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**Capability and Development Risk Management in  
System-of-Systems Architectures: A Portfolio  
Approach to Decision-Making**

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# Preface & Acknowledgements

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Welcome to our Ninth Annual Acquisition Research Symposium! This event is the highlight of the year for the Acquisition Research Program (ARP) here at the Naval Postgraduate School (NPS) because it showcases the findings of recently completed research projects—and that research activity has been prolific! Since the ARP's founding in 2003, over 800 original research reports have been added to the acquisition body of knowledge. We continue to add to that library, located online at [www.acquisitionresearch.net](http://www.acquisitionresearch.net), at a rate of roughly 140 reports per year. This activity has engaged researchers at over 60 universities and other institutions, greatly enhancing the diversity of thought brought to bear on the business activities of the DoD.

We generate this level of activity in three ways. First, we solicit research topics from academia and other institutions through an annual Broad Agency Announcement, sponsored by the USD(AT&L). Second, we issue an annual internal call for proposals to seek NPS faculty research supporting the interests of our program sponsors. Finally, we serve as a “broker” to market specific research topics identified by our sponsors to NPS graduate students. This three-pronged approach provides for a rich and broad diversity of scholarly rigor mixed with a good blend of practitioner experience in the field of acquisition. We are grateful to those of you who have contributed to our research program in the past and hope this symposium will spark even more participation.

We encourage you to be active participants at the symposium. Indeed, active participation has been the hallmark of previous symposia. We purposely limit attendance to 350 people to encourage just that. In addition, this forum is unique in its effort to bring scholars and practitioners together around acquisition research that is both relevant in application and rigorous in method. Seldom will you get the opportunity to interact with so many top DoD acquisition officials and acquisition researchers. We encourage dialogue both in the formal panel sessions and in the many opportunities we make available at meals, breaks, and the day-ending socials. Many of our researchers use these occasions to establish new teaming arrangements for future research work. In the words of one senior government official, “I would not miss this symposium for the world as it is the best forum I've found for catching up on acquisition issues and learning from the great presenters.”

We expect affordability to be a major focus at this year's event. It is a central tenet of the DoD's Better Buying Power initiatives, and budget projections indicate it will continue to be important as the nation works its way out of the recession. This suggests that research with a focus on affordability will be of great interest to the DoD leadership in the year to come. Whether you're a practitioner or scholar, we invite you to participate in that research.

We gratefully acknowledge the ongoing support and leadership of our sponsors, whose foresight and vision have assured the continuing success of the ARP:

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- Office of the Assistant Secretary of the Air Force (Acquisition)



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- Director of Open Architecture, DASN (RDT&E)
- Program Executive Officer, Littoral Combat Ships

We also thank the Naval Postgraduate School Foundation and acknowledge its generous contributions in support of this symposium.

James B. Greene Jr.  
Rear Admiral, U.S. Navy (Ret.)

Keith F. Snider, PhD  
Associate Professor



## Panel 3. New Approaches to Reducing Risk in Acquisition Programs

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Wednesday, May 16, 2012	
11:15 a.m. – 12:45 p.m.	<p><b>Chair: Dr. Jomana Amara</b>, Associate Professor, Naval Postgraduate School</p> <p><b><i>Data-Driven Monetization of Acquisition Risk</i></b> Katherine Morse and David L. Drake <i>The John Hopkins University Applied Physics Laboratory</i></p> <p><b><i>Capability and Development Risk Management in System-of-Systems Architectures: A Portfolio Approach to Decision-Making</i></b> Navindran Davendralingam, Muharrem Mane, and Daniel DeLaurentis <i>Purdue University</i></p> <p><b><i>Addressing Risk in the Acquisition Lifecycle With Enterprise Simulation</i></b> Doug Bodner, <i>Georgia Institute of Technology</i></p>

**Jomana Amara**—Dr. Amara, PhD, PE, is an associate professor of economics at the Defense Resources Management Institute at the Naval Postgraduate School in Monterey, California and a Fulbright Scholar. Dr. Amara worked with Shell Oil before joining the Naval Postgraduate School. She currently researches and publishes on international economics, defense economics, health economics, and the economics of the public sector. She has addressed various national and international academic organizations, institutions and conferences. Dr. Amara is the author of the forthcoming book *Economic Development and Post Conflict Reconstruction* and co-editor of *Military Medicine: From Pre-Deployment to Post-Separation*. She has published in numerous peer-reviewed journals. Dr. Amara is a member of the American Economic Association (AEA) and the International Institute of Strategic Studies (IISS). [jhamara@nps.edu]



# Capability and Development Risk Management in System-of-Systems Architectures: A Portfolio Approach to Decision-Making

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## Abstract

In a capability-centered acquisition paradigm, with many interacting and interdependent systems, new approaches are needed for addressing the architecting and acquisition of individual systems to achieve capability targets. Prior research work has explored the use of a Computational Exploratory Model (CEM; Mane & DeLaurentis, 2011) and a Markov network model (Mane, DeLaurentis, & Frazho, 2011) to evaluate complex development networks of system-of-systems (SoS) architectures. The present paper complements this line of work with a portfolio management approach as a decision tool in the acquisition and integration of systems within an SoS context. The approach leverages potential SoS-level capability gains from the integration of individual systems against cost and developmental risks due to system interdependencies. An example application using the Littoral Combat Ship is provided to demonstrate the approach. Congruence of the method in relation to potential benefits of system vendor-level competition in light of open architecture (OA) considerations is also addressed.

## Introduction

A system-of-systems (SoS) consists of a network of operationally and managerially independent systems that work synergistically in achieving an overarching capability (Maier, 1998). This confluence of multiple entities gives rise to an emergent behavior that may not be explicitly apparent from the development of its individual constituents. The establishment of an SoS paradigm has motivated the development of new acquisition strategies and integration of individual systems to better address the issue of achieving a set of overall capabilities instead of requirement-specific metrics. Acquisition efforts of SoS capabilities for projects such as the U.S. Army's Future Combat Systems (FCS; Gilmore, 2006) and the U.S. Coast Guard's Deepwater System have been met with a large degree of developmental difficulties. This includes a combination of vulnerabilities in the developmental stage and poor management oversight that often leads to costly schedule overruns and, ultimately, cancellations. These large-scale systems are often developed incrementally with system-level requirements being the immediate focus of attention. Implicit consideration is given to the overarching objectives of the intended SoS capability. The

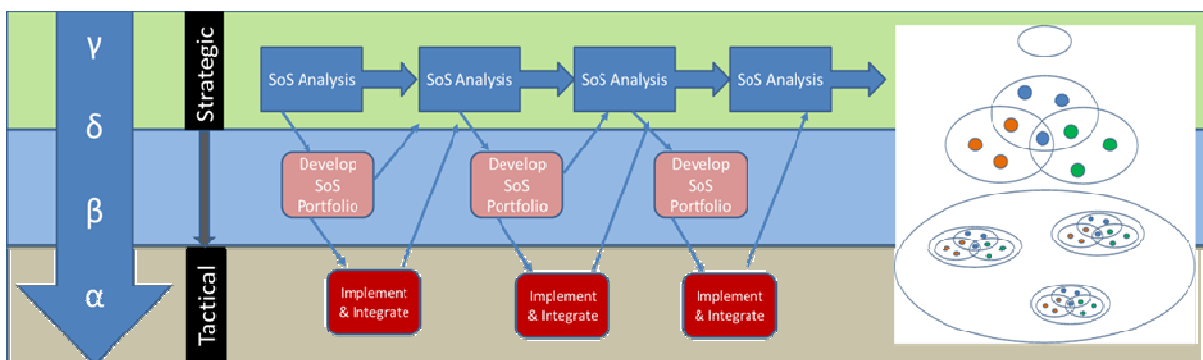


decoupled and decentralized nature of architecting SoS across multiple hierarchies of interdependencies has given rise to a range of inefficiencies and warranted the adaptation of current SE practices to now encompass SoS principles.

The recognition of the need for improved methods in architecting and acquiring systems that comprise an SoS has led to further research in developing frameworks that maximize SoS-wide capabilities while minimizing cost and mitigating risk. Prior funded efforts have introduced the concept of a Computational Exploratory Model (CEM)—a discrete event simulator for the development and acquisition process. The CEM is based on the 16 basic technical management and technical system-engineering processes outlined in the *Defense Acquisition Guidebook*—referred to as the 5000-series guide. The method also considers the modified processes in accordance with the *Systems Engineering Guide for System-of-Systems* (SoS-SE) that adapts the 5000-series guide processes to the SoS framework. Research work in this paper provides a complementary decision-making tool that provides a means of balancing capability development against cost and interdependent risks through the use of modern portfolio theory (MPT).

Portfolio management techniques have been successfully used to address strategic-level asset acquisition and are extendable to include multi-period considerations. *Real options analysis*, for example, has shown effectiveness across various industries to evaluate discrete, long-term investment strategies. The work by Komoroski, Housel, Hom, and Mun (2006) has developed a methodology that addresses strategic financial decisions through an eight-phase process using a toolbox of financial techniques—including portfolio optimization techniques. Such frameworks are geared towards financial uncertainty considerations of strategic projects and do not explicitly address technical architecture and/or evolving SoS-wide capabilities.

Figure 1 is a simplified adaptation of a wave model structure from current literature (Lane, Dahmann, Rebovich, & Lowry, 2010; Dahmann et al., 2011) on SoS artifacts and their employment in the engineering of SoS architectures. The model is adapted here to include hierarchy and time scale that ranges from the broad, overarching objectives that are strategic in nature ( $\gamma$ -level) to the tactical aspects of individual system (and subsystem) acquisition ( $\alpha$ -level). Research in this paper addresses the  $\beta$ -level SoS portfolio development stage that evaluates candidate systems and, consequently, selects a portfolio of interdependent systems to fulfill overarching SoS capability objectives. The idea is to still maintain compliance with the “top-down integration, bottoms-up implementation” paradigm that is part of the wave model implementation. The portfolio method will better assist in the architecting of these SoS constructs at the evaluation phase so as to improve upon decision processes in expanding capabilities.



**Figure 1. SoS Acquisition Hierarchy**



The framework proposed in this paper does not attempt to replace, but rather to complement existing methodologies by more directly addressing issues of integration and acquisition from a *robust portfolio theory* standpoint. Robust methodologies have been widely used by financial engineering practitioners to manage portfolios in the face of market volatility and uncertainties. In the present context, such quantitative guidance is important for providing acquisition groups with the means of performing acquisition, integration, and development decisions in the midst of evolving capability requirements.

## **Development of an Investment Model**

### ***Acquisition Strategy: Investment Portfolio Approach***

A key component of this research is the development of an ability to balance capability and risk in acquiring systems in an SoS context. The investment portfolio approach presented in this section does not attempt to replace but rather complements existing methodologies by more directly addressing issues of integration and acquisition from a robust portfolio theory standpoint. Robust methodologies have been more recently used by financial engineering practitioners to manage portfolios in the face of market volatility and uncertainties. The developed approach in this section is also aimed at improved means of performing acquisition, integration, and development decisions while maintaining advantages in balancing systems acquisition against evolving capability requirements. The research work in this section also addresses more recent efforts in acquisition that have emphasized the implementation of open architectures and modularity to facilitate competition (to lower costs) and innovation.

### ***Open Architectures, Competition, and Modularity***

Open architecture (OA) involves the design and implementation of systems that conform to a common and unified set of technical interfaces and business standards. This form of architecture results in the development of modular systems and increases opportunities for innovation and rapid development of new technologies that can be readily integrated/swapped into current architectures. The Littoral Combat Ship (LCS) program, for example, has recognized the need for multi-vendor acquisitions and OA implementations to ensure greater technological adaptability. The LCS program exploits the benefits of dual-award contracting under fixed-price initiatives (FPI), along with rapid technology insertion processes and open architectures, to fulfill the evolving technological and mission requirements of littoral warfare. The combination of dual contracting and system modularity helps achieve the necessary cost reductions while maintaining a greater degree of adaptability towards changing mission requirements (“LCS,” 2011; GAO, 2007). Although the platform is not, strictly speaking, an SoS, it nevertheless is a representative microcosm of what constitutes an SoS and carries many comparable salient features, such as the confluence of multiple (sub) systems within it that work cohesively to achieve required capabilities.





**Figure 2. Littoral Combat Ship Layout**  
 (“LCS,” 2011)

The benefits of open architectures and competitive contracting are intuitively clear and have been shown to generate notable cost savings as exhibited in previous development projects such as Joint Direct Attack Munitions (JDAM; GAO, 2007). However, system integrators and program managers are often faced with the challenge of leveraging the potential benefits of introducing new and improved systems against potential risks associated with developmental disruptions and cost considerations. Although the LCS program had significant success through the dual-contracting scheme, it still experienced cost overruns due to a variety of problems. The problems included risks from a simultaneous design and build strategy due to schedule constraints, unrealistic budget expectations, and market risk from the greatly increased price of steel during the development period (O’Rourke, 2011). There have also been revisions in the requirements of fleet capabilities and refocusing of intended capabilities (O’Rourke, 2011).

***Concept Acquisition Portfolio: Littoral Combat Ship Example***

The littoral combat ships are designed and developed by two primary contractors—General Dynamics and Lockheed Martin—as a result of the Navy’s dual contract award strategy that seeks to minimize costs through competitive contracting. The ships are designed to serve as primary units in close coastal littoral warfare and take advantage of modularized onboard packages (systems) that are interchangeable for different operational requirements. These packages include the Anti-Submarine Warfare (ASW), Mine Counter Measure (MCM), and Surface Warfare (SUW) packages. More recent developments have seen the introduction of an irregular warfare package for assistance and general support missions. Although the LCS is not, strictly speaking, an SoS, it nevertheless exhibits striking resemblance to one where the conglomeration of systems provide the intended overarching capabilities. The ongoing work in this demonstration assumes a representative acquisition problem using the LCS acquisition case where the objective is to achieve desired combat effectiveness and operational capabilities while minimizing cost and development risk. The simple model inputs and characteristics are described in Table 1.

**Table 1. Individual System Information**

		System Capabilities					System Req.	Develop Time	Acq. Cost	
		Weapon Strike Range	Threat Detection Range	Anti Mine Detection Speed	Comm. Capacity	Air/Sea State Capacity	Air/Sea State	Comm.	(Years)	(\$)
<b>Package</b>										
ASW	Variable Depth	0	50	0	0	0	0	250	3	3000000
	Multi Fcn Tow	0	40	0	0	0	0	150	2	2000000
	Lightweight tow	0	30	0	0	0	0	100	4	4000000
MCN	RAMCS II	0	0	40	0	0	3	200	1	1000000
	ALMDS (MH-60)	0	0	30	0	0	4	100	2	2000000
SUW	N-LOS Missiles	25	0	0	0	0	0	200	3	3000000
	Griffin Missiles	3	0	0	0	0	0	100	4	4000000
<b>Seaframe &amp; Combat Management</b>	Package System 1	0	0	0	400	4	0	0	3	3000000
	Package System 2	0	0	0	300	4	0	0	4	4000000
	Package System 3	0	0	0	250	3	0	0	5	5000000

Table 1 is a hypothetical and simplified catalogue of individual systems available to the Navy in its pursuit of achieving desired capabilities. Although the numbers are hypothetical and do not explicitly illustrate real data, the salient features of considering capabilities, requirements, and risk in acquisition problems are still preserved. Table 1 lists systems that are available for each of the three mission packages—ASW, MCM, SUW—along with an individual rating of system capabilities and requirements for the systems to operate. Additionally, Table 1 provides the system development time and associated acquisition costs. Systems that are unable to provide a particular capability (or do not have a particular requirement) have a zero entry. Although the sea frame is typically a single system, the current sample problem couples the sea frame with battle management software as a base system that provides intra-system capabilities. The development of these systems is based on a projected time schedule that is inherently subject to overruns and risk. This element is captured in the covariance matrix shown in Table 2.

**Table 2. System Interdependency and Development Risk (Covariance)**

	Variable Depth	Multi Fcn Tow	Lightweight tow	RAMCS II	ALMDS (MH-60)	N-LOS Missiles	Griffin Missiles	Package System 1	Package System 2	Package System 3
Variable Depth	0.1	0	0	0	0	0	0	0	0	0
Multi Fcn Tow	0	0.6	0	0	0	0	0	0	0.1	0
Lightweight tow	0	0	0.2	0	0	0	0	0	0	0.2
RAMCS II	0	0	0	0.3	0.1	0	0	0	0.2	0
ALMDS (MH-60)	0	0	0	1	0.1	0	0	0	0	0.3
N-LOS Missiles	0	0	0	0	0	0.5	0.2	0	0.1	0
Griffin Missiles	0	0	0	0	0	0.2	0.3	0	0	0
Package System 1	0	0	0	0	0	0	0	0.5	0	0
Package System 2	0	0.1	0	0.2	0	0.1	0	0	0.3	0
Package System 3	0	0	0.2	0	0.3	0	0	0	0	0.2

Table 2 shows the risk and interdependency aspects of the decision process. The diagonal terms represent the variance (degree of deviation from expected time) in



development time. The off-diagonal terms are the variances due to interdependencies between individual systems that have commonly developed subsystems. For example, since the N-LOS and Griffin missile systems are both developed by Northrop Grumman, it is conceivable that they have common parts or undergo similar processes in development and manufacturing. The covariance value in the cross term therefore represents joint development risk due to interdependencies between two systems.

Estimation of these quantities can come directly from manufacturing and development data. In the case of new systems, the quantities can be estimated heuristically using basic rules similar to those used in project management techniques such as PERT and other CPM methods (Blanchard & Fabrycky, 2005). The entries of the matrix in Table 2 are typically inferred from data; in this case, the values are hypothetically developed for the concept example problem. Most of the individual systems do not bear many interdependencies, with the exception of the sea frame and combat management support systems that are interlinked more explicitly to other listed systems in Table 2.

### **Investment Model Formulation and Solution**

The problem statement for the given acquisition problem is formulated as a mathematical optimization problem, which requires the definition of two primary segments; these are the objective function and constraints. The objective function is the equation that describes the primary metric to be optimized. This typically translates to, for example, the maximization of profits or minimization of costs/risk in the commercial sense. The second important aspect deals with the formulation of constraints, which are equations that typically describe resource (e.g., time and cost) constraints on the system and can be manipulated to reflect the salient conditions of the problem to be solved. The investment portfolio problem presented in the formulation shown below (also known as the Markowitz formulation) seeks to maximize the aggregate capabilities of an SoS architecture, while minimizing the cumulative effect of cost, developmental time, and integration risks. The mathematical model for the concept problem can be written as follows:

$$\max \left( \sum_q \left( \frac{S_{qc} - R_c}{R_c} \cdot w \cdot X_q^B \right) - \lambda \left( X_q^F \right)^T \Sigma_{ij} X_q^F - \sum_q \left( C_q X_q^B \right) \right) \quad (1)$$

$$X_q^F = \frac{X_q^B C_q}{\text{Budget}} \text{ (Portfolio Fractions)} \quad (2)$$

$$\sum_q C_q X_q^B + \varepsilon = \text{Budget} \text{ (Budget Constraint)} \quad (3)$$

$$\sum_q S_{qC} X_q^B \geq \sum_q S_{qR} X_q^B \text{ (Satisfy All System Requirements)} \quad (4)$$

$$X_1^B + X_1^B + X_1^B = 1 \text{ (ASW System Compatibility)} \quad (5)$$

$$X_4^B + X_5^B = 1 \text{ (MCM System Compatibility)} \quad (6)$$

$$X_6^B + X_7^B = 1 \text{ (SUW System Compatibility)} \quad (7)$$

$$X_8^B + X_9^B + X_{10}^B = 1 \text{ (Package System Compatibility)} \quad (8)$$



$$X_q^B \in \{0,1\}(\text{binary}) \quad (9)$$

The mathematical model shown by Equations 1–9 represents the formulation of a traditional single-stage optimization problem that is typical of operations research and financial engineering circles. The current form for the portfolio model at hand is known as a quadratic integer program (QIP) and is based on the Markowitz formulation that seeks to generate optimal portfolios that balance potential expected rewards against risk. Equation 1 is the objective function. The objective is to maximize overall capability while minimizing cost and development risk. Equation 2 is the fraction of the budget invested in individual systems. Equation 3 is the budgeting constraints, where the sum of all investments in individual systems (and savings) must be equal to the total budget allotted. Equation 4 ensures that all requirements of individual systems must be met. Equations 5–7 are the individual system compatibilities. In Equations 5–7, this translates to the selection of one system from each mission package (ASW, MCM, SUW) and a sea frame and combat management package that services the mission modules. These packages are mutually exclusive and, therefore, warrant a total selection of summation equal to 1, which ensures that no two packages per category are selected to satisfy the respective requirements. The covariance matrix, as denoted by  $\Sigma_{ij}$ , represents variations in development time due to system interdependencies. The formulation is amenable to several methods of solution using both freeware and commercially available solvers that are written with system integration and IT considerations in mind. Models using these solver platforms are readily integrated into IT environments and enterprise systems, providing a model-centric environment for the decision-making process.

#### *Investment Portfolio Robustification*

It is well known in financial engineering circles that the Markowitz formulation, as used in the simplified LCS scenario, is sensitive to changes in estimated quantities of the covariance matrix (system interdependencies) and expected return (system performance). The sensitivity due to poor covariance estimations can result in highly inefficient portfolios due to errors in estimation or market shifts. Such sensitivity issues have prompted the development of a variety of robust methods in portfolio analysis to ensure that the chosen portfolio of assets is stable against potential changes in market conditions/expected volatility.

The current portfolio formulation in Equations 1–9 can be reformulated using robust optimization techniques; this includes semi-definite programming (SDP) approaches (Fabozzi, Kolm, Pachamanova, & Focardi, 2007; Tutuncu & Cornuejols, 2007) that are extensions of modern portfolio and control theory. The reformulation allows for possible changes in estimated quantities (e.g., due to market shifts in pricing, volatility, system interdependencies) to be accounted for more explicitly as uncertainty sets. The resulting portfolio allocation will not change appreciably even if salient estimated quantities or benefits change (within prescribed limits). In the context of an acquisition problem, the use of a robust formulation translates to reduced costs associated with capability estimation errors, development time volatility, and changing requirement conditions.

The general form of the portfolio problem in this research can be reposed as a robust optimization problem, given by the following form (Fabozzi et al., 2007; Tutuncu & Cornuejols, 2007):

$$\max_x \left\{ \min_{\mu \in U_\mu} \{ \mu_i x \} - \lambda \max_{\Sigma \in U_\Sigma} \{ x^T \Sigma x \} \right\} \quad (10)$$



$$\mathbf{AX} \geq \mathbf{B} \quad (11)$$

$$\mathbf{CX} = \mathbf{D} \quad (12)$$

$$\bar{\mathbf{\Lambda}} \geq \mathbf{0}, \underline{\mathbf{\Lambda}} \geq \mathbf{0} \quad (13)$$

$$U_{\delta}(\hat{\boldsymbol{\mu}}) = \{\boldsymbol{\mu} \mid \mu_i - \hat{\mu}_i \leq \delta_i, i = 1, \dots, N\} \quad (14)$$

$$\boldsymbol{\Sigma}_{ij}^L \leq \boldsymbol{\Sigma} \leq \boldsymbol{\Sigma}_{ij}^U \quad (15)$$

Although the complexity of the optimization problem increases, it is nevertheless very amenable to a collection of numerical methods that provide good computational performance for realistic portfolio problems, especially portfolios with high volatility (Fabozzi et al., 2007). Equation 10 denotes the robust form of the objective function in Equation 1. Equations 11 and 12 are the generalized linear form that represents the linear relationships in Equations 3–8. Equations 14 and 15 are the uncertainty bounds of the performance (capability) and operational risk due to interdependencies in each system. The portfolio formulation of the LCS sample problem, as shown in Equations 1–9, is rewritten to now incorporate the uncertainties that are associated with estimation of the covariance matrix and the effective capabilities of individual systems. The demonstration LCS problem assumes an uncertain covariance matrix and utilizes the conversion methodology as detailed in literature (Fabozzi et al., 2007) to convert the problem into an SDP. Equations 1–9 are adapted into the robust framework in Equations 10–15 to yield the following SDP:

$$\max \left( \sum_q \left( \frac{S_{qc} - R_c}{R_c} \cdot w \cdot X_q^B \right) - \lambda \left\{ \langle \bar{\mathbf{\Lambda}} \bar{\boldsymbol{\Sigma}} \rangle - \langle \underline{\mathbf{\Lambda}} \underline{\boldsymbol{\Sigma}} \rangle \right\} - \sum_q (C_q X_q^B) \right) \quad (16)$$

$$X_q^F = \frac{X_q^B C_q}{\text{Budget}} \quad (\text{Portfolio Fractions}) \quad (17)$$

$$\sum_q C_q X_q^B + \varepsilon = \text{Budget} \quad (\text{Budget Constraint}) \quad (18)$$

$$\sum_q S_{qC} X_q^B \geq \sum_q S_{qR} X_q^B \quad (\text{Satisfy All System Requirements}) \quad (19)$$

$$X_1^B + X_1^B + X_1^B = 1 \quad (\text{ASW System Compatibility}) \quad (20)$$

$$X_4^B + X_5^B = 1 \quad (\text{MCM System Compatibility}) \quad (21)$$

$$X_6^B + X_7^B = 1 \quad (\text{SUW System Compatibility}) \quad (22)$$

$$X_8^B + X_9^B + X_{10}^B = 1 \quad (\text{Package System Compatibility}) \quad (23)$$

$$\begin{bmatrix} \bar{\mathbf{\Lambda}} - \underline{\mathbf{\Lambda}} & \mathbf{X}_q^F \\ \mathbf{X}_q^F & \mathbf{1} \end{bmatrix} \pm \mathbf{0} \quad (\text{Linear Matrix Inequality}) \quad (24)$$



$$X_q^B \in \{0,1\}(\text{binary}) \tag{25}$$

Equations 16–25 retain most of their original form from Equations 1–9. The exceptions are that the uncertainty in the covariance matrix and absolute value bounds on the capability weighting vector,  $w$ , are reintroduced via exploitation of the dual form of the problem, now an SDP. The reparametrization of the problem as an SDP manifests as additional terms in the objective function and constraints, namely, Equations 24 and 25. Equation 24 is a linear matrix inequality and enforces the condition of positive definiteness due to the symmetry of the matrix and positive values of all variables in the problem.

The uncertainty set of the covariance, as defined in Equation 15, is assumed to be +/- 10% of each respective entry in the matrix. This is arbitrarily chosen for this LCS demonstration problem; however, real-world problems will require the estimation of these bounds from statistical measures, such as through the use of confidence intervals. Additional measures, such as *factor models*, can be used to estimate the values of the covariance with respect to relevant drivers; for an SoS problem, metrics such as the TRL and SRL that are shown to relate directly to project risk may be used.

**Littoral Combat Portfolio: Results**

The presented portfolio optimization problem is modeled and solved by varying the risk aversion parameter,  $\lambda$ , each time to generate the robust performance efficiency frontier. By changing this parameter, the portfolio's aversion to risk is increased as the penalty effect of  $\lambda$  is more pronounced with increasing value. The increase in  $\lambda$  forces the portfolio to select systems that are lower risk and, consequently, results in a lower performance SoS (Figure 3).

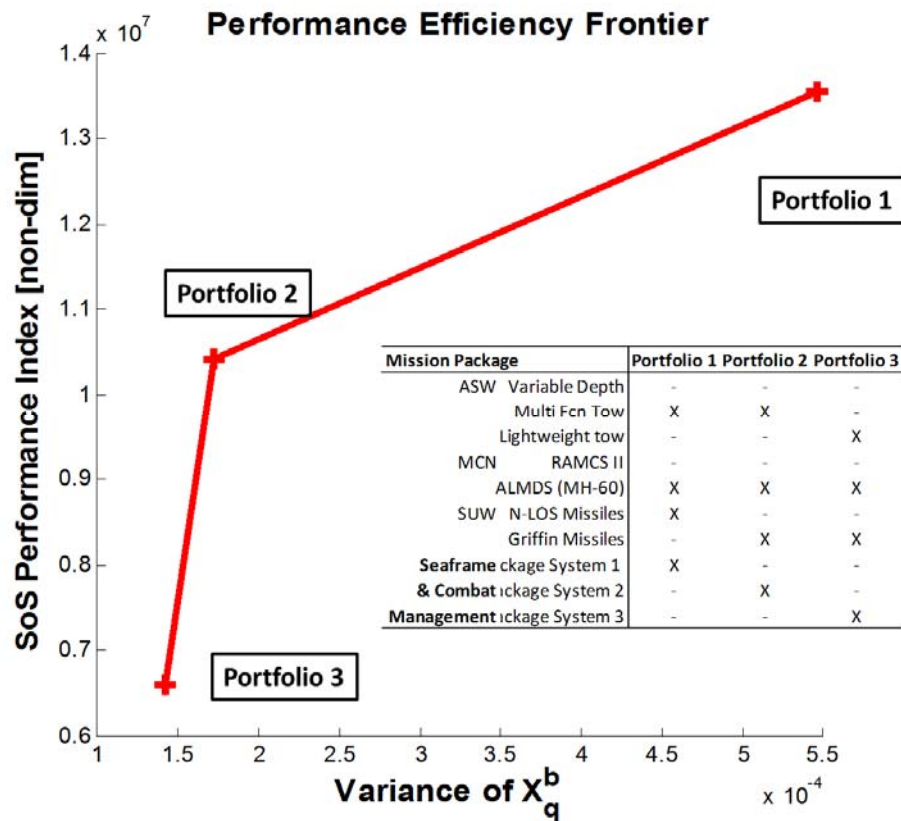
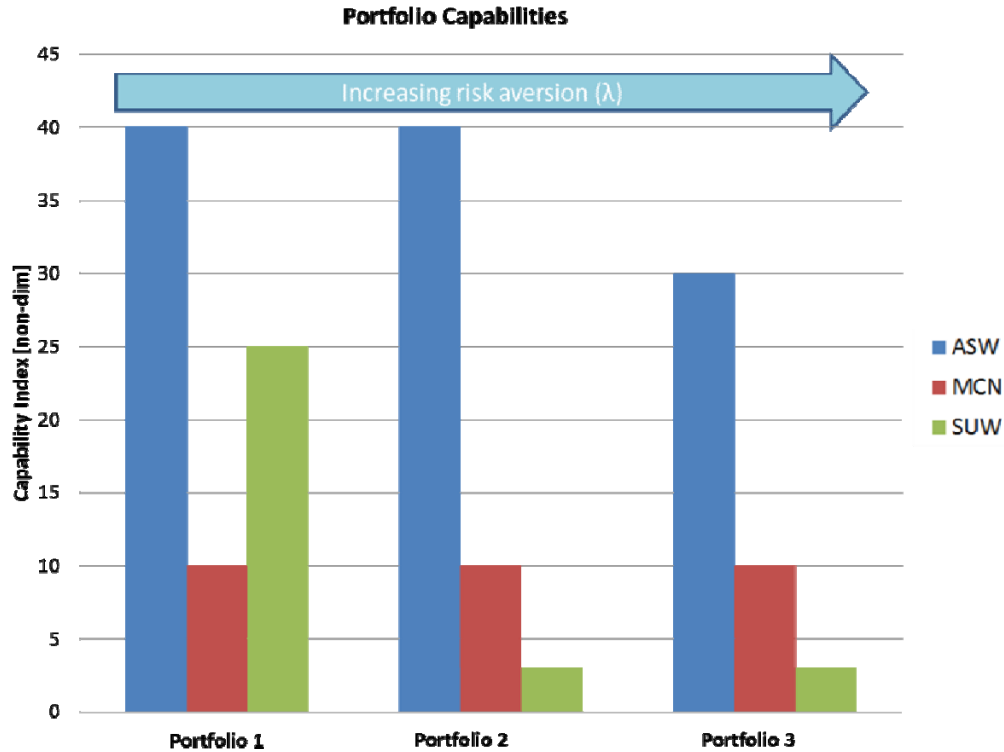


Figure 3. Robust Portfolio Efficiency Frontier





**Figure 4. System Capabilities**

Figures 3 and 4 show the robust efficiency frontier and individual portfolio of capabilities, respectively, for the LCS investment problem. In Figure 3, each point on the frontier is a portfolio that corresponds to a chosen level of risk aversion and shows the amount of variance associated with it. The higher the risk aversion, the lower the expected SoS performance due to the trade-off in choosing say, older, more reliable technology over newer technologies with lower TRL values. The table within Figure 3 shows the portfolio allocation for each of the three critical points on the frontier. Typical efficiency frontiers will have more points due to the combinatorial possibilities of systems available to the portfolio selection process. One system is common across all three portfolios, which indicates that this system has high performance and relatively low risk. Some systems, however, exhibit increased SoS-wide performance, but with added intersystem risk. The trade-off between individual SoS capabilities (ASW, MCN, SUW) and risk is shown in Figure 4. The analysis, as shown in Figures 3 and 4, is useful for acquisition practitioners to determine the appropriate balance of SoS-wide performance against developmental risk.

**Potential Extension: Multi-Period Investment Portfolio**

The general portfolio formulation in the current work considers a static portfolio approach without consideration for sequential, multi-period investment horizons. Strategic decisions are performed through sequential, shorter term acquisitions that incrementally expand the capabilities of an SoS architecture. These shorter term acquisitions need to account for their potential impact on future acquisitions, making it a multi-period investment problem.

The addition of multi-step considerations into the decision process makes the problem amenable to dynamic programming and control theory methods (Powell, 2011). This generally amounts to the objective of the optimization problem being rewritten as the following:





$$\max \left( \underbrace{\sum_q \left( \frac{S_{qc} - R_c}{R_c} \cdot w \cdot X_q^B \right) - \lambda \left( X_q^F \right)^T \Sigma_{ij} X_q^F - \sum_q \left( C_q X_q^B \right)}_A \right) + E \left( A_{t+1} \mid w_{t+1}, \Sigma_{t+1}, \lambda_{t+1} \right) \quad (26)$$

The objective function now reflects consequential effects where current acquisition decisions affect later decisions as denoted by the expectation term of the equation. The stochastic nature of the problem lends itself to being a stochastic optimization problem, which has a variety of efficient and industry-tested solutions, such as approximate dynamic programming (ADP). The current motivation is to, thus, bring these tools to bear upon the immediate acquisition problem, keeping enterprise, model-centric architectures of decision-making processes in mind.

## Conclusions and Future Work

The development of SoS architectures involves a complex process of identifying systems that fulfill mission objectives while mitigating risk. The cascading effects of system interdependencies in an SoS hierarchy requires effective tools to manage the uncertainties in risk estimation to allow for effective acquisition decisions to be made. The robust portfolio framework in this paper addresses the identification of portfolios of systems that can fulfill specified capabilities while taking these estimation uncertainties into account.

A simple representation of the LCS program as an acquisitions problem demonstrates the framework. The objective is to select a portfolio of interconnected (compatible) systems that fulfill overarching capability objectives under acceptable operational risk. The resulting efficiency frontier of three portfolios shows how certain systems are more prone to contributing to overall SoS risk than others due to interconnectivity, performance, and risk characteristics within each system. The analysis enables acquisition practitioners to select portfolios of systems that maximize performance at accepted levels of risk under conditions of uncertainty.

Further work is to be directed at extension of the current framework to multi-period considerations and alternate measures in the objective and constraints to account for additional SoS architecting metrics. The sequential nature of capability expansion, evolving requirements, and changing market conditions makes such extensions a natural complement to addressing these issues.

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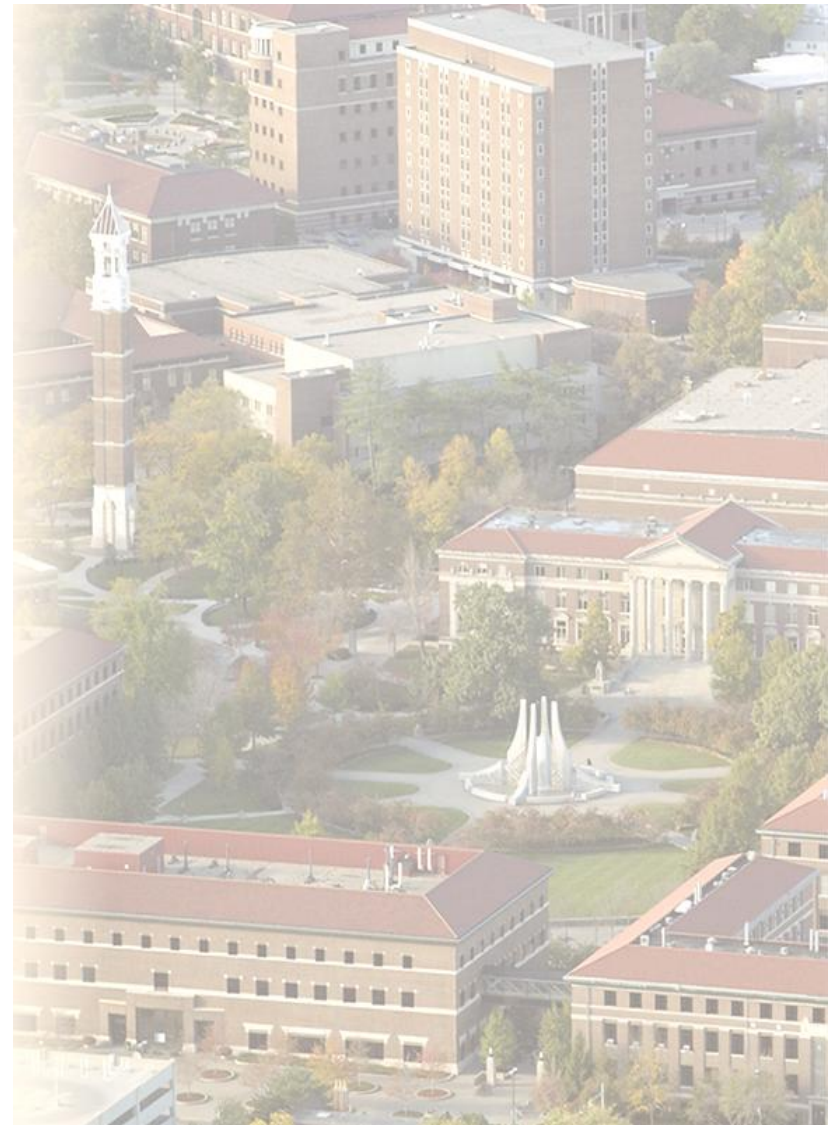
[www.acquisitionresearch.net](http://www.acquisitionresearch.net)

# Capability and Development Risk Management in System- of-Systems Architectures: A Portfolio Approach to Decision Making

**NPS Acquisition Research  
Symposium**  
16-MAY -2012

Dr. Daniel DeLaurentis , Dr. Muharrem Mane,  
Dr. Navindran Davendralingam

School of Aeronautics & Astronautics  
and System-of Systems Laboratory  
Purdue University

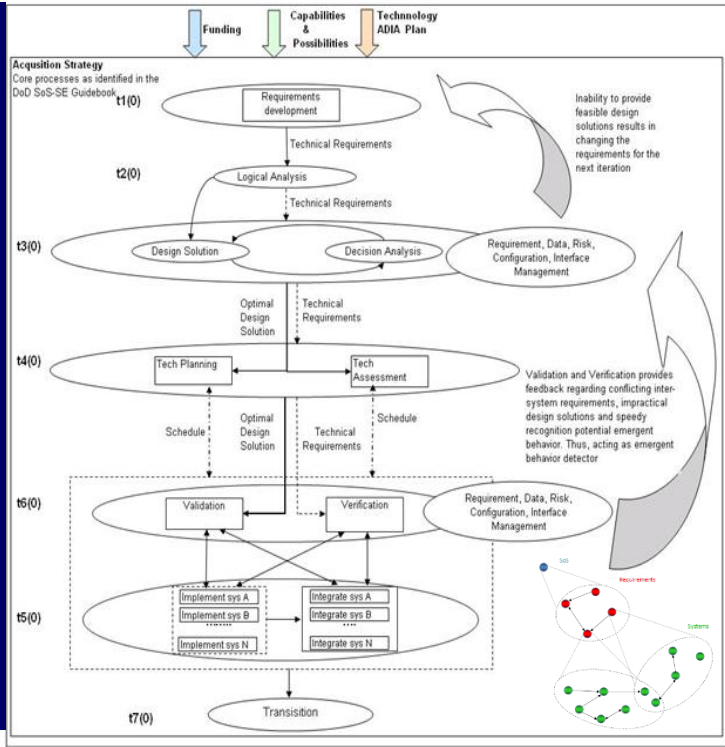


# Presentation Outline

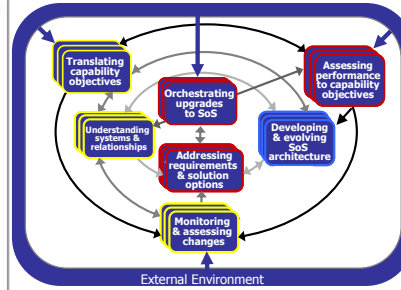
- The Big Picture
- SoS Architecting and Acquisition: Wave Model context
- An Investment Portfolio Approach
  - Mean Variance Approach
  - Mean-Variance: A Robust Version
- Concept Problem: Simple Littoral Combat Ship (LCS)
  - Robust Portfolio application
  - Multiple risk measures
- Future Work

# The Big Picture ...

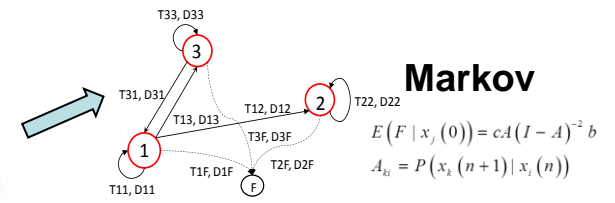
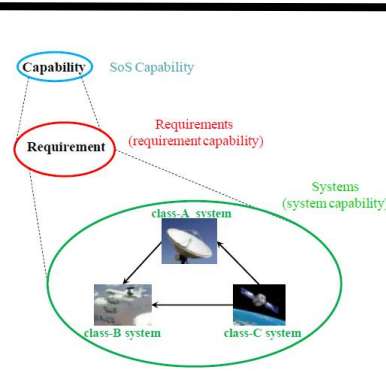
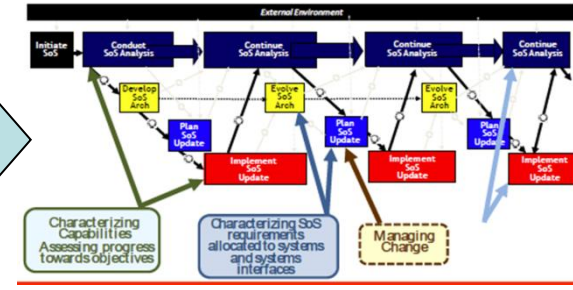
## DoD SoSE Guidebook



## Trapeze



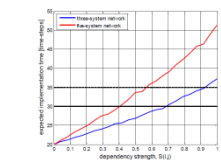
## Wave Model



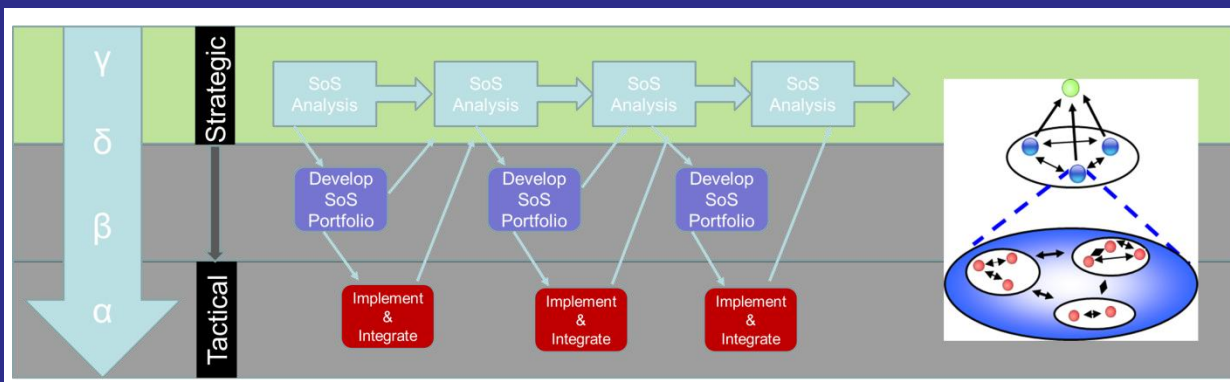
**Markov**

$$E(F | x_j(0)) = cA(I - A)^{-2} b$$

$$A_{ij} = P(x_i(n+1) | x_j(n))$$

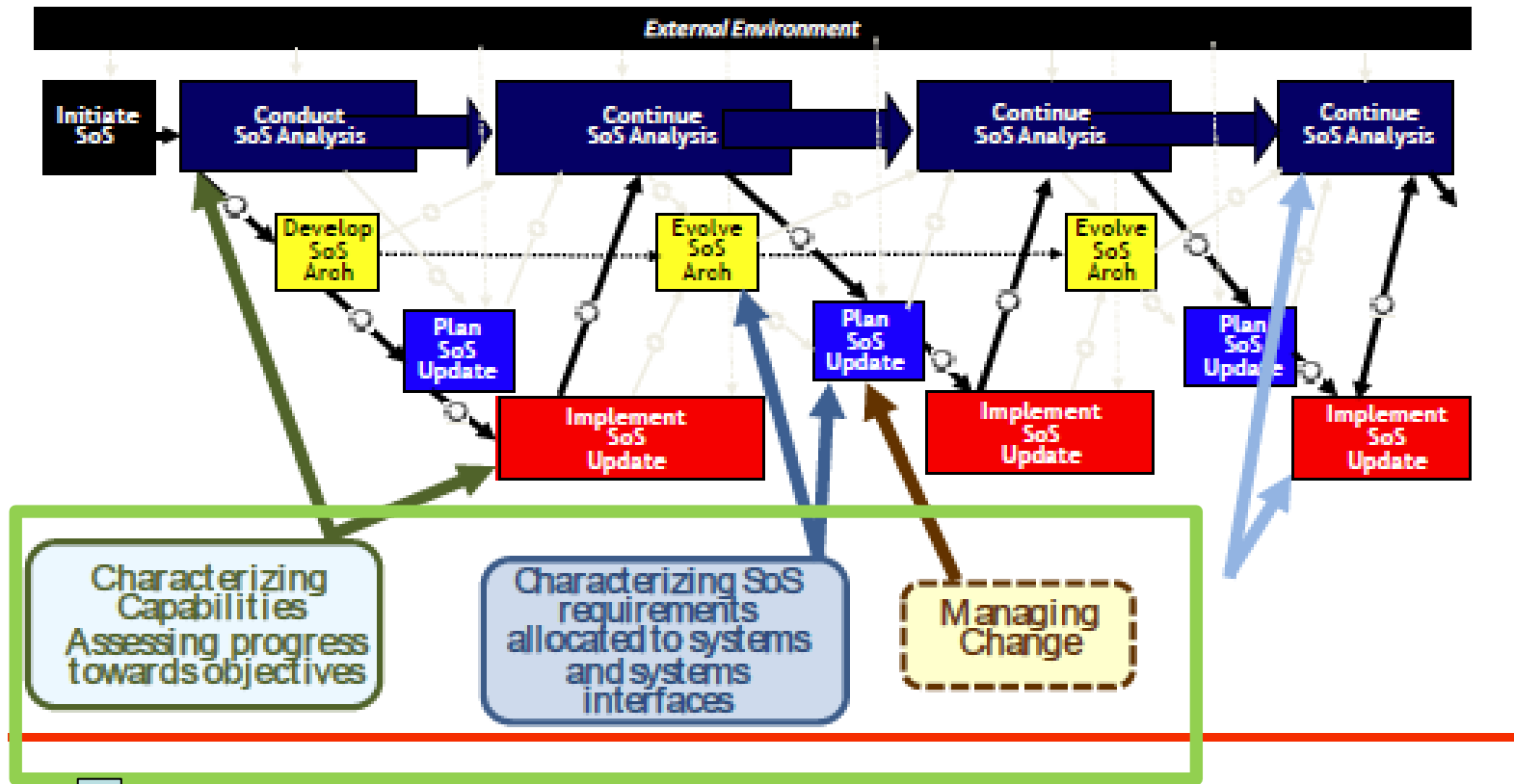


## CEM



Methods	Nature	Inputs
CEM	Discrete Event Simulation	Probabilities, Connectivities
Markov	Probabilistic Graphical	Probabilities, Connectivities
Bayesian Network (BN)	Probabilistic Graphical	Conditional Distribution Connectivity
Portfolio Approach	Decision/Analysis based	Capabilities, Requirements, Connection rules

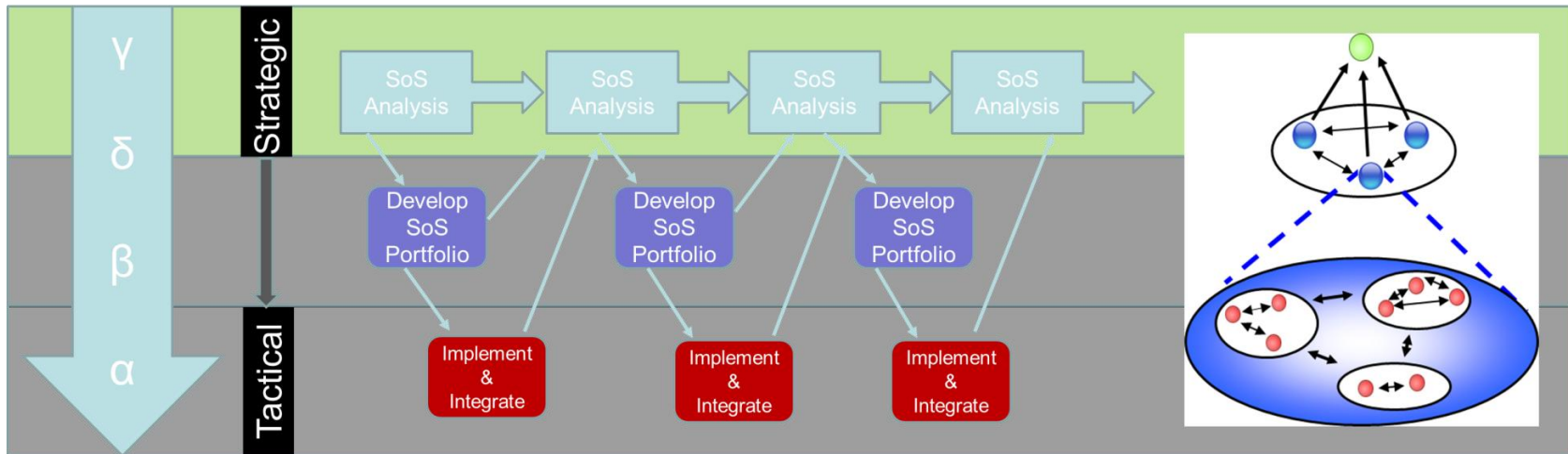
# Wave Model\*: SoS Architecture Development



How do we support these actions for SoS acquisitions?

\*adapted from Dahmann et. al, "Integrating Systems Engineering and Test & Evaluation in System of Systems Development" IEEE Vancouver, 2011

# Wave Model: Acquisition and Architecture

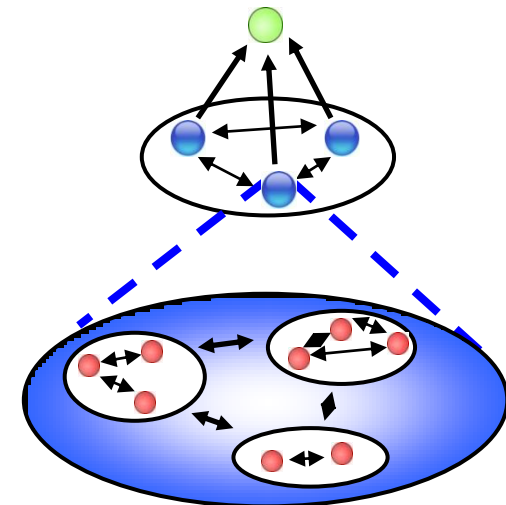
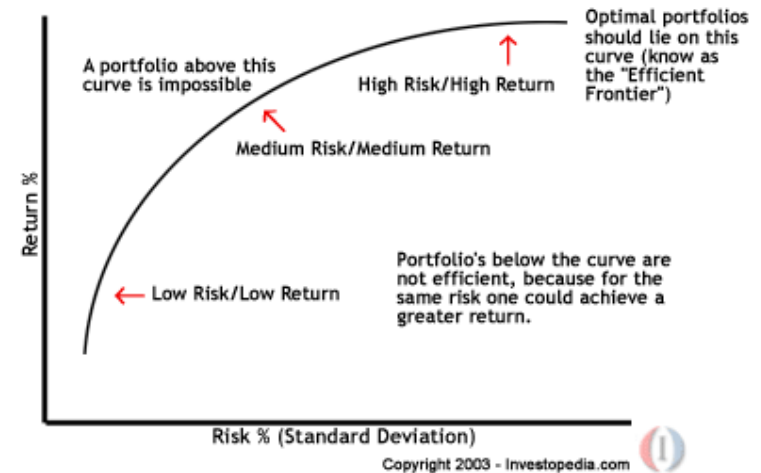


- How to leverage acquiring capabilities against associated risk?
  - Evolving requirements, Open Architectures (OA)
- What about system interdependencies?
- What about acquisition uncertainty considerations?
  - SRL, TRL, operational/developmental characteristics



# A Portfolio Approach: Background

- Classical **Mean-Variance optimization** among techniques adopted by financial engineering and operations research.
- Balance **expected profit (performance) against risk (variance)** in investments
- Generates efficiency frontier of optimal portfolios given investor risk averseness
- Extends current frameworks (Housel, Mun, et.al)
- Systems (nodes) can be modeled as potential investment assets → how do we invest?



Nodes = systems

# Mean-Variance Portfolio Approach

## Objective

Maximize Performance Index

$$\max \left( \sum_q \left( \frac{S_{qc} - R_c}{R_c} \cdot w \cdot X_q^B \right) - \lambda \left( X_q^F \right)^T \Sigma_{ij} X_q^F - \sum_q \left( C_q X_q^B \right) \right)$$

Capability
Risk
Cost

Portfolio Fraction

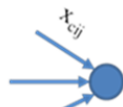
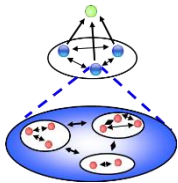
$$X_q^F = \frac{X_q^B C_q}{\text{Budget}} \text{ (Portfolio Fractions)}$$

Portfolio Total Budget

$$\sum_q C_q X_q^B + \varepsilon = \text{Budget} \text{ (Budget Constraint)}$$

Requirements Satisfaction

$$\sum_q S_{qc} X_q^B \geq \sum_q S_{qR} X_q^B \text{ (Satisfy All System Requirements)}$$



Capability

Requirement

Selection Rules (Compatibility)

$$X_1^B + X_1^B + X_1^B = 1 \text{ (ASW System Compatibility)}$$

$$X_4^B + X_5^B = 1 \text{ (MCM System Compatibility)}$$

$$X_6^B + X_7^B = 1 \text{ (SUW System Compatibility)}$$

$$X_8^B + X_9^B + X_{10}^B = 1 \text{ (Package System Compatibility)}$$

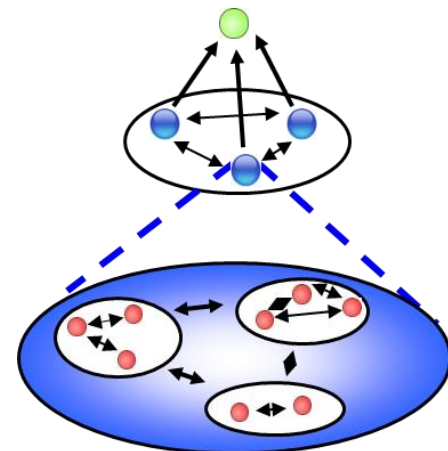
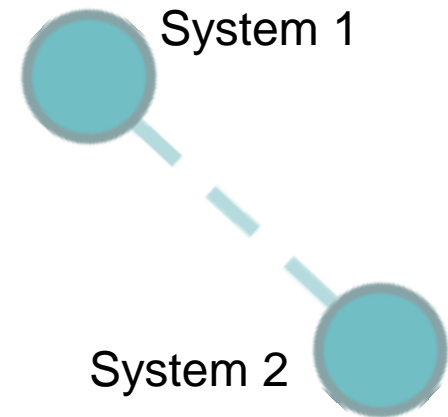
Uncertainty in Covariance  
(Interdependencies)

$$\Sigma_{ij}^L \leq \Sigma \leq \Sigma_{ij}^U$$

Constraints

# Portfolio Uncertainty

- Sources of uncertainty
  - **System Capability:** Actual performance of system individually and as a whole SoS entity
  - **System Interdependence:** Interdependency variances/covariances?
  
- Addressing uncertainty
  - Operations Research/Financial Engineering Methods to address uncertainty measures
  - Introduce uncertainty in interdependencies and individual asset performances
  - Introduce SoS connectivity in portfolio space



# Mean-Variance Portfolio: Robust Approach

## Objective

Maximize Performance Index

Capability	Risk	Cost
$\max \left( \sum_q \left( \frac{S_{qc} - R_c}{R_c} \cdot w \cdot X_q^B \right) - \lambda \{ \langle \bar{\Delta \Sigma} \rangle - \langle \Delta \Sigma \rangle \} - \sum_q (C_q X_q^B) \right)$		

Portfolio Fraction

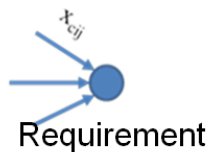
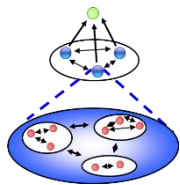
$$X_q^F = \frac{X_q^B C_q}{\text{Budget}} \text{ (Portfolio Fractions)}$$

Portfolio Total Budget

$$\sum_q C_q X_q^B + \varepsilon = \text{Budget} \text{ (Budget Constraint)}$$

Requirements Satisfaction

$$\sum_q S_{qc} X_q^B \geq \sum_q S_{qR} X_q^B \text{ (Satisfy All System Requirements)}$$



Selection Rules (Compatibility)

$$X_1^B + X_1^B + X_1^B = 1 \text{ (ASW System Compatibility)}$$

$$X_4^B + X_5^B = 1 \text{ (MCM System Compatibility)}$$

$$X_6^B + X_7^B = 1 \text{ (SUW System Compatibility)}$$

$$X_8^B + X_9^B + X_{10}^B = 1 \text{ (Package System Compatibility)}$$

Reformulate as SDP  
(Tutuncu & Koenig 2004)

$$\begin{bmatrix} \bar{\Lambda} - \Delta & X_q^F \\ X_q^F & 1 \end{bmatrix} \succeq 0 \text{ (Linear Matrix Inequality)}$$

$$X_q^B \in \{0,1\} \text{ (binary)}$$

Constraints

# Robust Portfolio Case Study: Simple LCS Portfolio

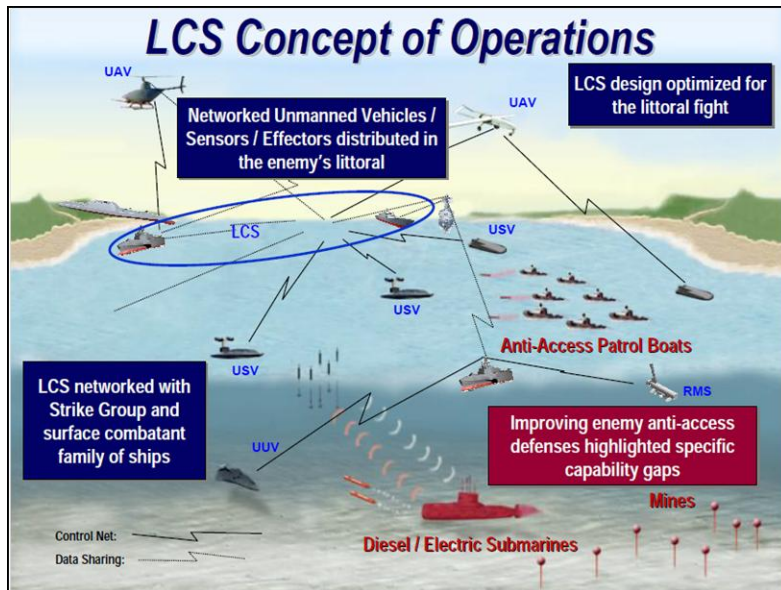


Table 2: System interdependency and development risk (covariance)

	able Depth	ti Fcn Tow	lightweight tow	CS II	IDS (MH-60)	OS Missiles	fin Missiles	age System 1	age System 2	age System 3
Package System 2	0	0.1	0	0.2	0	0.1	0	0	0.3	0
Package System 3	0	0	0.2	0	0.3	0	0	0	0	0.2

Diagonal : System Variance  
Off Diagonal : System Interdependency

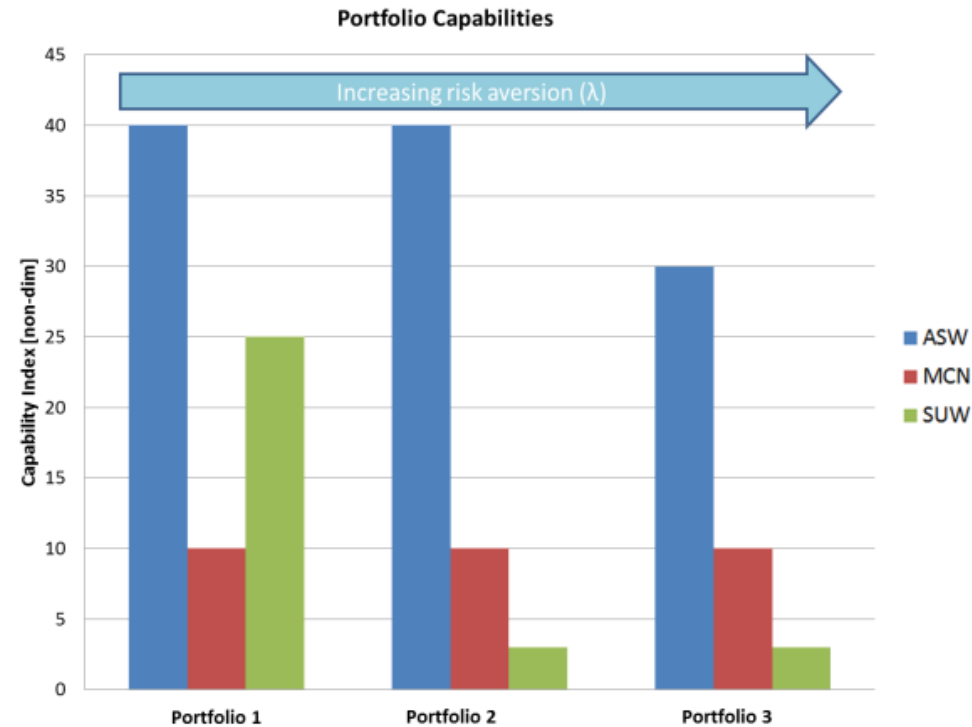
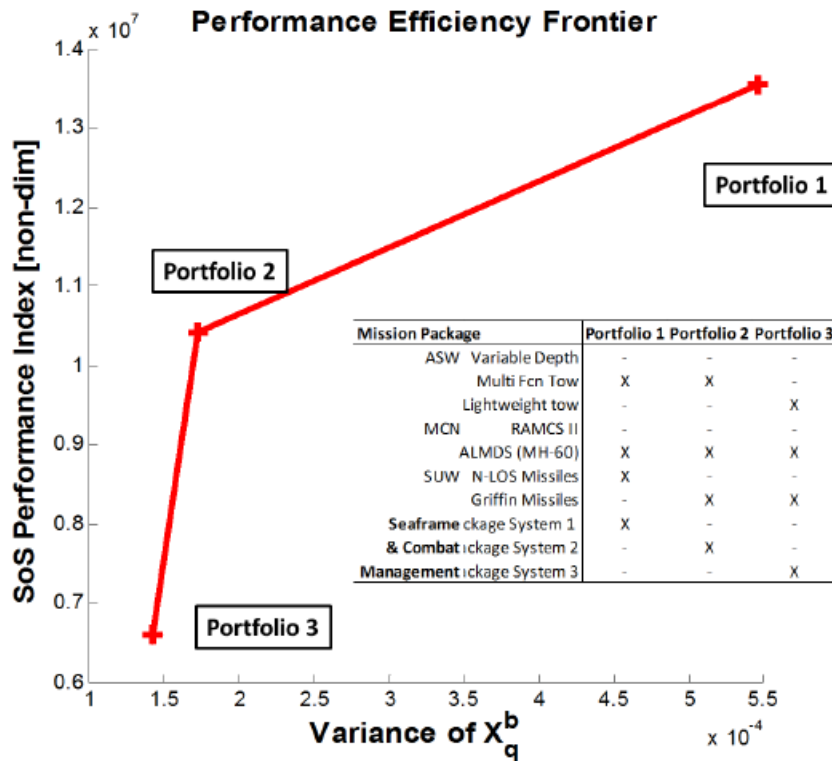
Table 1: Individual system information

Package	System Capabilities			System Req			Develop. Time (Years)	Acq. Cost (\$)
	Weapon Strike Range	Threat Detection Range	Anti Mine Detection Speed	Comm. Capacity	Air/Sea State Capacity	Air/Sea State		
ASW	Variable Depth	0	50	0	0	0	3	3000000
	Multi Fcn Tow	0	40	0	0	150	2	2000000
	Lightweight tow	0	30	0	0	100	4	4000000
MCN	RAMCS II	0	40	0	0	200	1	1000000
	ALMDS (MH-60)	0	30	0	0	100	2	2000000
SUW	N-LOS Missiles	25	0	0	0	200	3	3000000
	Griffin Missiles	3	0	0	0	100	4	4000000
Seaframe	Package System 1	0	0	0	4	0	3	3000000
& Combat	Package System 2	0	0	0	4	0	4	4000000
Management	Package System 3	0	0	0	250	3	5	5000000

Capability (highlighted in red box)  
Requirement (highlighted in blue box)

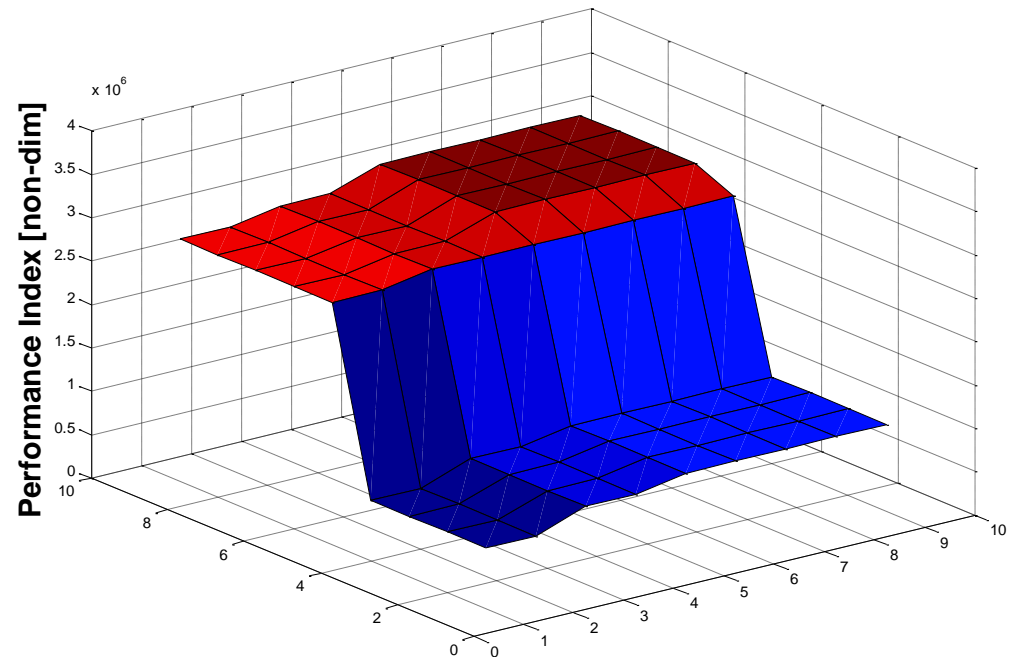
Image from: Presentation slides by RDML Vic Guillory of OPNAV at Mine Warfare Association Conference (titled "Littoral Combat Ship", 08-May-07)

# Robust Portfolio Case Study: Simple LCS Portfolio



# Portfolio Approach: LCS Multiple Risk Measures

- Layered measure of risk (e.g. weapons vs. communications layer).
- Separate covariance for each measure of risk



Variance Risk Measure (Comm)

Variance Risk Measure (Weapons)

Comm. Variance (Risk) Constraint

Weapon Variance (Risk) Constraint

$$\sqrt{(X_i^B)^T \Sigma_{ij}^{comm} X_i^B} \leq \sigma_{comm}$$

$$\sqrt{(X_i^B)^T \Sigma_{ij}^{weapons} X_i^B} \leq \sigma_{weapon}$$

# Summary/Conclusion

- RMVO promising framework to leverage SoS performance against risk
- Considers uncertainty and system interdependencies explicitly in portfolio construction
- Needs more realistic data (performance, interdependencies) for real world application and verification



# Portfolio Approach: Future Work

- Extend to multi-period considerations
  - How do I make investment decisions in changing environments?
  - Can I hedge my bets for future anticipations?
    - (e.g. price of steel in LCS program?)
  - Do my decisions now allow me to learn for the future?
    - Similar technologies, frameworks → knowledge space?

$$\max \left( \underbrace{\sum_q \left( \frac{S_{qc} - R_c}{R_c} \cdot w \cdot X_q^B \right)}_A - \lambda \left( X_q^F \right)^T \Sigma_{\tilde{y}} X_q^F - \sum_q \left( C_q X_q^B \right)}_t + E(A_{t+1} | w_{t+1}, \Sigma_{t+1}, \lambda_{t+1}) \right)$$

Capability vs. Risk now

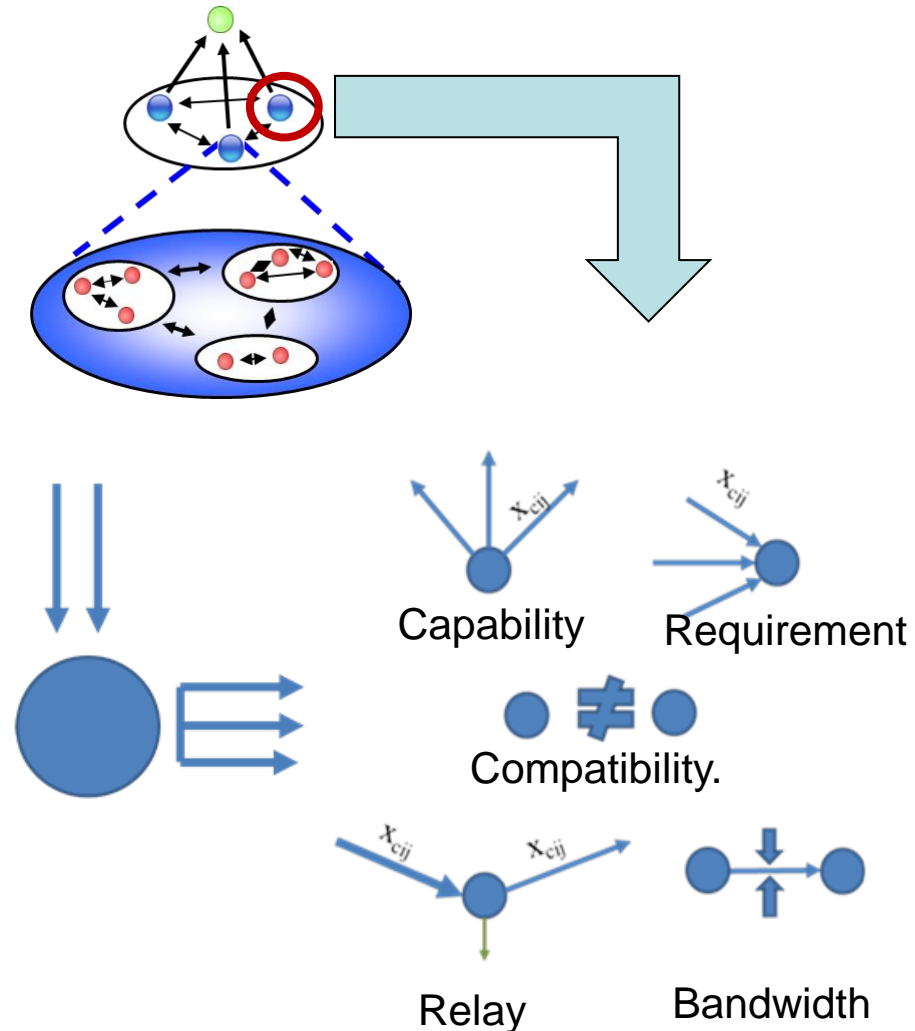
Effect on Capability Later

- Application to more realistic world SoS portfolio problems

# Extra/Backup Slides

# Portfolio Approach: SoS Modelling Additions

- Model individual system as 'nodes'
  - Functional & Physical representation
- Rules for node connectivity (this is currently not addressed elsewhere, e.g., RT-18)
  - Compatibility between nodes
  - Bandwidth of linkages
  - Supply (Capability)
  - Demand (Requirements)
  - Relay capability



# Extension to SoS Interconnectivities

Maximize Capability Performance Index



$$\max \left( \frac{\sum_i S_{ic} \cdot w \cdot X_i^B - R_c}{R_c} \right)$$

Sufficient Capabilities Supplied



s.t.

$$\sum_i X_{cij} \geq X_j^B S_{rj}$$

Individual System Requirements met



$$\sum_i X_{cij} \geq X_j^B S_{rj}$$

Connectivity Rules Obeyed  
(Big-M formulation)



$$\begin{aligned} X_1 + \square + X_n &= 0 \\ \sum_c X_{cij} - X_{ij} M &\leq 0 \\ M \sum_c X_{cij} - X_{ij} &\geq 0 \\ \sum_i X_{cij} - \sum_j X_{cij} - X_j^B S_{rj} &= 0 \end{aligned}$$

Risk Tolerance (per measure of risk)



$$\sqrt{(X_i^B)^T \Sigma_{ij} X_i^B} \leq \sigma_{critical}$$

$$X_{cij} \leq \text{Limit}_{cij}$$

$$X_{cij} = 0 \quad c \in \text{capability}$$

$$\Sigma_{ij}^L \leq \Sigma_{ij} \leq \Sigma_{ij}^U$$

$$X_{cij}, X_j^B \in \text{binary } \{0,1\}$$

