A Novel Filtering Approach for the General Contact Lens Problem with Range Rate Measurements*

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Abstract – *This paper proposes a Novel filtering algorithm for the general contact lens problem, where the measurement uncertainty region takes a thin, curved, contact lens-like shape in the states' Cartesian coordinates. Such problems have severe measurement nonlinearity and will lead to consistency problems for existing nonlinear filtering techniques such as the extended Kalman filter (EKF) and the unscented Kalman filter (UKF). This problem is very ill-conditioned, which makes it extremely hard and expensive to use a particle filter (PF). In this paper, a General Measurement Adaptive Covariance rule (GMACR) is proposed, for which the consistency of EKF is guaranteed. This leads to a new filtering approach for the general contact lens problem — the General Measurement Adaptive Covariance Extended Kalman Filter (GMAC-EKF). Simulation results show that GMAC-EKF is consistent and has superior tracking accuracy. When the state estimate becomes sufficiently accurate, GMAC-EKF is equivalent to EKF and has the optimal tracking performance.*

The only drawback of the filter is that it has loss in accuracy at the early stage of the filtering due to the artificially enlarged measurement covariance. A hybrid filter combining the alternative extended Kalman filter and GMAC-EKF is also proposed, which yields the best filtering performance.

Keywords: Tracking, nonlinear filtering, general contact lens problem.

1 Introduction

The Kalman Filter (KF) is widely used for various kinds of state estimation problems. It is well known that KF is optimal for Linear Gaussian (LG) systems. For nonlinear systems with non-Gaussian noises, variations of KF, e.g., the Converted Measurement Kalman filter (CMKF) [3], the Extended Kalman Filter (EKF) and the Unscented Kalman Fitler (UKF) have been very successful. However there are exceptions, such as the very long range tracking problem investigated in [11], which belongs to the so called the "contact lens problem" named after the thin, curved, contact lens-like shape of the measurement uncertainty region in the Cartesian coordinates. The curvature of the nonlinear measurement uncertainty region and its impact on various filters was also discussed in [8, 9]. When range rate measurements are also involved, the problem becomes more challenging because the contact lens issue exists not only in the position but also in the velocity. The problem is thus named as the general contact lens problem, which is not covered by the Measurement Covariance Adaptive Extended Kalman Filter (MCAEKF) in [11]. Note that nonlinearities in state estimation problems come from two sources: system dynamics nonlinearity and measurement nonlinearity. In this paper we focus on measurement nonlinearity, which is directly related to the filter update and is the prime cause of filter inconsistency and divergence.[1]

The MCAEKF is proposed for the problem of very long range tracking using range and direction sine measurements from a phased array radar, which is a typical "contact lens problem". In the presence of the severe measurement nonlinearity, the filters above (except CMKF) failed to produce consistent filtering results. CMKF, however, has significant loss in range accuracy. MCAEKF successfully solved the consistency and accuracy problems simultaneously by adaptively changing the measurement covariance matrix to guarantee the consistency of EKF. However, it is limited to the contact lens problem in position and can not be used with range rate measurements.

An important observation from MCAEKF is that, to guarantee the consistency of EKF, the measurement

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[1]The nonlinearity in system dynamics, which is involved only in the track prediction, has been successfully treated in the existing filtering approaches, e.g., the UKF.

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14. ABSTRACT

This paper proposes a Novel filtering algorithm for the general contact lens problem, where the measurement uncertainty region takes a thin, curved contact lens-like shape in the states? Cartesian coordinates. Such problems have severe measurement nonlinearity and will lead to consistency problems for existing nonlinear filtering techniques such as the extended Kalman filter (EKF) and the unscented Kalman filter (UKF). This problem is very ill-conditioned, which makes it extremely hard and expensive to use a particle filter (PF). In this paper, a General Measurement Adaptive Covariance rule (GMACR) is proposed, for which the consistency of EKF is guaranteed. This leads to a new filtering approach for the general contact lens problem ? the General Measurement Adaptive Covariance Extended Kalman Filter (GMAC-EKF). Simulation results show that GMAC-EKF is consistent and has superior tracking accuracy. When the state estimate becomes sufficiently accurate, GMAC-EKF is equivalent to EKF and has the optimal tracking performance. The only drawback of the filter is that it has loss in accuracy at the early stage of the filtering due to the artificially enlarged measurement covariance. A hybrid filter combining the alternative extended Kalman filter and GMAC-EKF is also proposed, which yields the best filtering performance.

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covariances need to be modified to have a less curved uncertainty region in the Cartesian coordinates. In this paper a *general measurement adaptive covariance rule* (GMACR) is proposed, where the measurement is not required to be convertible to the Cartesian coordinates. This leads to a General Measurement Adaptive Covariance Extended Kalman filter (GMAC-EKF) which is shown to be consistent and have superior tracking accuracy. When the track estimate becomes sufficiently accurate GMAC-EKF is equivalent to EKF and has the optimal tracking performance.

The Alternative Extended Kalman Filter (AEKF)[4, 7] was proposed for the state update with range rate measurements. It uses an approximate linearization of the observation function of the range rate measurement, which implicitly assumes that the position errors do not contribute to the errors of the estimated range rate. AEKF, although heuristic, is shown to be much more robust than EKF and has small tracking errors. However, as shown later in this paper, AEKF has consistency problems as the state estimate becomes accurate and it can cause significant loss in range accuracy. Further investigations indicate that it may cause also large errors in crossrange. The strength of AEKF is at the initial stage of the filtering, when the use of range rate measurements will significantly improve the tracking accuracy in the crossrange directions. In this paper, a hybrid filter combining the advantage of AEKF at the early stage and the strength of GMAC-EKF at later stage of the filtering is also proposed, which yields the best filtering performance.

The paper is organized as follows. Sec. 2 illustrates the contact lens problem and shows the challenges in solving it. The performance of EKF, UKF, and AEKF are investigated in a 2-D tracking scenario using range, azimuth and range rate measurements, which show that beyond certain limits these filters will have consistency problems. The necessity of adaptively modifying the measurement covariance for EKF update is also justified. Sec. 3 proposes the GMACR and the GMAC-EKF. Simulation results are presented in Sec. 4. Concluding remarks are given in Sec. 5.

2 The General Contact Lens Problem and Its Challenges

The contact lens problem occurs when states in the Cartesian coordinates are updated with nonlinear measurements from a different coordinate system e.g., the polar or the range–direction-sine (r-u-v) coordinates, where the measurement uncertainty region has (because of the very accurate range) a very thin, curved, contact lens-like shape in the states' Cartesian coordinates. Fig. 2 shows an illustrative example of the uncertainty region of such a measurement.

From basic geometry, the crossrange uncertainty increases with a larger range. However, the uncertainty in

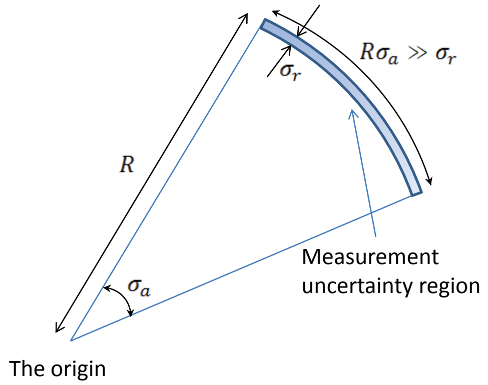


Figure 1: Example: A curved measurement uncertainty region which is very accurate in the range direction, but has large uncertainty in the crossrange.

the range direction is fixed. As the range increases, the measurement uncertainty region takes a curved shape like a contact lens which is increasingly non-Gaussian (non-elliptical) in the Cartesian coordinates. When the range of such measurements is over certain limit, conventional filters, such as EKF, UKF, have been observed to have significant consistency problems [9, 11]. The CMKF is consistent but it has a loss in range accuracy [11]. This is because, to be consistent, the measurement conversion needs to significantly decrease the accuracy in the range direction.[2] The Particle filter (PF) is not suitable for the contact lens problem because of the thiness of the measurement uncertainty region, which will cause severe particle depletion.

Range rate measurements are often available from radars. Like for range measurements, the measurement nonlinearity becomes more severe as the range rate accuracy increases and it can also cause consistency problems for EKF and UKF. To show this, consider a 2-D tracking scenario where the radar obtains range r, azimuth a and range rate $\dot{r}$ measurements of a target. The target follows the continuous white noise acceleration (CWNA) motion model [3] with process noise intensity (PSD) $\tilde{q}$. The state of the target is defined as

$$x = [\xi \ \ \dot{\xi} \ \ \zeta \ \ \dot{\zeta}]' \tag{1}$$

which evolves as

$$x(k+1) = Fx(k) + v(k) \tag{2}$$

where T is the sampling time and

$$F = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

[2]See [11] for an example of this in the measurement conversion from the r-u-v coordinates to the Cartesian coordinates.

The covariance of the process noise in (2) is

$$E[v(k)v(k)'] = \begin{bmatrix} \frac{1}{3}T^3 & \frac{1}{2}T^2 & 0 & 0 \\ \frac{1}{2}T^2 & T & 0 & 0 \\ 0 & 0 & \frac{1}{3}T^3 & \frac{1}{2}T^2 \\ 0 & 0 & \frac{1}{2}T^2 & T \end{bmatrix} \tilde{q} \qquad (4)$$

The nonlinear measurements functions are given by

$$r = \sqrt{\xi^2 + \zeta^2} + w_r \qquad (5)$$

$$a = \arctan \frac{\zeta}{\xi} + w_a \qquad (6)$$

$$\dot{r} = \frac{\xi\dot{\xi} + \zeta\dot{\zeta}}{\sqrt{\xi^2 + \zeta^2}} + w_{\dot{r}} \qquad (7)$$

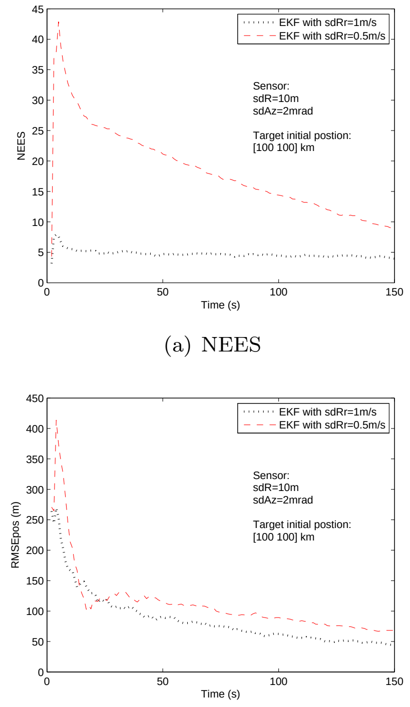
The measurement noises w_r, w_a and $w_{\dot{r}}$ are assumed to be zero mean white Gaussian with standard deviations σ_r, σ_a and $\sigma_{\dot{r}}$ respectively, and for the sake of simplicity it is assumed that the noises are mutually uncorrelated.

Assume that the sensor is at the origin which takes measurements of the target every $T = 1\,\mathrm{s}$, with measurement standard deviations $\sigma_r = 10\,\mathrm{m}$, $\sigma_a = 2\,\mathrm{mrad}$ and $\sigma_{\dot{r}}$ to be specified. The target has initial velocity $[-9\ {-2}]\,\mathrm{m/s}$ and process noise intensity $\tilde{q} = 10^{-3}\,\mathrm{m}^2/\mathrm{s}^3$. The initial position is to be specified in the simulations.

In the first case, EKF (1st order) is used for tracking. Fig. 2 shows the filtering results including the Root Mean Square position Error (RMSposErr) and the Normalized Estimation Error Squared (NEES) [3], which are obtained using 100 Monte Carlo runs. The target starts from [100 100] km. Fig. 2 shows that, when the standard deviation of the range rate (sdRr) measurements is $\sigma_{\dot{r}} = 1\,\mathrm{m/s}$, the EKF already shows inconsistency and has large position errors. When the error standard deviation decreases to $\sigma_{\dot{r}} = 0.5\,\mathrm{m/s}$, the consistency problem became much worse, which leads to the larger errors in position.

For the UKF, the same tracking scenario is used, the target starts further away from the radar at [600 600] km.[3] From Fig. 3 it can be seen that the UKF has consistency problems when $\sigma_{\dot{r}} = 1\,\mathrm{m/s}$. The filter inconsistency becomes more significant as $\sigma_{\dot{r}}$ decreases to $0.5\,\mathrm{m/s}$. In both cases, the UKF has loss in position accuracy compared to the UKF without the range rate measurements. The reason, similar to that in the contact lens problem discussed earlier in this section, is that in the filter update the highly accurate range rate measurement is overly "selective", like the PF, "neglecting" the crossrange position estimates.

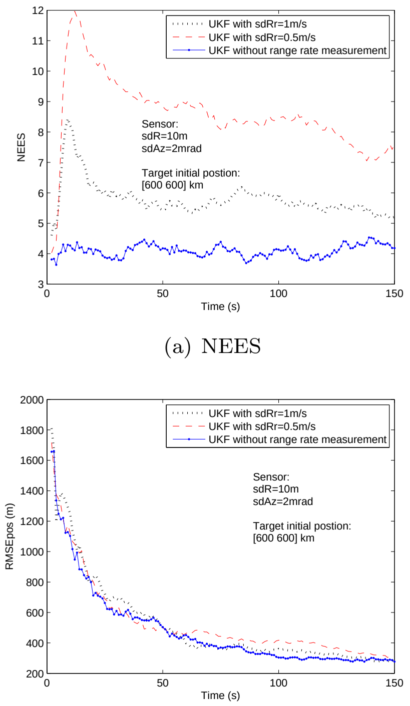
Another filter using the range, azimuth and range rate measurements is the Alternative Extended Kalman filter (AEKF) proposed in [4]. For range and azimuth it uses the converted measurement approach as in CMKF; for the range rate measurement, an approximate lin-



(a) NEES

(b) RMSpos

Figure 2: Case 1: EKF filtering results for various range rate accuracies



(a) NEES

(b) RMSpos

Figure 3: Case 2: UKF filtering results for various range rate accuracies

3 For the radar accuracy considered, the UKF has no consistency problem below this range.

earization of the observation matrix is used. The resulting observation matrix is

$$H_a = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \frac{\mathrm{d}\dot{r}}{\mathrm{d}\dot{x}} & 0 & \frac{\mathrm{d}\dot{r}}{\mathrm{d}\dot{y}} \end{bmatrix} \tag{8}$$

In comparison, the standard observation matrix is

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{\mathrm{d}\dot{r}}{\mathrm{d}x} & \frac{\mathrm{d}\dot{r}}{\mathrm{d}\dot{x}} & \frac{\mathrm{d}\dot{r}}{\mathrm{d}y} & \frac{\mathrm{d}\dot{r}}{\mathrm{d}\dot{y}} \end{bmatrix} \tag{9}$$

Next, the AEKF is compared to the UKF in the same 2-D tracking scenario as above.

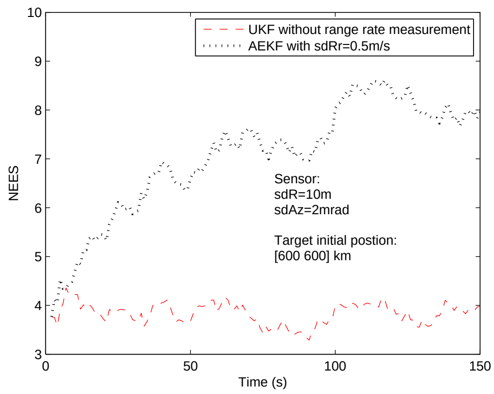


Figure 4: AEKF vs. UKF: consistency test

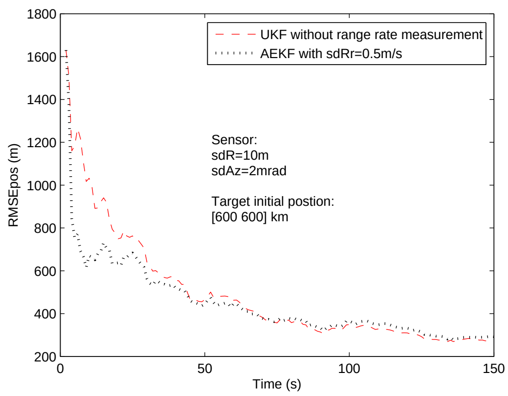


Figure 5: AEKF vs. UKF: RMS position errors

Fig. 4 shows that AEKF shows some degree of inconsistency. In contrast, a standard EKF will quickly diverge in the same tracking scenario, which is not shown in the figure for the sake of clarity. The key for AEKF to be much more robust than EKF is that the cross-correlation between the predicted state estimates and the innovation is calculated using $P(k|k-1)H_a(k)$ instead of $P(k|k-1)H(k)$. This effectively prevents the range rate measurement from being "overly selective" on the position estimates, and improves the consistency of the filter. However, as shown in Fig. 6, the heuristic AEKF has significant loss in range accuracy (which is even worse than the UKF without using the range rate measurements). Further simulations show that the consistency problem of AEKF may lead to large errors in crossrange.

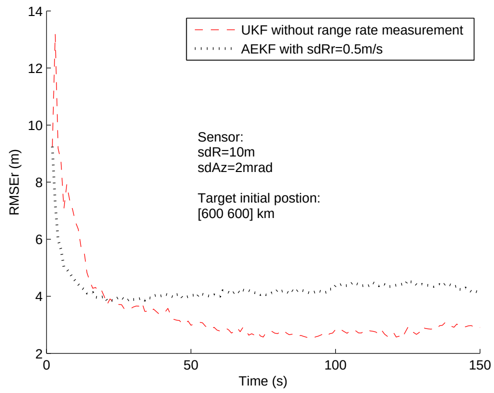


Figure 6: AEKF vs. UKF: range accuracy

Fig. 5 shows that the AEKF has smaller position errors than the UKF (without using range rate measurements)[4] before 70 s. AEKF's fast convergence at the initial stage of the filtering is consistently observed in various simulations. This is because the use of range rate measurements helps reduce the crossrange errors in both velocity and position.

From the above discussions, for the general contact lens problem, the EKF, UKF and AEKF all have their working limits beyond which they will show consistency problems and have loss in accuracy. In the next section the GMACR, which guarantees the consistency of the linearized EKF, is presented. This is then used to develop the GMAC-EKF for the general contact lens problem.

3 The Solution to the General Contact Lens Problem: GMAC-EKF

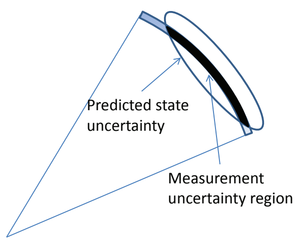
In [11] it has been shown that EKF works very well when the state estimate becomes accurate, and unlike CMKF, a consistent EKF has no loss in accuracy. In fact, the consistency problem of EKF occurs only in the early stage of the filtering when the state estimates are not accurate enough. Fig. 7 gives the geometric explanation for this phenomenon.

[4] As shown in Fig. 3, in the tracking scenario considered, UKF with the range rate measurements has consistency problems and position errors larger than the UKF without the range rate measurements.

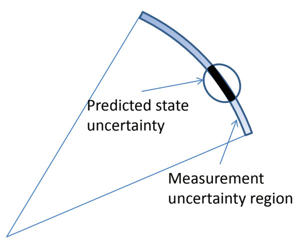
In Fig. 7(a) the intersection of the measurement uncertainty and the predicted state uncertainty, indicated by the dark region, is significantly curved and non-Gaussian (non-elliptical). In this case, the actual posterior uncertainty of the state will also be very different from Gaussian, which is, thus, impossible to be represented by a Gaussian density. For the above reason, in order to keep the "Gaussian based" filters, e.g., the EKF and UKF, consistent, it is necessary to somehow thicken the uncertainty region to make it more Gaussian like. The CMKF does this in its measurement conversion, which significantly decreases the accuracy in the range direction [11]. However, one feature of the linearized EKF is trying to preserve the measurement accuracy and this will cause consistency problems in this case.

On the other hand, if the predicted state estimate is accurate enough, as shown in Fig. 3, the intersection of the uncertainties is not very curved, thus the posterior distribution of the state can be well approximated by a Gaussian density. In this case, EKF has no consistency problem and will yield the optimal filtering performance.

Like MCAEKF, the idea of GMAC-EKF is to allow EKF to thicken the posterior uncertainty when necessary by artificially increasing elements in the measurement covariance matrix before the update. The general Measurement Adaptive Covariance rule (GMACR) presented in the sequel is able to guarantee the consistency of EKF and is applicable to both range and range rate measurements.



(a) Large predicted uncertainty region in the crossrange direction (initial stage of the filtering)



(b) Small predicted uncertainty region in the crossrange direction (when the state estimates become accurate.)

Figure 7: A geometric illustration of the reason of the divergence of EKF

3.1 GMACR

From the above discussion, it can be seen that the key for the general measurement adaptive covariance rule (GMACR) is to characterize and quantify the curvature of the posterior uncertainty region. To illustrate how this can be done, Fig. 8 shows a curved uncertainty region and the Gaussian uncertainty with the same center and thickness. The impact of the curvature at a point in the curved uncertainty region can be characterized by the "distance" D_C from the point to the corresponding Gaussian uncertainty.

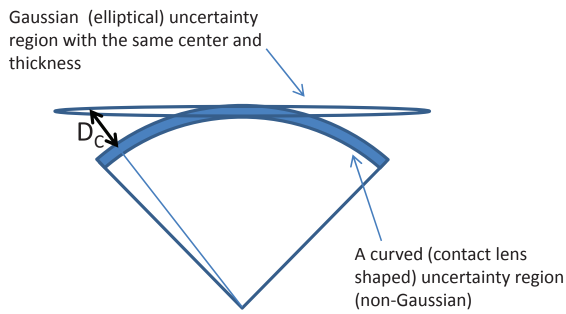


Figure 8: The quantification of the curvature of a curved (contact lens shaped) uncertainty region

To be specific, consider the previously investigated 2-D tracking problem. Let $\hat{x} = [\hat{\xi}\ \hat{\dot{\xi}}\ \hat{\zeta}\ \hat{\dot{\zeta}}]$ denote the center of the two uncertainty regions (the elliptical and the contact lens). Next, the measures of the curvature of the uncertainty regions are defined.

Range measurement

For a point $x = [\xi\ \dot{\xi}\ \zeta\ \dot{\zeta}]$ in the curved uncertainty region, from simple geometry one has

$$r^e = \sqrt{\hat{\xi}^2 + \hat{\zeta}^2} \quad (10)$$

$$r^p = \frac{\xi\hat{\xi} + \zeta\hat{\zeta}}{\sqrt{\xi^2 + \zeta^2}} \quad (11)$$

where r^e is the range at $\hat{x}$, r^p is the range r^e projected on x. From simple geometry one has

$$D_C^r(x) = \frac{(r^e)^2}{r^p} - r^e \quad (12)$$

Range rate measurement

Suppose $\hat{x}$ contains the true velocity. As the position moves across the uncertainty region, the projected range rate also changes. One has,

$$\dot{r}^e = \frac{\hat{\xi}\hat{\dot{\xi}} + \hat{\zeta}\hat{\dot{\zeta}}}{\sqrt{\xi^2 + \zeta^2}} \quad (13)$$

$$\dot{r}^p = \frac{\xi\hat{\dot{\xi}} + \zeta\hat{\dot{\zeta}}}{\sqrt{\xi^2+\zeta^2}} \quad (14)$$

$$D_C^{\dot{r}}(x) = |\dot{r}^p - \dot{r}^e| \quad (15)$$

where $\dot{r}^e$ is the range rate at $\hat{x}$, $\dot{r}^p$ is the range rate being projected on x.

The distance $D_C^*(x)$ (where $*$ stands for r or $\dot{r}$ for range and range rate respectively) can also be interpreted as the error introduced by the corresponding measurement nonlinearity. The curvature of the uncertainty region can be quantified using

$$c_a^* = \frac{\max_{x\in\Phi_a}\{D_C^*(x)\}}{\sigma_*} \quad (16)$$

where $*$ stands for r or $\dot{r}$, Φ_a denotes the predicted uncertainty region that contains most ($100a\%$, with typical $a = 0.95$) of the prior probability. For the posterior uncertainty region to be Gaussian like, its curvature, (w.r.t. range or range rate) should not be higher than a threshold C^*, namely

$$c_a^* \le C^* \quad (17)$$

Formally, the general measurement adaptive covariance rule (GMACR) is as follows:

- Calculate the minimum allowable standard deviation of the measurement noise $\sigma_*^{\min}$ from (17)

$$\sigma_*^{\min} = \frac{1}{C^*}\max_{x\in\Phi_a}\{D_C^*(x)\} \quad (18)$$

- To guarantee the consistency of the linearized EKF, the standard deviation used by the filter update should not be smaller than σ_*^M (19), which is the larger one between $\sigma_*^{\min}$ (18) and the actual measurement standard deviation σ_*.

$$\sigma_*^M = \max\{\sigma_*^{\min}, \sigma_*\} \quad (19)$$

3.2 GMAC-EKF

For the problem of tracking using range and range rate measurements, the GMAC-EKF operates as follows:

- Generate N (e.g., $N = 100$) random samples from the predicted distribution of the states, which yields approximately the set Φ_a.
- Calculate $D_C^r(x)$ (12) and $D_C^{\dot{r}}(x)$ (15) for all the samples.
- Find $\sigma_r^{\min}$ and $\sigma_{\dot{r}}^{\min}$ according to (18). Based on GMACR $\sigma_r^M = \max\{\sigma_r^{\min}, \sigma_r\}$ and $\sigma_{\dot{r}}^M = \max\{\sigma_{\dot{r}}^{\min}, \sigma_{\dot{r}}\}$. Then one has the modified measurement covariance matrix as

$$R_M = \begin{bmatrix} (\sigma_r^M)^2 & & \\ & \sigma_a^2 & \\ & & (\sigma_{\dot{r}}^M)^2 \end{bmatrix} \quad (20)$$

- Do the measurement update using EKF with the modified measurement covariance matrix (20).

Note that the N samples can be generated more efficiently at more desirable locations, which is illustrated in the appendix.

4 Simulation Results

The performance of GMAC-EKF is evaluated using the same tracking scenario as in Sec. 2. The target starts from [600 600] km; the sensor accuracies in range, azimuth, and range rate are $\sigma_r = 1\,\mathrm{m}$, $\sigma_a = 2\,\mathrm{mrad}$ and $\sigma_{\dot{r}} = 0.5\,\mathrm{m/s}$. The threshold (see (17)) for range and range rate are chosen as[5]

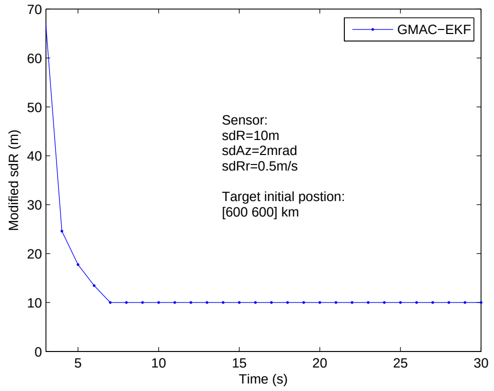
$$C^r = C^{\dot{r}} = \frac{1}{4} \quad (21)$$

Fig. 9 shows the modified range and range rate standard deviations σ_r^M and $\sigma_{\dot{r}}^M$ (19) from the first 30 s in one run of simulation. At the initial stage of the filtering large standard deviations were used because the state estimates were very inaccurate in crossrange. This may temporarily cause some loss of tracking accuracy, however, as shown in Sec. 3, this is necessary for the linearized EKF to be consistent. The modified standard deviations decease sharply as the accuracy of the state estimate increases. After 12 s the actual range and range rate measurement standard deviations are used for the filtering and GMAC-EKF is equivalent to the standard EKF and its performance is practically optimal.

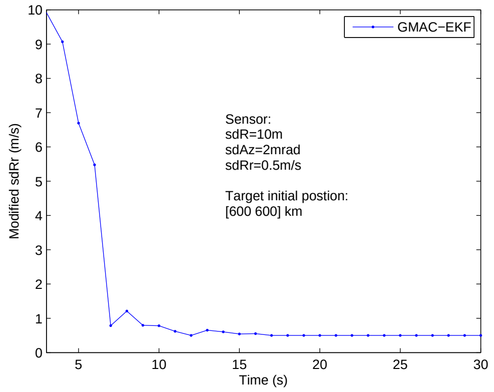
The following simulation results (obtained from 100 Monte Carlo runs) compare the performance of GMAC-EKF, UKF and AEKF. Fig. 10 shows that the GMAC-EKF is consistent, while the other two filters show different levels of inconsistency. Fig. 11 shows that, at the early stage of the filtering (before 30 s), the AEKF has the smallest position errors. This is because that the GMAC-EKF uses artificially increased standard deviations for the range rate measurements (see Fig. 4), which causes the filter to converge slower than the AEKF. Note that the use of the increased range standard deviations causes little loss of accuracy in crossrange. After 30 s, the GMAC-EKF has the smallest position errors, because it is consistent and equivalent to the EKF, thus has no loss in accuracy. Fig. 12 compares the accuracy of the filters in range, before 20 s the GMAC-EKF has some loss in range accuracy due to the increased measurement standard deviations in range, after that it has high accuracy as the UKF. The AEKF, however, has much larger range errors.

From the above discussions it can be seen that the only drawback of GMAC-EKF is the loss of accuracy due to the enlarged measurement covariance at the

[5] For a different problem c_s need to be fine tuned in the filter design process.



(a) Modified range standard deviation



(b) Modified range rate standard deviation

Figure 9: GMAC-EKF: modified standard deviations in range and range rate from the first 30 s of one round of the simulations.

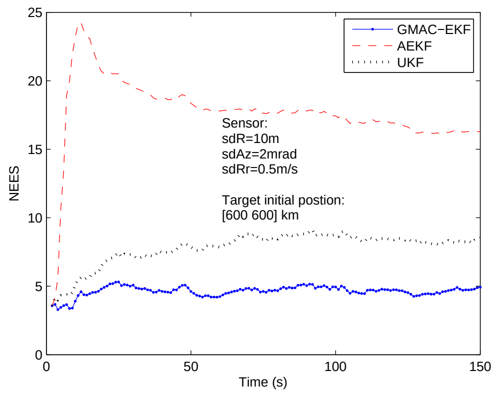


Figure 10: GMAC-EKF vs. UKF and AEKF: Consistency Test

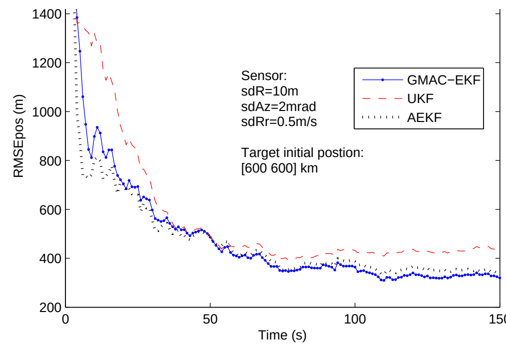


Figure 11: GMAC-EKF vs. UKF and AEKF: Position Errors

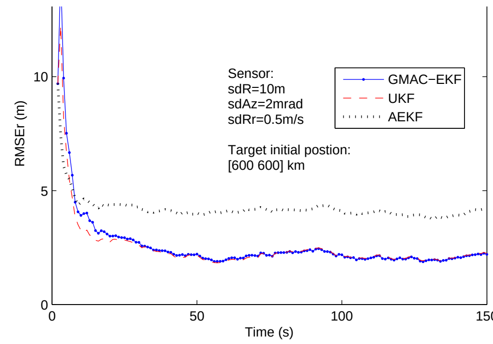


Figure 12: GMAC-EKF vs. UKF and AEKF: range accuracy

early stage of the filtering. While AEKF, although not performing very well in the late stage of the filtering, has the strength of fast convergence in the early filtering stage. Note that the GMACR allows the switching from an arbitrary filter to EKF, thus combine the advantages of the filters. A simple switching rule

- Use AEKF if $\sigma_{\dot r}^{\min} \geq \sigma_{\dot r}$, where $\sigma_{\dot r}^{\min}$ is given in (18); otherwise, use GMAC-EKF.

can be used to combine AEKF and GMAC-EKF. The resulting hybrid filter yields the best filtering results, which is not shown in the paper for the sake of conciseness.

5 Conclusions

This paper investigates the problem of tracking in the presence of severe measurement nonlinearity, which is referred to as the general contact lens problem because of the thin, curved and contact lens-like shape of measurement uncertainty region. One typical problem of this kind is the problem of tracking using highly accurate range and range rate measurements. For this problem, existing nonlinear filters including the extended Kalman filter (EKF), the unscented Kalman (UKF) filter and the particle filter (PF) are shown to have consistency problems. The alternative extended Kalman filter (AEKF) for the fusion of range rate measurements is also investigated. It is shown that AEKF converges very fast at the early stage of the filtering, however, it does not perform very well as the state estimate becomes more accurate.

For the general contact lens problem, a General Adaptive Covariance Rule is proposed, which guarantees the consistency of the linearized EKF. This leads to the General Adaptive Covariance Extended Kalman Filter (GMAC-EKF), which is shown to be consistent and has the optimal performance when it converges to EKF. A drawback of GMAC-EKF is that it may have some loss in accuracy at the initial stage of the filtering due to the artificially enlarged measurement covariance matrix. To fix this, a hybrid filter based on the GMACR, which combines the strength of AEKF and GMAC-EKF, can be used.

This paper answers the key question of how the linearized EKF can be used for nonlinear measurement update without causing consistency problems, which greatly extends the usability of EKF in nonlinear filtering problems.

6 Appendix — An Efficient Way for Generating Samples for the Φ_a Uncertainty Region

Let $\hat{x}_P$ and P_P denote the predicted position and its covariance (Note that for the problem considered velocity states are not needed for the calculation of D_C from (12) and (15).). The sample points are generated as follows:

- Find the orthogonal basis of the hyperplane that is perpendicular to $\hat{x}_P$. (suppose $\hat{x}_P$ is in a space with dimension L, then the dimension of the hyperplane is $L-1$.)
- Generate N random samples $n_s(i), i = 1, ..., N$ in this hyperplane.
- For each $n_s(i)$ do

$$\tau(i) = n_s(i)' P_P^{-1} n_s(i) \tag{22}$$

$$n_s(i) = \frac{4(L-1)}{\tau(i)} n_s(i) \tag{23}$$

  Then the samples are given by $x_s(i) = \hat{x}_P + n_s(i), i = 1, ..., N$.

The above sampling procedure will put the samples at the cross range boundary of the Φ_a uncertainty region, where the impact of the nonlinearity is the most significant. For the special case when L=2, there are only two such $x_s(i)$ which can be directly calculated without using random samples.

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