THE STRATEGIC LEVEL OPTIMIZATION OF AIR TO GROUND MISSILES FOR TURKISH AIR FORCE DECISION SUPPORT SYSTEM

THESIS

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Abstract

Inventory Management is one of the most important elements in military systems. Especially, if it is an ammunition inventory, because of its vital role in war, it might change a country's independence. To be able to find the optimum types and levels of ammunition, the probable needs of each type has to be calculated in peace time. Although the Turkish Air Force has Strategic Plans which show detailed war scenarios, there are many different ways to accomplish a mission in a scenario with different ammunition usages. Because of this, a model is needed to solve this problem with a different perspective.

In this research, the needs of air-to-ground missiles are calculated by using a Weapon Target Assignment algorithm with cost minimization, bomb usage minimization and effect maximization objectives. The model finds different combinations of bombs for each objective and it shows the main tradeoffs between many cheap dumb bombs and a few expensive smart bombs, the total cost of the operation and the total effects of the operation on targets with the current inventory. The preferences of the Decision Makers will shape this inventory due to these tradeoffs. To aid in this modeling, the number of strategies that can be created with the inventory is calculated using multinomial theory.

To my family who has sacrificed their life for me and to my wife who always loves me

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Nurdinc SENAY

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I. Introduction

In this chapter, Section 1.1 summarizes the problem background. Section 1.2 draws the scope of this research. Section 1.3 describes the main research question. Section 1.4 presents all of the assumptions that are considered in this research. Section 1.5 draws the outline of the thesis by giving information about the other chapters. Section 1.6 gives a brief summary of this chapter.

1.1 Problem Background

Logistics Management in operational planning is one of the most important factors that affect the result of a war. A good logistics plan in the peace time will increase the reaction time at first. Then, it will support the operational side of the war. Finally, while the countries in war are getting weakened, it will put an end to the war by maintaining the fighting force of one of the countries. Through the ages, the people who conquered very wide territories had powerful logistics systems. They moved thousands of people to the countries which were thousands of miles away from their homeland, had them fight and brought them back home. Although it is easier than the previous logistics managements by the help of technology, Logistics Management still has a lot of areas waiting to be improved.

As a part of Logistics Management, Inventory Management has a vital importance for parties like armed forces of a country which are trying to maximize its level of readiness or a company which is trying to maximize its profit while minimizing its inventory costs. Of course, it is not easy to forecast the need and usage for both type of parties, but, when the results of the decisions made by these parties to reach their objectives are compared, the decisions for forecasting the warfare needs might seem that they have to be perfect, since they might be more catastrophic.

After World War II, the expansion of the area of Operations Research and improvement in technology helped both these military and civilian societies so much to approach the logistics management in a measurable way and for better decisions in planning (Kress 2002). And, in this approach, "some of the typical logistics parameters and problems include:

1) Quantitative parameters such as volume of fuel, tonnage of ammunition and number of spare-parts;
2) Time parameters such as force accumulation time and order-and-ship time;
3) Forecasting attrition and projecting demands for resources;
4) Optimization of logistics processes such as transportation, inventory and storage." (Kress 2002).

In this research, one of these problems, optimization of inventory level of ammunition, is examined. While doing this, the usage of ammunition will be calculated by using one of the techniques for the assignment matching problem.

On the other hand, the assignment matching problem is a very common problem that various types of researchers from different areas are trying to solve. This problem can be applied to find the best solution for assigning numbers of people to numbers of jobs or numbers of weapons to numbers of targets and etc. (Kleeman and Lamont 2007).

So, since there are various types of weapons and targets on the battlefield, the war-time need for ammunition will be optimized by finding the best weapon target assignment to be able to increase the readiness.

1.2 Scope of the Research

According to the Turkish Air Force official website:

> "Main duty of the Air Force Command is to deter the enemy from its aggressive intention via its arms and means with superior velocity and brisance, to counteract enemy aircraft rapidly as soon as they enter Turkish airspace in case of an attack against the country, to discourage and dishearten from maintaining the war by destroying the vital military targets of the enemy country and to ensure that war is won within the shortest time possible with least casualties." (www.hvkk.tsk.tr 2010)

To be able to perform this duty, the Turkish Air Force has to be well-prepared to any kinds of threats. As a natural thing, the Turkish Air Force has some Strategic Plans. These plans show what to do in each period of war. They give the targets that will be destroyed and desired damage levels for these targets. On the other hand, the decision maker (DM) should think about the inventory levels of each weapon to be allocated to the targets to manage the power of fight during the war. Since it might take immense time during the war, all of these factors should be considered during peace time.

In this research, the strategic level of air to ground missiles is studied for achieving the optimum inventory level of these missiles according to optimum cost effective target weapon allocation.

1.3 Research Questions

In this research, the main research question is:

"*What is the optimum strategic level of air to ground missiles according to optimum allocation of weapons to targets due to the strategic plans of the Turkish Air Force in each day during the war?"*

To be able to answer this main question, there are two other sub level questions to be answered.

First of them is:

"How many variables will be considered in the model according to the given data?"

Since large scale models can be difficult to solve exactly, the solution algorithm may be changed if there are too many variables in the model, so we should learn about the number of variables.

Then,

"What is the optimum weapon-target allocation according to cost, effectiveness and bomb usage?"

Since it is easier to evaluate the inventory level of the air to ground missiles, the optimum usage should be calculated before the evaluation.

All of these questions will be answered one by one in a logical order.

1.4 Assumptions

As it is expected, operational planning is a very challenging issue. There are some assumptions made in this research to outline the problem.

1. In operational planning, air-to-air, air-to-ground and ground-to-ground missiles are considered. But in this research, only air-to-ground missiles are planned to be used.
2. Since all of the aircraft carry different kinds of ammunition with different kinds of combination according to their carrying capacity, the model is designed to calculate all of the scenarios according to each number of bombs and platforms. Eight types of bombs and five types of carrying platforms are considered in the model.
3. The probability of damage for each target by each bomb is known by the DM.

4. If a bomb is assigned to a target, it is assumed that it will certainly give damage according to its damage value. So, the conditions that the bomb is not sent to the target because of the pilot's fault or deficiency of the bomb are not considered in this research.
5. The targets that are damaged on each day are assumed to be rebuilt by the enemy on other days.
6. The depot levels of each bomb on each day are not considered to be able to see the actual needs of bombs on each day.
7. Since this research doesn't consider operational planning, the numbers of sorties on each day and aircrafts to fly are assumed to be planned by operational planners according to bomb-target assignments.
8. The needs of bombs for unplanned missions are not considered in this research.
9. All types of bombs can be launched without any differences like geological distances of bomb depots or time for being available.
10. There is no strategy that sending aircrafts without any bombs to targets. So at least one bomb will be assigned to any target on any day of war.

1.5 Organization

In Chapter 2, the literature review for the calculation of numbers of scenarios in the model and weapon target assignment problem will be discussed. In Chapter 3, the methodology used in this research will be discussed. Also, the mathematical model will be built in this chapter. In Chapter 4, the application of the mathematical model and its analysis will be discussed. Finally, in Chapter 5, the conclusions and the future works related to the same research topic will be discussed.

1.6 Summary

In this Chapter, the reason for this research and the problem background are mentioned in Section 1.1. The scope of the research is drawn in Section 1.2. The main and sub level research questions are specified in Section 1.3. The assumptions that are considered in this research are given in Section 1.4. Finally, the information about the other chapters are mentioned in Section 1.5 to give reader a quick look at what will discussed in the next chapters.

II. Literature Review

In this chapter, Section 2.1 reminds the reader of the research questions. Section 2.2 represents the binomial theorem, the multinomial theorem and their mathematical formulations. Section 2.3 gives information about assignment problems in a general way. Section 2.4 explains the weapon-target assignment (WTA) problem and its variations like static and dynamic WTA problems. Additionally, this section represents the previous works in operational planning with different WTA approaches. Finally, Section 2.5 explains the summary of this chapter.

2.1 Research Questions

In this section, the research questions discussed in Section 1 will be represented again.

In this research, the main research question is:

"*What is the optimum strategic level of air to ground missiles according to optimum allocation of weapons to targets due to the strategic plans of the Turkish Air Force in each day during the war?"*

To be able to answer this main question, there are two other sub level questions to be answered.

First of them is:

"How many variables will be considered in the model according to the given data?"

Because the solving algorithm may be changed if there are too many variables in the model, the DM should learn the number of variables.

Then,

"What is the optimum weapon-target allocation according to cost, effectiveness and bomb usage?"

Since it is easier to evaluate the inventory level of the air to ground missiles, the optimum usage should be calculated before the evaluation.

These research questions will be answered in Chapter 3 by giving formulations and mathematical models, but in Chapter 2, the related previous research will be represented.

2.2 Binomial and Multinomial Theorems

According to Gallier (2011):

For all $n \in \mathbb{N}$ and all $k \in \mathbb{Z}$, if $\binom{n}{k}$ symbolizes the number of subsets of cardinality *k* of a set of cardinality *n*, then

$$\binom{0}{0} = 1 \tag{2.1}$$

$$\binom{n}{k} = 0 \ if \ k \notin \{0, 1, \dots, n\} \tag{2.2}$$

$$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1} \quad (n \geq 1, 0 \leq k \leq n) \tag{2.3}$$

The numbers $\binom{n}{k}$ are named as binomial coefficients, since they appear in the expansion of the binomial expression$(a+b)^n$. And the binomial coefficients can be found by the right side of the Equation (2.3).

For all $n \in \mathbb{N}$ and for all reals,$a, b \in \mathbb{R}$,

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots$$
$$+ \binom{n}{k} a^{n-k} b^k + \cdots + \binom{n}{n-1} a b^{n-1} + b^n \tag{2.4}$$

Shortly,

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k \qquad (2.5)$$

Equation (2.5) is called as binomial formula and in general representation,

$$(a+b)^r = \sum_{k=0}^{\infty} \binom{r}{k} a^{r-k} b^k, \quad r \in \mathbb{N}\ or\ |b/a| < 1 \qquad (2.6)$$

Equation (2.6) is called the binomial theorem.

For all , m, k_1, ... , $k_m \in \mathbb{N}$, with $k_1 + \cdots + k_m = n$ and $m \geq 2$, $\binom{n}{k_1 \ldots k_m}$ represents the multinomial coefficient which gives the number of ways to split a set of *n* elements into an ordered sequence of m disjoint subsets, the *i*th subset having $k_i \geq 0$ elements.

Since the number of multinomial coefficients is used to find the number of combinations of a group of elements, this technique is used to calculate the number of decision variables or strategies that can be created according to the number of bomb types and the number of platforms in this research.

For all n, m ∈ ℕ with m ≥ 2, for all pairwise commuting variables $a_1, \ldots, a_m$,

$$(a_1 + \cdots + a_m)^n = \sum_{\substack{k_1,\ldots,k_m \geq 0 \\ k_1+\cdots+k_m = n}} \binom{n}{k_1 \ldots k_m} a_1^{k_1} \ldots a_m^{k_m} \qquad (2.7)$$

Equation (2.7) represents the multinomial theorem and the number of finite multisets of size $n \geq 0$ whose elements come from a set of size $m \geq 1$ can be calculated with $\binom{m+n-1}{n}$.

2.3 Assignment Problem

The assignment problem is a very popular subject in many areas, especially in Operations Research (OR) and there is a lot of research for the different kinds of optimum assignment. (Kleeman and Lamont 2007)

The previous research has looked at the military personnel assignment problem (Cimen 2001), personnel-course assignment problem (Malyemez 2011), naval personnel assignment problem (Paul 1990), multiple assignment problem (Walkup and MacLaren 1964), three-index assignment problem (Balas and Saltzman 1988), generalized weapon target assignment problem (Yucel, et al. 2005), dynamic weapon target assignment problem (Hosein and Athans 1989), traffic assignment problem (Larsson and Patriksson 1991), pilot-mission assignment problem (Durkan 2011), etc. with different techniques.

The assignment problem is originally a special case of the transportation problem. (Bazaraa, Jarvis and Sherali 2010). "The transportation problem may be represented mathematically,

$$Minimize \qquad z = \sum_{i=1}^{m_1} \sum_{j=1}^{m_2} c_{ij} \, . x_{ij} \tag{2.8}$$

Subject to

$$\sum_{j=1}^{m_2} x_{ij} = s_i \qquad i = 1,2,\dots,m_1 \tag{2.9}$$

$$\sum_{i=1}^{m_1} x_{ij} = d_j \qquad j = 1,2,\dots,m_2 \tag{2.10}$$

$$x_{ij} \geq 0 \qquad all\ i,j \tag{2.11}$$

given

m_1 source nodes (i=1,2, … , m_1) , each with s_i units of supply,

m_2 destination nodes (j=1,2, … , m_2) ,each with d_j units of demand,

c_{ij} is the unit flow cost from node i to j,

x_{ij} is the amount of arc flow to be determined." (Chen, Batson and Dang 2010).

In this model, Objective function (2.8) tries to minimize the total cost of the transportation, since each flow from node i to j has a cost. Constraint (2.9) is the constraint which gives the sources' amount of supply. Constraint (2.10) is the constraint which represents the destinations' amount of demand. To be able to solve the transportation problem as a network flow problem, the demand and supply must be equal to each other. If they are not equal, some dummy nodes can be created. (Bazaraa, Jarvis and Sherali 2010).

Figure 1 is the representation of this problem with dummy nodes. There is a procedure to change the network flow problem as a transportation problem when the demand and supply are not equal.

1. "Renumber m_1 source and m_2 destination nodes using a common index i = 1, 2, …, m_1, $m_1 + 1$, $m_1 + 2$, … , m, where $m = m_1 + m_2$. The unit transportation costs c_{ij} are also renumbered accordingly.
2. Set $b_i = s_i$ for i = 1, 2, …, m_1 and set $b_{m_1+j} = -d_j$ for j = 1, 2, …, m_2. Note that $\sum_i s_i = \sum_j d_j$ implies $\sum_{i=1}^{m_1} b_i + \sum_{i=1}^{m_2} b_{m_1+i} = \sum_{i=1}^{m} b_i = 0$.
3. Create a dummy source node (say node 0) with $b_0 = \sum_{i=1}^{m} b_i$ and connect arcs (0,i) for i = 1, 2, …, m_1 with unit cost $c_{0i} = 0$.
4. Create a dummy sink node (say node m+1) with $b_{m+1} = \sum_{i=m_1+1}^{m} b_i$ and connect arcs $(m_1 + i, m + 1)$ for i = 1, 2, …, m_2 with unit cost $c_{i,m+1} = 0$.
5. Add a return arc $(m + 1{,}0)$ with $c_{m+1,0} = 0$." (Chen, Batson and Dang 2010).

After following this procedure, the demand and supply will be balanced and the transportation problem will be able to be solved by linear programming (LP).

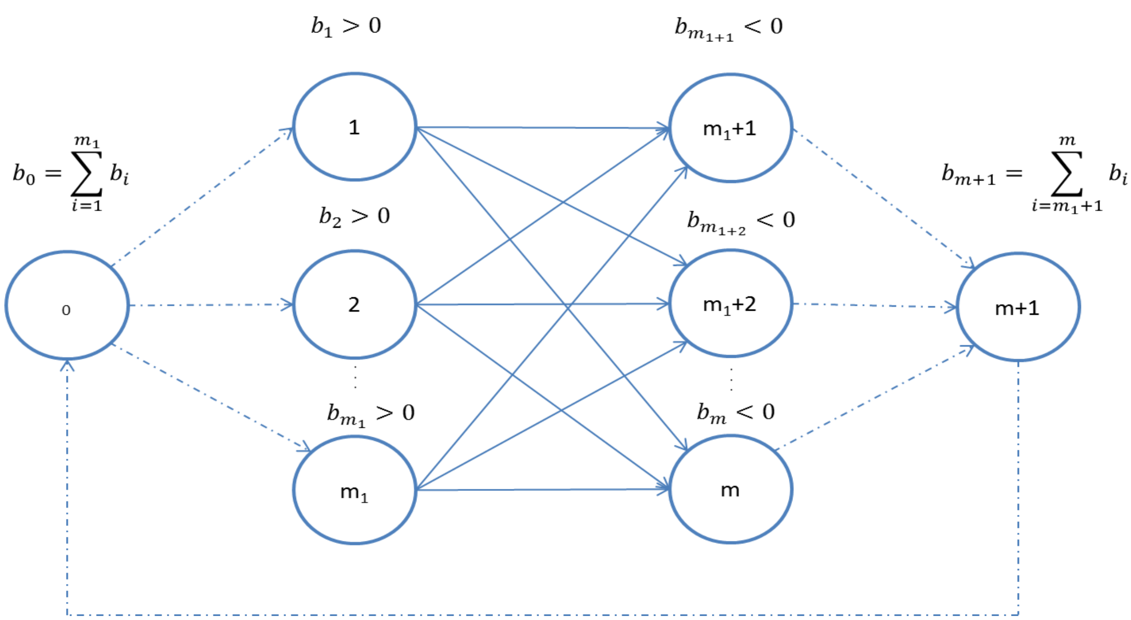

Figure 1. Representation of the Transportation Problem as a Network Flow Problem (Chen, Batson and Dang 2010) .

On the other hand, we know that the assignment problem is also a special case of the transportation problem. The difference between them is that the supply and demand in the transportation problem are men or personnel and job or machines in the assignment problem respectively and they are equal to one (Jeong 2010). For example, according to Gass (1975), if it is a personnel-job assignment problem, the mathematical formulation will be:

$$Maximize \qquad z = \sum_{i=1}^{m_1} \sum_{j=1}^{m_2} c_{ij} \,.\, x_{ij} \tag{2.12}$$

$$\sum_{j=1}^{m_2} x_{ij} = 1 \qquad i = 1,2,\dots,m_1 \tag{2.13}$$

$$\sum_{i=1}^{m_1} x_{ij} = 1 \qquad j = 1,2,\dots,m_2 \tag{2.14}$$

$$x_{ij} \geq 0 \qquad all\ i,j \tag{2.15}$$

where

m_1 is the number of personnel that will be assigned to a job,

m_2 is the number of jobs that will be assigned to a personnel,

c_{ij} is the value of assigning personnel i to job j,

x_{ij} is the decision variable that represents the assignment of personnel i to job j.

To be able to see this matching clearly, the bipartite graph representation may be used. Here, the nodes can be portioned into two sets of nodes like individuals (V_1) and jobs (V_2) and each node in the first set can be connected to a node in the second set by an arc or link. The arcs show the cost or value of assigning personnel i to a job j. (Goemans 2007). If all of the possible arcs or links are represented on the graph, it is called as complete bipartite graph. (Bazaraa, Jarvis and Sherali 2010).

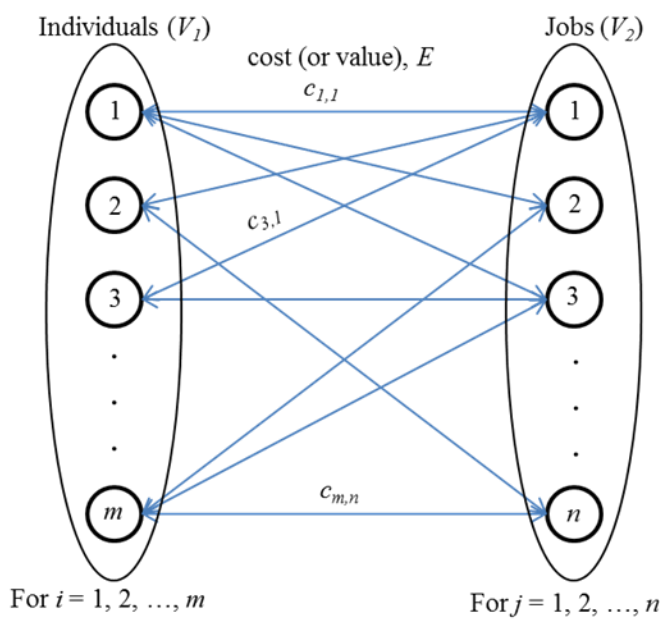

Figure 2. The Bipartite Graph Representation of Personnel-Job Assignment Problem (Jeong 2010)

According to Goemans, (2007), the mathematical model for the bipartite graph personnel-job assignment problem is:

$$Maximize\ or\ Minimize \qquad z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} \,.\, x_{ij} \tag{2.16}$$

$$\sum_{j=1}^{n} x_{ij} = 1 \qquad i \in V_1 = 1,2,\ldots,m \tag{2.17}$$

$$\sum_{i=1}^{m} x_{ij} = 1 \qquad j \in V_2 = 1,2,\ldots,n \tag{2.18}$$

$$x_{ij} \geq 0 \qquad all\ i,j \tag{2.19}$$

The Objective Function (2.16) tries to minimize or maximize the objective value. If the objective of the DM is minimizing the total cost of assignment process, then c_{ij} will be the cost of assigning the individual i to job j. On the other hand, if the objective of the DM is maximizing the total value of assignment process, then c_{ij} will be the value of assigning the individual i to job j. The decision variables, x_{ij}'s, will be equal to 0 or 1 since they are the pairwise matching. Constraint (2.17) requires that each individual can be assigned to only one job. Also, Constraint (2.18) requires that each job can be assigned to one individual.

2.4 Weapon-Target Assignment (WTA) Problem

Weapon-target assignment problem is a special kind of assignment problem. According to the hierarchy between WTA and the other problem types mentioned in the previous section, Figure 3 is drawn.

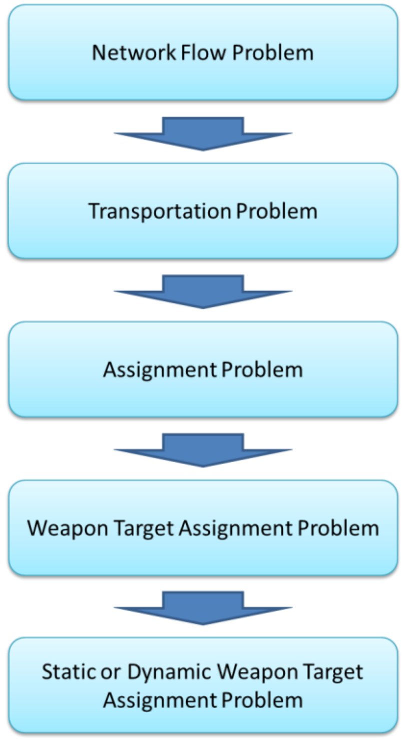

Figure 3. The hierarchical relationship between the problems

It is a very common subject in the defense-related applications. (Ni et al., 2011). There are different kinds of definitions and models for this problem, since it is solved for different objectives. In general, it tries to find an optimum assignment of weapons to target while maximizing or minimizing the objective value.

Since it is a special case of assignment problem, WTA may be defined as maximization of benefit of assigning weapons to targets. The mathematical formulation for this type WTA is defined by Bogdanowicz et al., (2004):

$$Maximize \qquad z = \sum_{i=1}^{n_1} \sum_{j=1}^{n_2} a_{ij} \, . \, x_{ij} \tag{2.20}$$

$$\sum_{j=1}^{n_2} x_{ij} \leq 1 \qquad i = 1,2, \dots, n_1 \tag{2.21}$$

$$\sum_{i=1}^{n_1} x_{ij} \leq 1 \qquad j = 1,2,\dots,n_2 \tag{2.22}$$

where

n_1 is the number of weapons,

n_2 is the number of targets,

a_{ij} is the benefit value of assigning weapon i to target j,

x_{ij} is the decision variable which indicates the assignment of weapon i to target j.

The Objective Function (2.20) tries to maximize the total benefit value of the assignment of $n_1$weapons to n_2 targets. Constraint (2.21) shows that each weapon can be assigned to one target at most. Constraint (2.22) shows that each target can be assigned by one weapon at most.

Another WTA problem is a problem that searches for the best weapon target matching which will maximize the total expected damage of the targets. (Ni et al., 2011). For this type of WTA, the mathematical model which is non-linear can be represented like:

$$maximize \sum_{j=1}^{T} u_j \left(1 - \prod_{i=1}^{W} \left(1 - P_{ij}\right)^{x_{ij}}\right) \tag{2.23}$$

Subject to:

$$\sum_{j=1}^{T} x_{ij} \leq w_i \qquad i = 1, 2, \dots, W \tag{2.24}$$

$$\sum_{i=1}^{W} x_{ij} \geq t_j \qquad j = 1, 2, \dots, T \tag{2.25}$$

$$x_{ij} \geq 0, integer, i = 1,2, \dots, W, j = 1, 2, \dots, T \tag{2.26}$$

where

W is the number of weapons,

T is the number of targets to attack,

u_j is the value of target j,

w_i is the number of weapons of type i available to be assigned to targets,

t_j is the minimum number of weapons needed for target j,

P_{ij} is the killing probability of weapon i on target j,

(1-P_{ij}) is the survival probability of target j attacked by weapon i.

x_{ij} is the decision variable which represents the number of weapons of type i assign to target j. (Ni et al., 2011).

In this model, the Objective Function (2.23) tries to maximize the total expected damage on targets by changing the survival probability to killing probability. The weapons to be assigned are restricted by Constraint (2.24) which prevents selecting weapons more than available and Constraint (2.25) which requires that the total number of weapons of type i used for target j must be greater than or equal to the minimum number of weapons required for target j.

Another formulation of WTA is based on minimization of the total expected value of the surviving targets. Hosein et al. (1988) define the mathematical formulation for this problem:

$$minimize \sum_{i=1}^{N} V_i \prod_{j=1}^{W} \left(1 - P_{ij}\right)^{x_{ij}} \tag{2.27}$$

subject to:

$$\sum_{j=1}^{M} x_{ij} = 1 \qquad i = 1, 2, \dots, N \qquad (2.28)$$

$$x_{ij} \in \{0,1\} \qquad (2.29)$$

where

N is the number of targets,

M is the number of weapons,

V_i is the value of target i,

P_{ij} is the killing probability of weapon j on target i,

x_{ij} is the decision variable which represents the number of weapons of type j assigned to target i.

In this model, the Objective function (2.27) tries to minimize the total expected value of surviving targets. Constraint (2.28) forces the model to attack to only one target by each weapon. Constraint (2.29) shows that the decision variable, x_{ij}, is a binary variable. So if it is equal to zero, the weapon j will not attack to target i. But, if it is equal to one, then weapon j will attack target i in the model.

Another type of WTA problem is studied by Dirik (2010). His model solves the problem in two phases. The first phase is supposed to find the optimum strike package by minimizing the difference between achievable damage and the desired damage. The second phase is supposed to solve the optimum weapon target assignment by minimizing the total cost of configurations and the distance flown by aircrafts. His model calculates the distance flown with Great Circle Distance equation. The detailed information about his model and the mathematical formulation might be reviewed in his research paper. (Dirik 2010).

There are two types of WTA problem in the literature. These are the static and dynamic WTA problems.

In the static version of WTA, the DM knows all of the inputs in the model including weapons, targets and the killing probabilities. Also, the targets are attacked by weapons in only one stage (Ahuja, et al. 2003). The problems similar to static WTA take place very commonly in logistics management and operations research areas (Wacholder 1989). The formulation for this kind of model is same as the general WTA problem. There are some properties of the static WTA problem:

1. It is Np-Complete because it has to be named one by one completely to find an optimal solution.
2. It is discrete since there is no fractional or partial weapon-target assignment.
3. It is stochastic because the weapon-target assignment results are based on the killing probabilities in the problem.
4. It is nonlinear since the objective function has a convexity property. (Johansson and Falkman 2009), (Hosein and Athans 1990).

According to Hosein and Athans (1990), there are three types of the static WTA problem. These are:

1. A single class of weapons.
2. Weapons with limited target coverage.
3. One weapon per target.

In the first case, the killing probabilities for each weapon are not changing due to the weapon type and they are equal to each other. So, if P_{ij} is the killing probability of weapon i on target j, then $P_{ij} = P_j$. (Hosein and Athans 1990).

In the second case, some weapons cannot be assigned to some targets and if some weapons are assigned to some targets, it will change according to target type. It means that the killing probability of a weapon-target matching can be zero or some value depends on the target. ($P_{ij} \in \{0, P_j\}$). (Hosein and Athans 1990)

In the third case, each target can be assigned to at most one weapon. So, more than one weapon will not be assigned to one target. (Hosein and Athans 1990)

However, in the dynamic WTA problem, the problem has multiple time stages. After each stage, the result of the stage is evaluated and the weapon-target assignments for the new stage are revised according to losses or wins at the previous stage. Each result for each stage can be observed by the DM. This process may be called "shoot-look-shoot strategy". (Hosein and Athans 1990)

There are some properties of the dynamic WTA problem:

1. It is Np-Complete because it has to be named one by one completely to find an optimal solution.
2. It is discrete since there is no fractional weapon-target assignment.
3. It is dynamic because the result in the previous stage is evaluated to find the new weapon-target assignment for the current stage.
4. It is nonlinear due to the convexity of the objective function.
5. It is stochastic because the weapon-target assignment results are based on the killing probabilities in the problem.

6. It is large-scale since the problem has very large number of weapons and targets (Hosein and Athans 1990)

The mathematical model for the dynamic WTA problem is defined by Hosein and Athans, (1990) :

$$minimize\ F_1 = \sum_{\vec{\omega} \epsilon \{0,1\}^N} \Pr[\vec{u} = \vec{\omega}] \,.\, F_2^* \,(\vec{\omega}, \vec{w}) \tag{2.30}$$

Subject to

$$x_{ij} \in \{0, 1\}\,, \qquad i = 1, 2, \dots, N \qquad j = 1, 2, \dots , M, \tag{2.31}$$

$$w_j = 1 - \sum_{i=1}^{N} x_{ij} \tag{2.32}$$

$$\Pr[u_i = k] = k \prod_{j=1}^{M} \left(1 - p_{ij}(1)\right)^{x_{ij}} + [1 - k] \left\{1 - \prod_{j=1}^{M} \left(1 - p_{ij}(1)\right)^{x_{ij}}\right\} \tag{2.33}$$

where

N is the number of targets,

M is the number of weapons,

T is the number of time stages,

V_i is the value of target i, for $i = 1, 2, \dots, N$,

$p_{ij}(t)$ is the kill probability of weapon j on target i in stage t, for $i = 1, 2, \dots, N$ and $j = 1, 2, \dots, M$,

$q_{ij}(t) = 1 - p_{ij}(t)$ is the corresponding survival probability,

x_{ij} is the decision variable that will be equal to one when weapon j is assigned to target i in stage 1 and otherwise it will be equal to zero,

u_i is the N-dimensional binary vector $\overrightarrow{(u \epsilon \{0,1\}^N})$ which represent the target state of the system in stage 2 and it also can be defined as the set of targets which survive in stage 1. So, it will be equal to one, when target i survives stage 1, otherwise it will be equal to zero which means it is destroyed in stage 1. (Hosein and Athans 1990)

On the other hand, the cost in the Objective function (2.30) which is represented as $F_2^*(\vec{\omega},\vec{w})$ will be the optimal cost of a (T-1) stage problem with initial target state $\vec{u}$ and initial weapon state $\vec{w}$. (Hosein and Athans 1990)

For detailed information about the mathematical formulation of the dynamic WTA problem, the paper written by Hosein and Athans, (1990) may be reviewed.

2.5 Summary

In Section 2, the previous research is mentioned to give information about background of the methodology used for solving the research questions. Section 2.1 reminds the research questions since the literature review is made according to order of the research questions. Section 2.2 represents the binomial and multinomial formulas which will be used to calculate the number of decision variables in this research after some modifications. Section 2.3 gives the mathematical formulations for different types of problems in a hierarchical order. Section 2.4 demonstrates the properties and mathematical formulations for the two types of WTA problem.

When the previous research is reviewed, it can be easily seen that the mathematical model of WTA problem is generally used for operational planning purpose and they are supposed to solve the best attack strategies for the homeland. However, the WTA problem is used for logistics planning and it is supposed to find ammunition needs after best weapon-target assignment in this research. Section 3 will demonstrate the model used for the optimal weapon-target matching with similar mathematical representation if it is compared to the ones in the literature. It is like a multi-day static WTA problem. Because, it is solved for seven days and there is no reevaluation after each assignment. On the other hand, the effect and costs of each strategy are calculated in the spreadsheet model before using in the model since the objective function would be nonlinear.

III. Methodology

In this research, the strategic level of the air-to-ground missiles is supposed to be solved by using an operational usage approach. Section 3 explains how this approach helps to find answers to the research questions. In Section 3.1, the research questions are represented. In Section 3.2, the definitions and parameters are given. Section 3.3 shows the formulation to find the total number of decision. Section 3.4 shows the mathematical model to solve the optimum weapon-target assignment problem. And, Section 3.5 gives a brief summary about Section 3.

3.1 Research Questions

In this section, the research questions discussed in Section 1 will be represented again.

In this research, the main research question is:

"*What is the optimum strategic level of air to ground missiles according to optimum allocation of weapons to targets due to the strategic plans of the Turkish Air Force in each day during the war?"*

To be able to answer this main question, there are two other sub level questions to be answered.

First of them is:

"How many variables will be considered in the model according to the given data?"

Because the solving algorithm may be changed if there are too many variables in the model, the DM should learn the number of variables.

Then,

“What is the optimum weapon-target allocation according to cost, effectiveness and bomb usage?”

Since it is easier to evaluate the inventory level of the air to ground missile level, the optimum usage should be calculated before the evaluation.

All of these questions will be answered one by one in a logical order.

3.2 Definitions and Parameters

Because it is important to see how many variables will be in the model for deciding about the solving algorithm, it has to be calculated before trying to solve the problem. To be clear, the terms and indices related to model should be defined.

The number of bomb types (b): The number of air to ground bomb types in the inventory.

$B_{i:}$ *The* i^{th} *bomb type used.*

The number of platforms (c): The total number of air to ground bombs that can be assigned to each target.

C_j*: j number of bombs will be assigned from of certain type of air to ground bomb.*

The number of days in war (d): The number of days considered to estimate the total bomb demand during war.

D_m*; The* m^{th} *day in war.*

The number of targets (t): The number of targets to be destroyed by any type of air to ground bomb in the inventory.

T_n: The n^{th} target that will be destroyed by the air to ground bombs according to the strategy.

$Mineffect_{n,m}$: The minimum damage effect required for target n on war day m.

$Cost_i$: The cost of bomb type i.

Budget: The maximum money amount that can be spent for this war.

So, the decision variables can be shown as;

By using only one type of bombs

$D_mT_nB_iC_j$ *where* $m \leq d,\ n \leq t,\ i \leq b$ *and* $j \leq c$,

By using only two types of bombs

$D_mT_nB_iC_jB_kC_p$ *where* $m \leq d,\ n \leq t, (i, k) \leq b,\ j+p \leq c$ *and* $i \neq k$,

By using only three types of bombs

$D_mT_nB_iC_jB_kC_pB_hC_s$ *where* $m \leq d,\ n \leq t,\ (i, k, h) \leq b,\ j+p+s \leq c$ *and* $i \neq k \neq h$,

By using b types of bombs

$$D_mT_n\underbrace{B_iC_j}_{1}\underbrace{B_kC_p}_{2}\underbrace{B_hC_s}_{3}\ \ldots\ \underbrace{B_eC_f}_{(b-2)}\underbrace{B_rC_x}_{(b-1)}\underbrace{B_yC_z}_{b}$$

where $m \leq d,\ n \leq t,\ (i, k, h, \ldots, e, r, y) \leq b,\ j+p+s \leq c$ *and* $i \neq k \neq h$.

Strategy (S): Since each of the decision variables shows an attack strategy to a target with specific combinations of bomb type or bomb types on each war day, we can rename them as strategy.

Total number of decision variables or strategies (TS): This number represents the total number of strategies that we can have according to the number of bombs in the inventory, the number of targets to attack, the number of platforms and the number of war days.

TS: This number represents the total number of strategies that we can have according to the number of bombs in the inventory when there is only one target and one war day. This number will be used for simplifying the formulation.*

As it is seen in the decision variable representation, if there are many bomb types that are used in the attack strategy, the decision variable name will be too long to describe in the mathematical and the spreadsheet model. So, if we use another variable name to represent the decision variables which have lots of characters, it will be easier to use and understand. These new notations are:

S_z*: The* z^{th} *strategy or decision variable where z=1,2,…, TS.*

$Strategyday_z$ *: The day that the specific target is attacked by the* z^{th} *strategy.*

$Strategyeffect_z$ *: The total effect of the* z^{th} *strategy.*

$Strategycost_z$ *: The total cost of the* z^{th} *strategy.*

$Tstnum_z$*: The target that will be attacked in* z^{th} *strategy.*

$T'_{n,m} = \{z : Tstnum_z = n \text{ and } Strategyday_z = m\}$

C_{zi}*: The matrix that shows the number of bombs will be assigned from of bomb type i in* z^{th} *strategy.*

$P_{i,Tstnum_z}$ *: The matrix that shows the killing probability of bomb type i on* $Tstnum_z$. (See Table 3)

3.3 How to calculate the number of decision variables in the model?

All of the decision variables must be included in the model for being totally exhaustive. For example, if there are three types of bombs, then the decision variables in the models that have only one type of bomb and two types of bombs must be in the model, too. Table 1 shows a model with three types of bombs, two platforms, one war day and one target. There are nine decision variables.

Table 1. Decision variables for a model with three types of bombs, two platforms, one target and one day

Decision Variable		Description
D1T1B1C1	(S_1)	1 x Bomb Type 1
D1T1B1C2	(S_2)	2 x Bomb Type 1
D1T1B2C1	(S_3)	1 x Bomb Type 2
D1T1B2C2	(S_4)	2 x Bomb Type 2
D1T1B3C1	(S_5)	1 x Bomb Type 3
D1T1B3C2	(S_6)	2 x Bomb Type 3
D1T1B1C1B2C1	(S_7)	1 x Bomb Type 1 and 1 x Bomb Type 2
D1T1B1C1B3C1	(S_8)	1 x Bomb Type 1 and 1 x Bomb Type 3
D1T1B2C1B3C1	(S_9)	1 x Bomb Type 2 and 1 x Bomb Type 3

Although there are only nine variables in the model above, 84 characters must be typed. So, some visual basic codes are written in Microsoft Excel to be able to build the model without any error and in an easy way. On the other hand, it helps to see a pattern according to number of bomb types, number of platforms, the number of days in war and the number of targets. Since the bomb and platform parts will be repeated for each target and day, it is easy to see that the number of variables is getting multiplied with the number of days in war and the number of targets. So, for a main formulation that will calculate the number of decision variables in the model without typing them, it is assumed that the number of days in war and the number of targets are equal to one. It will be multiplied with the number of days in war and the number of targets to find the exact number of decision variables for the model.

After running the program while changing the number of bomb types and the number of platforms one at a time, the numbers of decision models are shown at Table 2.

Table 2. The numbers of decision variables for different numbers of bomb types and platforms

		The Numbers of Bomb Types									
		10	9	8	7	6	5	4	3	2	1
The Numbers of Platforms	1	10	9	8	7	6	5	4	3	2	1
	2	65	54	44	35	27	20	14	9	5	2
	3	285	219	164	119	83	55	34	19	9	3
	4	1000	714	494	329	209	125	69	34	14	4
	5	3002	2001	1286	791	461	251	125	55	20	5
	6	8007	5004	3002	1715	923	461	209	83	27	6
	7	19447	11439	6434	3431	1715	791	329	119	35	7
	8	43757	24309	12869	6434	3002	1286	494	164	44	8
	9	92377	48619	24309	11439	5004	2001	714	219	54	9
	10	184755	92377	43757	19447	8007	3002	1000	285	65	10

Since the numbers are seen as repeating in some ways, Table 2 is represented in a different shape like Pascal Triangle and is shown in the red triangle above.

1
2 2
3 5 3
4 9 9 4
5 14 19 14 5
6 20 34 34 20 6
7 27 55 69 55 27 7
8 35 83 125 125 83 35 8
9 44 119 209 251 209 119 44 9
10 54 164 329 461 461 329 164 54 10
11 65 219 494 791 923 791 494 219 65 11
12 77 285 714 1286 1715 1715 1286 714 285 77 12
13 90 363 1000 2001 3002 3431 3002 2001 1000 363 90 13

Figure 4. The number of strategies triangle

When it is seen that it resembles Pascal Triangle, the pattern can be found by adding one to the numbers just above it. So, to be able to build this triangle, it should be started by one at top. And then, since there is no other number near to one on the same line, the numbers at the left and right sides below should be equal to summation of zero, one itself and extra one. There will be two numbers which are equal to two on the below line of one at top. (Step 1).

Step 1:

$$\begin{array}{ccccc} 0 & & 1 & & 0 \\ & 0+1+1 & & 1+0+1 & \end{array} \rightarrow \begin{array}{ccc} & 1 & \\ 2 & & 2 \end{array}$$

For the third line, the first number will be equal to summation of zero, two and extra one. The second number will be equal to summation of two, two and extra one. The third number will be equal to summation of two, zero and extra one (Step 2).

Step 2:

$$\begin{array}{ccccccc} & & & 1 & & & \\ 0 & & 2 & & 2 & & 0 \\ & 0+2+1 & & 2+2+1 & & 2+0+1 & \end{array}$$

$$\rightarrow \begin{array}{ccccc} & & 1 & & \\ & 2 & & 2 & \\ 3 & & 5 & & 3 \end{array}$$

For the fourth line, the first number will be equal to summation of zero, three and extra one. The second number will be equal to summation of three, five and extra one. The third number will be equal to summation of five, three and extra one. The fourth number will be equal to summation of three, zero and extra one (Step 3).

Step 3:

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 2 & & 2 & & & \\ 0 & & 3 & & 5 & & 3 & & 0 \\ & 0+3+1 & & 3+5+1 & & 5+3+1 & & 3+0+1 & \end{array} \rightarrow$$

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 2 & & 2 & & \\ & 3 & & 5 & & 3 & \\ 4 & & 9 & & 9 & & 4 \end{array}$$

A visual example is shown on the triangle below.

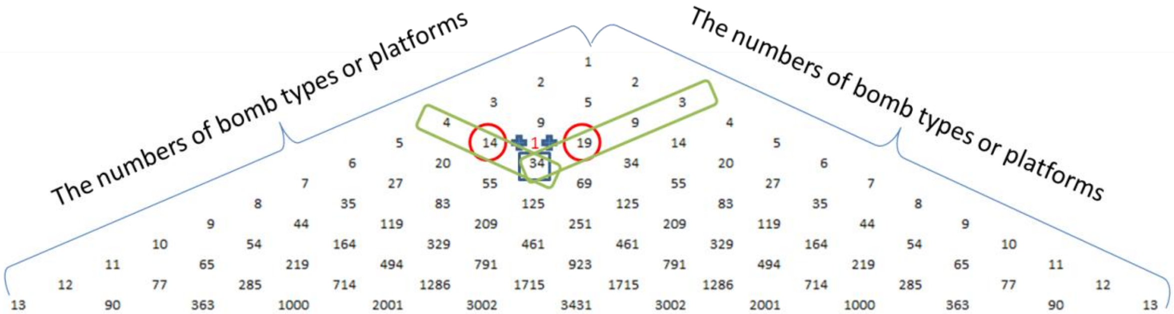

Figure 5. The representation of calculating the number of strategies with triangle method

In this example, after getting the numbers 14 and 19, an extra one must be added to these numbers to be able to calculate the number of decision variables in the model which has four types of bombs and three platforms. On the other hand, the model which has three types of bombs and four platforms will have the same numbers of decision variables because of the symmetry of the triangle.

Although this triangle finds the numbers of the decision variables for all of the models, it may take some time to calculate the summations to get all of the numbers on the triangle. So, as it is mentioned in Section 2, the multinomial coefficient calculation formula is used after modification.

According to Gallier (2010):

"For all n, m ∈ ℕ with m ≥ 2, for all pairwise commuting variables $a_1, \dots, a_m$,

$$(a_1 + \cdots + a_m)^n = \sum_{\substack{k_1,\dots,k_m \geq 0 \\ k_1+\cdots+k_m=n}} \binom{n}{k_1 \dots k_m} a_1^{k_1} \dots a_m^{k_m} \tag{3.1}$$

The number of finite multisets of size n ≥ 0 whose elements come from a set of size m≥1 is

$$\binom{m+n-1}{n}.” \quad (3.2)$$

On the other hand, if we want to find the number of elements in the expression which is equal to $(1 + a_1 + \cdots + a_m)^n$.

$$(1 + a_1 + \cdots + a_m)^n = \sum_{\substack{k_0, k_1, \ldots, k_m \geq 0 \\ k_0 + k_1 + \cdots + k_m = n}} \binom{n}{k_0 k_1 \ldots\ k_m} a_1^{k_1} \ldots a_m^{k_m} \quad (3.3)$$

In the research problem, since we add another element to the left side of Equation (3.1), we don’t need to subtract one from (m + n). But, because we will have (1^n) in the expansion of the left side at the end, we have to subtract one from the calculation result of combination formula. Besides that, since we are combining the bomb combinations and the multinomial coefficients, we can say that the element (1^n) is the representation of sending an aircraft without any bombs. According to the assumptions in Chapter 1, there cannot be any strategy that sending aircrafts without any bombs. So, we will take out this strategy from the set of strategies and the final formula to calculate this number will be like that:

$$TS^* = \binom{m+n}{n\ or\ m} - 1 \ \rightarrow \quad TS^* = \binom{b+c}{b\ or\ c} - 1 \quad (3.4)$$

where

b is the number of bomb types,

c is the number of platforms.

For example, if there are ten types of bombs and nine platforms, the total decision variable number will be calculated like:

$$TS^* = \binom{10+9}{10\ or\ 9} - 1 = \binom{10+9}{10} - 1 = \binom{10+9}{9} - 1 = 92378 - 1 = 92377 \quad (3.5)$$

By using this formula, the number of decision variables is calculated for each number of bomb types and platforms. According to these calculations, Figure 6 is drawn. And, it is easy to see that the number of decision variables will increase exponentially and rapidly. Also, if the number of platforms and the number of bomb types are so high, they will make the problem very hard to solve.

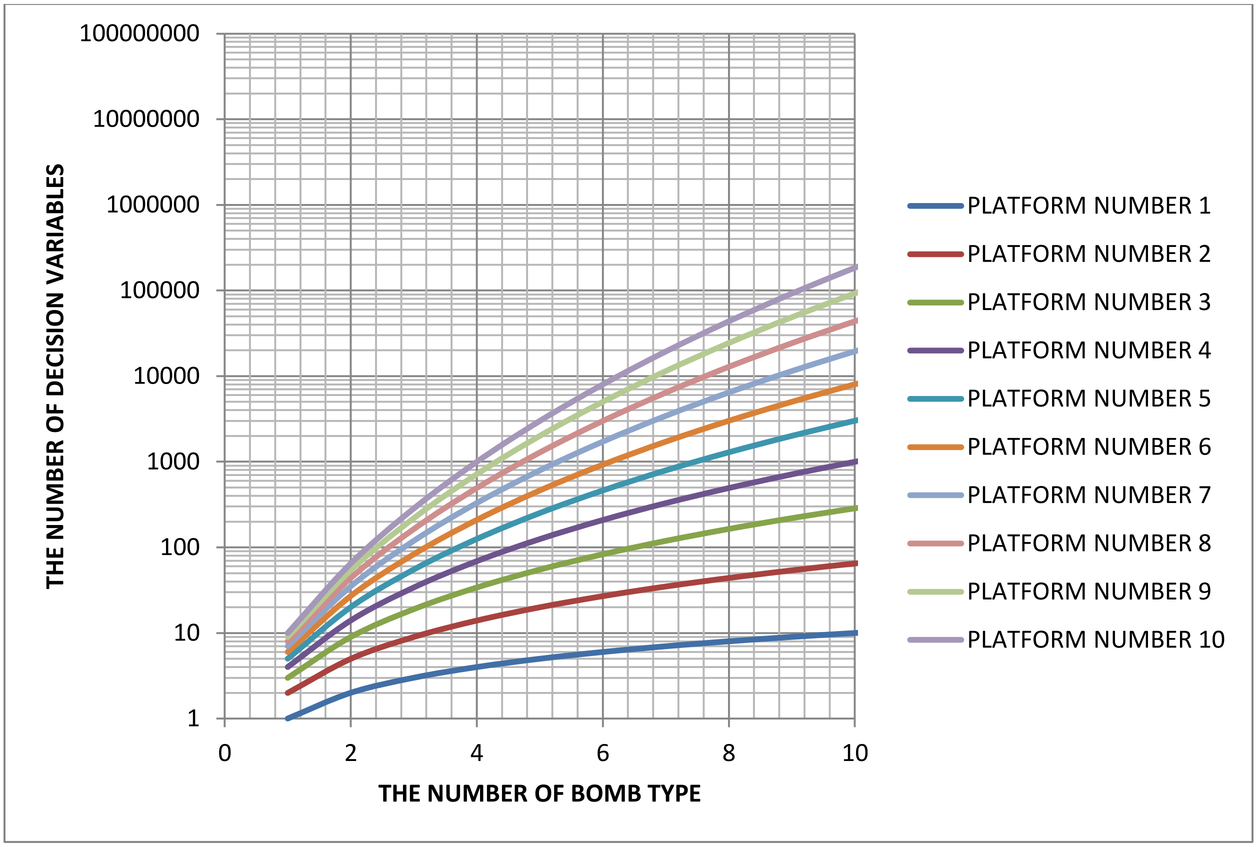

Figure 6. The graph of number of decision variables due to the number of bomb types and platforms

Because it is assumed that the number of targets and the number of days in war are equal to one to simplify the problem formulization, the main formula to calculate the number of strategies or decision variables for all of the problems can be represented as Equation (3.6).

$$TS = \left(\binom{b+c}{b\ or\ c} - 1\right) * t * d \tag{3.6}$$

where

b is the number of bomb types,

c is the number of platforms,

t is the number of targets,

d is the number of days in war.

3.4 What is the optimum weapon-target allocation according to cost, effectiveness and the bomb usage?

To be able to answer this question, three models are built in Microsoft Excel and some Visual Basic codes are written to make it easy to transfer to LINGO 13.0. These codes will be available at Appendix A. The mathematical formulation for solving this problem will be represented as cost, effect and bomb usage optimization.

The decision variables will be attack strategies that will be binary and they will change according to the number of bomb types, the number of platforms, the number of days in war and the number of targets.

Since the models are linear, some values have to be calculated before building the models. Equations (3.7), (3.8) and (3.9) represent these preprocess formulations.

$$Strategycost_z = \sum_{i=1}^{b} C_{z,i} \,.\, Cost_i \qquad for \ \ \forall\, z \tag{3.7}$$

$$Strategybomb_z = \sum_{i=1}^{b} C_{z,i} \qquad for \ \ \forall z \tag{3.8}$$

$$Strategyeffect_z = 1 - \prod_{i=1}^{b} \left(1 - P_{i,Tstnum_z}\right)^{C_{z,i}} \qquad for \ \ \forall z \tag{3.9}$$

Then, the objective functions are:

$$\min \sum_{z=1}^{TS} (Strategycost_z * S_z) \qquad for\ minimizing\ the\ total\ war\ cost \qquad (3.10)$$

$$\min \sum_{z=1}^{TS} (Strategyeffect_z * S_z) \qquad for\ maximizing\ effects\ on\ targets \qquad (3.11)$$

$$\min \sum_{z=1}^{TS} (Strategybomb_z * S_z) \qquad for\ minimizing\ the\ total\ bomb\ usage \quad (3.12)$$

Although the objective functions for the cost-based, effect-based and bomb usage-based models are different, the constraints will be similar.

Subject to

$$\sum_{z=1}^{TS} (Strategycost_z * S_z) \ \le budget \qquad (3.13)$$

$$\sum_{z \,\in\, T'_{n,m}} S_z = 1 \qquad for\ \forall\ n, m \qquad (3.14)$$

$$\sum_{z \,\in\, T'_{n,m}} (Strategyeffect_z * S_z) \ \ge \ Mineffect_{n,m} \qquad (3.15)$$

$$\sum_{z \,\in\, T'_{n,m}} (Strategyeffect_z * S_z) \ \le \ Mineffect_{n,m} \qquad (3.16)$$

$$S_z \ \in \ \{0, 1\} \qquad for\ each\ decision\ variable\ or\ strategy \qquad (3.17)$$

Objective Function (3.10) shows that the model will try to minimize the total cost of the war, if it is used as an objective function in the model. Since each of the decision variables shows an attack strategy, they will have different costs according to bomb usages. For example, if the decision variable is D1T1B1C1B2C1, it is the seventh decision variable at Table 1. So z is equal to 7, b is equal to 3 since there are three types of bombs in the inventory of the example. $Strategyday_7 = 1$, $Tstnum_7 = 1$ and then the total cost will be:

$$Strategycost_z = \sum_{i=1}^{b} C_{z,i} \,.\, Cost_i \qquad for \;\; \forall\, z \tag{3.18}$$

$$Strategycost_7 = \sum_{i=1}^{3} C_{7,i} \,.\, Cost_i \tag{3.19}$$

$$Strategycost_7 = C_{7,1} \,.\, Cost_1 + \; C_{7,2} \,.\, Cost_2 + C_{7,3} \,.\, Cost_3 \tag{3.20}$$

$$Strategycost_7 = 1 \,.\, Cost_1 + \; 1 \,.\, Cost_2 + 0 \,.\, Cost_3 \tag{3.21}$$

$$Strategycost_7 = \; Cost_1 + \; Cost_2 \tag{3.22}$$

So, the cost of this decision variable will be equal to summation of the cost of bomb type one and the cost of bomb type two. Because the third bomb type is not used in this strategy, $C_{7,3}$ is equal to zero. On the other hand, each of the costs will be calculated in the model building process and then, these final costs will be used as coefficients in the objective function.

Objective Function (3.11) shows that the model will try to maximize the total effect or damage on the targets, if it is an objective function in the model. Because, each of the decision variables or attack strategies has different kinds of bomb usage and each bomb type has different effect values on the different types of targets, they will have different effects or damages on targets. These effects will be calculated in the model building process, but since they are probabilities, they have to be calculated in a different way. So, the formula to calculate these effects is:

$$Strategyeffect_z = 1 - \prod_{i=1}^{b} \left(1 - P_{i,Tstnum_z}\right)^{C_{z,i}} \qquad for \quad \forall z \tag{3.23}$$

where

z is the number that shows the order number of the strategy on the decision variable list,

$Strategyeffect_z$ *is the total effect of the z^th strategy,*

i is a bomb type,

b is the total number of bomb types,

$C_{z,i}$ is the number of bombs that are assigned to the target from each type of bombs by the z^{th} strategy,

$P_{i,Tstnum_z}$ *is the matrix that shows the killing probability of bomb type i on* $Tstnum_z$. (See Table 3)

If there is more than one bomb type in the decision variable, they will be multiplied together. For example, if the decision variable is D1T1B1C1B2C1, it is the seventh decision variable at Table 1. So z is equal to 7, b is equal to 3 since there are three types of bombs in the inventory of the example, $Tstnum_7$ will be equal to 1 because that strategy will attack target 1. $Strategyday_7 = 1$ and then the total effect will be:

$$Strategyeffect_z = 1 - \prod_{i=1}^{b} \left(1 - P_{i,Tstnum_z}\right)^{C_{z,i}} \qquad for \quad \forall z \tag{3.24}$$

$$Strategyeffect_7 = 1 - \prod_{i=1}^{3} \left(1 - P_{i,Tstnum_7}\right)^{C_{7,i}} \tag{3.25}$$

$$Strategyeffect_7 = 1 - \prod_{i=1}^{3} \left(1 - P_{i,1}\right)^{C_{7,i}} \tag{3.26}$$

$$Strategyeffect_7 = 1 - \left(\left(1 - P_{1,1}\right)^{C_{7,1}} * \left(1 - P_{2,1}\right)^{C_{7,2}} * \left(1 - P_{3,1}\right)^{C_{7,3}} \right) \tag{3.27}$$

$$Strategyeffect_7 = 1 - \left(\left(1 - P_{1,1}\right)^{1} * \left(1 - P_{2,1}\right)^{1} * \left(1 - P_{3,1}\right)^{0} \right) \tag{3.28}$$

$$Strategyeffect_7 = 1 - \left(\left(1 - P_{1,1}\right) * \left(1 - P_{2,1}\right) \right) \tag{3.29}$$

Before using as coefficients in the objective function for the effect-based model, these calculations have to be done in the model building process.

Objective Function (3.12) shows that the model is trying to find strategies with minimum bomb usages, if it is the objective function in the model. Because each type of bombs has different effects on targets, the same level damage might be reached by different bomb usages. For example, if a smart bomb has 0.59 damage power on a target, we can attain that damage level by using four dumb bombs with 0.2 damage power each. This is also important for the depot capacity that we have. If we don’t have opportunity to increase the depot capacity, we have to try to have minimum number bombs as we can. If the decision variable is D1T1B1C1B2C1, it is the seventh decision variable at Table 1. So z is equal to 7, b is equal to 3 since there are three types of bombs in the inventory of the example. $Strategyday_7 = 1, Tstnum_7 = 1$ and then the total bomb usage will be:

$$Strategybomb_z = \sum_{i=1}^{b} C_{z,i} \qquad for \quad \forall z \tag{3.30}$$

$$Strategybomb_7 = \sum_{i=1}^{3} C_{7,i} \tag{3.31}$$

$$Strategybomb_7 = C_{7,1} + C_{7,2} + C_{7,3} \tag{3.32}$$

$$Strategybomb_7 = 1 + 1 + 0 \tag{3.33}$$

$$Strategybomb_7 = 2 \tag{3.34}$$

Constraint (3.13) ensures that total cost will not be more than the budget the country has for the war. Since each bomb assigned to the targets will cost some money, the DM has to calculate the total estimated cost to prevent impossible attack scenarios. So, the DM can spend as much as the war budget. This constraint is not be used for the bomb usage-based model since we are trying to find the minimum bomb usage although it costs more.

Constraint (3.14) ensures that each target is attacked on each day. Since the decision variables show the attack scenarios to the targets, this constraint will prevent multiple attacks to each target on each day. On the other hand, it should be remembered that the sorties for this attack scenario will be planned after finding the total bomb need by operational planners.

Constraint (3.15) ensures that each target is damaged at least as much as the minimum level of damage required. Since the total bomb need is supposed to be solved in this research, the condition that targets may be rebuilt by the enemy should be considered. In the real world, the targets may be rebuilt in a shorter period of time, but it will be assumed that minimum recovery time is one day time period. This constraint is used for the cost-based and bomb usage-based models only.

Constraint (3.16) ensures that the model will try to reach to the minimum damage level as much as it can while using the minimum number of bombs. This constraint is used for the effect-based model only.

Constraint (3.17) ensures that the decision variables are binary which means that they are one or zero. Since the attack scenarios are decision variables in this research, their values show that they are used or not. So, there is no fractional decision variable.

3.5 Summary

Since large scale models can be difficult to solve exactly, the solution algorithm may be changed if there are too many variables in the model, Equation (3.6) will be used to calculate the total number of variables.

The optimum weapon-target allocation will be found by the model which will change according to the DM’s preferences like cost, effect or bomb usage. So, if the total cost is the main issue for the DM, Equation (3.10) will be used as an objective function. If the maximizing effect on targets is the main issue, Equation (3.11) will be used as an objective function. On the other hand, if the total bomb usage is the main issue, Equation (3.12) will be used as an objective function. Constraints (3.13), (3.14), (3.15), (3.16) and (3.17) will be used as constraints to restrict the models.

After answering these questions, the optimum strategic level of air to ground missiles will be calculated by just summing up the bomb usages in decision variables which are equal to one in the optimum weapon-target assignment problem solution.

Consequently, Section 3 represents the solution algorithm that is used in this research.

IV. Application, Results, and Analysis

In this section, the methodology mentioned in the previous section is put in practice and the results and analyses are represented. Section 4.1 gives the application assumptions. Section 4.2 represents the model inputs which are created to perform analyses. Section 4.3 demonstrates the results of the model which is using the methodology mentioned in Section 2. Section 4.4 shows the analyses related to the results. Finally, Section 4.5 gives the summary of this chapter.

4.1 Application Assumptions

The model assumptions are represented in Section 1.4. However, there are some assumptions only made in the application of the model.

1. There are eight types of bombs, five platforms, ten targets and the war is seven-days long,
2. The probabilities of killing for each type of bomb on each type of target are given by the operational planner.
3. The costs of each type of bomb are current and there is no discount according to the number of bomb bought.
4. The minimum damages required for each type of bomb are equal to each other and they change from 0.5 to 0.9.
5. The more expensive bomb doesn't mean that it is more effective on every target. Because, some bombs are designed for certain types of targets.

4.2 Model Inputs

The first information needed for building the model is the information of the number of bomb types, the number of platforms, the number of targets, the number of days in the war and the total budget that can be spent for this war. This information is given by the operational planner and it will be entered into the spreadsheet model.

In Figure 7, we can see some information entered into the model by the operational planner. These parameters are important to solve the problem, since the number of strategies or decision variables are changing according to these numbers.

NUMBER OF BOMBS	8
NUMBER OF PLATFORMS	5
NUMBER OF TARGETS	10
NUMBER OF DAYS IN WAR	7
BUDGET	14140
DATA ENTERING	DELETE THE DATA ABOVE

Figure 7. The picture of parameters that are entered into the model by the operational planner

For example, the number of decision variables for this model can be calculated by the Equation (4.1).

$$TS = \left(\binom{b+c}{b\ or\ c} - 1 \right) * t * d \tag{4.1}$$

where

b is the number of bomb types,

c is the number of platforms,

t is the number of targets,

d is the number of days in war.

$$TS = \left(\binom{8+5}{8\ or\ 5} - 1 \right) * 10 * 7 \tag{4.2}$$

$$TS = \left(\binom{13}{8\ or\ 5} - 1 \right) * 10 * 7 \tag{4.3}$$

$$TS = (1287 - 1\ ) * 10 * 7 \tag{4.4}$$

$$TS = 90020 \tag{4.5}$$

Then, the effects of each bomb on each target and the minimum damage level tables have to be built. Table 3 shows the effects of each bomb on each target with related costs and this information is known by the operational planner. This table is created according to the interviews with Ozdemir 2012.

Table 3. The effects of each bomb on each target with related costs

	EFFECTS OF EACH BOMB ON EACH TARGET										
BOMB COST	BOMB/TARGET	TARGET1	TARGET2	TARGET3	TARGET4	TARGET5	TARGET6	TARGET7	TARGET8	TARGET9	TARGET10
70	BOMB1	0.2	0.2	0.15	0.25	0.2	0.17	0.15	0.2	0.22	0.2
90	BOMB2	0.25	0.3	0.32	0.24	0.36	0.19	0.2	0.21	0.28	0.3
100	BOMB3	0.3	0.32	0.21	0.26	0.38	0.24	0.21	0.22	0.24	0.32
110	BOMB4	0.32	0.28	0.38	0.2	0.32	0.3	0.25	0.28	0.29	0.35
160	BOMB5	0.45	0.4	0.48	0.42	0.43	0.45	0.46	0.42	0.39	0.45
125	BOMB6	0.35	0.33	0.29	0.37	0.3	0.4	0.38	0.36	0.35	0.33
115	BOMB7	0.27	0.32	0.45	0.3	0.35	0.38	0.35	0.33	0.32	0.34
130	BOMB8	0.37	0.34	0.41	0.34	0.32	0.4	0.45	0.38	0.36	0.36

When we analyze Table 3, it might be easily seen that the cheapest bomb is Bomb1 and the most expensive bomb is Bomb5. On the other hand, although Bomb1 is cheaper than Bomb2, it is more effective on Target4, since some bombs are produced for some types of targets respectively.

The minimum damage levels are also needed in the model. They might vary due to each target and day, but it is assumed that they are equal to each other and they are changing between 0.5 and 0.9. Table 4 represents an example for the model where the minimum damage levels for each target on each day are equal to 0.6.

Table 4. The minimum effects required for each target on each day

MINIMUM EFFECTS REQUIRED FOR EACH TARGET ON EACH DAY										
DAY/TARGET	TARGET1	TARGET2	TARGET3	TARGET4	TARGET5	TARGET6	TARGET7	TARGET8	TARGET9	TARGET10
DAY1	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
DAY2	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
DAY3	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
DAY4	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
DAY5	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
DAY6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
DAY7	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6

After giving that information, the model will be built by Microsoft Excel. As it is mentioned in the methodology section, the effects of each scenario which has more than one bomb will be calculated by the model in the background. The formula used for this calculation can be shown as:

$$Strategyeffect_z = 1 - \prod_{i=1}^{b} \left(1 - P_{i,Tstnum_z}\right)^{C_{z,i}} \qquad for \quad \forall z \tag{4.6}$$

where

z is the number that shows the order number of the strategy on the decision variable list,

$Strategyeffect_z$ *is the total effect of the* z^{th} *strategy,*

i is a bomb type,

b is the total number of bomb types,

$C_{z,i}$ is the number of bombs that are assigned to the target from each type of bombs by the z^{th} strategy,

$P_{i,Tstnum_z}$ *is the matrix that shows the killing probability of bomb type i on* $Tstnum_z$. (See Table 3)

According to the DM's preferences, the objective function is selected. There are three kinds of objective in this problem. These are:

1. Minimization the cost of assignment.
2. Minimization the number of bombs used.
3. Maximization the total effect on targets.

After deciding about the objective, one of the options on the user form will be selected as shown in Figure 8.

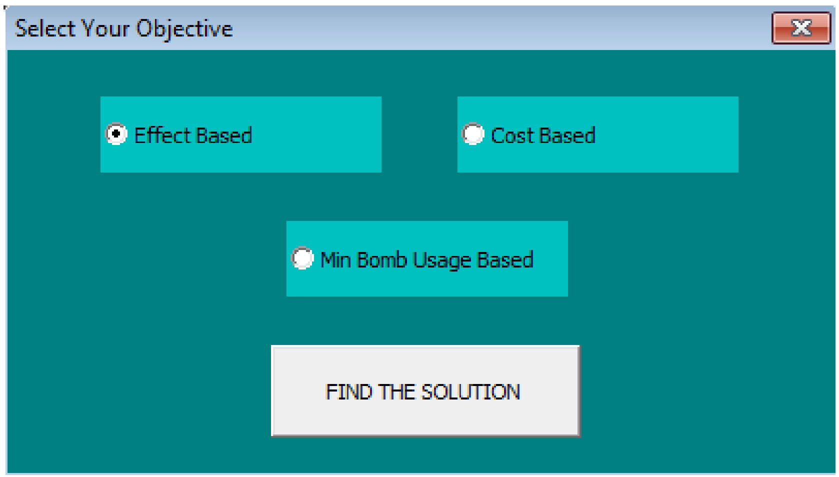

Figure 8. Selection of the model objective in the Spreadsheet model

The results for each run according to the three objectives, five minimum damage levels (0.5, 0.6, 0.7, 0.8, 0.9) and the assigned damage effects or killing probabilities for each bomb type on each target will be represented in Section 4.3.

4.3 Results of the Model Runs

First of all, the model is run for the cost minimization objective. The bomb needs on each day for each target and minimum damage level will be represented in Appendix

B separately, but the table of the total bomb needs for each bomb type and minimum damage level is shown in Table 5.

Table 5. The results of cost minimization objective

BOMB NEEDS						
		MINIMUM DAMAGE LEVELS				
		0.5	0.6	0.7	0.8	0.9
BOMB TYPES	BOMB1	42	21	28	42	0
	BOMB2	42	35	70	91	14
	BOMB3	21	14	7	14	35
	BOMB4	0	0	28	21	28
	BOMB5	0	14	14	21	70
	BOMB6	14	28	28	35	42
	BOMB7	0	28	14	21	70
	BOMB8	21	14	21	28	70
TOTAL USAGE		140	154	210	273	329
COST ($ K)		13300	16800	22120	28630	41440

Since the cheapest bombs are the Bomb1 and Bomb2, they are used for reaching the minimum damage level until the other bombs are more cost effective. The graph of bomb usages for each type of bomb and minimum damage levels are represented below.

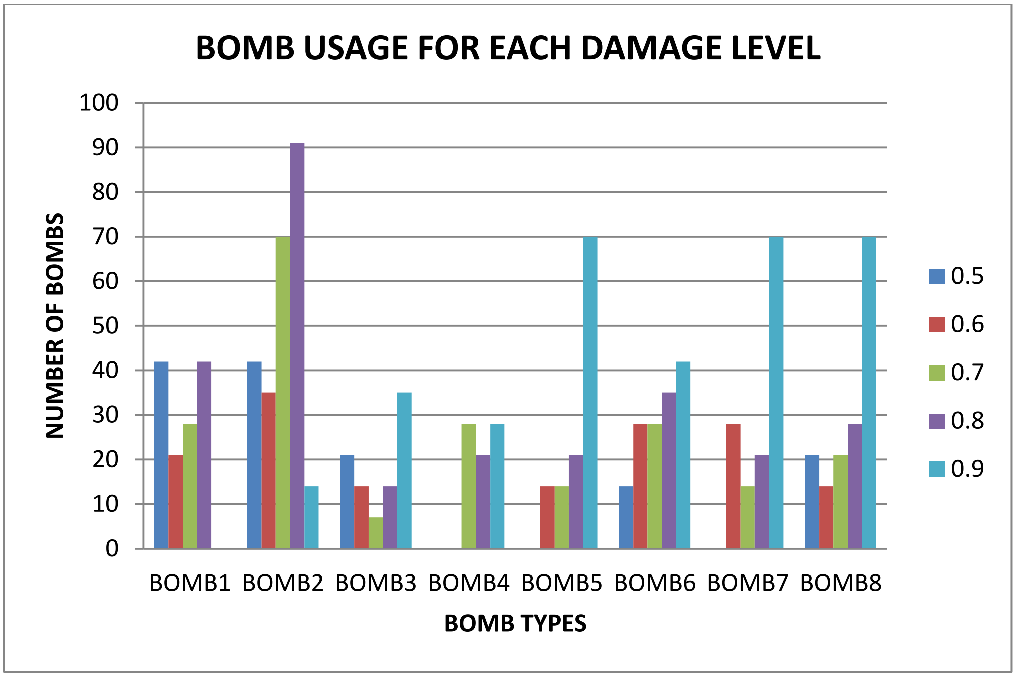

Figure 9. The bomb usage for each bomb type and minimum damage level for cost minimization objective

In Figure 9, it might be seen that the usage of the most expensive bomb which is Bomb 5 is increasing according to the minimum damage level. It is not used for reaching 0.5 damage level, since there are cheaper bombs for those levels like Bomb1 and Bomb2.

On the other hand, the platform number is an important factor while selecting the best bomb type, because the total effect of the selection should be over the minimum damage level according to the Equation (4.6).

Secondly, the model is run due to the minimization of total number of bombs used with the constraint that assures the total effect of the selection must be greater than or equal to the minimum damage level. With this objective, the budget constraint is not used to see the tradeoff between using few smart but expensive bombs and many dumb and cheaper bombs. So, the model is supposed to select as few bombs as possible while satisfying the minimum damage level without the budget constraint.

After running the model with the same inputs, the total bomb needs are found and represented in Table 6.

Table 6. The results of bomb usage minimization objective

BOMB NEEDS						
		MINIMUM DAMAGE LEVELS				
		0.5	0.6	0.7	0.8	0.9
BOMB TYPES	BOMB1	0	0	0	0	0
	BOMB2	56	0	21	0	0
	BOMB3	0	14	0	0	35
	BOMB4	0	0	0	7	0
	BOMB5	0	35	70	168	259
	BOMB6	0	7	28	7	7
	BOMB7	42	21	42	21	7
	BOMB8	42	63	35	21	7
TOTAL USAGE		140	140	196	224	315
COST ($ K)		15330	18480	25970	33670	47530

Since Bomb5 is the most effective bomb type in the inventory, it is used more than the other bombs for this objective except the lowest damage level requirement. The graph of bomb usages for each type of bomb and minimum damage levels are represented below.

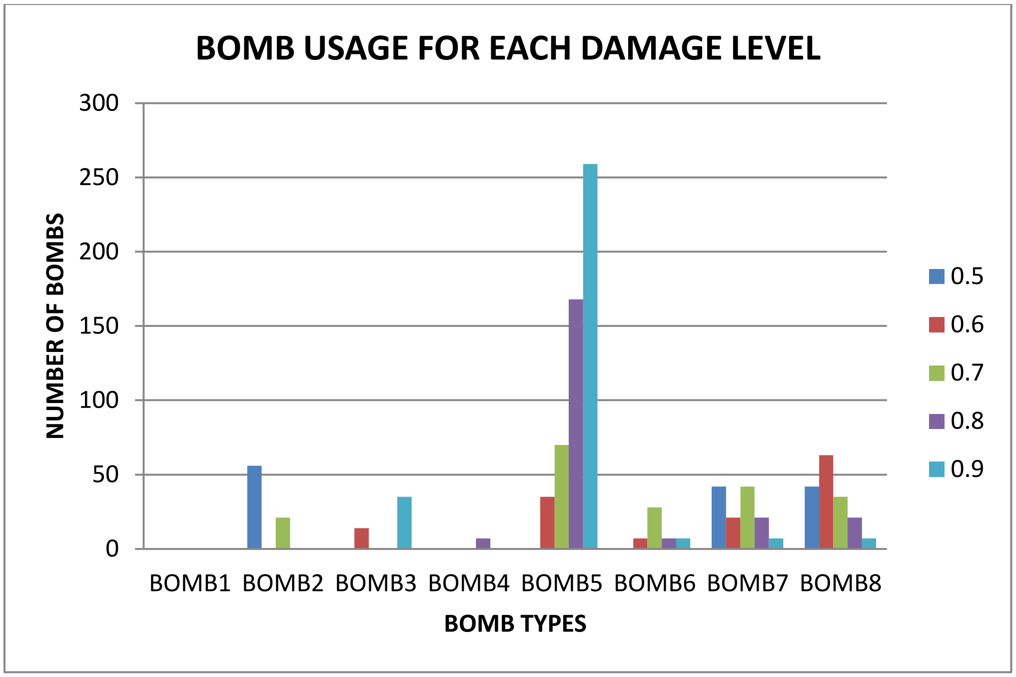

Figure 10. The bomb usage for each bomb type and minimum damage level for bomb usage minimization objective

In Figure 10, we can see that the total number of bombs used for the models with minimum damage levels 0.5 and 0.6 are equal to each other. But, the bomb combinations for each model are different. Bomb1 is not used for any damage levels, since it is the least effective bomb in the inventory, although it is the cheapest bomb.

On the other hand, the related costs are increasing according to the increasing minimum damage level. The total number of Bomb5 used is a significant factor on these costs, because it is the most expensive bomb.

Thirdly, the model is solved for the maximization of the total effects on the targets with different required damage levels. In this objective, the total effect of the strategies for each target on each day must be less than or equal to the minimum damage levels while satisfying the budget constraint, because, it will be restricted by an upper effect limit. The minimum costs are used as budgets for each damage levels to force the model to find a solution with cheapest combinations.

After running the model with the same inputs, the total bomb needs are found and represented in Table 7 for the effect maximization objective.

Table 7. The results of effect maximization objective

BOMB NEEDS						
		MINIMUM DAMAGE LEVELS				
		0.5	0.6	0.7	0.8	0.9
BOMB TYPES	BOMB1	42	50	18	63	13
	BOMB2	28	49	35	49	27
	BOMB3	1	0	16	28	30
	BOMB4	13	20	20	14	36
	BOMB5	0	3	0	21	75
	BOMB6	14	28	35	63	43
	BOMB7	8	10	49	21	53
	BOMB8	28	12	30	14	59
TOTAL USAGE		134	172	203	273	336
COST ($ K)		13300	16800	22120	28630	41440

In Table 7, it is easily seen that the number of the least effective bomb is higher than the other bombs for 0.5 damage level. The reason for this is the minimum damage level constraint with less than or equal to sign. If it were a greater than or equal to sign, it would try to use the most effective bombs as much as the budget permitted.

On the other hand, the number of bombs used and the total cost are increasing according to increase in the minimum damage levels. The graph of bomb usages for each type of bomb and minimum damage levels are represented below.

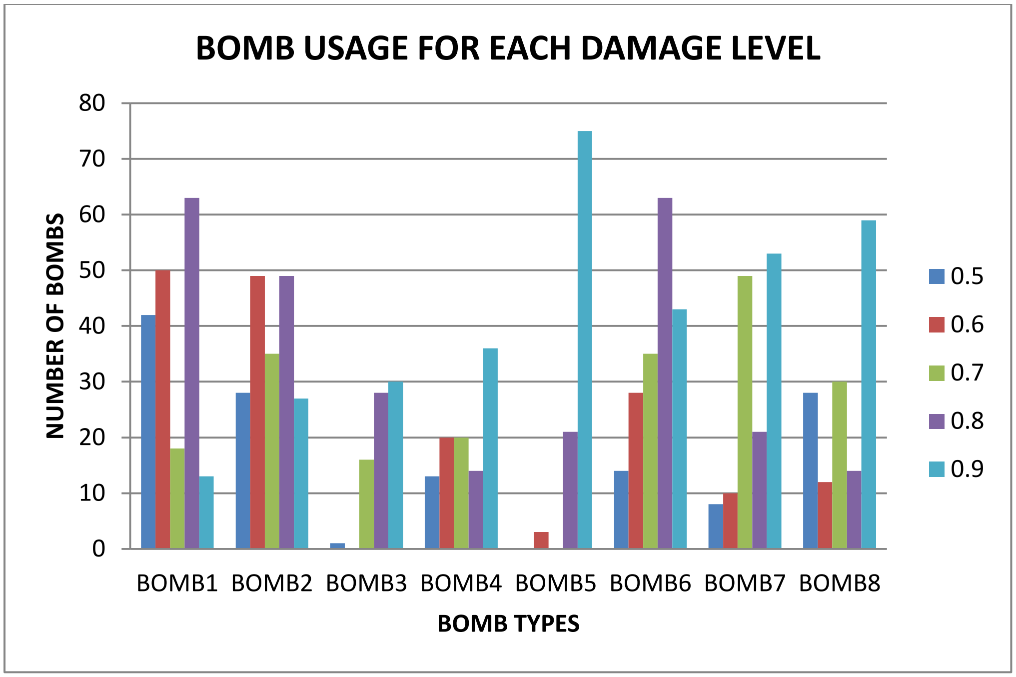

Figure 11. The bomb usage for each bomb type and minimum damage level for effect maximization objective

In this objective, Bomb1, Bomb2 and Bomb6 are used more than the other bombs for some damage levels since they are more cost effective than the other bombs for that damage levels. Besides that, the budget constraint forces the model to select the cheapest one.

To be able to see the differences and similarities between three models with different objectives, the results of the each model are represented together in Table 8.

Table 8. The results of three models with three objectives

		THREE OBJECTIVES					
		MIN COST		MIN BOMBS		MAX EFFECT	
		USAGE	COST ($K)	USAGE	COST ($K)	USAGE	COST ($K)
MINIMUM DAMAGE LEVELS	0.5	140	13300	140	15330	134	13300
	0.6	154	16800	140	18480	172	16800
	0.7	210	22120	196	25970	203	22120
	0.8	273	28630	224	33670	273	28630
	0.9	329	41440	315	47530	336	41440

According to Table 8, it is easily seen that there are different ways for getting same damage level on targets with different inventory, cost and depot levels. For example, if the goal is reaching a damage level 0.7, the DM might choose an alternative with 210 bombs which include some smart bombs and $22,120 K cost and an alternative with 196 bombs which include more smart bombs and $25,970 K cost or an alternative with 203 bombs which include lots of dumb bombs and $22,120 K cost. The graph that shows the total bomb usages in three objectives is represented below.

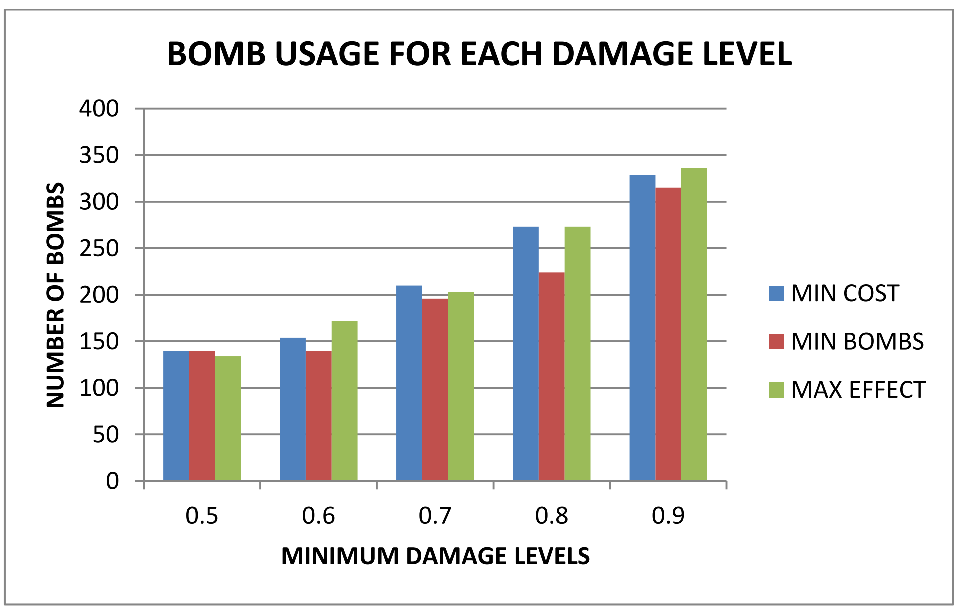

Figure 12. The bomb usage for each minimum damage level and for each objective

Although the number of bombs used in the model with the bomb usage minimization objective is greater than the number of bombs used in the model with effect maximization, this is because of the constraint that is binding the model with effect maximization objective with less than equal to sign. Besides that, the increasing minimum damage level cause an increase on the number of bombs used since there is no bomb type which can have damage up to 0.9 by itself.

The graph that shows the total costs for each damage level and each objective is represented below.

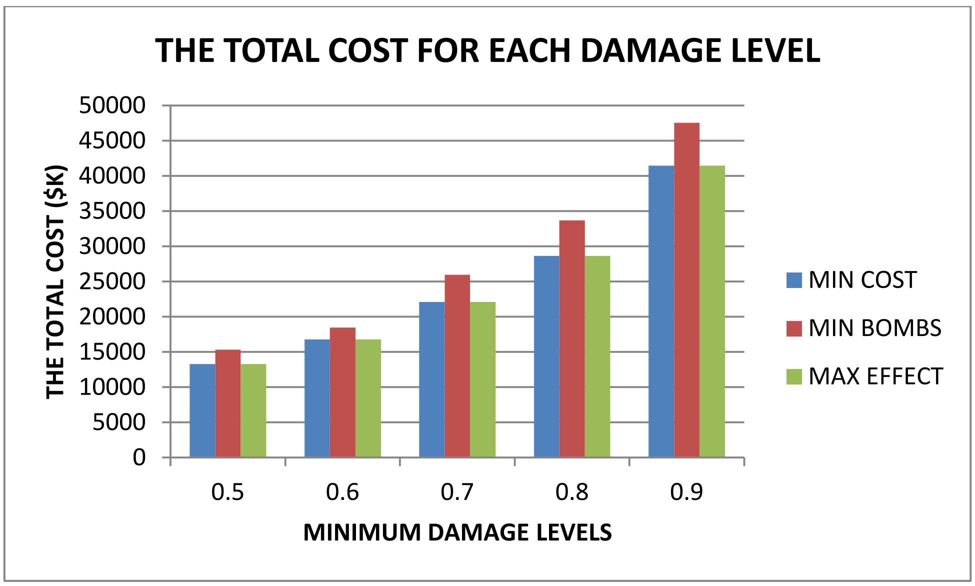

Figure 13. The total cost for each minimum damage level and for each objective

The increase in cost for each model due to increase in the minimum damage levels is what we might expect since the model is trying to reach to required damage level with the bombs that are not enough to kill a target themselves. So, they have to use more bombs to reach to the higher damage levels. In the model with bomb usage minimization, the cost is always higher than the other models, because that model is trying to use most effective bombs without the budget limit. The cost of the model with effect maximization objective is equal to the model with the cost minimization objective since we use the minimum cost value as a budget constraint.

In Figure 14, the graph that shows the relationship between the number of bombs used and the total cost of the model is represented. There are five markers on each model line and they represent the five minimum damage levels. To make it more clearer, the three points of each objective for the same damage levels are encircled with red ovals.

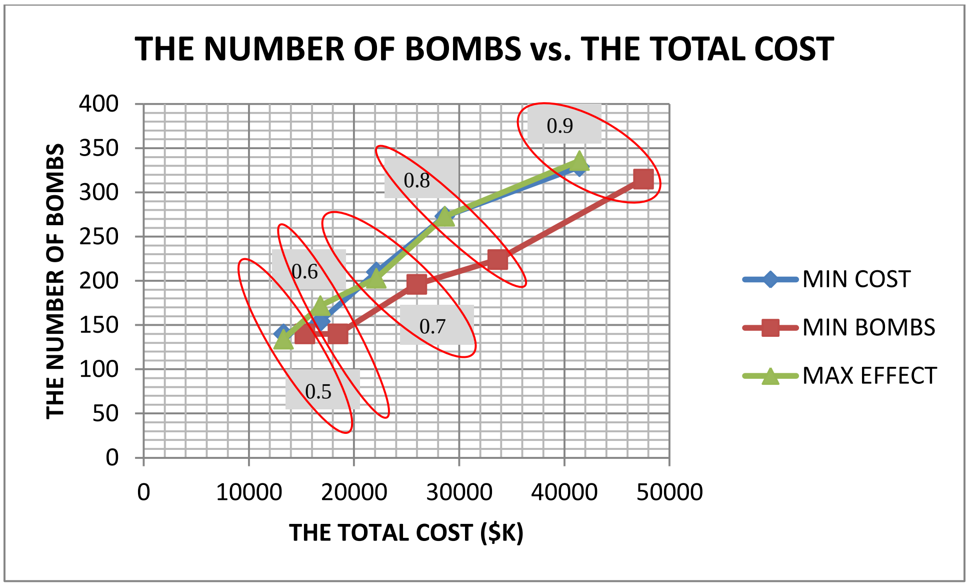

Figure 14. The bomb usage vs. the total cost for each objective

For the 0.5 minimum damage level, the model with cost minimization objective and the model with bomb usage minimization objective are dominated by the model with effect maximization objective since they use more bombs to reach that damage level although they cost more. The model with effect maximization objective is only one on the efficient frontier for this damage level.

For the 0.6 minimum damage level, the model with effect maximization objective is dominated by the model with cost minimization objective since it uses more bombs to reach that damage level although they have same cost value. So, the model with bomb usage minimization objective and the model with cost minimization objective are on the efficient frontier for this damage level. On the other hand, the DM might choose paying \$1680 K more to reduce the bomb usage from 154 to 140 by selecting the model with bomb usage minimization objective instead of the model with cost minimization objective.

For the 0.7 minimum damage level, the model with cost minimization objective is dominated by the model with effect maximization objective since it uses more bombs to reach that damage level although they have same cost value. So, the model with bomb usage minimization objective and the model with effect maximization objective are on the efficient frontier for this damage level. On the other hand, the DM might choose paying \$3850 K more to reduce the bomb usage from 203 to 196 by selecting the model with bomb usage minimization objective instead of the model with effect maximization objective.

For the 0.8 minimum damage level, all of the models are on the efficient frontier because any of the models cannot dominate others. Also, if the DM would like to reduce the bomb usage from 273 to 224, he or she has to accept \$5040K to do that. Although the bomb usage and cost values are same for the model with cost minimization objective and the model with effect maximization objective, they are using different bomb combinations to reach this damage level. So, there might be more than one optimum solution.

For the 0.9 minimum damage level, the model with effect maximization objective is dominated by the model with cost minimization objective since it uses more bombs to reach that damage level although they have same cost value. So, the model with bomb usage minimization objective and the model with cost minimization objective are on the efficient frontier for this damage level. On the other hand, the DM might choose paying $6090 K more to reduce the bomb usage from 329 to 315 by selecting the model with bomb usage minimization objective instead of the model with cost minimization objective.

Consequently, the best alternative will change according to the DM's preferences for the cost, effect and bomb usages for each damage level.

V. Summary, Conclusions and Future Work

In this chapter, Section 5.1 gives the summary of this research briefly, Section 5.2 represents the conclusions of this research and Section 5.2 gives recommendations for future work.

5.1 Research Summary

In Chapter 1, after explaining the problem background, the scope of the problem is given and the assumptions made are represented. Also, the research questions that are supposed to be solved are explained.

In Chapter 2, the previous research and solving methodologies for different kinds of problems are demonstrated in a hierarchical way since the problems mentioned are special cases of other problems to make the methodology that is used in this research more understandable.

In Chapter 3, the methodology that is used in this research is explained while answering the research questions. The model parameters and definitions are given to explain the notation used in the mathematical formulation. Then, three objective functions and related constraints are represented with their explanations.

In Chapter 4, the application of the model is represented in an example with created data. The assumptions used only for the application are mentioned and the results of each model with different objectives are given with their graphical representation and explanations. Then, the three models with different objectives are compared and discussed.

In this chapter, the summary of this research, the conclusions after the model results and recommendations for future work are discussed.

5.2 Research Conclusions

In this research, the main and two sub-level research questions are supposed to be solved with the models represented in Chapter 3. In the literature, the similar models are used to solve WTA problem for operational usage only. But, it is used to calculate the optimum bomb levels for logistics planning purpose in this research. Since it is not taking care of the operational side of war, the strategic level of air to ground missile inventory is supposed to be solved.

The mathematical model is structured according to three different types of objectives. The first model with cost minimization objective is tried to find the inventory with the minimum cost without including the cost of new depots or other related costs except the cost of bombs. Of course, the cost is not so important for governmental parties because they are not trying to make profit, but the budgets are getting restricted day by day in these days. So, the DM should think about the budget that might be spent for war preparation while he or she is deciding about the inventory.

The second model with effect maximization is tried to find the inventory as close as possible to the desired damage level while satisfying the budget constraint. In this model, if we don't restrict the budget constraint, it will select the inventory with most effective bombs which are the most expensive bombs generally. So, if the DM wants to maximize the damage level with a limited budget, this model may be suitable for him or her.

The third model with bomb usage minimization is supposed to find the inventory as small as possible. In this model, the budget limit is not used. With a limited depot area, the DM might want to store as few bombs as possible. On the other hand, since a few smart bombs might have same damage level on targets with many dumb bombs, the DM should think about the cost of these smart bombs.

The selection of the model objective is the one of the most important decision that the DM has to make. There are some tradeoffs between these models. If the DM wants to minimize the total cost and there is no problem with having a huge inventory of dumb bombs, he or she might select to have the dumb bombs instead of fewer smart bombs. In this situation, if something happens to some type of bombs related to manufacturing defects, since there are many of them, it might affect lots of bombs but the loss may not be expensive as the smart bombs with same issue.

Also, in this research, we assume that each bomb that is sent will hit the target, but it is not so simple in the real world. There might some problems with the bomb, the aircraft that will send the bomb and the pilot that will push the button while killing the target. In case that there are more bombs similar to the bomb that couldn't reach to the target, the DM might send another aircraft with same type of bombs to the same target.

On the other hand, the ammunition depots are very dangerous places and they have to be well protected and their temperatures must be between bombs' upper and lower limits. So, the maintenance costs might be very high if there are many bomb depots because of the number of bombs we have to have according to our future logistics planning.

In the Chapter 4, these tradeoffs are represented on graphs and tables according to the created data and they might be so different for different data. After running the model for each objective with same damage levels, the DM might see the dominant alternatives with related costs and bomb usages and be aware of the risks related to the inventory.

So, the DM should consider all of the risks related to type of the inventory and see the pros and cons for each strategy. The combination of both smart and dumb bombs may reduce these kinds of risks, although it might cost more than having an inventory with same type of bombs. In addition, the proportion of inventory would be another important decision of the DM.

Consequently, we try to show insights of the decisions of the DM by changing his or her preferences to cost, bomb usage, effectiveness, and the related risks in this research. Since it is a strategic decision, it might effect a country's future and make the DM a good leader or the one of the worst leaders in its history. So, this research provides a good support and important information for decision support systems of the defense related organizations, especially the Turkish Air force.

5.3 Recommendations for Future Work

Although there are many models for the WTA problem in the literature, it is suitable to derive new types of models by changing objectives and constraints.

In this research, we assume that the levels of each bomb type given by the model are the best alternatives and we make the analysis according to these levels. But as a future research, a new algorithm might be used to find other optimum alternatives very close to optimum alternative by changing the desired damage levels, the effects of each bomb type on each target type and the total budget. And, this may help the DM to get more information before his or her final decision.

In this research, although the attack strategies for each target are optimized for multiple days, they are calculated for a single state since it is a static WTA problem. As a future research, this model might be solved as a dynamic WTA problem.

Since there are three types of objectives in this research model, problem might be solved as a multicriteria optimization problem by giving some weights to each objective and unifying them.

On the other hand, the same problem might be solved with more qualitative methods like the Value Focused Thinking method.

Appendix A. VBA Codes for the Model

To get a copy of the VBA codes contact Dr. Jeffery D. Weir, Civ, USAF (ENS) by phone or email. (Telephone Number: (937) 255-3636, ext 4523; E-mail: Jeffery.Weir@afit.edu)

Appendix B. Additional Figures and Tables

Appendix B contains the tables that show model solutions for each model with different objective and the desired minimum damage levels.

Table 9. The needs of each bomb on each day for 0.5 minimum damage level by minimizing the total cost

NEEDS OF EACH BOMB ON EACH DAY (MIN DAMAGE LEVEL = 0.5)								
BOMB/DAY	DAY1	DAY2	DAY3	DAY4	DAY5	DAY6	DAY7	TOTAL
BOMB1	6	6	6	6	6	6	6	42
BOMB2	6	6	6	6	6	6	6	42
BOMB3	3	3	3	3	3	3	3	21
BOMB4	0	0	0	0	0	0	0	0
BOMB5	0	0	0	0	0	0	0	0
BOMB6	2	2	2	2	2	2	2	14
BOMB7	0	0	0	0	0	0	0	0
BOMB8	3	3	3	3	3	3	3	21
							TOTAL BOMB USAGE	140
							TOTAL COST	13300

Table 10. The needs of each bomb on each day for 0.6 minimum damage level by minimizing the total cost

NEEDS OF EACH BOMB ON EACH DAY (MIN DAMAGE LEVEL = 0.6)								
BOMB/DAY	DAY1	DAY2	DAY3	DAY4	DAY5	DAY6	DAY7	TOTAL
BOMB1	3	3	3	3	3	3	3	21
BOMB2	5	5	5	5	5	5	5	35
BOMB3	2	2	2	2	2	2	2	14
BOMB4	0	0	0	0	0	0	0	0
BOMB5	2	2	2	2	2	2	2	14
BOMB6	4	4	4	4	4	4	4	28
BOMB7	4	4	4	4	4	4	4	28
BOMB8	2	2	2	2	2	2	2	14
							TOTAL BOMB USAGE	154
							TOTAL COST	16800

Table 11. The needs of each bomb on each day for 0.7 minimum damage level by minimizing the total cost

NEEDS OF EACH BOMB ON EACH DAY (MIN DAMAGE LEVEL = 0.7)								
BOMB/DAY	DAY1	DAY2	DAY3	DAY4	DAY5	DAY6	DAY7	TOTAL
BOMB1	4	4	4	4	4	4	4	28
BOMB2	10	10	10	10	10	10	10	70
BOMB3	1	1	1	1	1	1	1	7
BOMB4	4	4	4	4	4	4	4	28
BOMB5	2	2	2	2	2	2	2	14
BOMB6	4	4	4	4	4	4	4	28
BOMB7	2	2	2	2	2	2	2	14
BOMB8	3	3	3	3	3	3	3	21
							TOTAL BOMB USAGE	210
							TOTAL COST	22120

Table 12. The needs of each bomb on each day for 0.8 minimum damage level by minimizing the total cost

NEEDS OF EACH BOMB ON EACH DAY (MIN DAMAGE LEVEL = 0.8)								
BOMB/DAY	DAY1	DAY2	DAY3	DAY4	DAY5	DAY6	DAY7	TOTAL
BOMB1	6	6	6	6	6	6	6	42
BOMB2	13	13	13	13	13	13	13	91
BOMB3	2	2	2	2	2	2	2	14
BOMB4	3	3	3	3	3	3	3	21
BOMB5	3	3	3	3	3	3	3	21
BOMB6	5	5	5	5	5	5	5	35
BOMB7	3	3	3	3	3	3	3	21
BOMB8	4	4	4	4	4	4	4	28
							TOTAL BOMB USAGE	273
							TOTAL COST	28630

Table 13. The needs of each bomb on each day for 0.9 minimum damage level by minimizing the total cost

NEEDS OF EACH BOMB ON EACH DAY (MIN DAMAGE LEVEL = 0.9)								
BOMB/DAY	DAY1	DAY2	DAY3	DAY4	DAY5	DAY6	DAY7	TOTAL
BOMB1	0	0	0	0	0	0	0	0
BOMB2	2	2	2	2	2	2	2	14
BOMB3	5	5	5	5	5	5	5	35
BOMB4	4	4	4	4	4	4	4	28
BOMB5	10	10	10	10	10	10	10	70
BOMB6	6	6	6	6	6	6	6	42
BOMB7	10	10	10	10	10	10	10	70
BOMB8	10	10	10	10	10	10	10	70
							TOTAL BOMB USAGE	329
							TOTAL COST	41440

Table 14. The needs of each bomb on each day for 0.5 minimum damage level by minimizing the total bomb usage

NEEDS OF EACH BOMB ON EACH DAY (MIN DAMAGE LEVEL = 0.5)								
BOMB/DAY	DAY1	DAY2	DAY3	DAY4	DAY5	DAY6	DAY7	TOTAL
BOMB1	0	0	0	0	0	0	0	0
BOMB2	8	8	8	8	8	8	8	56
BOMB3	0	0	0	0	0	0	0	0
BOMB4	0	0	0	0	0	0	0	0
BOMB5	0	0	0	0	0	0	0	0
BOMB6	0	0	0	0	0	0	0	0
BOMB7	6	6	6	6	6	6	6	42
BOMB8	6	6	6	6	6	6	6	42
							TOTAL BOMB USAGE	140
							TOTAL COST	15330

Table 15. The needs of each bomb on each day for 0.6 minimum damage level by minimizing the total bomb usage

NEEDS OF EACH BOMB ON EACH DAY (MIN DAMAGE LEVEL = 0.6)								
BOMB/DAY	DAY1	DAY2	DAY3	DAY4	DAY5	DAY6	DAY7	TOTAL
BOMB1	0	0	0	0	0	0	0	0
BOMB2	0	0	0	0	0	0	0	0
BOMB3	2	2	2	2	2	2	2	14
BOMB4	0	0	0	0	0	0	0	0
BOMB5	5	5	5	5	5	5	5	35
BOMB6	1	1	1	1	1	1	1	7
BOMB7	3	3	3	3	3	3	3	21
BOMB8	9	9	9	9	9	9	9	63
							TOTAL BOMB USAGE	140
							TOTAL COST	18480

Table 16. The needs of each bomb on each day for 0.7 minimum damage level by minimizing the total bomb usage

NEEDS OF EACH BOMB ON EACH DAY (MIN DAMAGE LEVEL = 0.7)								
BOMB/DAY	DAY1	DAY2	DAY3	DAY4	DAY5	DAY6	DAY7	TOTAL
BOMB1	0	0	0	0	0	0	0	0
BOMB2	3	3	3	3	3	3	3	21
BOMB3	0	0	0	0	0	0	0	0
BOMB4	0	0	0	0	0	0	0	0
BOMB5	10	10	10	10	10	10	10	70
BOMB6	4	4	4	4	4	4	4	28
BOMB7	6	6	6	6	6	6	6	42
BOMB8	5	5	5	5	5	5	5	35
							TOTAL BOMB USAGE	196
							TOTAL COST	25970

Table 17. The needs of each bomb on each day for 0.8 minimum damage level by minimizing the total bomb usage

NEEDS OF EACH BOMB ON EACH DAY (MIN DAMAGE LEVEL = 0.8)								
BOMB/DAY	DAY1	DAY2	DAY3	DAY4	DAY5	DAY6	DAY7	TOTAL
BOMB1	0	0	0	0	0	0	0	0
BOMB2	0	0	0	0	0	0	0	0
BOMB3	0	0	0	0	0	0	0	0
BOMB4	1	1	1	1	1	1	1	7
BOMB5	24	24	24	24	24	24	24	168
BOMB6	1	1	1	1	1	1	1	7
BOMB7	3	3	3	3	3	3	3	21
BOMB8	3	3	3	3	3	3	3	21
							TOTAL BOMB USAGE	224
							TOTAL COST	33670

Table 18. The needs of each bomb on each day for 0.9 minimum damage level by minimizing the total bomb usage

NEEDS OF EACH BOMB ON EACH DAY (MIN DAMAGE LEVEL = 0.9)								
BOMB/DAY	DAY1	DAY2	DAY3	DAY4	DAY5	DAY6	DAY7	TOTAL
BOMB1	0	0	0	0	0	0	0	0
BOMB2	0	0	0	0	0	0	0	0
BOMB3	5	5	5	5	5	5	5	35
BOMB4	0	0	0	0	0	0	0	0
BOMB5	37	37	37	37	37	37	37	259
BOMB6	1	1	1	1	1	1	1	7
BOMB7	1	1	1	1	1	1	1	7
BOMB8	1	1	1	1	1	1	1	7
							TOTAL BOMB USAGE	315
							TOTAL COST	47530

Table 19. The needs of each bomb on each day for 0.5 minimum damage level by maximizing the total effect

NEEDS OF EACH BOMB ON EACH DAY (MIN DAMAGE LEVEL = 0.5)								
BOMB/DAY	DAY1	DAY2	DAY3	DAY4	DAY5	DAY6	DAY7	TOTAL
BOMB1	6	6	6	6	6	6	6	42
BOMB2	4	4	4	4	4	4	4	28
BOMB3	0	0	0	0	0	0	1	1
BOMB4	2	2	2	2	2	1	2	13
BOMB5	0	0	0	0	0	0	0	0
BOMB6	2	2	2	2	2	2	2	14
BOMB7	1	1	1	1	1	1	2	8
BOMB8	4	4	4	4	4	5	3	28
							TOTAL BOMB USAGE	134
							TOTAL COST	13300

Table 20. The needs of each bomb on each day for 0.6 minimum damage level by maximizing the total effect

NEEDS OF EACH BOMB ON EACH DAY (MIN DAMAGE LEVEL = 0.6)								
BOMB/DAY	DAY1	DAY2	DAY3	DAY4	DAY5	DAY6	DAY7	TOTAL
BOMB1	8	8	6	6	8	8	6	50
BOMB2	7	7	7	7	7	7	7	49
BOMB3	0	0	0	0	0	0	0	0
BOMB4	3	3	2	3	3	3	3	20
BOMB5	0	0	1	1	0	0	1	3
BOMB6	4	4	4	4	4	4	4	28
BOMB7	1	1	2	2	1	1	2	10
BOMB8	2	2	2	1	2	2	1	12
							TOTAL BOMB USAGE	172
							TOTAL COST	16800

Table 21. The needs of each bomb on each day for 0.7 minimum damage level by maximizing the total effect

NEEDS OF EACH BOMB ON EACH DAY (MIN DAMAGE LEVEL = 0.7)								
BOMB/DAY	DAY1	DAY2	DAY3	DAY4	DAY5	DAY6	DAY7	TOTAL
BOMB1	3	3	2	3	2	2	3	18
BOMB2	5	5	5	5	5	5	5	35
BOMB3	3	4	1	3	1	1	3	16
BOMB4	2	2	4	2	4	4	2	20
BOMB5	0	0	0	0	0	0	0	0
BOMB6	5	5	5	5	5	5	5	35
BOMB7	7	4	8	7	8	8	7	49
BOMB8	4	6	4	4	4	4	4	30
							TOTAL BOMB USAGE	203
							TOTAL COST	22120

Table 22. The needs of each bomb on each day for 0.8 minimum damage level by maximizing the total effect

NEEDS OF EACH BOMB ON EACH DAY (MIN DAMAGE LEVEL = 0.8)								
BOMB/DAY	DAY1	DAY2	DAY3	DAY4	DAY5	DAY6	DAY7	TOTAL
BOMB1	9	9	9	9	9	9	9	63
BOMB2	7	7	7	7	7	7	7	49
BOMB3	4	4	4	4	4	4	4	28
BOMB4	2	2	2	2	2	2	2	14
BOMB5	3	3	3	3	3	3	3	21
BOMB6	9	9	9	9	9	9	9	63
BOMB7	3	3	3	3	3	3	3	21
BOMB8	2	2	2	2	2	2	2	14
							TOTAL BOMB USAGE	273
							TOTAL COST	28630

Table 23. The needs of each bomb on each day for 0.9 minimum damage level by maximizing the total effect

NEEDS OF EACH BOMB ON EACH DAY (MIN DAMAGE LEVEL = 0.9)								
BOMB/DAY	DAY1	DAY2	DAY3	DAY4	DAY5	DAY6	DAY7	TOTAL
BOMB1	2	2	2	2	2	2	1	13
BOMB2	4	4	4	4	4	4	3	27
BOMB3	4	4	4	4	4	4	6	30
BOMB4	5	5	5	5	5	5	6	36
BOMB5	10	11	10	12	12	10	10	75
BOMB6	7	7	7	4	4	7	7	43
BOMB7	7	7	7	9	9	7	7	53
BOMB8	9	8	9	8	8	9	8	59
							TOTAL BOMB USAGE	336
							TOTAL COST	41440

Appendix C. Storyboard

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14. ABSTRACT

Inventory Management is one of the most important elements in military systems. Especially, if it is an ammunition inventory, because of its vital role in war, it might change a country's independence. To be able to find the optimum types and levels of ammunition, the probable needs of each type has to be calculated in peace time. Although the Turkish Air Force has Strategic Plans which show detailed war scenarios, there are many different ways to accomplish a mission in a scenario with different ammunition usages. Because of this, a model is needed to solve this problem with a different perspective.

In this research, the needs of air-to-ground missiles are calculated by using a Weapon Target Assignment algorithm with cost minimization, bomb usage minimization and effect maximization objectives. The model finds different combinations of bombs for each objective and it shows the main tradeoffs between many cheap dumb bombs and a few expensive smart bombs, the total cost of the operation and the total effects of the operation on targets with the current inventory. The preferences of the Decision Makers will shape this inventory due to these tradeoffs. To aid in this modeling, the number of strategies that can be created with the inventory is calculated using multinomial theory.

15. SUBJECT TERMS

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