Australian Government
Department of Defence
Defence Science and
Technology Organisation

Drop Calibration of Accelerometers for Shock Measurement

Andrew Krelle

Maritime Platforms Division
Defence Science and Technology Organisation

DSTO-TR-2584

ABSTRACT

This report details the Drop Calibration Method for determining the sensitivity of shock accelerometers. The Drop Calibration Method requires the accelerometer under calibration and either a traceable acceleration standard or a traceable force transducer standard to undergo a series of drop impacts. The sensitivity of the accelerometer is determined by comparing the outputs of the accelerometer and the standard. By appropriate characterisation of both the accelerometer's and the standard's data channels, prior to the calibration drops, the Drop Calibration Method ensures that the resultant sensitivity is traceable to National Association of Testing Authorities, Australia (NATA) standards.

RELEASE LIMITATION

Approved for public release

Published by

DSTO *Defence Science and Technology Organisation*
506 Lorimer St
Fishermans Bend, Victoria 3207 Australia

Telephone: (03) 9626 7000
Fax: (03) 9626 7999

AR-015-057
August 2011

APPROVED FOR PUBLIC RELEASE

Drop Calibration of Accelerometers for Shock Measurement

Executive Summary

Shock accelerometers are used to both validate predictions and gather primary data on the motion of various test items due to explosive shockwaves. In Maritime Platforms Division (MPD), accelerometers are extensively used to measure shock induced motion on a variety of test platforms as well as in full ship trials.

To ensure accurate results the accelerometers are calibrated both prior to and after any test or trial by a procedure known as the Drop Calibration Method. The accelerometer sensitivities are determined by comparing the output of the accelerometer under calibration to that of a known secondary standard. By mounting the accelerometer to a drop weight and dropping the weight onto a force transducer standard, or mounting the accelerometer directly to an acceleration standard and dropping the standard onto a solid base, the output of each device can be directly compared.

This report provides a brief description of the two classes of accelerometer used within the Division. It illustrates the differences between calibrating accelerometers with the acceleration standard and the force transducer standard, and details the steps taken to ensure the calibrations are traceable to National Association of Testing Authorities, Australia (NATA) standards. Furthermore, the report provides an error analysis of the drop calibration procedure and details the problems associated with this method of calibrating accelerometers.

This page is intentionally blank

Author

Andrew Krelle
Maritime Platforms Division

Andrew Krelle graduated from Swinburne Institute of Technology with a Bachelor of Engineering Degree specialising in computer systems in 1989. Since joining the laboratory in 1990 he has been involved with the high speed instrumentation of a large variety of experiments. This includes shockwave measurements involving pressure, acceleration and strain.

This page is intentionally blank

Contents

1. Introduction

Shock accelerometers are used to both validate predictions and gather primary data on the motion of various test items due to explosive shockwaves. In Maritime Platforms Division (MPD) accelerometers are extensively used to measure the shock motion on various test rigs as well as in full ship trials.

The prime objective of this report is to explain and summarise the drop calibration technique used within MPD for determining the sensitivity of accelerometers [1-3]. The primary class of accelerometers used within the Division are piezoelectric type, with ranges from 50 g up to 100000 g. Less commonly used are a number of piezoresistive accelerometers, with ranges from 2000 g to 200000 g. Using the drop calibration method it is possible to calibrate against two different categories of secondary standards. The first category is an acceleration standard and the second is a force transducer.

To ensure the accuracy of reported results the sensitivity of the accelerometer must be determined with reference to a defined standard. Therefore all instruments used as primary sources within the calibration process, including the acceleration standard, are certified to National Association of Testing Authorities, Australia (NATA) standards on an annual basis. One exception is the force transducer, which is calibrated in Switzerland and is traceable to the Swiss Federal Office of Metrology.

Besides detailing the drop calibration method, the report includes a brief discussion of the classes of shock accelerometers used within MPD and the two categories of standards used to perform the calibrations. Furthermore the report provides an error analysis of the drop calibration procedure and details the problems associated with this method of calibrating accelerometers.

2. Basic Accelerometer Calibration Concepts

2.1 Piezoelectric Accelerometers

The piezoelectric accelerometer utilises a piezoelectric element, such as a quartz crystal or a purpose built ceramic. When a force is applied to it this element becomes bipolar and produces an electric charge. Piezoelectric accelerometers have two modes of operation. The first is charge mode whereby the transducer, consisting primarily of the piezoelectric element, produces a charge proportional to the applied force. This charge is converted, using a charge amplifier, into a voltage signal.

The second mode of operation is the piezoelectric transducer with built in amplifier [4-6]. This form of accelerometer contains not only the piezoelectric element but also electronic circuitry to produce a more rugged and stable device. These devices contain a MOSFET transistor connected to the piezo crystal all fully contained within the accelerometer body, Figure 1.

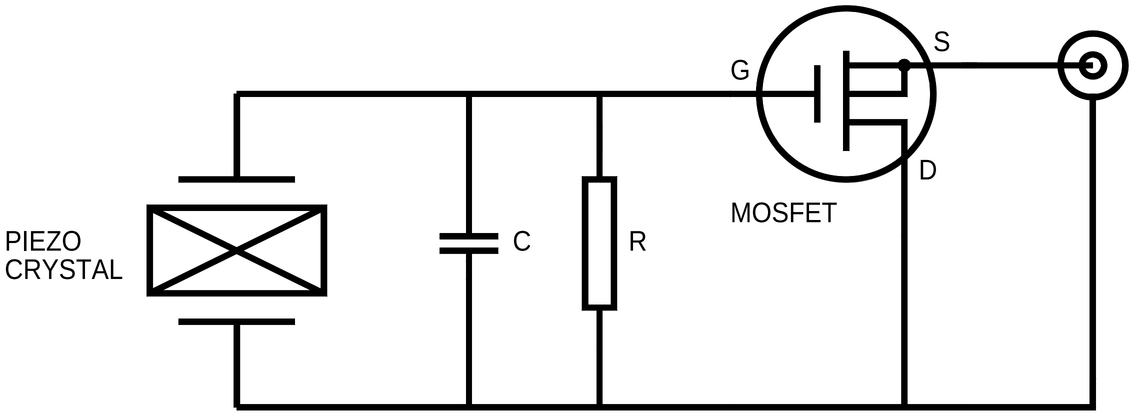


Figure 1 A typical schematic of a low impedance voltage mode transducer

In this schematic, the capacitance C represents the total capacitance of the crystal and any shunt capacitance, whilst R is the bias resistance. The dissipation of the charge (q) through the capacitor (C) produces a voltage (V) such that $V=q/C$, which is input into the MOSFET. The MOSFET is designed to provide unity gain and provides low output impedance. The *RC* combination also determines the Time Constant (TC) of the transducer commonly measured in minutes. However, as these accelerometers are used in a shock environment where the excitation is a high speed transient, the TC has little impact on these results.

Charge mode transducers are high impedance devices. Any small change in the quality of the connection to the transducer will affect the output impedance. While the capacitance of the cable has no effect on the gain, this being an effect solely of the charge amplifier, it does affect the noise level into the amplifier and a poor connection or low quality cable can dramatically affect the quality of the signal.

In a piezoelectric transducer with built in amplifier, the MOSFET provides low output impedance, this means that the quality of the cable and connection has less impact than that for charge mode operation. It is still good practice to use the highest quality cable available, to ensure clean connectors and provide good connector contact. While the low output impedance helps, poor connections do lead to poor results.

An external line power supply providing constant current powers the MOSFET. The constant current is typically, 4 -12 mA, with longer cables requiring the higher currents to compensate for greater losses. The MOSFET typically operates with a quiescent output voltage of approximately 11V and is connected to the power unit using a coaxial cable or twisted pair. The acceleration signal is measured as a variance around the quiescent voltage. Typically the Division uses a PCB F482A10 Line Power Supply to power this mode of accelerometer [7].

The technology used in the production of piezoelectric transducers with built in amplifier are known by various trademarked names from different companies. For example, ICP® is the trademark of PCB Electronics while Piezotron® is registered to Endevco, and there are others as well. When referring to accelerometers of this type, the term ICP® will be used throughout this report. All piezoelectric shock accelerometers currently used within MPD are of the built in amplifier type. MPD has a variety of ICP® type accelerometers ranging from 50 *g* up to high shock accelerometers with ranges of 100000 *g*.

It should also be noted that most ICP ® type transducers used within MPD have built-in mechanical filters. These mechanical filters isolate the crystal and circuitry from severe high frequency shock components which would otherwise destroy the accelerometer regardless of its range.

2.2 Piezoresistive Accelerometers

In simplest terms the piezoresistive accelerometer is a Wheatstone Bridge device, as shown in Figure 2, where the four arms of the bridge are piezoresistive elements [8]. The piezoresistive elements are typically a single, solid state, silicon resistor whose electrical resistance changes proportionally to an applied force. When an acceleration (force) is applied it produces a distortion in the sensing elements. This distortion changes the resistance in the arms of the bridge thus producing a change in output voltage.

The piezoresistive accelerometer is a passive device and it requires four connections, two for the source voltage (V_s+ & V_s-) and two for the output voltage (V_o+ & V_o-). Typically the source voltage is ± 5 V, limiting in turn the output voltage to ± 5 V. During calibration the piezoresistive accelerometer is powered by a differential amplifier such as the Endevco 4423 signal conditioner [9].

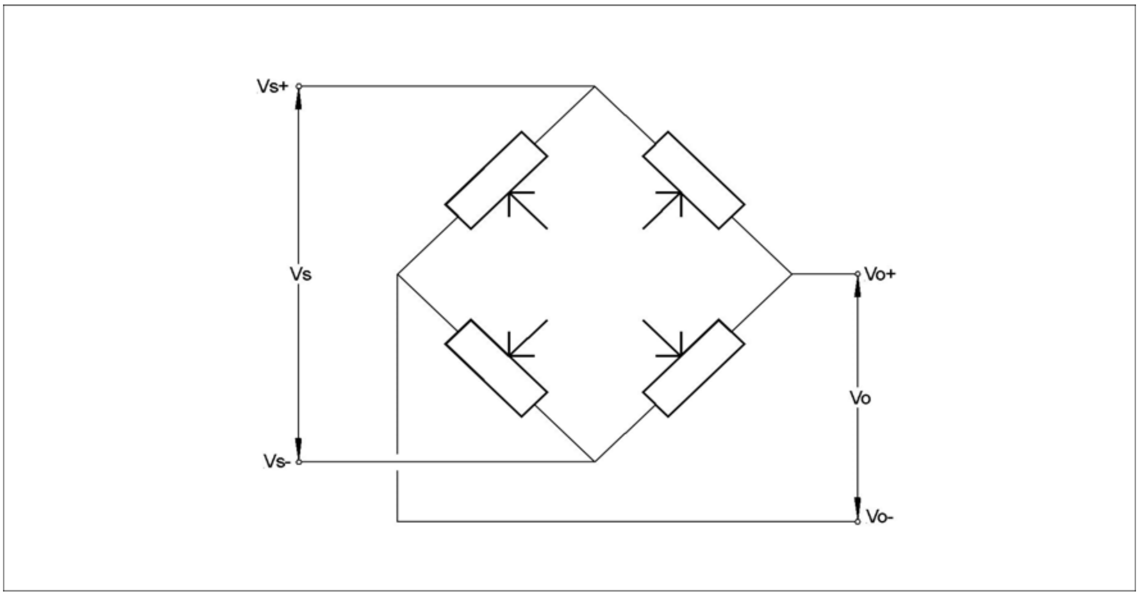


Figure 2 A Schematic of a typical Piezoresistive accelerometer

2.3 Two Drop Calibration Standards

The Division has two standards with which to perform drop calibration of accelerometers. The first is an Acceleration Standard the second is a Force Transducer Standard and each standard has its own calibration procedure. Both drop calibration methods are fundamentally similar in that they compare the output from a certified transducer of known sensitivity with the output from the transducer under calibration. However the two procedures approach the calibration process with slightly different principles.

Drop calibration with the Acceleration Standard requires the accelerometer to be mounted directly onto the standard and the standard dropped vertically onto a steel base. Drop calibration with the force transducer requires the accelerometer to be mounted to a drop weight and dropped vertically onto a force transducer. The difference in these two methods is discussed below.

2.4 Acceleration Standard

When using the acceleration standard the accelerometer to be calibrated is attached, using the recommended torque, to the acceleration standard "piggy back" style as shown in Figure 3. The standard with accelerometer attached is then dropped within a vertical tube (drop tower) onto a solid steel base. Depending on the amplitude and time base of the output signals, rubber pads may be used to provide both damping and filtering. The acceleration standard has a NATA certified sensitivity measured in mV/g. By capturing and comparing the voltage outputs from each device the sensitivity of the accelerometer can be determined by a direct comparison.

Figure 3 The PCB 301AO3 acceleration standard (bottom) with a PCB ICP ® accelerometer attached (top)

The acceleration standard system used within the Division is manufactured by PCB (model 394A03). The 394A03 comes with an acceleration standard (model 301A03), a power supply (model F482A10) and low noise, miniature coaxial cable (microdot). The 301A03 acceleration standard is itself an ICP ® accelerometer and operates in the same manner as the accelerometers being calibrated. This 394A03 standard is NATA certified to have a linear acceleration sensitivity when operating at up to 500 *g*. The sensitivity of the 394A03 is NATA certified using its own component items and all drop tests are conducted using these items [10].

2.5 Force Transducer

The Force Transducer Standard that the Division uses for calibration of shock accelerometers is a Kistler (model 894F) Shock Calibration System [11]. The 894F includes a force transducer (model 902A) and a miniature coaxial cable. The model 902A force transducer is a piezoelectric charge device. That is, it produces an electrical charge relative to the force of the impact. It contains a piezoelectric element but unlike the acceleration standard it contains no internal MOSFET electronics instead requiring connection to a charge amplifier. The charge amplifier converts the charge into a voltage. The force transducer has a certified sensitivity measured in pCb/N.

Figure 4 shows a PCB accelerometer attached to a drop weight and ready for calibration with the force transducer. The drop weight is shaped like an hourglass to reduce friction during its fall within the drop tower.

Figure 4 An ICP® accelerometer (top) mounted to a drop weight and ready for calibration with the Force Transducer

The mechanics of using the force transducer is similar to that of the acceleration standard. The accelerometer is "piggy backed" to a drop weight using the appropriate torque. The weight with accelerometer attached is dropped within a drop tower onto the force transducer. The impact of the drop weight produces an output from both the force transducer and the accelerometer.

Since the charge amplifier produces an output in N/V, a simple ($f = ma$) conversion needs to be performed to convert the output into mV/g. The conversion process requires the total mass of the drop weight and accelerometer to be known.

An electronic scale is used to measure the mass of the drop weight and accelerometer. The scales are first calibrated by using a series of NATA certified weights. By calculating the ratio between the actual and the measured values of the certified masses and applying this to the measured mass of the accelerometer and drop weight the actual mass of the drop weight and accelerometer is determined.

The drop tower tubing has a vertical slit cut into it running along its length from the top to just above the impact point on the bottom. This slit which is approximately 5 mm wide is to allow the drop weight handle and the cables from the accelerometer and the acceleration standard to pass through. This slit, while required for both calibration methods, is most important when using the force transducer as the drop weight is required to fall with the constant acceleration of gravity. Once released it is important to not allow accelerometer cable to retard the fall of the drop weight. To this end the cable should be held loosely in one hand and lowered to keep pace with, but not move ahead of, the fall of drop weight. In order to further minimise drag the drop weight is ground into an hour glass shape so as to minimise friction.

Figure 5 The Kistler force transducer with Perspex drop tower attached. The acceleration standard and a drop weight are shown each with a PCB accelerometer attached at the top.

Figure 5 shows the relative size of the force transducer and the acceleration standard. Also shown is a drop weight for use with the force transducer. The metal base plate on which the acceleration standard is dropped is of the same dimensions as the force transducer.

2.6 Filtering

The metal to metal impact from both forms of drop calibration is capable of producing high frequency ringing. This ringing can interfere with the calibration signals; to remove this interference, filters are used. The filtering can be mechanical such as the inbuilt filters in the piezoelectric accelerometers or a rubber disc mounted on top of the force transducer/metal base. Filtering can also be performed electrically using electronic filters or in software. In practice a combination of all these techniques are used to reduce the ringing.

Most ICP ® accelerometers used within the Division contain inbuilt mechanical filters. However, the piezoresistive accelerometers used within the Division have no inbuilt mechanical filters.

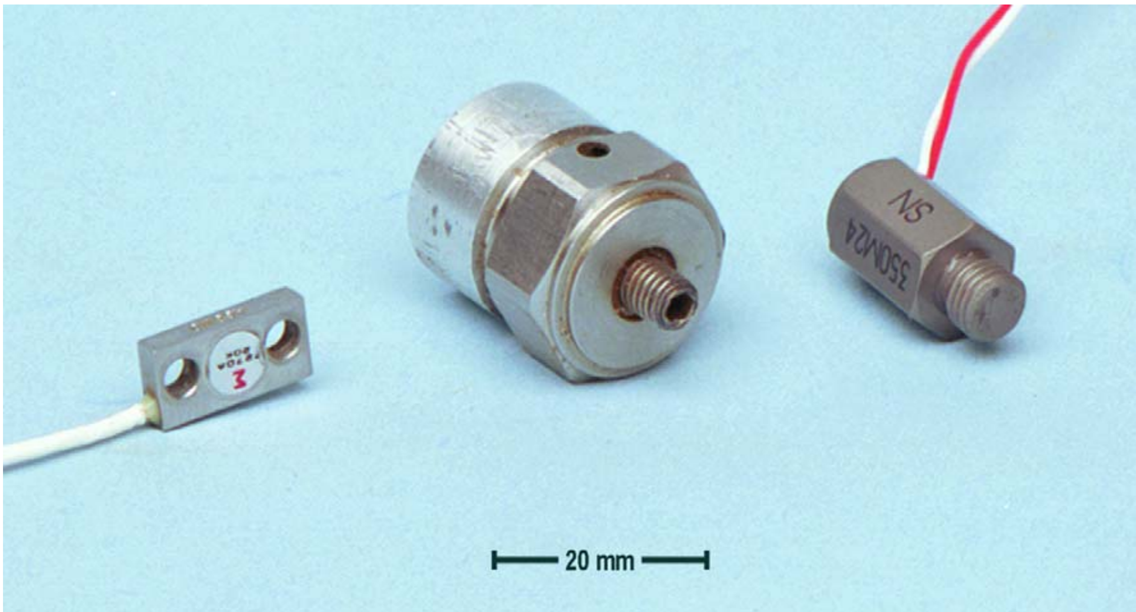


Figure 6 An Endevco piezoresistive accelerometer (left), a Bruel and Kjaer mechanical filter with mounting adapter (central) and a PCB, ICP ® accelerometer (right).

In normal applications it is generally advisable to mount the accelerometer directly to the test piece. In high shock environments however, some external mechanical filtering may be required for those accelerometers without inbuilt mechanical filters. These external mechanical filters are not used in the calibration process.

Figure 6 illustrates the relative size of the compact varieties of the two accelerometer classes. Even though the piezoresistive accelerometer is small, when attached to the Bruel and Kjaer mechanical filter its overall mass becomes much greater than that of the piezoelectric accelerometer with its inbuilt filter. This is important in those tests where the mass distribution of a test sample is critical. It is also noticeable that the mechanical filter requires the fitting of an adapter in order to allow this model of piezoresistive accelerometer to be mounted.

In order to decrease high frequency ringing during the calibration process, rubber pads are placed on the impact surface of the force transducer or the steel base. The placing of rubber pads also produces other effects such as decreasing the peak acceleration and increasing the time duration of the pulse. Various rubber pads are used to produce different effects.

Low pass, electrical RC filters with a 26 kHz cutoff frequency are used in the output lines from the amplifiers. By putting the filters inline on both the accelerometer and calibrator channels it prevents phase shifting while applying the same transfer function to both signals. The low pass electrical filters are used on all calibrations, including those accelerometers with inbuilt mechanical filters. Figure 7 shows a photo of a purpose built RC filter box.

Figure 7 A box containing four simple, 26 kHz lowpass RC filters

2.7 Digital Storage Oscilloscope

Each drop calibration is treated as a single shot event and a digital storage oscilloscope (DSO) is used to both capture the signals and to provide analysis of them. In order to provide a high accuracy record, a DSO with a high sample rate, a large vertical resolution and a high analogue bandwidth is required. It is also important that the screen is clear, the records displayed are crisp and values are easily read.

The current DSO, used within the Division, in the accelerometer calibration process is a Yokogawa DL708E. This DSO records through a 12 bit analogue to digital converter (ADC) at 10 million samples per second with a bandwidth of 1 MHz.

The memory length and time base of the DL708E are configurable and each is set by the user to provide the best possible visual record while still keeping the sample rate at 10million samples per second. The DL708E is also capable of applying low pass filtering at 50 kHz. The Yokogawa DL708E is shown in Figure 8.

Figure 8 A Yokogawa DL708E digital storage oscilloscope

2.8 A Note on Choosing Accelerometers

The piezoresistive accelerometers are now rarely used within MPD in the field shock-experiment environment. The ease of installing and cabling the ICP® accelerometer along with the electrical characteristics and the inclusion of a mechanical filter make it the preferred accelerometer for shock measurements. However, the Endevco piezoresistive accelerometer is relatively small and light when used without the Bruel and Kjaer mechanical filter and has a different physical profile, such factors may have a bearing on whether it is used.

3. Preparing the Calibration Process

3.1 Choosing a Calibration Method

Before the calibration process begins, the appropriate calibration method must be determined. This is based on the relative advantages and disadvantages of both calibration systems in relation to the requirements of the accelerometers needed for the intended field trial application. Several advantages and disadvantages are listed in Table *1*.

Table 1 The advantages and disadvantages of the two drop calibration systems

Acceleration Standard	Force Transducer
Advantages	Advantages
Simple setup	Capable of accommodating all accelerometers
Simple operation	Capable of up to 10000 *g*
Simple comparison of outputs	
Disadvantages	Disadvantages
Limited to 500g	Need to know mass of drop weight
Limited to accelerometers with a 10-32 mounting thread	Need to convert force signal to acceleration
	More complicated setup using charge amplifier
Higher error than associated with force transducer	High precision capacitor not NATA certified

The simplicity of using the acceleration standard makes it the preferred method. This method of calibration, however, has a limited range of 500 *g* and can only accept accelerometers with a 10-32 thread mounting stud. Any accelerometer that requires calibration over a range greater than 500 *g* or has a different mounting technique requires the use of the force transducer. The force transducer can calibrate up to 10000 *g* and can use custom made drop weights to accept any mounting technique.

3.2 Attaching Accelerometer to Drop Weights

The accelerometers within the Division primarily have one of three mounting techniques.

- Centrally mounted with a fixed 0.25-inch thread.
- Centrally mounted with a fixed 10-32 thread.
- Mounted with a 3 mm screw at either end. (This only applies to the Division's Endevco 7270 piezoresistive series)

All accelerometers are mounted with a thin film of silicon grease smeared onto the bottom of the accelerometer to aid in high frequency data transmission. While the accelerometer calibration drops are considered to produce a high frequency signal, it should be noted that these signals will generally be of a lower frequency than those produced by an explosively generated shock.

All accelerometers are centrally located, attached directly to the drop weight or acceleration standard and mounted using the torques specified in that particular accelerometer's specification sheets.

3.3 Connecting the Instrumentation

3.3.1 Piezoelectric Accelerometers and Acceleration Standard

The Piezoelectric accelerometers are connected to a constant current supply (also known as a line power supply) using a microdot cable. Typically this constant current supply is also an amplifier (PCB model F482A10). From the output of the amplifier a conventional coaxial cable is used to connect to a 26 kHz low pass filter and then to the DSO.

The PCB acceleration standard is connected in the same way using the line power supply and microdot provided.

3.3.2 Piezoresistive Accelerometers

Piezoresistive accelerometers are connected to a differential amplifier using a 4-core cable. The amplifier is connected to the DSO via a 26 kHz low pass filter by conventional coaxial cable.

3.3.3 Force Transducer

The force transducer, being a charge device, is connected to a charge amplifier using a microdot cable. The charge amplifier is connected to the DSO via a 26 kHz low pass filter using conventional coaxial cable.

When connecting any cable to an input or altering any switches on the charge amplifier the GROUND button is depressed. This ground button does two things. Firstly it protects the input stage of the charge amplifier by grounding the input of its FET, grounding the output of the connected transducer and removing any accumulated charge. Secondly it restores the amplifier output to zero.

The Division typically uses a Kistler model 504D dual mode amplifier or a PCB model F464A dual mode charge amplifier. Both these amplifiers are capable of operating in either charge or voltage modes [12-13]. The Kistler 504D is shown in Figure 9.

Figure 9 A Kistler 504D dual mode amplifier

The Kistler 504D has two dials, one labelled "Transducer Sensitivity" and the other "Range". The transducer sensitivity is adjusted to the latest NATA certified sensitivity value for the force transducer while the range switch determines the output sensitivity measured in N/V. The range switch is set according to the desired range of output voltages, the larger the value selected the lower the voltage output will be for any given force.

In Figure 9 the output range is set for 1000 N/V. However, while the set value of these switches enable quick calculations of approximate force levels and voltages, their settings, while indicative, are not actually necessary for calibrating the accelerometer. Section 3.4 describes how the transducer channels are characterised.

3.4 Characterising Instrumentation Channels

Once the instrumentation is connected, both the accelerometer and the standard channels are characterised. Characterising each channel determines the total gain of each channel due to the equipment used in each particular channel. A NATA certified AC voltage standard is applied to each channel and the corresponding output signal is measured. The gain of each channel is determined as a ratio of the applied and measured voltages. This is also known as channel calibration.

3.4.1 The Digital Storage Oscilloscope

The DSO is the primary instrument for capturing and analysing the calibration signals. As it forms part of the calibration process it is included in the characterisation of the channels. The individual DSO channels are each characterised as though they are part of that channels circuitry. That is to say the total gain of each channel is determined as the ratio of the captured voltage to the calibration voltage.

$$G_t = \frac{V_{dso}}{V_{ac}}$$

where
G_t = the total gain of the channel
V_{ac} = the calibration voltage
V_{dso} = the measured voltage.

By determining the total gain of the system and referencing it back to a NATA certified standard, any systematic errors in each channel are eliminated. This process is fundamental to all channel calibrations though the procedures differ for the various transducer types.

3.4.2 ICP ® Type Piezoelectric Accelerometers

The gain within these accelerometer channels is determined by connecting the AC standard to the input of the constant current power unit using a piezoelectric calibration adaptor. The output signal is captured by the DSO and the gain of the channel is calculated. The calibration adaptor as shown in Figure 10, is a simple device consisting of a high tolerance capacitor and resistor and allows an AC voltage source to connect into the input of the constant current power unit. The adaptor is checked using the AC standard and a NATA certified voltmeter connected across the output of the adaptor to ensure it applies unity gain to the system.

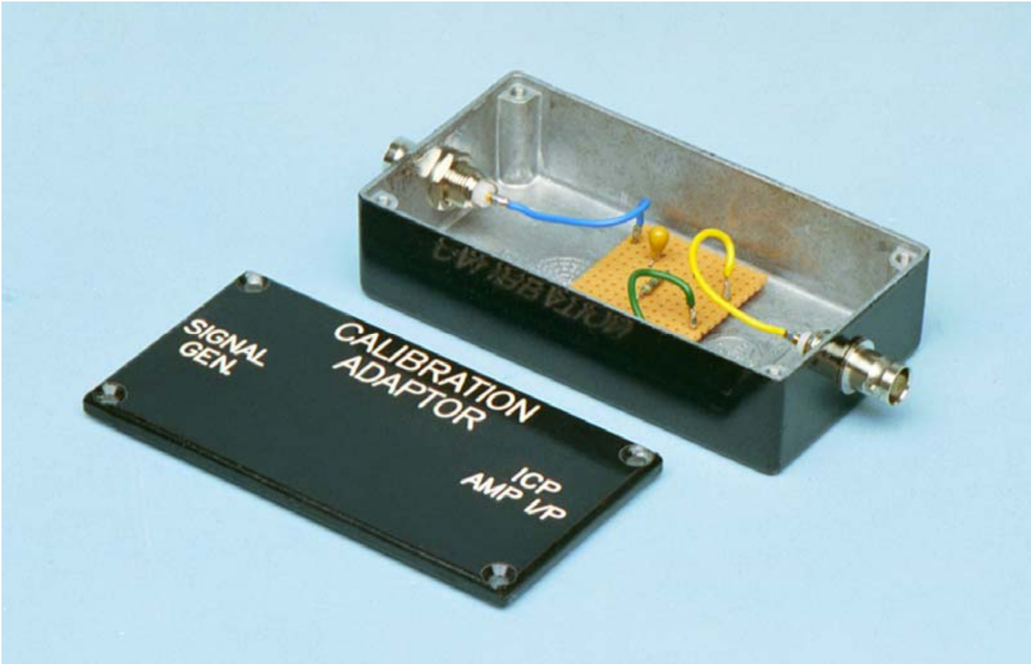


Figure 10 A constant current, calibration adaptor

Figure 11 shows the schematic of a piezoelectric calibration adaptor. The piezoelectric calibration adaptor contains a 10 µF capacitor and a 3 kΩ resistor, forming a high pass filter with a –3 dB lower cutoff frequency of 5 Hz. When monitoring the constant current power unit input, the voltmeter measures the quiescent voltage when it is set to measure DC volts or monitors the AC calibration voltage, when it is set to measure AC Volts.

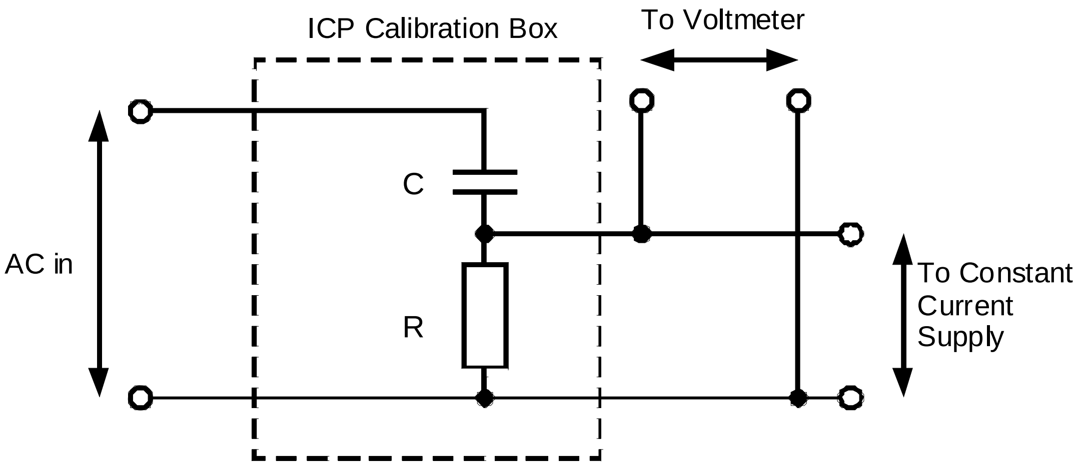


Figure 11 A schematic diagram of the constant current, calibration adaptor

3.4.3 Piezoresistive Accelerometers

A slightly different method is applied in determining the gain when piezoresistive accelerometers are connected to the DSO channels. These accelerometers are bridge devices and as such require an appropriate amplifier. For this task the Division typically uses an Endevco 4423 signal conditioner. The Endevco 4423 amplifier has a gain button and a balance button on the front face. The gain button amplifies the signal and the balance button is adjusted to balance the output signal to zero prior to accelerometer calibration.

Channel calibration begins by connecting a piezoresistive calibration adaptor, as shown in Figure 12, to the Endevco 4423 amplifier. The quiescent output level of the amplifier is adjusted to zero using a DC voltmeter. The AC voltage standard is connected to the adaptor and the output from the amplifier is recorded using the DSO. The gain of the channel is then determined. The amplifier output is set to zero upon reconnection of the accelerometer; this has no effect on the gain as it simply balances the bridge.

Figure 12 A piezoresistive calibration adaptor

The schematic of the piezoresistive calibration adaptor is shown in Figure 13. The four resistors simulate the full bridge configuration of the accelerometer. The AC calibration signal is connected directly into the differential amplifier's inverting and non-inverting inputs. The resistors in the adaptor have no effect on the gain of the system.

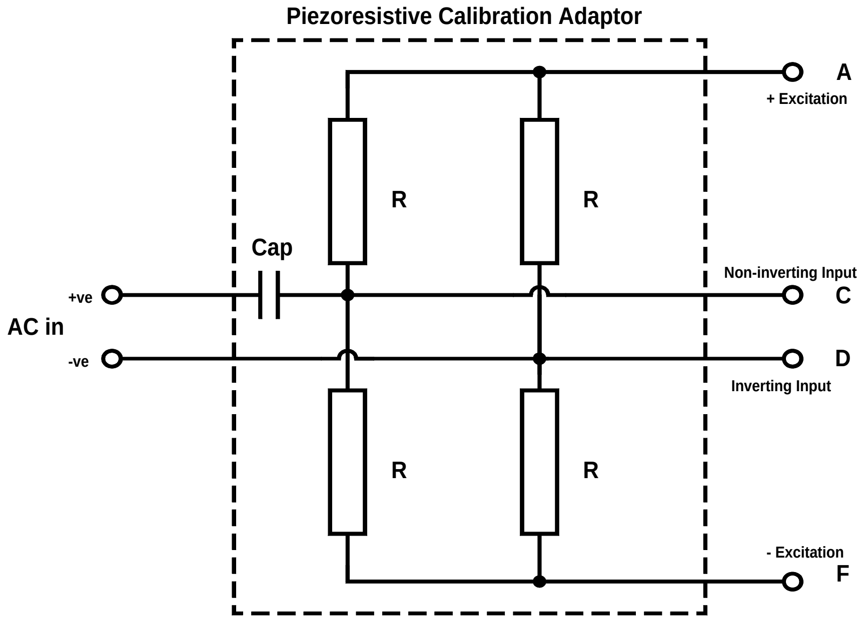


Figure 13 A schematic diagram of the piezoresistive calibration adaptor

3.4.4 Accelerometer Standard

The accelerometer standard, miniature coaxial cable and constant current amplifier are treated as a single device and the accelerometer standard's sensitivity applies to all three parts of the system combined. Thus, in order to characterise this particular channel, the AC voltage standard is connected directly to the DSO using the same coaxial cable to be used between the constant current amplifier and the DSO.

3.4.5 Force transducer

The force transducer channel is characterised by connecting the AC voltage standard to the calibration input of the charge amplifier. This applies the AC signal to the charge amplifier by way of a precision capacitor of 1000 pF. By comparing the output voltage to the input voltage the gain of the channel is determined.

$$G_{ft} = \frac{V_{so}}{V_{si}} \tag{3.1}$$

Applying a voltage V_{si} across the 1000 pF capacitor is effectively applying $V_{si}.1000$ pCb of charge into the charge amp.

$$Q_{so} = V_{si}.1000 \tag{3.2}$$

This is the equivalent of supplying an equal amount of charge from the force transducer, which produces charge at the rate of its sensitivity ie: S_{ft} [pCb/N].

This can be rewritten as

$$F = \frac{Q}{S_{ft}} \tag{3.3}$$

Combining Equations (3.1), (3.2) & (3.3), and noting that both Equations (3.2) & (3.3) use units of pCb, then:

$$F = \frac{1000.V_{si}}{S_{ft}}$$

$$\Rightarrow F = (\frac{V_{so}}{G_{ft}}).(\frac{1000}{S_{ft}})$$

Hence the force required to produce the output voltage V_{so} is

$$F = \frac{1000.V_{so}}{S_{ft}.G_{ft}} \qquad [\mathrm{N}] \tag{3.4}$$

Applying Equation (3. 4) to any output voltage V_o

$$\Rightarrow F = \frac{1000.V_o}{S_{ft}.G_{ft}} \tag{3.5}$$

But $F = ma$, Hence

$$\Rightarrow m.a = \frac{1000.V_o}{S_{ft}.G_{ft}} \tag{3.6}$$

$$\Rightarrow a = \frac{1000.V_o}{S_{ft}.G_{ft}.m} \qquad [\mathrm{m/s^2}] \tag{3.7}$$

Convert acceleration from m/s^2 to g

$$\Rightarrow a_g = \frac{1000.V_o}{S_{ft}.G_{ft}.m.g_n} \qquad [g] \tag{3.8}$$

It can be seen from Equation (3.8), that the acceleration level is determined independently of the settings on the dials on the charge amplifier. While the dials alter the gain of the amplifier and therefore the amplitude of the output, the total gain of the amplifier is calculated solely by the input and output values of the channel calibration signal.

As long as no dial is altered during the accelerometer calibration, the numerical settings of these dials are of little consequence to the calibration procedure. However, by setting the sensitivity switch to the force transducer calibration value and selecting a suitable range value, the output voltage level can be used to quickly determine the approximate force produced by each drop.

3.4.6 Charge Amplifier Time Constant

Each charge amplifier has a time constant switch. The range setting in conjunction with the time constant setting does have an effect on the total time constant of the amplifier. If a low "range" is selected and the short time constant position is also selected the time constant of the amplifier can indeed be relatively short. However, with a range of 500 to 5000 N/V the amplifier time constant is > 10 s when on the short time constant setting and even longer in the medium or long time constant setting. The 1 kHz to 5 kHz frequency used for calibration provides a timebase significantly shorter than the time constant. Tests have shown that when using an AC calibration signal in this frequency range there is little difference in the calibration regardless of the time constant switch being in the long, medium or short position. With this in mind the switch is left in the short position.

3.4.7 The Overall Accelerometer Characterisation Configuration

Figure 14 shows a typical configuration for a drop calibration of an ICP accelerometer with the force transducer. A similar configuration applies to other accelerometer calibrations. This provides an indication of the simplicity of this calibration method.

If no changes to the instrumentation settings are made, the channel characterisations are only performed once per calibration session. As many accelerometers as required can be calibrated where the only necessary new measurements are the mass of each accelerometer. It should be noted that the instrumentation should be powered for a sufficient time before the calibration in order to reach a stable operating temperature, typically this is of the order of an hour.

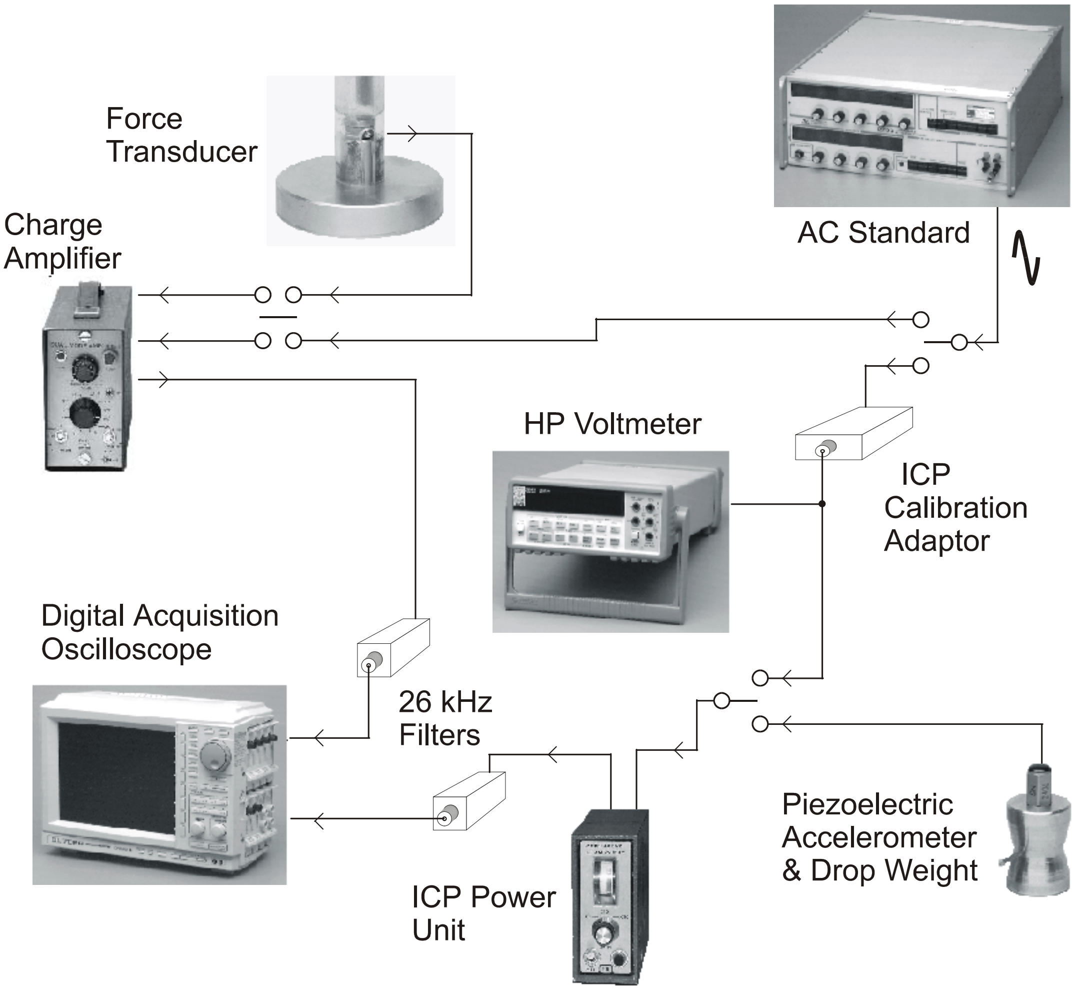


Figure 14 A schematic for calibrating an accelerometer with the force transducer

4. Performing the Drop Calibrations

4.1 Preparing For The Drop Tests

Prior to the transducer calibration, the expected peak voltage from the accelerometer is calculated so the maximum range of the device is not exceeded.

Example: An Endevco accelerometer model 226A-2000, with a 2000 *g* range has a nominal sensitivity of 0.5 mV/*g*. Therefore, with an amplifier gain of 5 the max expected output is:

2000 x 0.5 x 5 = 5000 mV

During the calibration process the accelerometer output is monitored to ensure the voltage does not exceed this maximum calculated value.

Unless a specific number of drops are required, all calibrations are usually carried out to a maximum of 18 drops over the total range of the accelerometer or to 10000 *g*. With the current force transducer and accelerometer setup it has been found that 10000-11000 *g* is the maximum acceleration obtainable for reliable calibration.

4.2 Performing the Drop Tests

The voltage from each drop is monitored on the DSO. It is customary to record the drops in ascending magnitude though it is not a necessity.

The DSO is configured with a reasonable proportion of the record as a pretrigger, typically a third or a quarter of the record. The pretrigger needs to be smooth and constant, this allows the peak output value of each drop to be referenced to this baseline. It is necessary that the data records allow enough time after the drop to confirm that the pulse returns to zero appropriately. Poor output decay may indicate a faulty accelerometer.

In order to make full use of the 12 bit ADC, the DSO range is altered to ensure the captured signals use as much vertical resolution as possible.

If the force transducer is used, the ground switch on the charge amplifier is pressed prior to each drop. As previously mentioned this removes any residual charge from the force transducer and zeros the output from the charge amplifier.

Figure 15 shows an example of a good calibration response. Both the force transducer (top trace) and the accelerometer show a smooth curve. In order to better scrutinise the signal the time has been expanded by a factor of four. The record also clearly shows the total time taken for the impact to occur is approximately 1 ms.

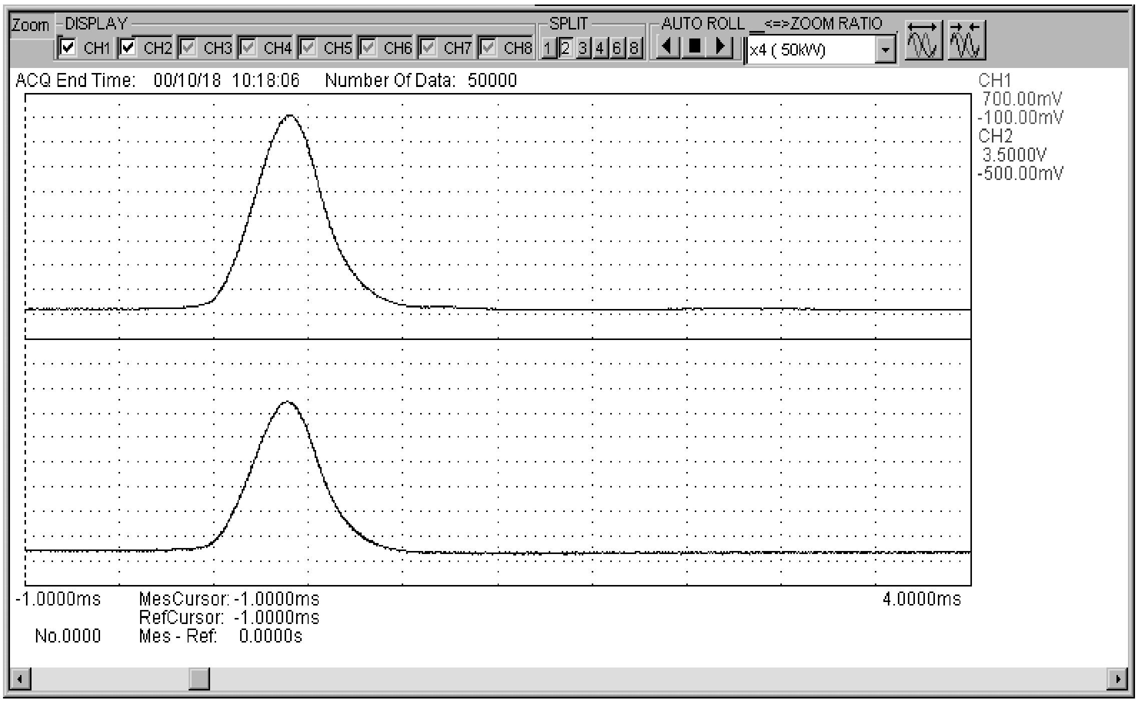


Figure 15 An example of a successful calibration drop

4.3 Errors with the Drop Tests

The most common problem associated with drop calibration of accelerometers is poor drops. Causes of poor drops can vary from overdriving the accelerometer or instrumentation through to lack of filtering or unsound cable connections. These poor quality records can exhibit a variety of problems including unexpected pulse shapes, ringing, unstable baselines and signal discontinuity. If the shape of the pulse does not exhibit a smooth half sinusoidal pulse with smooth and level baselines prior to and after the impact pulse then the record is discarded. If continual poor records occur and no reason can be found then the accelerometer is regarded as suspect.

A second problem may be poor operator technique. This is not as easily identifiable as a poor drop as the record itself may appear fine. Poor technique covers a lot of ground but can take in items such as flawed drop practice or drop mass preparation through to poor judgement on whether a record is acceptable or not. An example of this type of error may be retarding the accelerometer cable as it falls. The impact may produce a fine record but the drop mass would not have impacted freely under gravity which is required. The only way to overcome human error is practice and diligence. This type of error does not include what Cook [14] describes as "blunders" and includes such things as transcription errors, reading errors and other mistakes of this kind, these need to be simply eliminated with diligence.

A typical poor drop record is shown in Figure 16. This is an example of a drop where the pulse from the accelerometer (the bottom trace) exhibits a poor profile and the record from the force transducer output has some ringing. The degradation of the signals was caused by metal-to-metal contact between the drop weight and the force transducer and illustrates the effects of removing the rubber pads. In Figure 15 the rubber pads have dampened the high frequency components.

In order to produce a record suitable for inspection, the time scale in Figure 16 has been expanded by a factor of 20 and shows that the initial pulse is within 150 μs compared to the 1 ms pulse in Figure 15. This shortening of the pulse demonstrates a secondary effect of removing the rubber damping pad. Rubber pads characteristically increase the width of the pulse but decrease its amplitude. The decrease in peak acceleration is only of concern if higher acceleration levels are required and cannot be reached with the use of rubber pads.

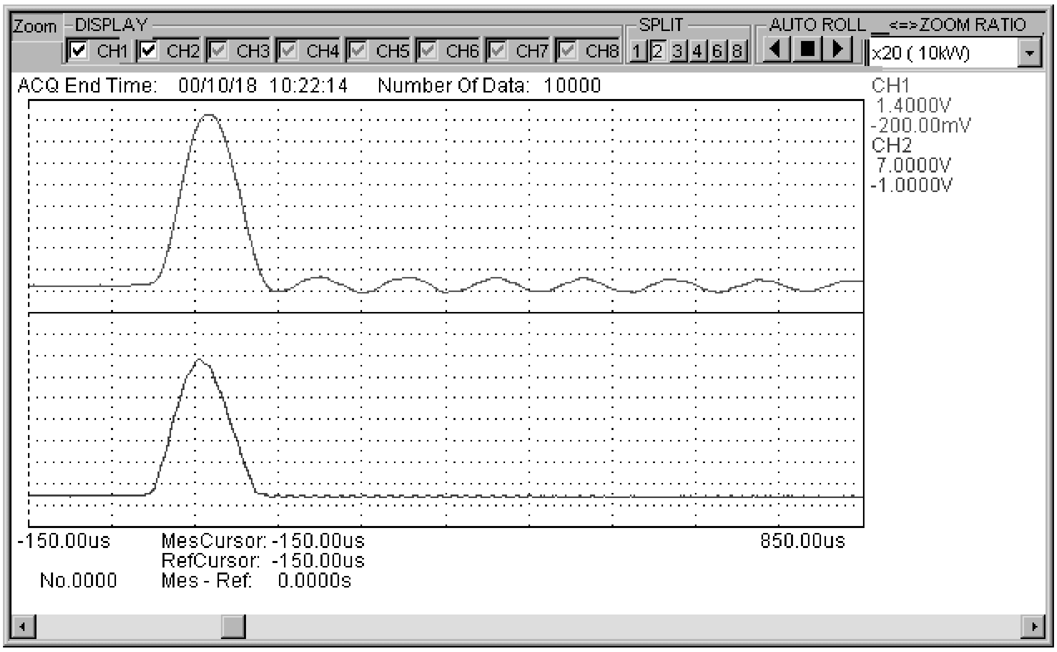


Figure 16 An example of a calibration drop resulting in an unacceptable record

Another common problem that can occur are clipped records. A clipped record is a record where the signal has exceeded one or more instrumentation limits, this causes the signal to 'flatten' off, at this limit, reducing the expected value. There are a few common causes, poor or incorrect amplitude and zero offset settings on the DSO or from over-ranging the output from the amplifier are examples. Figure 17 shows a clipped record, this particular record is from an incorrect DSO setting. An over-ranged amplifier record may not be as obvious. For clearer inspection the time base of the record in Figure 17 has been expanded by a factor of 4.

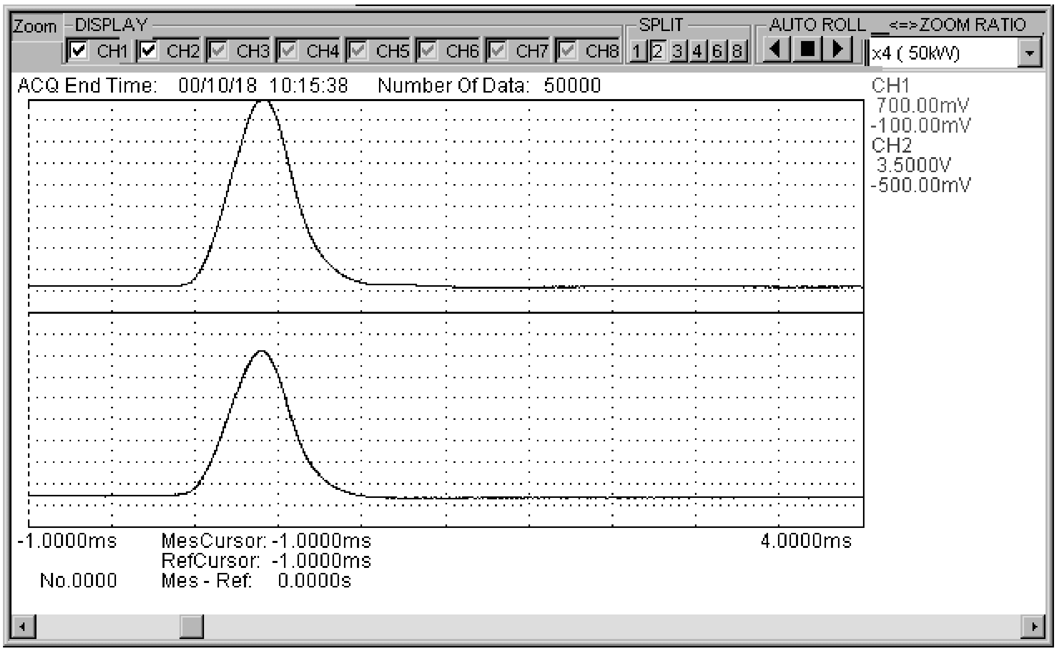


Figure 17 An example of a calibration drop resulting in a clipped record

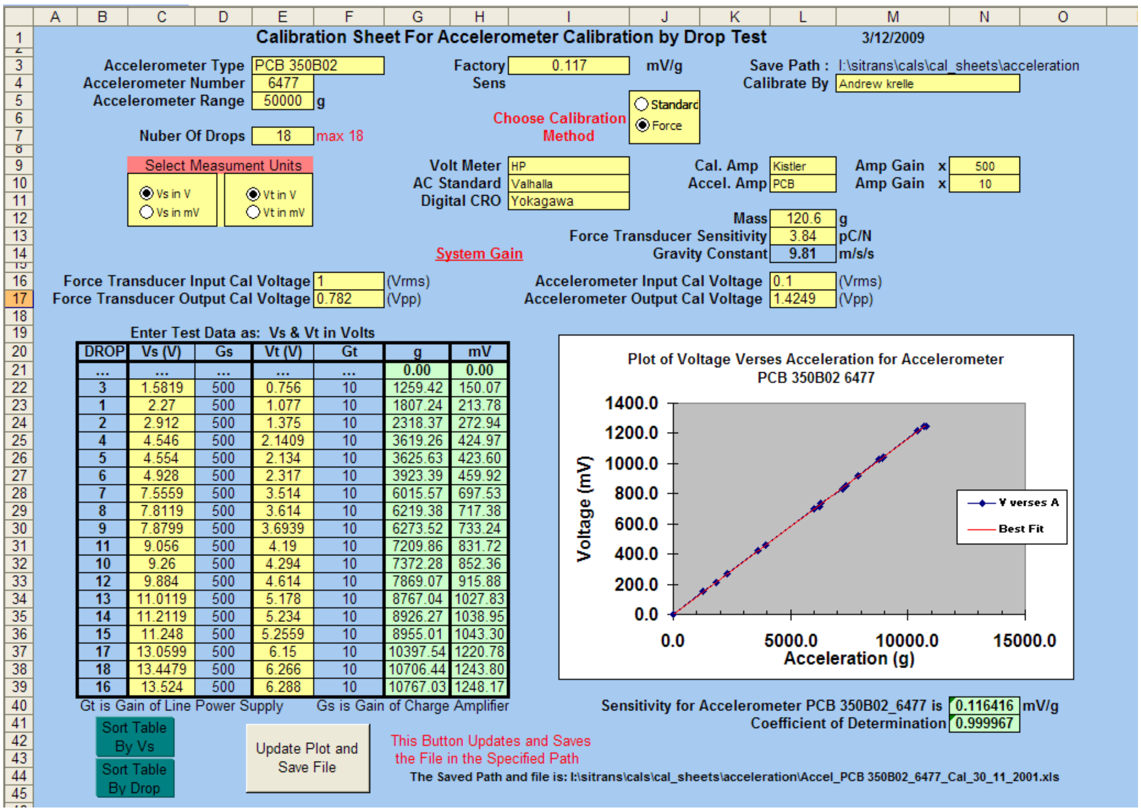
It is also possible to over-range the accelerometer or the standard. This may produce a clipped signal, though the pulses are also likely to exhibit other problems as described at the beginning of this section. Severe over-ranging of either device may damage them.

5. Determining the accelerometer sensitivity

5.1 The Accelerometer Calibration Spreadsheet

The results from the drop calibrations along with all relevant data concerning the calibration process are tabulated into a Microsoft Excel spreadsheet as shown in Figure 18. The spreadsheet produces a graph of the drop calibrations as well as a linear regression on the data to return both the sensitivity of the accelerometer and the coefficient of determination, which is a measure of the accelerometer's linearity.

The spreadsheet also contains a cell to record the factory calibration values. If the drop calibration sensitivity differs from the factory calibration value or from previous calibration values by too great a margin, the calibration is repeated. Other items monitored are the linearity of the plot and the closeness of the coefficient of determination to unity. If poor calibration results persist then the accelerometer is regarded as suspect and discarded.

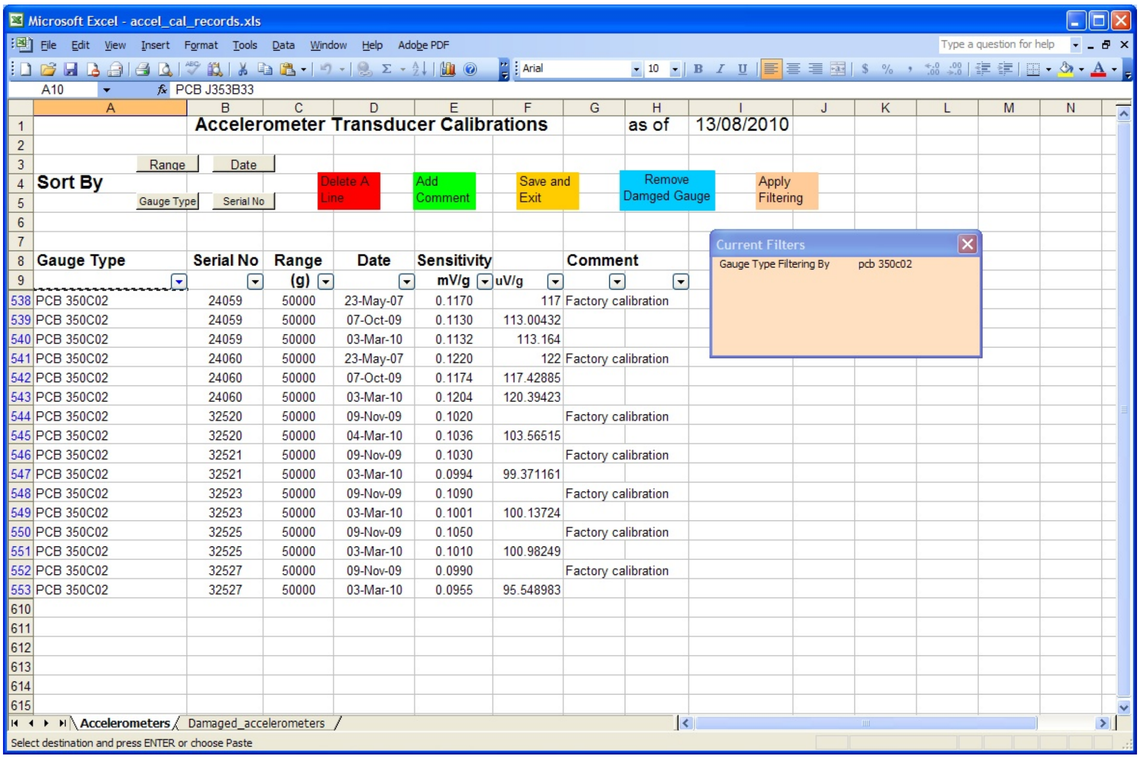


DROP	Vs (V)	Gs	Vt (V)	Gt	g	mV
...	...	...	...	...	0.00	0.00
3	1.5819	500	0.756	10	1259.42	150.07
1	2.27	500	1.077	10	1807.24	213.78
2	2.912	500	1.375	10	2318.37	272.94
4	4.546	500	2.1409	10	3619.26	424.97
5	4.554	500	2.134	10	3625.63	423.60
6	4.928	500	2.317	10	3923.39	459.92
7	7.5559	500	3.514	10	6015.57	697.53
8	7.8119	500	3.614	10	6219.38	717.38
9	7.8799	500	3.6939	10	6273.52	733.24
11	9.056	500	4.19	10	7209.86	831.72
10	9.26	500	4.294	10	7372.28	852.36
12	9.884	500	4.614	10	7869.07	915.88
13	11.0119	500	5.178	10	8767.04	1027.83
14	11.2119	500	5.234	10	8926.27	1038.95
15	11.248	500	5.2559	10	8955.01	1043.30
17	13.0599	500	6.15	10	10397.54	1220.78
18	13.4479	500	6.266	10	10706.44	1243.80
16	13.524	500	6.288	10	10767.03	1248.17

Figure 18 The accelerometer calibration spreadsheet, set for use with the force transducer

5.2 The Accelerometer Calibration Histories.

Each time an accelerometer calibration is performed the results of the calibration are logged to a separate spreadsheet. This spreadsheet contains the calibration history of all accelerometers and forms an archive of the calibration date and the sensitivity of all accelerometers within the system. This database enables individual accelerometers or entire accelerometer class histories to be tracked. Any change to the sensitivity over time can be detected, as can poor individual calibrations where a single calibration value does not correlate with other calibrations. The accelerometer calibration record spreadsheet enables filtering by gauge type, gauge number, range, calibration date and finally a filter for the most recently performed calibration based on any of the other filters, for example the most recent calibration for all 5000 *g* accelerometers. It also allows for sorting by these categories. Figure 19 shows the calibration history of various accelerometers based on filtering for PCB 350C02 accelerometers with a 50000 *g* range. The results are sorted according to accelerometer number and date of calibration.



Accelerometer Transducer Calibrations as of 13/08/2010

Gauge Type	Serial No	Range (g)	Date	Sensitivity mV/g	uV/g	Comment
PCB 350C02	24059	50000	23-May-07	0.1170	117	Factory calibration
PCB 350C02	24059	50000	07-Oct-09	0.1130	113.00432	
PCB 350C02	24059	50000	03-Mar-10	0.1132	113.164	
PCB 350C02	24060	50000	23-May-07	0.1220	122	Factory calibration
PCB 350C02	24060	50000	07-Oct-09	0.1174	117.42885	
PCB 350C02	24060	50000	03-Mar-10	0.1204	120.39423	
PCB 350C02	32520	50000	09-Nov-09	0.1020		Factory calibration
PCB 350C02	32520	50000	04-Mar-10	0.1036	103.56515	
PCB 350C02	32521	50000	09-Nov-09	0.1030		Factory calibration
PCB 350C02	32521	50000	03-Mar-10	0.0994	99.371161	
PCB 350C02	32523	50000	09-Nov-09	0.1090		Factory calibration
PCB 350C02	32523	50000	03-Mar-10	0.1001	100.13724	
PCB 350C02	32525	50000	09-Nov-09	0.1050		Factory calibration
PCB 350C02	32525	50000	03-Mar-10	0.1010	100.98249	
PCB 350C02	32527	50000	09-Nov-09	0.0990		Factory calibration
PCB 350C02	32527	50000	03-Mar-10	0.0955	95.548983	

Figure 19 The accelerometer calibration record spreadsheet

Any accelerometer that fails the calibration procedure is not removed from the accelerometer database; rather all calibrations for this accelerometer are placed in a separate worksheet devoted to damaged accelerometers. This aids in keeping track of all accelerometers used within the Division.

Along with the spreadsheet containing the history of the accelerometer calibrations, each time an accelerometer is calibrated the individual calibration sheet is also saved with the date and accelerometer number included in the file name. Any calibration performed with this system can be substantiated at any time should a query about an individual calibration be raised.

Paper copies of each calibration can be automatically produced and placed on the file pertaining to the experiment.

6. Error Analysis

6.1 The Uncertainty in the Calibration Equipment

6.1.1 General Conventions Used in the Error Analysis

As per NATA standards it will be assumed that unless stated otherwise all uncertainties listed on the calibration results are to a 95% confidence limit, [14]. In other words the uncertainties are assumed to be stated to a level of two standard deviations (2σ). However, to simplify the calculation, the error analysis in the following sections is performed for an uncertainty of one σ, a level of 68% confidence, and it is these values that are listed in Table 2. This uncertainty level is then expanded out to a level of 95% confidence at the end of the calculation.

Convention states that experimental uncertainties should be stated to only one significant figure though an extra digit can be carried through for calculation purposes [15]. This convention is used in the following error analysis.

In order to determine the uncertainty of the system, the relative random uncertainties are added in quadrature, that is, the total random uncertainty is the root of the sum of the squared random uncertainties. The total systematic uncertainty will be determined by direct addition of the systematic uncertainties. This will produce a maximum systematic error, [15].

6.1.2 Error Levels in the Equipment

There are several items that need to be taken into consideration when determining the accuracy of the calibrations. Of prime importance are the uncertainty of the acceleration secondary standard and force transducer standard. Next is the confidence in the instrumentation AC calibration signal and the uncertainty in the voltage measurements. Also to be considered is the uncertainty in the masses and the tolerance of the precision capacitor used in the charge amplifier to convert the calibration signal from a voltage source into a charge input.

The DSO operating with 12 bits has 4096 digitisation levels. For every measurement where a minimum (or baseline) reading and a maximum reading are taken together it incurs a two digitisation level error. The uncertainty in this measurement is therefore 2 in 4096 or 0.049%. However, it is reasonable to assume that the DSO will not be recording signals at full scale all the time but recording signals somewhere between half and full scale. If we assume the worst case of half full scale the relative error would increase by 2 and become 0.098%.

The DSO does not need to be NATA certified because during the channel characterisation it, along with all the other components in the channel, is directly characterised with the AC standard.

The total random uncertainty in the drop mass must include the uncertainty in the measured value of the drop mass and the uncertainty in the measured value of the certified mass. These are typically 0.1 g in 120 g (0.083%) and 0.1 g in 100 g (0.1%) respectively and are included in Table 2.

At this point it should be noted that the tolerance in the capacitance and the certified mass are not random errors but are systematic errors, as such there is not a uniform method of handling them. It can be argued that direct addition would lead to the maximum error and this method is used in this report [15].

In the case of the capacitor the tolerance is a manufacturing limit and while the spread of values during production may indeed be random and Gaussian, for a single capacitor the value is fixed. Similarly for the certified mass. While the uncertainty of the mass may follow a random, Gaussian probability during its own calibration, the actual mass is fixed, though within the stated limit. This makes sense, as the actual mass of the certified mass won't change between one measurement and the next. Any difference is due to the measuring device.

The precision capacitor in the charge amplifier has a typical tolerance of 0.01%, which is comparable to the NATA certified uncertainty of much of the equipment. The certified mass has a tolerance of 0.0015%, which is an order of magnitude better than the uncertainty in the other NATA certified standards.

Table 2 Typical uncertainties in the calibration instrumentation, stated with a 68% confidence level

Process or Item	Uncertainty	Symbol	Comment
Acceleration Standard	≤ ± 1.5%	σ_{as}	From cal sheet reduced to one σ
Force Transducer Standard	≤ ± 0.05 %	σ_{ft}	From cal sheet reduced to one σ
AC Voltage Standard	≤ ± 0.025%	σ_{vs}	From cal sheet reduced to one σ
12 bit ADC in DSO	≤ ± 0.098 %	σ_{dso}	2 in 4096 (2^{12}) × 2, half full scale
Precision Capacitor	≤ ± 0.01%	ξ_c	Tolerance of capacitor
Drop Mass Reading	≤ ± 0.083 %	σ_{dm}	0.1g over 120g (typically)
Reference Mass Reading	≤ ± 0.1 %	σ_{rm}	0.1g over 100g
Reference Mass Error	≤ ± 0.0015 %	ξ_{rme}	From cal sheet reduced to one σ

6.2 The Uncertainty of a Calibration Drop

6.2.1 Uncertainty in the Acceleration Standard Channel

The uncertainty in the data channel containing the accelerometer standard (σ_{asc}) consists of the acceleration standard sensitivity uncertainty (σ_{as}), the uncertainty in the data channel (σ_{da}) and the uncertainty in the drop calibration reading (σ_{dc})

$$\sigma_{asc} = \sqrt{\sigma_{as}^2 + \sigma_{da}^2 + \sigma_{dc}^2} \quad (6.1)$$

where

$$\sigma_{da}^2 = \sigma_{vs}^2 + \sigma_{dso}^2 \quad (6.2)$$

and

$$\sigma_{dc}^2 = \sigma_{dso}^2 \quad (6.3)$$

Combining Equations (6.1), (6.2) & (6.3) leads to,

$$\sigma_{asc} = \sqrt{\sigma_{as}^2 + \sigma_{vs}^2 + 2 \times \sigma_{dso}^2} \quad (6.4)$$

Substituting the relative uncertainties contained in Table 2.

$$\sigma_{asc} = \sqrt{1.5^2 + 0.025^2 + 2 \times 0.098^2} = 1.507\%$$

Ordinarily this would reduce to 1 significant figure, however as this error along with subsequent errors will be used for calculating purposes it is rounded off to 2 significant figures giving

$$\sigma_{asc} = 1.5\%$$

6.2.2 Uncertainty in the Force Transducer Standard Channel

The random uncertainty in data channel containing the force transducer standard (σ_{ftc}) consists of the force transducer sensitivity uncertainty (σ_{ft}), the uncertainty in the gain of the system (σ_{gs}), the uncertainty of the mass (σ_m) and the uncertainty in the drop calibration reading (σ_{dc}). The systematic uncertainty consists of the uncertainty in the capacitor (ξ_c) and the uncertainty in the reference mass (ξ_{rme}).

$$\sigma_{ftc} = \sqrt{\sigma_{ft}^2 + \sigma_{gs}^2 + \sigma_m^2 + \sigma_{dc}^2} \quad (6.6)$$

σ_{gs} has uncertainty components due to σ_{vs}, and σ_{dso} but also has a systematic error component ξ_c.

The random component is

$$\sigma_{gs}^2 = \sigma_{vs}^2 + \sigma_{dso}^2 \tag{6.7}$$

The mass of the drop weight is determined using the following equation. $M_{total} = M_{measured} \times \frac{m_{actual}}{m_{measured}}$
Where ***M*** is the mass of the drop weight, and ***m*** is the mass of the standard.
Therefore σ_m has uncertainty components due to σ_{rm} , σ_{dm} and ξ_{rme}.

The random component is

$$\sigma_m^2 = \sigma_{rm}^2 + \sigma_{dm}^2 \tag{6.8}$$

and once again

$$\sigma_{dc} = \sigma_{dso} \tag{6.9}$$

Combining Equations (6.6), (6.7), (6.8) & (6.9) leads to,

$$\sigma_{ftc} = \sqrt{\sigma_{ft}^2 + \sigma_{vs}^2 + \sigma_{rm}^2 + \sigma_{dm}^2 + 2 \times \sigma_{dso}^2} \tag{6.10}$$

Substituting the relative uncertainties contained in Table 2.

$$\sigma_{ftc} = \sqrt{0.05^2 + 0.025^2 + 0.1^2 + 0.083^2 + 2 \times 0.098^2} = 0.1980\%$$

Rounding off to 2 significant figures gives:

$$\sigma_{ftc} = 0.20\%$$

We now need to add the uncertainty due to the tolerances of capacitor and the reference mass. There is no easy way to add the systematic errors, however, as worst case scenario we can add them directly [15].

$$\xi_{ftc} = \sigma_{ftc} + \xi_c + \xi_{rme}$$

$$\xi_{ftc} = 0.1980 + 0.01 + 0.0015 = 0.1980\% + 0.0115\%$$

Rounding off to 2 significant figure gives:

$$\xi_{ftc} = 0.20\% + 0.01\%$$

While the uncertainties due to the capacitor and the reference mass do make a small difference in the overall uncertainty of the force transducer channel, their effect is an order of magnitude less than the random uncertainties. When significant figures are taken into account they can be ignored and it is reasonable to claim that σ_{ftc} = 0.2% at the 68% confidence level.

6.2.3 Uncertainty in the Accelerometer Channel

The uncertainty in the data channel containing the accelerometer under calibration (σ_{acc}) consists of the uncertainty in the gain of the accelerometer channel (σ_{gac}) and the uncertainty in the drop calibration reading (σ_{dc})

$$\sigma_{acc} = \sqrt{\sigma_{gac}^2 + \sigma_{dc}^2} \qquad (6.11)$$

where

$$\sigma_{gac}^2 = \sigma_{vs}^2 + \sigma_{dso}^2 \qquad (6.12)$$

and once again

$$\sigma_{dc} = \sigma_{dso} \qquad (6.13)$$

Combining Equations (6.11), (6.12) & (6.13) leads to,

$$\sigma_{acc} = \sqrt{\sigma_{vs}^2 + 2 \times \sigma_{dso}^2} \qquad (6.14)$$

Substituting the relative uncertainties contained in Table 2.

$$\sigma_{acc} = \sqrt{0.025^2 + 2 \times 0.098^2} = 0.141\%$$

Once more rounding off to 2 significant figures gives

$$\sigma_{acc} = 0.14\%$$

6.2.4 Relative Uncertainty for a Calibration Drop Using the Acceleration Standard

The relative uncertainty in any one reading for a calibration drop using the acceleration standard (σ_{das}) is

$$\sigma_{das} = \sqrt{\sigma_{asc}^2 + \sigma_{acc}^2} \qquad (6.15)$$

Substituting in the calculated values

$$\sigma_{das} = \sqrt{1.5^2 + 0.14^2} = 1.51\%$$

Rounding off to 2 significant figures gives:

$\sigma_{das} = 1.5\%$

6.2.5 Relative Uncertainty for a Calibration Drop Using the Force Transducer

The relative uncertainty in any one reading for a calibration drop using the force transducer (σ_{dft}) is

$$\sigma_{dft} = \sqrt{\sigma_{ftc}^2 + \sigma_{acc}^2} \quad (6.16)$$

Substituting in the calculated values

$\sigma_{dft} = \sqrt{0.2^2 + 0.14^2} = 0.244\%$

Rounding off to 2 significant figures gives:

$\sigma_{dft} = 0.24\%$

6.2.6 Comparison of the Uncertainties

Comparing the final uncertainties of the drop calibration methods it is clear that the uncertainty for calibration using the acceleration standard is significantly greater than using the force transducer. This is primarily due to the relatively large uncertainty of the acceleration standard itself.

Secondly, the error in the accelerometer channel has a minor effect on the total uncertainty of the drop regardless of whether the calibration was performed using the acceleration standard or the force transducer. In both cases, when rounding off the final uncertainty, at one standard deviation, to a single significant figure, it remains the same as the uncertainty in the channel of the standard alone.

Thirdly, the systematic uncertainties associated with the precision capacitor and the reference mass, when combined, are an order of magnitude lower than the random uncertainty in force transducer channel. When rounding to the correct number of significant figures they have a negligible effect.

The errors for each data channel are given in Table 3 with the uncertainty at 95% confidence limit rounded off to 1 significant figure.

Table 3 Uncertainties in each data channel

Item	Uncertainty, 68% confidence	Uncertainty, 95% confidence
σ_{asc}	1.5%	3%
σ_{ftc}	0.20%	0.4%
σ_{acc}	0.14%	0.3%

The errors for the drop calibration method are given in Table 4 with the uncertainty at 95% confidence limit rounded off to 1 significant figure.

Table 4 Uncertainties in each drop calibration method

Item	Uncertainty, 68% confidence	Uncertainty, 95% confidence
σ_{das}	1.5%	3%
σ_{dft}	0.24%	0.5%

6.3 The Uncertainty Associated with Linear Regression

The error analysis carried out in the previous sections only refers to a single drop; the actual calibration process involves a series of drops, which are then subjected to a linear regression. The uncertainties for this calculation, which can be found in several references, including [15-17], are described by Equations (6.17) to (6.25), below.

If a linear regression (least squares fit) is performed on a set of data it can be shown that for the function

$$y(x) = a + bx \tag{6.17}$$

The values ***a*** and ***b*** will be the following

$$a = \frac{1}{\Delta}(\sum \omega_i x_i^2 \sum \omega_i y_i - \sum \omega_i x_i \sum \omega_i x_i y_i) \tag{6.18}$$

$$b = \frac{1}{\Delta}(\sum \omega_i \sum \omega_i x_i y_i - \sum \omega_i x_i \sum \omega_i y_i) \tag{6.19}$$

where

$$\Delta = \sum \omega_i \sum \omega_i x_i^2 - (\sum \omega_i x_i)^2 \tag{6.20}$$

and ω is the weighted average, such that

$$\omega_i = \frac{1}{\sigma_i^2} \tag{6.21}$$

and σ_i represents the uncertainty in a data point.

When performing a standard linear regression it is customary that the uncertainty is in the dependent variable y and that it is expected to be uniform for all points. The drop calibration procedure has uncertainties in both y and x and they are not uniform for all points but are relative to the size of the y and x values. Since the errors are not uniform, the linear regression requires the use of the weighting function ω, Equation (6.21).

The Equations (6.18), (6.19) & (6.20) relate to errors in the y value. Fortunately since the relationship between x and y is linear, the uncertainties in x can be ascribed to y by using the gradient b of the function as shown in Equation (6.22).

$$\sigma_i^2 = \sigma_{yi}^2 + (b\sigma_{xi})^2 \qquad (6.22)$$

A problem arises, however, in that the gradient b can't be determined until the error is known. In this case, the assumption is made that the gradient b will be close to the gradient B of a linear regression assuming no error, where B is determined by Equation (6.23) [15].

$$B = \frac{N\sum xy - \sum x \sum y}{N\sum x^2 - (\sum x)^2} \qquad (6.23)$$

Having evaluated the values of a and b it can be shown that the uncertainties associated with these constants are determined by Equations (6.24) and (6.25).

$$\sigma_a = \sqrt{\frac{1}{\Delta}\sum \omega_i x_i^2} \qquad (6.24)$$

$$\sigma_b = \sqrt{\frac{1}{\Delta}\sum \omega_i} \qquad (6.25)$$

6.4 Examples of Error Analysis

6.4.1 Examples of Error Analysis

To illustrate the error analysis method, the formulas listed in Section 6.3 are applied to the data listed in the calibration shown in Figure 18. The value of B from Equation (6.23) is simply the value returned by the calibration spreadsheet and equals 0.1164.

The error analysis calculations are listed in Table 5.

Table 5 Error analysis applied to the data contained in Figure 18

Y(mV)	σ_{mV} ±0.14%	X(*g*)	σ_g ±0.2%	ω_i	$\omega_i x_i^2$	$\omega_i x_i$	$\omega_i y_i$	$\omega_i x_i y_i$
150.07	0.210	1259.42	2.519	7.6864	1.219E+07	9.680E+03	1153.461	1.453E+06
213.78	0.299	1807.24	3.614	3.7511	1.225E+07	6.779E+03	801.923	1.449E+06
272.94	0.382	2318.37	4.637	2.2867	1.229E+07	5.302E+03	624.138	1.447E+06
424.97	0.595	3619.26	7.239	0.9399	1.231E+07	3.402E+03	399.449	1.446E+06
423.60	0.593	3625.63	7.251	0.9398	1.235E+07	3.407E+03	398.078	1.443E+06
459.92	0.644	3923.39	7.847	0.8007	1.233E+07	3.142E+03	368.283	1.445E+06
697.53	0.977	6015.57	12.031	0.3431	1.241E+07	2.064E+03	239.304	1.440E+06
717.38	1.004	6219.38	12.439	0.3221	1.246E+07	2.003E+03	231.039	1.437E+06
733.24	1.027	6273.52	12.547	0.3138	1.235E+07	1.969E+03	230.089	1.443E+06
831.72	1.164	7209.86	14.420	0.2396	1.246E+07	1.728E+03	199.306	1.437E+06
852.36	1.193	7372.28	14.745	0.2289	1.244E+07	1.687E+03	195.068	1.438E+06
915.88	1.282	7869.07	15.738	0.2000	1.238E+07	1.574E+03	183.174	1.441E+06
1027.83	1.439	8767.04	17.534	0.1604	1.233E+07	1.406E+03	164.818	1.445E+06
1038.95	1.455	8926.27	17.853	0.1554	1.238E+07	1.387E+03	161.481	1.441E+06
1043.30	1.461	8955.01	17.910	0.1543	1.238E+07	1.382E+03	161.015	1.442E+06
1220.78	1.709	10397.54	20.795	0.1139	1.231E+07	1.184E+03	139.040	1.446E+06
1243.80	1.741	10706.44	21.413	0.1082	1.240E+07	1.158E+03	134.544	1.440E+06
1248.17	1.747	10767.03	21.534	0.1071	1.242E+07	1.153E+03	133.688	1.439E+06
sum								
12268.04	17.18	105265.30	210.53	18.74	2.100E+08	4.925E+04	5784.21	2.453E+07

These calculations in turn produce the following results:

$b = 0.115810$, $\sigma_b = 0.000111$

Rounding the error off to one significant figure:

$b = 0.1158$, $\sigma_b = 0.0001$

While this seems a very small error it is approximately 0.09 % of the sensitivity, which is comparable to the errors associated with the instrumentation used in the calibration.

This error level is for a confidence level of 68%, to expand the confidence level to 95% and using only 1 significant figure gives:
$b = 0.1158 \pm 0.0002$

An example of the error analysis calculations performed on the calibration results of a 50 *g* accelerometer calibrated with the acceleration standard over 15 drops is shown in Table 6.

Table 6 Error analysis applied to a 50 g accelerometer calibration

Y(mV)	σ_{mV} ±0.14%	X(*g*)	σ_g ±1.5%	ω_i	$\omega_i x_i^2$	$\omega_i x_i$	$\omega_i y_i$	$\omega_i x_i y_i$
530.71	0.743	5.25	0.079	0.41163	0.07846	7.93676	41.63958	0.41163
821.48	1.150	8.09	0.121	0.41161	0.05085	5.16019	41.77116	0.41161
1114.19	1.560	11.14	0.167	0.41171	0.03694	3.69331	41.16088	0.41171
1281.03	1.793	12.77	0.192	0.41169	0.03223	3.23263	41.28984	0.41169
1407.10	1.970	14.03	0.211	0.41169	0.02934	2.94133	41.27826	0.41169
1525.03	2.135	15.13	0.227	0.41166	0.02720	2.74167	41.48741	0.41166
1882.91	2.636	18.79	0.282	0.41170	0.02191	2.19487	41.24853	0.41170
2139.11	2.995	21.32	0.320	0.41169	0.01931	1.93831	41.31544	0.41169
2566.12	3.593	25.46	0.382	0.41166	0.01617	1.62924	41.48591	0.41166
3102.93	4.344	30.87	0.463	0.41168	0.01333	1.34008	41.37428	0.41168
3574.68	5.005	35.47	0.532	0.41166	0.01161	1.16957	41.48591	0.41166
3826.82	5.358	38.16	0.572	0.41169	0.01079	1.08215	41.29039	0.41169
4331.09	6.064	43.08	0.646	0.41168	0.00956	0.96087	41.39126	0.41168
4920.77	6.889	48.89	0.733	0.41167	0.00842	0.84737	41.43122	0.41167
5429.12	7.601	53.94	0.809	0.41167	0.00763	0.76820	41.43570	0.41167
sum								
38453.09	53.83	382.41	5.74	0.0352	6.1751	0.37375	37.6366	621.0858

This produces the following results,
b = 100.36, σ_b = 0.672

Reducing the uncertainty to a single significant figure and expanding it to the 95% confidence level, gives:
b = 100 ± 1

6.4.2 Comparison of Linear Regression Results With And Without Error Analysis

It is now possible to compare the sensitivity produced with error analysis to that calculated without it.

In the first example the sensitivity of the calibration as shown in Figure 18, where there was no error analysis, is 0.1164. This is outside the range of values obtained by doing an error analysis on the linear regression. However, if the two values of the sensitivity are directly compared the difference in them is

$$\left|\frac{0.1158-0.1164}{0.1158}\times 100\right| \approx 0.5\% .$$

In the second example the stated sensitivity for this calibration was 100.57 mV/*g*. This value is within the limits of the uncertainty of the determined result when calculating errors even when considering a single significant figure.

Whether the type of outcome as shown in the first example, where the sensitivity obtained by a standard linear regression falls outside of a fully analysed value and error limit, is acceptable must be determined by the operator. However, experimental experience would dictate that the difference of values in this particular case, which is less than 2 orders of magnitude smaller than the sensitivity value, is within an acceptable level when dealing with explosive related experiments.

6.4.3 Limitations associated with the error analysis

The main limitation associated with the error analysis performed using linear regressions is that it requires the use of the result from a general linear regression without error analysis. This assumes that the sensitivity produced by the linear regression without error analysis is close to the actual value produced with error analysis.

A second consideration not directly concerned with the error calculation but related to it is operator error, this was discussed in section 4.3. It can be argued that operator error is often systematic in nature and therefore adds directly to the overall uncertainty of the system [15]. These errors may be impossible to quantify and might only be overcome with experience. When operator error is present, it would be hoped that checks on items such as the linearity of the calibration or the comparison to previous sensitivity values would help to indicate its presence.

7. Limitations and Recommendations

While the drop calibration method used within MPD for determining the sensitivity of accelerometers produces good results there are a number of limitations when using this system and they are summarised below.

Firstly, this calibration process has an acceleration limit of approximately 10000 *g*. Several of the shock accelerometers used within MPD range to 200000 *g*, this means that the calibration range is restricted to only 5% of the maximum acceleration level of the highest rated accelerometers.

Secondly, the tolerance of the calibration capacitor used in the charge amplifier is specified but not NATA certified. Also, neither the tolerance of the capacitor nor the certified mass is random. Fortunately these systematic errors are smaller, by an order of magnitude, compared to the other errors in the data channels. This allows us to neglect them for the most part especially when dealing with only one significant digit in the calibration uncertainty.

Thirdly, the resolution of the electronic balance is limited. A balance with greater resolution, say 0.01 g, would improve the overall random uncertainty markedly.

A fourth limitation of the current process is the spreadsheet does not take into account the uncertainty in the data points when calculating the sensitivity, nor does it return the associated uncertainty in the sensitivity. While it is recognised that this is a limitation in the calibration process, there are no immediate plans to change the spreadsheet to produce an uncertainty. Primarily, this is because of the relatively small difference between the sensitivity calculated without error analysis to that calculated with error analysis, when compared to the size of the experimental uncertainty of shock experiments. Instead, if the resultant sensitivity is acceptable, when compared to previous calibrations, and the correlation coefficient is close to one and there are no visible anomalies in the plot, then the value produced by the spreadsheet is accepted. If the accelerometer uncertainty is required or if the accelerometer produces anomalies, the sensitivity and the error can be calculated independently at a latter date.

Another limitation of this calibration procedure is that there is little regard to the frequency response of the accelerometer [1-2]. This calibration procedure supplies only a single sensitivity and assumes that the variation in the sensitivity over the frequencies of interest is negligible. Although the drop process occurs over a short time duration it is significantly longer and the frequency is considerably lower than the shock response commonly produced by shock waves due to explosive detonations. Despite the fact this calibration process is not a shock response test of the accelerometers, it is worth keeping in mind that accelerometers that calibrate successfully with the drop calibration method are not guaranteed to function correctly under a shock loading.

Limitations to the peak acceleration when using the drop calibration procedure could be overcome by employing a Hopkinson Bar to calibrate the accelerometers. This would replace the current method of accelerometer calibration with a new procedure which may also provide a function test of the accelerometers by applying higher acceleration levels over shorter time durations approaching those produced by explosive loading. A drawback to changing the calibration system to a Hopkinson Bar calibration system is the introduction of a new set of error analysis requirements.

While changing the calibration process would not necessarily overcome any limitations inherent in calculating the sensitivity of the accelerometers, an automated system may help eliminate operator errors.

8. Conclusion

This report has outlined the process within Maritime Platforms Division of DSTO for calibrating accelerometers for shock measurements, known as the drop calibration method. This calibration procedure provides accelerometer sensitivities that are traceable to NATA certified standards.

In describing the concepts and the methodology of the drop calibration process for accelerometers this report illustrated the ease of use and traceability of this process. The report differentiated between two procedures that can be used with this method and an analysis of the uncertainty of both procedures was presented. The limitations of the drop calibration method were discussed and a Hopkinson Bar calibration system was recommended if calibration to a level of acceleration greater than 10000*g* is required.

9. References

1. Richard Lally 1995, 'Accelerometer Calibration for the Test Engineer', Test Engineering & Management, April/May.
2. Raymond R. Bouche 1979, 'Calibration of Shock and Vibration Measuring Transducers', Bouche Laboratories, Tujunga, California.
3. PCB Piezotronics, Inc 2001, 'Model 350B02 ICP Accelerometer Installation and Operating Manual', Depew, New York.
4. James W. Dally, William F. Riley & Kenneth G. McConnell 1984, *Instrumentation For Engineering Measurements*, John Wiley & Sons, Inc. New York.
5. Klass B. Klaassen 2002, *Electronic Measurement and Instrumentation*, Cambridge University Press, United Kingdom
6. PCB Piezotronics, Inc, 'General Guide to ICP Instrumentation', Depew, New York.
7. PCB Piezotronics, Inc, 'Transducer Instrumentation Model F482A10 Line Power Supply', Buffalo, New York.
8. Endevco Corporation 1978, 'Instruction Manual for Endevco Piezoresistive Accelerometers', San Juan Capistrano, California.
9. Endevco Corporation 1979, 'Instruction Manual, Models 4422 and 4423 Signal Conditioners', San Juan Capistrano, California.
10. PCB Piezotronics, Inc, 'Transducer Instrumentation, Model F394A03 Vibration Calibration System', Depew, New York.
11. Kistler Instrument Company 1971, 'Model 894K3 Shock Calibration System' Amherst, New York.
12. Kistler Instrument Company 1971, 'Models 504D and 504D3 Dual Mode Amplifiers', Amherst, New York.
13. PCB Piezotronics, Inc, 'Transducer Instrumentation, Model F464A Dual Mode Charge Amplifier', Depew, New York.
14. R. R. Cook, 2002, 'Assessment of Uncertainties of Measurement for Calibration & Testing Laboratories', Second edition, National Association of Testing Authorities, Australia.
15. John R. Taylor 1997, *An Introduction to Error Analysis - The Study of Uncertainties In Physical Measurements*, Second Edition, University Science Books, Sausalito, California.
16. Philip R. Bevington & D Keith Robinson 2003, *Data Reduction and Error Analysis for the Physical Sciences*, Third Edition, McGraw-Hill Inc, New York.

17. John Mandel 1984, *The Statistical Analysis of Experimental Data*, Dover Publications, Inc. New York.

DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION DOCUMENT CONTROL DATA	1. PRIVACY MARKING/CAVEAT (OF DOCUMENT)

2. TITLE	3. SECURITY CLASSIFICATION (FOR UNCLASSIFIED REPORTS THAT ARE LIMITED RELEASE USE (L) NEXT TO DOCUMENT CLASSIFICATION)
Drop Calibration of Accelerometers for Shock Measurement	Document (U) Title (U) Abstract (U)
4. AUTHOR(S)	5. CORPORATE AUTHOR
Andrew Krelle	Defence Science and Technology Organisation 506 Lorimer St Fishermans Bend Victoria 3207 Australia

6a. DSTO NUMBER	6b. AR NUMBER	6c. TYPE OF REPORT	7. DOCUMENT DATE
DSTO-TR-2584	AR-015-057	Technical Report	August 2011

8. FILE NUMBER	9. TASK NUMBER	10. TASK SPONSOR	11. NO. OF PAGES	12. NO. OF REFERENCES
510/207/1194	LRR 07/246	CMPD	40	17

DSTO Publications Repository	14. RELEASE AUTHORITY
http://dspace.dsto.defence.gov.au/dspace/	Chief, Maritime Platforms Division

15. SECONDARY RELEASE STATEMENT OF THIS DOCUMENT

Approved for public release

OVERSEAS ENQUIRIES OUTSIDE STATED LIMITATIONS SHOULD BE REFERRED THROUGH DOCUMENT EXCHANGE, PO BOX 1500, EDINBURGH, SA 5111

16. DELIBERATE ANNOUNCEMENT

No Limitations

17. CITATION IN OTHER DOCUMENTS Yes

18. DEFTEST DESCRIPTORS

Accelerometers, Calibration, Standards, Error Analysis

19. ABSTRACT

This report details the Drop Calibration Method for determining the sensitivity of shock accelerometers. The Drop Calibration Method requires the accelerometer under calibration and either a traceable acceleration standard or a traceable force transducer standard to undergo a series of drop impacts. The sensitivity of the accelerometer is determined by comparing the outputs of the accelerometer and the standard. By appropriate characterisation of both the accelerometer's and the standard's data channels, prior to the calibration drops, the Drop Calibration Method ensures that the resultant sensitivity is traceable to National Association of Testing Authorities, Australia (NATA) standards.