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**LARGE SCALE OPTIMIZATION VIA REDUCED SUB-SPACE  
MULTIPOINT APPROXIMATIONS AND CONTINUOUS SENSITIVITY**

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# LARGE SCALE OPTIMIZATION VIA REDUCED SUB-SPACE MULTIPOINT APPROXIMATIONS AND CONTINUOUS SENSITIVITY

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## Abstract

The objective of this research was to derive, demonstrate, and apply new computational algorithms for the aeroelastic design of coupled structural and aerodynamic models of aircraft. An approximate response surface was derived to accelerate convergence to an optimal design using a trust-region-based sequential multipoint approximation algorithm (MCA). It was shown reduce the number of numerical simulations needed to solve nonlinear optimization problems that entail for a large number of design variables. Continuous sensitivity equations (CSE) for fluid-structure interaction (FSI) were also developed. The CSE and their associated sensitivity boundary conditions were derived for a built-up joined beam structure under transient aerodynamic loads. The local formulation was compared to the total form CSE in terms of derivation, implementation, and results. For built-up structures with stress discontinuity at the joints, the total derivative (domain velocity) formulation of CSE was shown to be easier to implement than the local derivative (boundary velocity) formulation. Finally, the coupled fluid-structure physics and CSE for gust response of a nonlinear joined beam with an airfoil model was posed and solved, using both local and total derivative formulations. The latter was shown to be more accurate. Accuracy of the results was verified by the finite difference method.

## Summary

AFOSR awarded the grant in May 2009. This grant builds upon research begun in an AFOSR/AFIT MOA grant in FY08 that initiated the research topic. The two-year follow-on research effort has further developed the computational algorithms for efficient analysis and design optimization of aeroelastic models in two areas. In the first research area, a multipoint cubic approximation (MCA) for non-linear functions was developed in order to realize computational savings in solving optimization problems [1]. The second research area involved the development of CSE using the least-squares finite element method for the first time [2].

Function and gradient information accumulated from multiple design points during the iteration was used in estimating a reduced Hessian matrix and purely cubic terms in a design sub-space. A numerical optimization procedure was developed based on the sequential MCA, in conjunction with a trust region algorithm. The implementation proved successful in solving several numerical examples and the algorithm appeared scalable to large numbers of variables; however, solution of the intermediate optimization sub-problem proved ineffective, leading to investigation of accuracy [3]. The resulting algorithm was applied to a design problem of interest to the AFRL Air Vehicles Directorate—the minimum drag design of a wing, using continuous control surface deflections, which is being investigated in ongoing research of morphing aircraft. In the

underdetermined case MCA was found to be no more accurate than Kriging or a linear Taylor series [4], although it was accurate along a line from the expansion point to any previous design point. Characterization of MCA accuracy for benchmark problems was completed at the end of the first year [5]. The most effect implementation was a two-step algorithm that first calculates a linear programming (LP) step, followed by a reduced design space optimization that incorporates the LP point in an MCA with appropriate filters to ensure linearly independent basis vectors for the reduced space.

In the second research area, a least-squares, continuous sensitivity equation (LS-CSE) method was developed for shape sensitivity of steady FSI problems [6]. The CSE system and boundary conditions were derived and the problem was posed in first-order, local derivative form. The local derivative formulation marks a significant improvement over previous work and avoids the requirement to calculate computationally expensive mesh sensitivities for shape variation problems. A least-squares finite element method was used to solve both the FSI and sensitivity systems. Results were presented for both the local and total material derivatives and compared to gradients obtained by finite-difference methods. The nonlinear, transient aeroelastic gust response, traceable to a joined wing, was posed and solved [7], marking the first solution for a coupled, nonlinear gust problem. The CSE system for shape parameter variation was also posed and solved using the same computational framework. The FSI and sensitivity solutions were validated using typical section aerodynamics. Additionally, a technique for using nonlinear “black box” solvers to also solve the linear CSE system was developed. This can be done without access to the FSI solver source code, however certain conditions on the system operator must be met. These conditions were described and demonstrated using a least-squares finite element FSI solver.

During the first year of the research effort the LS-CSE was extended from steady to transient problems [8]. In the second year, lessons learned from the least-squares formulation were applied to a Galerkin formulation, which was used to compare alternative CSE formulations [9]. The CSE method in both local derivative form and the total derivative form were derived and solved using  $p$ -version Galerkin finite element method that was also used to solve the analysis problems. For nonlinear problems, when the Newton-Raphson method is used, the tangent stiffness matrix was shown to yield the desired coefficient matrix for solving the linear sensitivity equations in the Galerkin finite element formulation. The CSE method was derived in both local derivative form and in total derivative form for a representative FSI problem. Benchmark examples with analytical sensitivity results were solved with CSE for the purpose of verification. Local derivative form requires the solution of the analysis problems only at the boundaries where the solutions tends not to be as accurate, if the finite element method is used for solving the analysis problems. Also, higher order derivatives of the analysis solution at the boundaries are needed, which are also not as accurate, since these derivatives are obtained by post-processing of the solution. Refining the mesh or increasing the order of shape functions can improve the accuracy of local form sensitivity solution. On the other hand, the total derivative form CSE make use of the information of the analysis solution in the whole domain where the solution is more accurate, and the order of derivative of the solution required is lower than local form. The total form CSE using the domain integration method was demonstrated to be more accurate than the local form. Another

advantage demonstrated was that the total derivative form CSE is more applicable for built-up structures in that its implementation is more general, since no special treatment of the interface conditions is required as shown for the local form. This advantage comes at the expense of requiring the geometric sensitivity of the design velocity throughout the domain.

## **Accomplishments**

Several significant contributions to the state of knowledge were made for the application of continuous sensitivity methods. Although they were posed in support of the sensitivity analysis of FSI problems, some findings are more general and more fundamental than the application to FSI. One is the recognition that the sensitivity system is simpler to pose in local derivative form than in total (material) derivative form. The local sensitivity solution may be transformed to total derivative form as needed. Another contribution is the detailed explanation of how to employ the same code for continuous sensitivity analysis as was used to solve the original, parent analysis problem. This may be done in a “black box” manner without access to the source code. These findings should allow a more widespread adoption of the CSE method. First, because the system of equations is simpler to state for a certain class of problems; second, because special analysis software is unnecessary to solve the sensitivity problem. Other original contributions also were made that advance the state of knowledge.

- 1) Local derivative form of the fluid-structure CSEs avoids the need to calculate and invert fluid mesh sensitivities.
- 2) Explicit explanation was provided to show that local sensitivities for shape variation problems are not unique, but the material derivative is unique.
- 3) Simpler sensitivity boundary condition expression for 2D elasticity problems was derived.
- 4) Most detailed application of continuous sensitivity to the solution of an elasticity problem to appear in the literature was published [2].
- 5) Demonstrated a condensation and recovery method for improving the condition number of least-squares finite element weakly enforced boundary data [10].

The FSI sensitivity analysis is the first known in the literature that studies the nonlinear response to a transient gust load. The ultimate application and goal of this research is that it will permit a computationally efficient analysis method to calculate nonlinear, aeroelastic design gradients that may someday ultimately be used to design and optimize very flexible aerospace structures. The analysis and sensitivity for a series of FSI models was accomplished for a basic example traceable to the joined-wing configuration, which exhibits nonlinear gust characteristics to include buckling. A steady solution, for which an analytic solution exists, was considered, and transient solutions, which were validated using typical section aerodynamics, as well as compressible and incompressible solutions using linear potential and nonlinear full potential flow models [10].

For the first time, CSE were derived and solved for a built-up structure (with discontinuous strain) using both local derivative (boundary velocity) and total derivative

(domain velocity) formulations [9]. Whereas the local derivative formulation lends itself to a simpler derivation of sensitivity partial differential equations, enforcing the CSE boundary conditions at structural interfaces was shown to be more difficult due to the discontinuous sensitivity variables at the interface. For the displacement finite element method this implies that sensitivity variables cannot be assembled at nodes. Rather, the CSE boundary conditions must be treated as additional constraint equations. In contrast, the total derivative formulation allows assembly of sensitivity variables at nodes, since the total derivative variables are continuous. Comparison of sensitivities calculated by the two formulations demonstrated that the total derivative formulation consistently yielded more accurate solutions than the local derivative formulation. The increased accuracy comes at the expense of requiring geometric sensitivity throughout the domain, not only at the boundaries.

Another contribution was the calculation of the CSE solution using the same code as was used to solve the original, parent FSI problem. The local form CSE were implemented and solved in a “black box” solver without access to the source code to numerically solve the sensitivity systems of a linear FSI problem involving a determinate structure. In this initial case, the black box solver was MD Nastran in conjunction with its OpenFSI interface [11] to couple the beam finite element model a typical section airfoil calculation of the aeroelastic loads. Work continues to elucidate the requirements for using a black box solver for nonlinear FSI with indeterminate structures.

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