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STOCHASTIC-INTEGRAL MODELS FOR PROPAGATION-OF-UNCERTAINTY PROBLEMS IN NONDESTRUCTIVE EVALUATION (PREPRINT)

Jeremy Knopp and Mark P. Blodgett
Materials State Awareness & Supportability Branch
Structural Materials Division

Elias H. Sabbagh, R. Kim Murphy, Harold A. Sabbagh
Victor Technologies, LLC

John C. Aldrin
Computational Tools

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AIR FORCE RESEARCH LABORATORY
MATERIALS AND MANUFACTURING DIRECTORATE
WRIGHT-PATTERSON AIR FORCE BASE, OH 45433-7750
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14. ABSTRACT
Generalized polynomical chaos (gPC) and the probabilistic collocation method (PCM) are finding considerable application to problems of interest to engineers in which random parameters are an essential feature of the mathematical model. So far the applications have been mainly to stochastic partial differential equations, but we extend the method to volume-integral equations, which have met great success in electromagnetic nondestructive evaluation (NDE), especially with eddy-currents. The problems of main interest to the NDE community in this connection are concerned with the issue of 'propagation of uncertainty' when the relevant parameters are not well characterized, or are known simply as random variables. We demonstrate the ideas by considering a metallic surface that has undergone a shot-peening treatment to reduce residual stresses, and has, therefore, become a random conductivity field.

15. SUBJECT TERMS
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Stochastic-Integral Models for Propagation-of-Uncertainty Problems in Nondestructive Evaluation

Elias H. Sabbagh, R. Kim Murphy, Harold A. Sabbagh
ehs@sabbagh.com, kimmurphy1@aristotle.net, has@sabbagh.com

Victor Technologies, LLC
PO Box 7706
Bloomington, IN 47407-7706 USA

John C. Aldrin
aldrin@computationaltools.com

Computational Tools
Gurnee, IL 60031 USA

Jeremy Knopp
jeremy.knopp@wpafb.af.mil

Air Force Research Laboratory (AFRL/RXLP)
Wright-Patterson AFB, OH 45433-7817 USA

Mark P. Blodgett
mark.blodgett@wpafb.af.mil

Air Force Research Laboratory (AFRL/RXLP)
Wright-Patterson AFB, OH 45433-7817 USA

Abstract: Generalized polynomical chaos (gPC) and the probabilistic collocation method (PCM) are finding considerable application to problems of interest to engineers in which random parameters are an essential feature of the mathematical model. So far the applications have been mainly to stochastic partial differential equations, but we extend the method to volume-integral equations, which have met great success in electromagnetic nondestructive evaluation (NDE), especially with eddy-currents. The problems of main interest to the NDE community in this connection are concerned with the issue of 'propagation of uncertainty' when the relevant parameters are not well characterized, or are known simply as random variables. We demonstrate the ideas by considering a metallic surface that has undergone a shot-peening treatment to reduce residual stresses, and has, therefore, become a random conductivity field.

Keywords: volume-integral equations, electromagnetic nondestructive evaluation, generalized polynomial chaos, probabilistic collocation method

1. Introduction

Critical structural components are often treated by shot-peening or low-plasticity burnishing to reduce residual tensile stresses. This process often leaves the surface of the component rough with a profile that can only be defined as a random process. It is necessary to characterize the surface, and the most common method is through the use of eddy-current nondestructive evaluation (NDE). This method measures the conductivity variation, which, because it simulates the rough surface, becomes itself a random field. This paper describes an approach to modeling electromagnetic interactions with a random conductivity field in the context of NDE. The computational engine is based on stochastic volume-integral equations.

2. Volume-Integral Equations

We have introduced the notion of volume-integral equations and their application in eddy-current NDE in a recent series of papers [1-3]. We will remind the reader that the essential feature, for our purpose, is that the 'anomalous region' (the 'flaw') is decomposed into a regular grid of voxels, each of which has uniform conductivity. Because the surface that this grid simulates is random, so, too, is the conductivity variation from voxel to voxel. In other words we have defined a random conductivity field, with each voxel conductivity being a random variable. The discretization process, of course, transforms an infinite-dimensional field (the 'true surface') into a finite-dimensional field, but still with a very large number of random variables. It is this feature that presents a numerical challenge in modeling stochastic integral equations.

3. Theoretical Background

If there are N voxels, then the impedance computed by the volume-integral equation can be written $Z(l, \sigma_1, \cdots, \sigma_N)$, where l is the position of the probe coil, and the average value of the impedance can be written

$$\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} Z(l, \sigma_1, \cdots, \sigma_N) p(\sigma_1, \cdots, \sigma_N) d\sigma_1 \cdots d\sigma_N \ , \tag{1}$$

where $p(\sigma_1, \cdots, \sigma_N)$ is the joint probability density of the conductivity field. N will generally be large, so we must consider a mapping into a lower-dimensional random space. One such mapping is the Karhunen-Loève expansion [4,5]. The discrete version of the K-L expansion states that every random vector, $\mathbf{V}$, has a decomposition

$$\mathbf{V} = \sum_{n=1}^{N} \lambda_n \xi_n \mathbf{v}_n \ , \tag{2}$$

where $\{\lambda_n\}$ and $\{\mathbf{v}_n\}$ are the eigenpair solutions of the covariance matrix of $\mathbf{V}$, and $\{\xi_n\}$ are uncorrelated random variables, $E\xi_m \xi_n^* = \delta_{mn}$. It is hoped that this expansion converges after M terms, where $M << N$ in (1). The $\{\xi_n\}_{n=1}^{M}$ then become the new random variables in expressions such as (1).

We will apply the K-L expansion to the double-exponential measured correlation function, $C(x, x') = \exp(-|x - x'|/L)$, which arises from shot-peened copper for two different peening intensities [6]. The discrete covariance matrix, G_{mn}, for this function is given by [7]:

$$
\begin{aligned}
G_{mn} &= -2L^2 \left(1 - e^{-\delta/L} - \delta/L\right), & m = n \\
&= -2L^2 e^{-(n-m)\delta/L} \left(1 - \cosh(\delta/L)\right), & m < n \\
&= -2L^2 e^{-(m-n)\delta/L} \left(1 - \cosh(\delta/L)\right), & m > n \ ,
\end{aligned}
\tag{3}
$$

where L is the correlation length of the process, and δ is the length of a one-dimensional cell.

Figure 1 shows the normalized eigenvalue spectrum for a 32×32 matrix for three values of the ratio, L/δ, and sample functions for the anomalous conductivity profiles are shown in Figures 2-4 for these same values of L/δ. Sample functions for the case, $L/\delta = 0.1$, are shown in Figure 2, while Figure 3 shows three sample functions for the conductivity profile when $L/\delta = 1$, and Figure 4 shows three sample functions for the conductivity profile when $L/\delta = 3$.
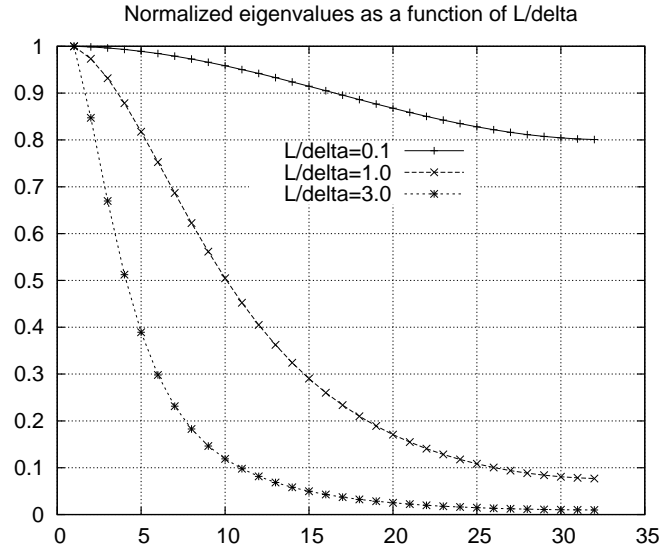
Figure 1: Normalized eigenvalue spectrum for the double-exponential covariance function for three values of the critical ratio, L/δ.
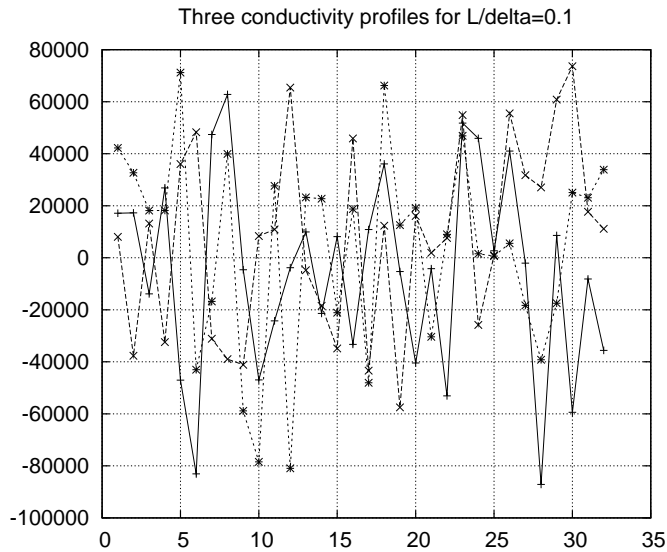

Figure 2: Three sample functions for the conductivity profile when $L/\delta = 0.1$. These functions are the departure from the mean value of $\sigma_{\mathrm{host}} = 3.02 \times 10^5$ S/m. We assume a uniform probability density function, centered at zero and with variance = 1, for the random variables, $\{\xi_i\}$, in the Karhunen-Loève expansion.
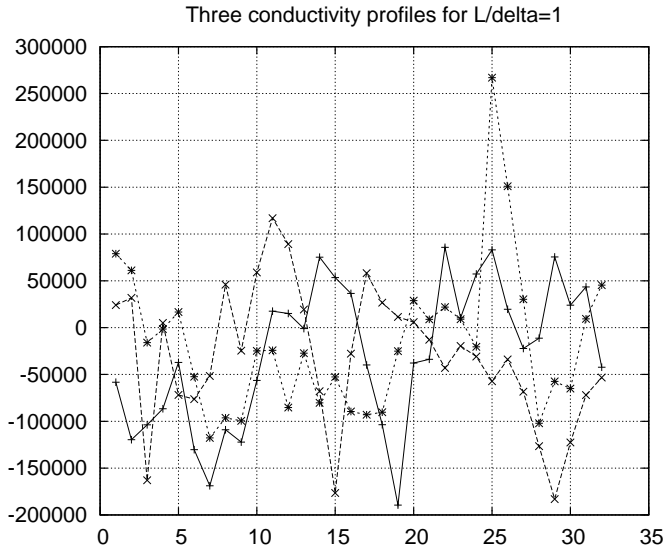
Figure 3: Three sample functions for the conductivity profile when $L/\delta = 1$. These functions are the departure from the mean value of $\sigma_{\text{host}} = 3.02 \times 10^5$ S/m. We assume a uniform probability density function, centered at zero and with variance $= 1$, for the random variables, $\{\xi_i\}$, in the Karhunen-Loève expansion.
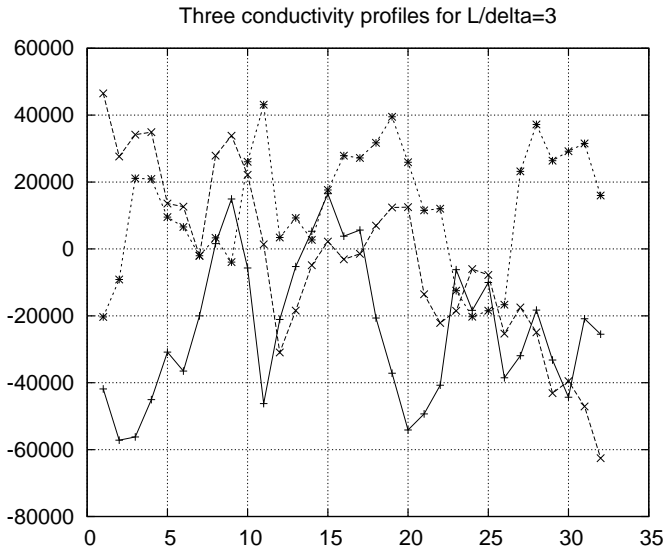

Figure 4: Three sample functions for the conductivity profile when $L/\delta = 3$. These functions are the departure from the mean value of $\sigma_{\text{host}} = 3.02 \times 10^5$ S/m. We assume a uniform probability density function, centered at zero and with variance $= 1$, for the random variables, $\{\xi_i\}$, in the Karhunen-Loève expansion.

4. gPC and PCM

We will assume that the new random variables, $\{\xi_1, \cdots, \xi_M\}$, that are the result of the K-L expansion, are independent, identically distributed random variables (iid), with the common density function, $\pi(\xi)$. The generalized polynomial chaos (gPC) expansion of degree N for $Z(l, \xi_1, \cdots, \xi_M)$ is given by [5]:

$$Z(l, \xi_1, \cdots, \xi_M) = \sum_{0 \leq |\mathbf{i}| \leq N} \hat{Z}_{i_1}(l) \cdots \hat{Z}_{i_M} \phi_{i_1}(\xi_1) \cdots \phi_{i_M}(\xi_M) , \tag{4}$$

where $|\mathbf{i}| = i_1 + \cdots + i_M$ and $\{\phi_k(\xi_i)\}_{k=0}^N$ is the set of univariate gPC basis functions in ξ_i of degree $0 \leq k \leq N$. They are orthogonal polynomials associated with the density function, $\pi(\xi)$, in the sense that

$$E[\phi_m(\xi_i)\phi_n(\xi_i)] = \int \phi_m(\xi)\phi_n(\xi)\pi(\xi)d\xi = \delta_{mn}\gamma_m, \;\; 0 \leq m, n \leq N , \tag{5}$$

where γ_m is a normalizing constant.

With this expansion in hand, we can calculate various statistical properties of $Z(l, \xi_1, \cdots, \xi_M)$ [5]. For example, $[\hat{Z}_0(l)]^M$ is the average value of Z, and the variance and covariance are given by:

$$\begin{aligned}
\text{Var}[Z(l, \xi_1, \cdots, \xi_M)] &= \sum_{0 < |i| \leq N} \gamma_{i_1} \cdots \gamma_{i_M} \hat{Z}_{i_1}^2(l_1) \cdots \hat{Z}_{i_M}^2(l_1) \\
C_Z(l_1, l_2) &= \sum_{0 < |i| \leq N} \gamma_{i_1} \cdots \gamma_{i_M} \hat{Z}_{i_1}(l_1) \cdots \hat{Z}_{i_M}(l_1) \hat{Z}_{i_1}(l_2) \cdots \hat{Z}_{i_M}(l_2) ,
\end{aligned} \tag{6}$$

where the sums exclude $|i| = 0$.

5. HDMR and ANOVA

High-dimensional model representation (HDMR) and analysis of variance (ANOVA) seek to reduce the complexity of problems with a large number of dimensions, and are the subject of considerable contemporary research in computational methods [8-11].

Instead of the gPC expansion of (4), we consider an ANOVA decomposition as (we suppress the scan variable, l, to simplify notation):

$$\begin{aligned}
Z(\xi_1, \cdots, \xi_M) &= Z_0 + \sum_{j_1}^M Z_{j_1}(\xi_{j_1}) + \sum_{j_1 < j_2}^M Z_{j_1, j_2}(\xi_{j_1}, \xi_{j_2}) \\
&+ \sum_{j_1 < j_2 < j_3} Z_{j_1, j_2, j_3}(\xi_{j_1}, \xi_{j_2}, \xi_{j_3}) + \cdots + Z_{j_1, \cdots, j_M}(\xi_{j_1}, \cdots, \xi_{j_M}) ,
\end{aligned} \tag{7}$$

where Z_0 is a constant function, the $\{Z_{j_1}\}$ are one-dimensional functions, $\{Z_{j_1, j_2}\}$ are two-dimensional functions, and so on, yielding 2^M different terms [9]. The various functions are orthogonal to each other (stochastically uncorrelated), which means that Z_0 is the mean value of Z, and the remaining terms are partial variances, whose sum is the total variance of Z.

ANOVA and PCM will be used to compute the stochastic response of the volume-integral equation given random parameters of the anomalous region that evolves from shot-peening or other surface treatment methods. This response will be expressed in terms of average values and variances, or even probability densities. These will have further ramifications in computing probability-of-detection (POD) for various testing setups, as well as inverting measured data to estimate flaw parameters, etc.

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