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14. ABSTRACT

This report details the research accomplishment of the funded project, most notably:
(1) a finite deformation elasticity based derivation of the reduced order modeling equations for cold structures delineating clearly the broad applicability of the approach,
(2) the selection strategy of an appropriate and efficient basis for the representation of the full displacement field of metallic and functionally graded panels, and the extended validation of the concepts to flat and curved panels,
(3) the extension of (1) in the presence of a temperature field itself represented by a modal expansion leading to a combined structural-thermal reduced order model, the latter capturing the heat convection, the selection strategy of an appropriate basis for the representation of the temperature distribution, and the validation of these concepts to steady and unsteady temperature problems,
(4) the inclusion of structural uncertainty in the reduced order model.

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NONLINEAR TRANSIENT THERMOELASTODYNAMICS OF FUNCTIONALLY GRADED PANELS SUBJECTED TO STRONG ACOUSTIC EXCITATIONS

AFOSR Contract FA9550-07-1-0031

FINAL REPORT

The focus of this investigation was on the understanding and prediction of the nonlinear geometric (large displacements) dynamic response of functionally graded panels subjected to the severe acoustic, thermal, and mechanical/aerodynamic excitations expected for hypersonic vehicles. The phenomenological analysis centered more specifically on the differences observed in the response of metallic panels and those functionally graded while a pre-existing reduced order modeling strategy was drastically enhanced to provide the needed, accurate prediction methodology.

The dynamic analysis of hypersonic aircraft panels is a particularly challenging problem owing to (1) the severity of the loading (acoustic, thermal, aerodynamic) that induces large, geometrically nonlinear motions of the structure, and (2) the multi-disciplinary, structural - thermal - aerodynamic coupling present. Each of these two peculiarities of the problem implies that a large computational effort will in general be necessary when applying standard, full order modeling approaches such as finite element methods.

The few years preceding the start of this investigation saw the appearance of structural reduced order modeling methods applicable in the presence of large deformations and build from finite element models developed with standard software (e.g. Nastran and Abaqus) [1-4]. Clearly, these techniques appeared rich in prospect but significant enhancements were needed to invoke their application in the context of functionally graded panels with complex through thickness properties. This observation was particularly true of the P.I.'s original approach [4] which was based on the representation of the full displacement field of the panel, i.e. transverse and inplane, using both linear transverse modes and inplane basis functions referred to as "dual modes" which were extracted from a limited set of nonlinear static displacement fields. The results obtained in [4] were encouraging but clearly lacked the needed accuracy, even for metallic panels.

Given the complex structural makeup of functionally graded panels, it was first questioned whether the *form* of the reduced order model governing equations, i.e. the set of nonlinear ordinary differential equations for the generalized coordinates, used in prior efforts was indeed still applicable. Indeed, these equations with cubic nonlinearity were traditionally justified with the von Karman strain definition and assuming a homogenous and isotropic material. To properly address this first concern, a fresh derivation of these equations was performed, from the equations of finite deformation elasticity in the undeformed configuration and seeking a solution of their week form as an expansion in a time-invariant basis with time-dependent generalized coordinates. This effort [5] demonstrated in particular that the cubic nonlinear differential equations used in earlier investigations were indeed applicable, even to functionally graded structures, provided that the material was linear elastic in the *undeformed* configuration. This property is achieved when the second Piola-Kirchhoff stress tensor is proportional to the Green strain tensor. Such a condition is typically used in finite element computations based on a total Lagrangian formulation. In updated Lagrangian computations, a proportionality between the Almansi strain tensor and the Cauchy stress tensor is often used which implies a nonlinear relation between the second Piola-Kirchhoff stress tensor and the Green strain tensor. In such cases, the differential equations for the generalized coordinates exhibit a much more complex nonlinearity than the cubic one (see discussion in [5]). While this difference has been observed in cantilevered structures (e.g. aircraft wings, see [6]) it seems negligible for the clamped panels on which the present investigation was focused.

Having properly grounded the reduced order modeling strategy on finite deformation elasticity, the investigation proceeded to the second key issue in the successful development of such models, namely the selection of the basis. The "dual modes" introduced in [4], were reconsidered and it was observed that they formed an incomplete basis, not accounting for the inplane displacements induced when the plate motion involves two or more dominant modes. The dual mode basis was then extended [5] to include such responses. A series of validations of this revised reduced order methodology [5] demonstrated the accuracy and computational efficiency of this approach for the prediction of not only the displacements but also the stresses in both static and dynamic cases.

A key feature of the structural makeup of functionally graded panels is the asymmetry of the cross-sectional properties that induces both *linear* and nonlinear (as opposed to purely nonlinear in symmetric panels) coupling between the transverse and inplane components. This property created a new challenge for the transverse linear - inplane dual modes basis which was resolved [5] by keeping the linear modes (no longer purely transverse) and complementing them with dual modes obtained through a proper orthogonal decomposition of a series of static displacement fields induced by forces that produce a linear response only along the transverse modes. Validation examples once again demonstrated [5] the appropriateness of this basis for the accurate prediction of the static and dynamic responses (displacements and stresses) of functionally graded panels.

For simplicity, the above analysis was carried in [5] on flat structures and having obtained excellent results, the second phase of the investigation focused on extending these concepts to curved panels. Most notably, it was desired to demonstrate that reduced order models could be used to predict the occurrence and response in snap-through events, when the panel deflections becomes large enough that a change of sign of the curvature takes place. These snap-throughs lead to peak displacements which may be tens of thicknesses and thus are much larger than those that can be anticipated on flat panels.

After initial difficulties, it was recognized that the linear modes component of the basis was not well suited for predicting the displacement field in the nonlinear regime, even when no snap-through did occur, because it required a significant number of modes with components of the same order of magnitude. In hindsight, a proper orthogonal decomposition of a limited number of static displacements could have been used to replace the linear modes but an analysis of the deflections suggested that the linear modes of the straight panel with identical supports would indeed represent a good alternative. To these linear modes were added dual modes constructed as in [5] but with only component along the tangential direction to the panel.

These efforts, conducted in parallel with the AFRL Structural Science Center (Drs S.M. Spottswood and T.G. Eason), were very successful [7] and led to a very accurate prediction of all displacements in static and dynamic analysis extending in amplitude to post snap-throughs, the first ever such validation of nonlinear structural reduced order models.

These efforts, and those of the AFRL and NASA groups, demonstrated with no uncertainty that these models were indeed the needed, reliable, computational efficient strategy for the prediction of the structural dynamic response of panels. Yet, as briefly discussed above, the panel response (and ultimately fatigue life) is not a pure structural problem but rather a multi-disciplinary one involving aerodynamics, acoustics, and heat conduction/convection. The aerodynamic and acoustic coupling is achieved through the modal forces applied on the structure and thus leads to an expected increased dimensionality of the problem in which, however, the structural block is untouched, similar to its form for the uncoupled structure (e.g. see [8-10]). The coupling with the thermal problem is different and much more complex. Indeed, it is well recognized that temperature affects intimately the behavior of the structure (e.g. by inducing thermal buckling) in addition to being a potential source of excitation.

Practically, this interaction is reflected by a dependence of the parameters of the reduced order models (the linear, quadratic, and cubic coefficients involved in the set of differential equations for the generalized coordinates) on the temperature *distribution* since temperature is expected to vary along and through the panel. Further, this distribution is expected to vary with time due to variations of the aircraft altitude and speed as dictated by the mission profile. In this light, it would seem that the analysis of a particular panel would require the consideration of a large number of reduced order models, maybe with the same basis but with parameters evaluated for a broad array of temperature distributions.

Such a computational alternative is clearly not acceptable and the third phase of the investigation focused on the formulation of an efficient solution to this problem. The approach successfully developed in this investigation is based on the representation of the temperature field in a modal expansion form, similar to the one used for the representation of the structural displacements, with space-dependent time-invariant basis functions and generalized coordinates that only depend on time.

The determination of the combined structural-thermal reduced order model equations was achieved as in [5] for cold structures but with the equations of finite deformation thermoelasticity in the undeformed configuration. The constitutive behavior of the material was specified through the Helmholtz free energy which was postulated to obey a Duhamel-Neumann form in terms of the undeformed variables, i.e. the Green strain tensor, and the temperature. This derivation [11,12] not only provided the necessary differential equations for the structural and thermal generalized coordinates but also clearly highlighted the interactions of the thermal and structural problems.

Specifically, it was first found, as expected, that the parameters of the structural reduced order model depend on the thermal generalized coordinates but only linearly, and in fact only the linear stiffness terms vary with temperature if the unstressed undeformed configuration is selected as baseline for the representation of the displacements. This temperature effect is very significant as it accounts for the occurrence of thermal buckling. A second term, sometimes referred to as “thermal moment term”, was also obtained which acts only as an excitation term (on the right-hand-side of the equations).

While it is traditional to view the coupling between thermal and structural problems as a one-way interaction, the temperature distribution affecting the structural response, this is only approximately so. In fact, the derivation [11,12] demonstrated the presence of two effects of the structural deformations on the heat convection problem, the first one of which is the known latency effect, a small heat flux proportional to the strain rate. The second effect is associated with the deformation-induced change of geometry over which the heat conduction is to be carried out and which is explicitly captured in the present formulation since it is based on the undeformed configuration.

Although the full formulation of the reduced order model equations presented in [11,12] includes both structural effects on the heat convection problem, it can be shown that the latency effect is very small for most metallic and functionally graded materials. Further, the change of geometry effect appears to be small for most panel designs except those with significant undeformed curvature undergoing snap-throughs. Accordingly, these effects were neglected in the ensuing validations which involved first steady temperature distributions which were constant, varying only through thickness, and varying along and through the panel. Further, they were carried out on both metallic and functionally graded panels and in static and dynamic conditions (excited by a time varying acoustic pressure). In each validation case, the temperature and displacement fields predicted by the combined reduced order model were found to be in very close agreement with those predicted by full finite element analyses.

The thermal basis selected for these problems was composed of functions satisfying any imposed temperature boundary conditions complementing the eigenvectors of the linear heat conduction problem with all of them varying through thickness as the steady heat conduction problem. Thus, for purely metallic panels, the temperature basis functions exhibited a linear through thickness dependence but for the functionally graded panels, this distribution was highly nonlinear owing to the dramatic grading-induced variation of the heat convection tensor through the thickness. Note finally that the determination of the parameters of the combined thermal-structural reduced order model was accomplished by extending the approach used for cold structures as originally proposed in [13] and extended in [5].

The final validation study [12, 14] focused on unsteady thermal effects, more specifically on a “rapid” heating scenario in which the temperature distribution through the thickness does not match the steady distribution as in [11]. With an appropriate enrichment of the thermal basis (see discussion of [12,14]), the temperature field was nevertheless very well captured. In regards to the structural problem, it was noted that the time scale of this heating is smaller than the period of the first linear elastic mode and thus is a short pulse loading exciting a significant number of modes.

The fourth and smallest focus of the present investigation was on the introduction of structural uncertainty directly in the reduced order model. This effort was motivated by the inherent variability associated with joined structures and boundary conditions (e.g. of the panel on its stiffeners as in [5]). It was also recognized that functional grading is a new process that may lead to variability both along the surface and from panel to panel. Structural uncertainties can be broadly classified as data uncertainties and/or model uncertainties. Data uncertainties affect only the parameters of the computational model, e.g. the value/distribution of Young’s modulus, coefficient of thermal expansion, etc. in the finite element model. On the contrary, model uncertainties are those that affect the computational model itself, e.g. a difference in the curvature or wavyness of the panel. Since both types of uncertainties are typically present, it was desired to adopt a strategy that could consider both *at the level of the reduced order model* (to avoid the reconsideration of the full finite element model). To the P.I.'s knowledge, there is only one approach satisfying both of these conditions in the case of linear structures, i.e. the nonparametric approach initially devised by C. Soize [15]. A fruitful collaboration with Prof. Soize was developed and led to the extension of the nonparametric methodology for the present class of nonlinear reduced order models of structures [16]. This approach provides a convenient, computationally efficient methodology for the prediction of the effects of structural uncertainty on the nonlinear response of panels.

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