set may be governed by an automaton. We provide a necessary and sufficient convex condition for the existence of a linear output feedback controller that can uniformly exponentially stabilize such a system, and do the same for a related disturbance attenuation problem. Each condition is in terms of a nested sequence of semidefinite programs, where (a) feasibility to any element provides an explicit controller; and (b) infeasibility implies that a controller does not exist for a given exponential decay rate.

5 Decentralized Control and Finite Wordlength Channels

Also motivated by control over networks, the program considered a decentralized-control scenario in the setting of limited bandwidth sensing channels. Specifically, considered was a decentralized stabilization configuration involving a linear time-invariant(LTI) plant, regulated by multiple control stations that receive sensing information through rate-limited channels, and where the stations are not capable of communicating with each other directly. The main result of this part of the program is a sufficient condition on the data rate of respective channels to guarantee system stabilizability. In the research an explicit way to construct the associated stabilizing encoders, decoders, and controllers is also provided. In part of the work it is shown that the control algorithm is structurally well-posed to model mismatch. This research is detailed in the publication [10]. A detailed overview of the work is given below.

When building a geographically-distributed feedback control system that utilizes a communication network, issues of bandwidth limitation, latency, and packet loss become inevitable challenges, adding to the challenge already presented by structural constraints imposed by the communication graph. It is particularly important that these information exchange issues be systematically addressed when aggressive network control schemes are to be deployed. The particular situation investigated in this research is a stabilization problem involving a multi-station control system operating over rate-limited data links. The goal is for the sub-controllers to stabilize the plant using only their locally available information and local control variables. Namely, in the formulation studied the decentralized controller structure poses a *topological* constraint on information exchange; meanwhile, the limited bandwidth of the communication channels over which system measurements are transmitted to controllers (without latency or packet loss) give rise to a *non-topological* limitation. The goal of the research was a stabilizing algorithm simultaneously accommodating these two different constraint types.

In this work the system setup is depicted in Figure 1. A generic LTI plant G with no special structural assumption is controlled by v decentralized control stations. The state space representation of the system is given in Section 5.1.

Each local measurement y_i is quantized, encoded, and then sent to its respective control station over a noiseless digital memoryless channel with capacity R_i bits per step, without latency or packet loss. We derive coupled *upper bounds* on the data rates required on each channel to guarantee global asymptotic stabilizability. We also provide an explicit way to construct the associated stabilizing encoder, decoder, communication code, and controller. This unique feature of the work makes the developed algorithm easy to implement. Furthermore, the stabilizing algorithm is robust against model mismatch; namely, the stabi-
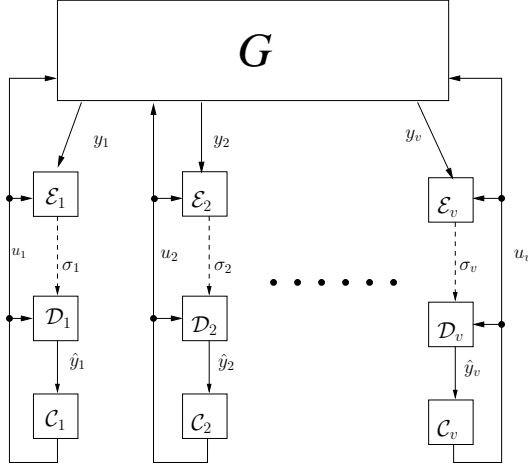
Figure 1: Decentralized system over communication channels with v control stations

lization is a well-posed. This is an important implementation issue since some model mismatch between plant and controllers is generally unavoidable.

5.1 Formulation

The infinity norm of a vector $x \in \mathbb{R}^n$ is defined as $\|x\|_\infty := \max_{1 \le i \le n} |x_i|$. The infinity norm of a matrix $A \in \mathbb{R}^{n \times m}$ is defined as $\|A\|_\infty := \max_{1 \le i \le n} \sum_{j=1}^m |A_{ij}|$. We denote $B_\infty^{p_i}(a, b)$ as a p_i-dimensional hypercube, which is centered at point a with edge length $2b$.

The decentralized control system with v control stations as shown in Figure 1 has the following state-space representation

$$
\begin{aligned}
x(t+1) &= Ax(t) + \sum_{i=1}^v B_i u_i(t), \quad \|x(0)\|_\infty \le E_0 \\
y_i(t) &= C_i x(t), \qquad i = 1, 2, \cdots, v
\end{aligned}
\tag{18}
$$

where state $x \in \mathbb{R}^n$, controls $u_i \in \mathbb{R}^{m_i}$, measurements $y_i \in \mathbb{R}^{p_i}$; and A, B_i, C_i are compatible real matrices.

We assume the system is unstable to make the problem non-trivial; that is, $1 \le \Lambda := \|A\|_\infty$.

Define $s_i := \mathtt{inf}\{m \text{ such that } \mathtt{dim}(\cap_{i=0}^m \ker CA^i) = n - \mathtt{dim}(\mathcal{K}_i^\perp)\}$, which is the *generalized observability index* of (C_i, A); in other words, the least number of steps needed to observe the state in $\mathcal{K}_i^\perp$ from measurements y_i.

Define $W_i := \begin{bmatrix} C_i^T & (C_i A)^T & \cdots & (C_i A^{s_i-1})^T \end{bmatrix}^T$, where $i = 1, \cdots, v$ and define W_i^+ as its generalized inverse.

We use $x_i(\cdot) = P_i x(\cdot)$ to denote the *projection* of the state vector in $\mathcal{K}_i^\perp$, where P_i is the projection matrix on $\mathcal{K}_i^\perp$ along $\mathcal{K}_i$. Notice that P_i here is not the natural projection since $x_i(\cdot)$ and $x(\cdot)$ are of the same size. We use $\tilde{x}_i(\cdot)$ and $\hat{x}_i(\cdot)$ to denote Station i's estimate of the state $x(\cdot)$ and its projection $x_i(\cdot)$ in $\mathcal{K}_i^\perp$ respectively.

The ith local encoder at time t is a map $\mathcal{E}_i(t) : \mathbb{R}^{p_i} \times \Sigma_i^{[0,t-1]} \times \mathbb{R}^{m_i t}$ to Σ_i, taking values $(y_i(t), \sigma_i[0, t-1], u_i[0, t-1]) \mapsto \sigma_i(t)$, the new codeword. Notation Σ_i is the ith codeword space and

$\Sigma_i^{[0,t-1]}$ denotes the union of all past codeword spaces up to time t. Notation $\sigma_i[0, t-1]$ is used to denote past codewords and $u_i[0, t-1]$ is used to denote all past local controls. The local encoder knows the local decoding policy $\mathcal{D}_i$ but not the local control policy $\mathcal{C}_i$.

The ith local decoder at time t is a map $\mathcal{D}_i(t) : \Sigma_i^{[0,t]} \times \mathbb{R}^{m_i t}$ to $\mathbb{R}^{p_i}$, taking values $(\sigma_i[0, t], u_i[0, t-1]) \mapsto \hat{y}_i(t)$, an estimate of the respective measurement y_i. The decoder knows the local encoding policy $\mathcal{E}_i$ but not the local control policy $\mathcal{C}_i$.

The ith local controller at time t is a map $\mathcal{C}_i(t) : \mathbb{R}^{p_i(t+1)}$ to $\mathbb{R}^{m_i}$, taking values $\hat{y}_i[0, t] \mapsto u_i(t)$, the local control, which depends causally on the local decoder's outputs.

The encoder and the decoder are assumed to have unlimited memory, therefore they can store and use all the past information. A digital noiseless memoryless channel connects each local encoder and decoder pair.

5.2 Stabilizing algorithm

In order to keep the notation clean and describe the algorithm more efficiently, the main idea behind the result is presented in the two-control-station setting, that is, system (18) with $v = 2$. The algorithm extends directly to the multi-station case. All technical details can be found in [10].

The control algorithm adopted here is divided into three phases: *observation, communication, and control*. First, both stations listen to the system with no controls applied in order to compute their own estimates of the initial state. Then these stations exchange their estimates by coding them into control signals. Finally, both controllers use their own noisy estimates of the initial state to design controls, and try to drive the state of the system back to zero.

The logic behind this algorithm is that if the data rate on each channel is high enough, then measurements y_i get sufficiently finely quantized so that each station will have quite accurate information about the initial state. Then the control action taken will at least bring the state to a smaller uncertainty set than the initial one and then eventually lead it to zero.

Without loss of generality, we assume $s_1 \geq s_2$.

5.2.1 Observation

For the observation stage, we have the following lemma.

Lemma 1. *Within at most s_i steps, Station i can estimate $x_i(0) = P_i x(0)$ with error bounded by*

$$\frac{\epsilon_i}{\sqrt[p_i]{N_i}} E_0$$

where $p_i = \dim(y_i)$, $\epsilon_i = \|W_i^+\|_\infty \|C_i\|_\infty \Lambda^{s_i-1}$, and $N_i = 2^{R_i}$ represents the number of quantization levels on channel i.

Let us consider the error in estimating $x_i(s_i)$ at time $t = s_i - 1$. This result will be used in later derivation. Let $\hat{x}_i(s_i)$ be the s_i-step ahead estimate, then

$$\|x_i(s_i) - \hat{x}_i(s_i)\|_\infty \leq \Lambda^{s_i} \|x_i(0) - \hat{x}_i(0)\|_\infty \leq \frac{\Lambda^{s_i} \epsilon_i E_0}{\sqrt[p_i]{N_i}} \tag{19}$$

17

5.2.2 Communication

At time $t = s_1$, each station has an estimate $\hat{x}_i(0)$ of the initial state. This information needs to be exchanged between them for stabilizing purposes. From Station i to Station j, we only need to send $(I - P_j)\hat{x}_i(0)$, where $i, j = 1, 2$ and $i \neq j$. The remaining part is directly available to Station j. Since direct communication between stations is not available, one station has to encode its estimate into control signals and the counter party has to decode it based on its local measurements. This is effectively to explore the *perfect* channel through the plant.

Recall that Station i's control action can be observed by Station j only if $\mathcal{R}_i \not\subseteq \mathcal{K}_j$. Assume this is true, then there exists a positive integer t_i such that

$$\alpha_i := \mathtt{rank} \left\{ C_j \sum_{\ell=0}^{t_i-1} A^{t_i-1-\ell} B_i B_i^T (A^T)^{t_i-1-\ell} \right\} \neq 0 \tag{20}$$

Let β_i be the smallest positive integer such that $\alpha_i \beta_i \geq n$, where $n = \dim(x)$.

Define the following *encoding matrix* from Station i to Station j

$$E_{ji} = \begin{bmatrix} C_j M_i(t_i, t_i) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ C_j M_i(\beta_i t_i, t_i) & \cdots & C_j M_i(t_i, t_i) \end{bmatrix} \tag{21}$$

where $M_i(p, q) = \sum_{\ell=0}^{t_i-1} A^{p-1-\ell} B_i B_i^T (A^{q-1-\ell})^T$. It is clear that $\mathtt{rank}\{E_{ji}\} \geq n$. Thus, there exist matrices S_i and T_i, such that $S_i E_{ji} T_i = I_n$. All these matrices can be pre-computed and stored in both local control stations.

We have the following lemma on information exchange between multiple stations when there are *no* non-topological constraints; in other words, all channels are perfect and are able to transmit real numbers precisely and instantaneously.

Lemma 2. *Given system (18) with no communication constraints, and assume $\mathcal{R}_i \not\subseteq \mathcal{K}_j$ for $i, j = 1, 2, \cdots, v$, then any vector $\phi \in \mathbb{R}^n$ can be encoded into a control sequence $u_i(\ell)$, $0 \leq \ell \leq \beta_i t_i - 1$ by Station i and decoded exactly by Station j from its measurements $y_j(k t_i)$, where $k = 1, 2, \cdots, \beta_i$.*

Now let us consider the information exchange in the two station system ($v = 2$) with the bandwidth constraint described in the previous sections. We have,

Lemma 3. *Given $\mathcal{R}_1 \not\subseteq \mathcal{K}_2$, Station 2 can reconstruct the initial state $x(0)$ at time $t = s_1 + \beta_1 t_1$ with error bounded by*

$$\left(\frac{\eta_1}{\sqrt[p_2]{N_2}} + \frac{\epsilon_1}{\sqrt[p_1]{N_1}} + \frac{\epsilon_2}{\sqrt[p_2]{N_2}} \right) E_0$$

where ϵ_1 and ϵ_2 are given in Lemma 1, and

$$\eta_1 = \|S_1\|_\infty \|C_2\|_\infty \left(\Lambda^{\beta_1 t_1} \left(\frac{\Lambda^{s_1} \epsilon_2}{\sqrt[p_2]{N_2}} \right) (1 + \sqrt[p_2]{N_2}) + \frac{1 - \Lambda^{\beta_1 t_1}}{1 - \Lambda} \|B_1\|_\infty \rho_1 \right) \tag{22}$$

$$\rho_1 = \|B_1^T\|_\infty \Lambda^{t_1-1} \|T_1\|_\infty \left(\frac{\epsilon_1}{\sqrt[p_1]{N_1}} + 1 \right)$$

During time $s_1 + \beta_1 t_1 \leq t \leq s_1 + (\beta_1 + 1)t_1 - 1$, Station 1 applies the following controls to the system to offset the previously accumulated control effects,

$$u_1(t) = -B_1^T (A^T)^{s_1 + (\beta_1+1)t_1 - 1 - t} z$$

$$z := M_1^+(t_1, t_1) \sum_{\ell=s_1}^{s_1+\beta_1 t_1 - 1} A^{s_1+\beta_1 t_1 - 1 - \ell} B_1 u_1(\ell)$$

where $M_i^+(\cdot, \cdot)$ is the generalized inverse of $M_i(\cdot, \cdot)$ defined in Equation (21).

Meanwhile, set $u_2(t) = 0$. Then at time $t = s_1 + (\beta_1 + 1)t_1$, the system state $x(t)$ is driven to

$$x(s_1 + (\beta_1 + 1)t_1) = A^{s_1+(\beta_1+1)t_1} x(0)$$

If $(h-1)s_1 < s_1 + (\beta_1 + 1)t_1 \leq hs_1 - 1$ for some $h \in \mathbb{N}_0$, then let $u_1(t) = u_2(t) = 0$ for time $s_1 + (\beta_1 + 1)t_1 \leq t \leq hs_1 - 1$ in order to take advantage of the repetitive estimation described below.

Both the encoder $\mathcal{E}_1$ and the decoder $\mathcal{D}_1$ know exactly the controls applied up to time hs_1, so they can run a simulating observation process and compute a tighter state estimate. The basic idea is to observe $\hat{y}_1(s_1), \cdots, \hat{y}_1(2s_1 - 1)$, and then estimate $x_1(s_1)$ using the same method as in the proof of Lemma 1. By repeating the process, we have the following error bound for s_1-*step ahead* estimate similar to that computed in Equation (19),

$$\begin{aligned}
\|x_1(ms_1) - \hat{x}_1(ms_1)\|_\infty &\leq \frac{\|W_1^+\|_\infty^m \|C_1\|_\infty^m \Lambda^{m(2s_1-1)} E_0}{(\sqrt[p_1]{N_1})^m} \\
&= \frac{\Lambda^{ms_1} \epsilon_1^m E_0}{(\sqrt[p_1]{N_1})^m}, \quad 1 \leq m \leq h
\end{aligned}$$

Then we start the information transmission from Station 2 to Station 1 at time $t = hs_1$. We have the following result,

Lemma 4. *Given $\mathcal{R}_2 \not\subseteq \mathcal{K}_1$, Station 1 can reconstruct the initial state $x(0)$ at time $t = hs_1 + \beta_2 t_2$ with error bounded by*

$$\left(\frac{\eta_2}{\sqrt[p_1]{N_1}} + \frac{\epsilon_1}{\sqrt[p_1]{N_1}} + \frac{\epsilon_2}{\sqrt[p_2]{N_2}} \right) E_0$$

where ϵ_1 and ϵ_2 are given in Lemma 1, and

$$\eta_2 = \|S_2\|_\infty \|C_1\|_\infty \left(\Lambda^{\beta_2 t_2} \left(\frac{\Lambda^{s_1} \epsilon_1}{\sqrt[p_1]{N_1}} \right)^h (1 + \sqrt[p_1]{N_1}) + \frac{1 - \Lambda^{\beta_2 t_2}}{1 - \Lambda} \|B_2\|_\infty \rho_2 \right)$$

$$\rho_2 = \|B_2^T\|_\infty \Lambda^{t_2 - 1} \|T_2\|_\infty \left(\frac{\epsilon_2}{\sqrt[p_2]{N_2}} + 1 \right)$$

Finally, we can also design controls $u_2(t)$, for $hs_1 + \beta_2 t_2 \leq t \leq hs_1 + (\beta_2 + 1)t_2 - 1$ to drive the state of the system back to $x(t_3) = A^{hs_1+(\beta_2+1)t_2} x(0)$ at time $t_3 = hs_1 + (\beta_2 + 1)t_2$.

5.3 Control

At time t_3, both stations have their estimates about the initial state, namely $\tilde{x}_1(0)$ and $\tilde{x}_2(0)$. They can then compute their own estimates of the state $x(t_3)$ and design the following controls independently to bring the state back to zero,

$$u_i(t) = -B_i^T (A^T)^{t_3+n-1-t} z_i, \quad t_3 \leq t \leq t_3 + n - 1$$

where

$$z_i := \theta_i M_i^+(n, n) A^{t_3+n} \tilde{x}_i(0)$$

with $M_i^+(\cdot, \cdot)$ defined as the generalized inverse of $M_i(\cdot, \cdot)$ in Equation (21) and $\theta_i = \|M_i\|_\infty (\sum_{i=1}^2 \|M_i\|_\infty)^{-1}$. The norm of $x(t_3 + n)$ can be easily bounded as follows,

For simplicity, let us omit the parameter n in $M_i(n, n)$, then we have

$$
\begin{aligned}
x(t_3 + n) &= A^{t_3+n} x(0) - \sum_{i=1}^2 \theta_i M_i M_i^+ A^{t_3+n} \tilde{x}_i(0) \\
&= \sum_{i=1}^2 \theta_i (I - M_i M_i^+) A^{t_3+n}(x(0) - \tilde{x}_i(0))
\end{aligned}
$$

Since M_i is positive semi-definite, from singular value decomposition, it is clear that $\|I - M_i M_i^+\|_\infty \leq 1$, therefore,

$$
\begin{aligned}
\|x(t_3 + n)\|_\infty &\leq \sum_{i=1}^2 \theta_i \|I - M_i M_i^+\|_\infty \Lambda^{t_3+n} \max_{i=1,2} \|x(0) - \tilde{x}_i(0)\|_\infty \\
&\leq \Lambda^{t_3+n} \max_{i=1,2} \|x(0) - \tilde{x}_i(0)\|_\infty
\end{aligned}
$$

Now, we can state the main result in the two station case.

Theorem 4. *If we follow the above control algorithm, then the two-station decentralized system (18) with bandwidth limited sensing channels can be asymptotically stabilized if the following inequality is satisfied for some $0 \leq \delta < 1$,*

$$
\max_{\substack{i,j=1,2 \\ i \neq j}} \left(\frac{\eta_j}{\sqrt[p_i]{N_i}} + \frac{\epsilon_i}{\sqrt[p_i]{N_i}} + \frac{\epsilon_j}{\sqrt[p_j]{N_j}} \right) < \delta \Lambda^{-(t_3+n)} \tag{23}
$$

The stabilizing channel data rate R_i is then given by $\log_2 N_i$.

5.4 Example

In this section, we use the algorithm developed in Section 5.2 to stabilize the following system and derive the corresponding bandwidth requirements,

$$
\begin{aligned}
x(t+1) &= \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u_1(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_2(t) \\
y_1(t) &= \begin{bmatrix} 0 & 1 \end{bmatrix} x(t) \\
y_2(t) &= \begin{bmatrix} 1 & 0 \end{bmatrix} x(t) \qquad \|x(0)\|_\infty \leq E_0
\end{aligned} \tag{24}
$$

This is a simplest non-trivial example. The system is jointly controllable and observable and it is strongly connected. Therefore it is stabilizable under the decentralized information structure. However, since the controllable subspace of any single station coincides with its own unobservable subspace, no station can stabilize any mode by itself. Therefore, collaboration is mandatory for stabilization.

The information lower bound for this system is $R \geq 1$ bit/step for stabilization purposes if *centralized* control is allowed.

Let us compute the upper bound on bandwidth requirement under the decentralized information structure using our algorithms: first the original one, then the enhanced version with simultaneous information exchange.

It is clear that $s_1 = s_2 = 1$ in this example. Thus, outputs $y_i(0), i = 1, 2$ are sufficient for stations to reconstruct their estimates of the initial state. If there is no quantization, then $y_i(0) = x(0)_j$, $i, j = 1, 2$ and $i \neq j$, where $x(0)_j$ denotes the jth component of the initial state $x(0)$. However, the output y_i is quantized with an N_i-level quantizer on the i-th channel. Then from Lemma 1, Station 1 and 2 have $\begin{bmatrix} 0 & \hat{x}_{20} \end{bmatrix}^T$ and $\begin{bmatrix} \hat{x}_{10} & 0 \end{bmatrix}^T$ respectively as their estimate of the initial state, where $|\hat{x}_{i0} - x(0)_i| \leq E_0/N_i$ for $i = 1, 2$.

Now we can exchange information about the initial state. Instead of constructing β_i, t_i, and E_{ij} as in Section 5.2, we simply choose $u_1(0) = \hat{x}_{20}$ and $u_2(0) = 0$ since the only difference would just be a constant coefficient. Following Lemma 3, we have,

$$|x(0)_2 - \bar{x}_{20}| \leq \frac{1}{N_2}(2 + \frac{1}{N_2} + \frac{1}{N_1})E_0 + \frac{1}{N_1}E_0$$

which is also the upper bound of $\|x(0) - \tilde{x}_2(0)\|_\infty$ due the structure of $\mathcal{K}_2$.

Now Station 1 applies a control $u_1(1) = -\hat{x}_{20}$ to drive the state $x(2)$ to $A^2 x(0)$.

During the same time, $\mathcal{E}_1$ and $\mathcal{D}_1$ run the simulation process and estimate $x(2)_2$ as $\hat{x}_{22}$ with error bounded by $|\hat{x}_{22} - x(2)_2| \leq \frac{4}{N_1^3}E_0$.

Now, let $u_1(2) = 0$ and $u_2(2) = \hat{x}_{10}$. We can compute the error between Station 1's estimate $\bar{x}_{10}$ and $x(0)_1$, which is bounded by

$$|\bar{x}_{10} - x(0)_1| \leq \frac{8}{N_1^4}E_0 + \frac{1}{N_1}(1 + \frac{1}{N_2})E_0 + \frac{8}{N_1^3}E_0 + \frac{1}{N_2}E_0$$

This is also the upper bound of $\|x(0) - \tilde{x}_1(0)\|_\infty$.

Station 2 now applies a control $u_2(3) = -2\hat{x}_{10}$ to drive the state back to $x(4) = A^4 x(0)$.

Now, both stations have a full estimate of the initial state, so we can design controls as follows:

$$\begin{bmatrix} u_1(4) \\ u_2(4) \end{bmatrix} = - \begin{bmatrix} 1 & 0 \\ 0 & 2^5 \end{bmatrix} \begin{bmatrix} \bar{x}_{10} \\ \bar{x}_{20} \end{bmatrix}$$

The condition of $\|x(5)\|_\infty < E_0$ is equivalent to

$$\max\left(\frac{8}{N_1^4} + \frac{1}{N_1}(1 + \frac{1}{N_2}) + \frac{8}{N_1^3} + \frac{1}{N_2}, \quad 2^5\left(\frac{1}{N_2}(2 + \frac{1}{N_1} + \frac{1}{N_2}) + \frac{1}{N_1}\right) \right) < 1$$

The feasible rate region is given in Figure 2(a).

There is a significant increase in the rate requirements compared to the centralized result. The upper bound of total required stabilizing data rate is $R_1 + R_2 \approx 15.0736$ bits/step.

We now proceed to use the enhanced algorithm. The first step of observation does not change. However, we can design control $u_1(0) = \hat{x}_{20}$ and $u_2(0) = \hat{x}_{10}$.

Both stations can compute their estimates $\bar{x}_{10}$ and $\bar{x}_{20}$ from their measurements and local controls. The errors are bounded by

$$\begin{aligned} |\hat{x}_{10} - \bar{x}_{10}| &\leq |y_1(1) - \hat{y}_1(1)| + 2|x(0)_2 - \hat{x}_{20}| \\ |\hat{x}_{20} - \bar{x}_{20}| &\leq |y_2(1) - \hat{y}_2(1)| + |x(0)_1 - \hat{x}_{10}| \end{aligned}$$

This time the control action will be completed at time $t = 3$. The inequality to satisfy is

$$\max\left(\frac{1}{N_1}(3 + \frac{1}{N_2} + 2\frac{1}{N_1}) + \frac{1}{N_2}, \quad 2^3\left(\frac{1}{N_2}(2 + \frac{1}{N_2} + \frac{1}{N_1}) + \frac{1}{N_1}\right) \right) < 1$$

The improved rate region is plotted in Figure 2(b). The upper bound of total required stabilizing data rate is dropped by around 2 bits/step to 13.0008 bits/step.
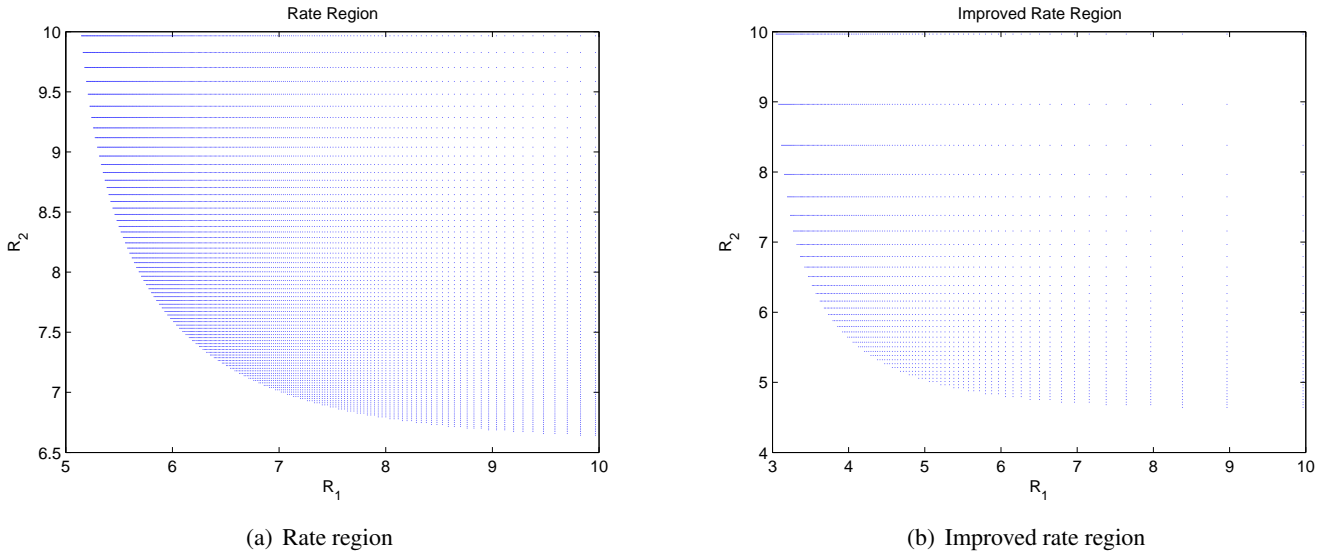
(a) Rate region

(b) Improved rate region

Figure 2: Admissible rate regions for system (24)

6 Distributed Control

Another thrust area for the project was distributed control over graphs, where the focus was on designing distributed controllers for interconnected systems in situations where the controller sensing and actuation topology is inherited from that of the plant. The main contribution of this research was to provide results on general graph interconnection structures in which the graphs have potentially an infinite number of vertices. This is accomplished by first extending our previous machinery which was developed for systems with spatial dynamics on integer lattices. From this we derived convex analysis and synthesis conditions for design in this setting. Furthermore, the methodology developed here provides a unifying viewpoint for our previous and related work on distributed control. The research is reported in [17].

In the recent work of [7,19] we have further built on these research tools and concepts. Utilizing a novel operator-pencil approach to represent the system dynamics we obtain stability conditions in the form of a generalized lyapunov inequality coupled with an inertia condition. A Kalman-Yakubovic-Popov(KYP)-type lemma is also presented to incorporate performance conditions in addition to stability. These results provide less conservative convex conditions for synthesis of control policies than have previously been available in the literature. This approach can also adapt this work to be applicable for infinite graphs.

7 General Hybrid Systems Theory and Modeling

The focus of the PI's overall research program is automated methods for control analysis and synthesis. The research described in the current section is aimed at answering fundamental questions about model types that are amenable to automated analysis, and in this work a particular new model class is introduced and investigated. The work is reported in the conference manuscripts [1,2,4,12] and the journal manuscripts [9,14,18].

With the wide use of embedded computing and control algorithms for complex systems it is important to have models for the interaction of computer software and the dynamic environment it deals with. A widely

known model for such systems is that of Hybrid automata (HA). Hybrid automata have both discretely and continuously varying states both whose dynamics may be tightly coupled. An analysis or verification problem asks if a given property is satisfied by a given model of a system. Due to the complexity allowed by the general class of Hybrid Automata models, even simple properties such as the reachability problem are known to be undecidable. Previous subclasses of Hybrid Automata that do admit algorithms for temporal property verification require either very simple dynamics for their continuous components or strong resets which decouple the discrete dynamics from the continuous dynamics. These observations, with some exceptions, reinforced the folklore impression that in order to achieve decidability a model has to have either restricted coupling between continuous and discrete dynamics or simple continuous dynamics

In this research component we have shown that decidability can be obtained even when simultaneously allowing complex continuous dynamics and strong coupling between the discrete-continuous interactions. This work utilizes the notion of boundedness in both discrete and continuous dynamics. Defining a class of models in this research it is proved that systems with a bounded-discrete-horizon that are definable in a so-called o-minimal structure admit a finite bisimulation. When the theory is decidable (e.g., semialgebraic FOL) there are algorithms to construct such a bisimulation. It has also be shown how an extended class of models suitable for embedded and realtime systems satisfies these specifications and hence verification of a CTL property in that class is equivalent to verifying a property on a finite automaton which is known to be decidable. This subclass comprised of so-called STORMED hybrid systems (with extensions) which satisfy the following constraints: They have the guards of two discrete transitions separable by some minimum positive distance. Next, they are definable in an order-minimal (o-minimal) theory. Furthermore, the flows (solutions of differential equations) of the continuous states have positive projections on some monotonic direction on which their guards have delimited-ends. It has been demonstrated that the constraints of this subclass are reasonably tight, as relaxations of any of the them yield undecidable models.

In addition to analysis of hybrid models, synthesis of control policies for such systems has also been considered. This was pursued by posing synthesis as a game, and the notion of STORMED hybrid games (SHG) was introduced. The control problem for SHG systems was reduced finding a bisimulation on finite game graphs. It can be shown that this generalizes to a greater family of games, which includes o-minimal hybrid games. Also solved, in the sense of reducing to a so-called effective algorithm, is the optimal-cost reachability problem for weighted SHG.

Acknowledgement/Disclaimer

8   Honors and Awards

During the performance period of this grant the PI, Geir E. Dullerud, received the following honors:

- Semi-Plenary, IEEE Chinese Conference on Decision and Control, Guilin, China, 2009
- Fellow, American Society of Mechanical Engineers (ASME), elected 2011.

9   Personnel Supported

Geir E. Dullerud   Professor, University of Illinois at Urbana-Champaign
Raymond Essick   PhD Student, University of Illinois at Urbana-Champaign
Anshuman Mishra   PhD Student, University of Illinois at Urbana-Champaign

10   References

[1] Vladimerou V., P. Prabhakar, M. Viswanathan, and G.E. Dullerud, "STORMED Hybrid Systems," Automata, Languages and Programming, Lecture Notes in Computer Science, Springer, 2009.

[2] Vladimerou, V., P. Prabhakar, M. Viswanathan, and G. E. Dullerud, "Games in STORMED Hybrid Systems", Hybrid Systems: Control and Computation, 2009.

[3] Zhang., C., K. Chen, and G. E. Dullerud, "Stabilization of Markovian Jump Linear Systems with Limited Information - A Convex Approach", Proceedings of the American Control Conference, 2009.

[4] Prabhakar, P., V. Vladimerou, M. Viswanathan, and G.E. Dullerud, "Tolerant Systems using Polynomial Approximations," Proceedings of the IEEE Real-Time Systems Symposium (RTSS), 2009.

[5] Zhang, C. and G.E. Dullerud, "Uniform Stabilization of Markovian Jump Linear Systems with Logarithmic Quantization - A Convex Approach," EEE Conference on Decision and Control, 2009.

[6] Lee, J.-W. and G.E. Dullerud, "Joint Synthesis of Switching and Feedback for Linear Systems in Discrete Time," Hybrid Systems: Computation and Control, 2011.

[7] Mishra A. and G.E. Dullerud, "On an Operator-pencil Approach to Distributed Control of Heterogeneous Systems," IEEE Conference on Decision and Control, 2011.

[8] Lee, J.-W. and G.E. Dullerud, "Supervisory Control and Measurement Scheduling for Discrete-Time Linear Systems," IEEE Transactions on Automatic Control, 56:873 - 879, 2011.

[9] Vladimerou, V., P. Prabhakar, M. Viswanathan, and G. E. Dullerud, "STORMED Hybrid Games", Theoretical Computer Science, 412:6770-6785, 2011.

[10] Zhang, C. and G.E. Dullerud, "Decentralized Control with Communication Bandwidth Constraints," Distributed Decision Making and Control, Lecture Notes in Control and Information Sciences, Springer-Verlag, 2011.

[11] J.-W. Lee and G.E. Dullerud, "Stabilization of Switched LPV Control Systems," Control of LPV Systems with Applications, Lecture Notes in Control and Information Sciences, Springer-Verlag, 2012; to appear.

[12] Prabhakar P., M. Viswanathan, and G.E. Dullerud, "Pre-orders for Reasoning About Stability," Hybrid Systems: Control and Computation, 2012; to appear.

[13] Essick, R., and G.E. Dullerud, "An Exact Convex Solution to Receding Horizon Control," Proceedings of the American Control Conference, 2012; to appear.

[14] Vladimerou, V., P. Prabhakar, M. Viswanathan, and G. E. Dullerud, "Verification of Bounded Discrete Horizon Hybrid Automata", IEEE Transactions on Automatic Control, 2011; to appear.

[15] Zhang, C. and G. E. Dullerud, "Stabilization of Markovian Jump Linear Systems with Limited Information", submitted to IEEE Transactions on Automatic Control; in revision.

[16] Lee, J.-W., G. E. Dullerud, and P. P. Khargonekar, "Path-by-Path LQG Control of Discrete-Time Switched and Markovian Jump Linear Systems, submitted to SIAM Journal of Control and Optimization.

[17] M. Farhood, Zhe Di, and G.E. Dullerud, "Distributed Control of Linear Time-varying Systems Interconnected over Arbitrary Graphs", submitted to International Journal of Robust and Nonlinear Control.

[18] P. Prabhakar, V. Vladimerou, M. Viswanathan, and G. E. Dullerud, "Hybrid System Verification by Polynomial Approximation", submitted to ACM Transactions on Embedded Computing Systems.

[19] Mishra, A., and G.E. Dullerud, "An Operator-pencil Approach to Distributed Control," in preparation for submission to Automatica.

[20] Essick, R., and G.E. Dullerud, "Receding Horizon Control and Markovian Jump Systems," in preparation for submission to IEEE Transactions on Automatic Control.