Development of a Two-Equation Turbulence Model for Mean Shear- and Internal Wave-Driven Mixing

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LONG-TERM GOALS

The long-term goal of the project is to develop realistic, functional parameterizations of stratified turbulent mixing usable in numerical circulation models of oceans and costal seas. Specifically, we aim at developing models which explicitly allow for the coexistence and three-way interaction of three major components of stratified geophysical flows: "mean" currents, internal inertia gravity waves (IGW) and smallscale, 3D turbulence. Here, "mean" loosely refers to any current that - in contrast to IGWs and turbulence - can be explicitly resolved in a variety of operational numerical models. Traditional turbulence closures, which our approach extends, only acknowledge the existence of two of the three flow components, turbulence and sheared mean currents. They are thus ignorant of the direct energy flux from "breaking" IGWs to turbulence. As to its importance, we note that this direct energy flux from IGWs powers the turbulent mixing in the bulk of the depths of the world ocean.

OBJECTIVES

The objectives of the project are to *(i)* develop a two-equation, closure-like turbulence model with turbulent kinetic energy (TKE) production and turbulence length and time scales resulting from mean shear *and* IGWs simultaneously and *(ii)* to test the new algorithm through comparisons with observed flows. Our chosen problem is rather challenging. Within the current project we intend to provide a "proof of concept."

APPROACH

This project is a joint effort of the PI and of *Helmut Baumert* of the Institute for Advanced Marine and Limnic Studies (IAMARIS), Hamburg, Germany (baumert@iamaris.de). Dr. Baumert is being funded by ONR Global / NICOP (Grant Number: N62909-10-1-7050). We have been collaborating for over 10 years on the topic of stratified turbulent mixing, combining the skills in theory and modeling of Baumert with the observational experience of Peters.

The current project is based on a series of published studies of stratified mixing by Baumert and Peters (*Baumert and Peters*, 2000, 2004, 2009; *Peters and Baumert*, 2007). Over the course of this work it became increasingly clear that, to be realistic, turbulence models can*not* ignore IGWs. This viewpoint

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is explicitly supported by the path-breaking analysis of Lagrangian stratified flow spectra of *D'Asaro and Lien* (2000a, b).

The base from which this project started is contained in our latest joint publication, *Baumert and Peters* (2009). It presents a two-equation model that contains the two limits of internal wave-driven mixing without mean shear and mean shear without internal waves. Our logical first step thus is to set up a physically sound two-equation system capable of handling mean shear and waves simultaneously. This requires theoretical and numerical as well as examinations and analyses of existing observations to prepare for the second step in our approach, testing the new model. Over the course of our work on the project, further detail to the general outline of our approach have emerged. These our outlined in the following section.

WORK COMPLETED

Organizational aspects
This project just started April/May 2010. In order to get jump-started, PI Peters traveled to Hamburg and worked with Baumert on a work plan from 7/12 to 7/19/2010. Work on theory and observations as outlined above began.

Observations for theory testing, analyses of data sets
We consulted a number of colleagues in search of observational data sets suitable for our purposes. Our requirements are substantial: we need data on mean flow, turbulent dissipation rate, preferably also turbulent length (overturning) scales, and at least *some* measure of internal waves. Additionally, we have to be able to utilize these data within 1D modeling in order to avoid the huge overhead and substantial extra problems associated with 2D and 3D modeling. This requirement means, among other things, that we have to have time series at a fixed location that span the relevant flow time scales and a sufficient depth range that allows for proper boundary conditions along with measurements of the latter. In other, more formal, words: the the spin-up and relaxation times for the physics of upper-ocean and estuarine spatial scales are of the same order of magnitude as tidal and circadian forcing signals so that tests of closures need to take the form of fully non-stationary simulations of at least one tidal or diurnal cycle in the sense of a periodically stationary process. Thus, our requirements exclude, e.g., transects taken from moving ships, isolated AUV paths, and, alas, D'Asaro's neutrally buoyant floats on their own. The following table lists the principal data sets considered.

Table 1:Observational data sets and their usability for purposes of the present project. Here ε is the dissipation rate of TKE and L is the overturning scale.

Observation	Principal "Owner"	Mean Flow	Turbulence	IGW
Hudson River	H. Peters	+	ε, L	no
Knight Inlet	D'Asaro, Gregg, others	+	+	+
1987 Pacific EUC	McPhaden	+	no	+
Tropic Heat I&II	Gregg	+	ε, L	no
TIWE	Moum, Gregg	+	ε, L	lost

The Knight Inlet data are extensive although the many components were not all taken simultaneously. Unfortunately, the flow regime at the site is too complex to be helpful for our purposes. Temperature sensors with 1 min sampling were added to the NOAA mooring at 0°,140°W in order to study the high-frequency internal waves typical of the site (*McPhaden and Peters*, 1992). For logistical reasons, the high-frequency T-measurements were not taken simultaneously with the 1987 Tropic Heat II cruise. We had high hopes for the TIWE observations, which are complete and usable for our purposes. However, thermistor chain measurements of IGW signals taken during TIWE were not retrievable either at COAS/OSU nor at APL/UW. We actually acquired and explored the data from Knight Inlet and TIWE. This leaves us currently without an ideal data set for testing of ideas and an eventual new closure. Our search continues.

However, we have found an easier and more elegant way of using observations to guide us in our first steps of setting up a closure that allows for mean shear *and* IGW energy input into TKE simultaneously. Specifically, this means extending the Ω-equation from *Baumert and Peters* (2009), which can only accommodate the limiting cases of (i) mean shear and no IGW TKE energy source and (ii) no mean shear but IGW energy input. (The extension of the TKE (k) equation is trivial.)

Our new approach is based on observations by *Mahrt* (2007) of the mean effect of non-stationary flow on the Monin-Obukhov scaling of the boundary layer. The flow variability is produced by submesoscale motions that include IGWs. After averaging over the flow variability, we have a statistical steady state problem that we can analyze analytically in our extended k- Ω system. In a bit more detail, *Baumert* (2005b) demonstrated in his Fig. 4.2 (see also Figs. 1 and 2 in *Businger et al*, 1971, and the upper curve in Fig. 3 of *Mahrt*, 2007) that for theoretical reasons the slope of the Monin-Obukhov linear curve is 4, as is observed most often. *Mahrt* (2007) shows that time-varying flow can flatten this curve to a slope as low as 1.

The analysis of the Monin-Obukhov scaling is currently being pursued. Once it is finished, we will proceed to fully non-stationary cases beginning with the previously exploited Hudson River data. The charge is to show that the extended k- Ω closure, after been tested in the a.m. MOM (Monin-Obukhov-Mahrt) steady-state situation, can simulate the observed turbulence without and with an assumed, “reasonable” IGW energy source.

Theory development

As laid out in *Baumert* (2009, 2011), Helmut Baumert has attempted to underpin our k-Ω approach in a fundamental way by considering a “particle” view of turbulence at infinite Reynolds number as an alternative to phenomenological eddy-viscosity modeling based on Friedman-Keller expansion of the Navier Stokes equations. Such a particle view was adopted earlier in hydrodynamic research by *Ludwig Prandtl* (1926, loc. cit. Baumert 2011). Starting from the principles of energy and circulation conservation and taking into account that the $Re \rightarrow \infty$ turbulent vortex dipoles (as microscopic elementary building blocks of the theory) must have zero circulation, the papers present a theory that has no adjustable constants and gives even, as a byproduct, insight into the universal constants of turbulent motions. E.g., von Kármán's constant is derived as $(2\pi)^{-1/2} \cong 0.399$. Inspired by *Herrmann*'s (1990) dynamic packing of circles, the theory was extended to wavenumber spectra and Lagrangian frequency spectra. In Kolmogorov's $\alpha\, \varepsilon^{2/3}\, k^{-5/3}$ and $\beta\, \omega^{-2}$, the new theory predicts values of α and β consistent with experimental values including those from the oceanographic observations of *D'Asaro and Lien* (2002a,b) and *Lien and D'Asaro* (2000).

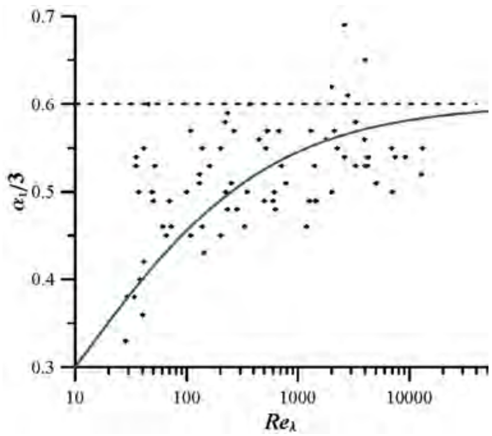

Fig. 1: Experimental and observational results for (one third of) the universal Kolmogorov constant for wavenumber spectra measured, collected from the literature, and analysed by Sreenivasan (1995; loc. cit. Baumert 2011).

The solid green line in Fig. 1 follows the somewhat arbitrary approximation, $0.6 * Re^{1/2}/(10+Re^{1/2})$, wherein 0.6, labelled by the black dotted horizontal line, is our theoretically derived asymptotic value of the Kolmogorov constant to which the data obviously converge for $Re \rightarrow \infty$.

Numerical modelling

We have prepared the numerical simulation of the periodically stationary, tidal Hudson river estuary based on a pre-existing code written in Mathematica. To achieve this it was necessary to transfer the pre-existing code from the older Mathematica version 5.2 to the most recent version 8.01. This was not so easy because the authors of Mathematica violated principles of backward compatibility so that it was necessary to re-programme certain modules and sub-programmes. A number of control runs were successfully completed.

RESULTS

As an ongoing project, results specific to the project objectives cannot be reported, yet. Results from the intermediate steps like theory development and useful byproducts like the universal numbers are presented above.

IMPACT/APPLICATIONS

Turbulence models that explicitly allow for the presence and effect of IGWs have the potential to make the representation of turbulent mixing in operational and research circulation models more realistic and thus to improve the model forecasts. For climate research and prediction this is essential.

RELATED PROJECTS

Numerous past and present projects either pursue traditional, wave-ignorant turbulence closure for stratified shear flows or internal wave-driven turbulent mixing based on observations and internal wave interaction theory. Links between mixing and waves in shear flows have been analyzed in the Equatorial Undercurrent [e.g., *Moum et al.* (1992), *Lien et al.* (1996)] and in Knight Inlet [*D'Asaro and Lien*, 2000a,b]. More recently, J. Nash, H. Peters and J. Pelegrí conducted pilot observations on the role of internal waves in the outflow from the Mediterranean in the Gulf of Cádiz. Two manuscripts analyzing these observations re still in preparation. In 2009 IAMARIS has sponsored an internal research project on fundamental questions of turbulence theory and modeling which has served as a preparatory phase of the present NICOP-supported engagement.

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