3-D Effects, Robustness and Model Validation Issues in Drifter Motion Planning for Optimal Surveillance of the Ocean (DRIMPOS)

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LONG-TERM GOALS

In recent years substantial progress, much of it resulting directly from ONR funding initiatives, has been made in understanding fundamental features of transport and mixing in oceans using methods derived from dynamical systems theory. The purpose of the current collaborative research is to extend these methods to the design of control algorithms for Drifter Motion Planning for Optimal Surveillance of the Ocean (DRIMPOS). This effort is a direct attempt to transition Lagrangian based dynamical systems methods from diagnostic, postdictive tools to essential and active components in the design of oceanographic and naval observing systems. The specific goals of the research project include the development of flow-based control algorithms for drifting autonomous sensing systems.

OBJECTIVES

Couple dynamical systems ideas and control-theoretic algorithms to produce real-time control of gliders based on the output from high resolution coastal ocean model forecasting systems. Specifically, use knowledge of Lagrangian ocean dynamics to develop readily computable, optimal control algorithms to (1) maximize the loitering time of autonomous surveillance platforms in a prescribed region under energy constraints (2) minimize the distance, over an extended time period, between a platform and a specified location in the flow (3) optimize sensor coverage of a given surveillance region by single or multiple platforms and (4) perform optimal path planning for an AUV platform by minimizing time to reach any number of specified way-points along a desired observational route.

APPROACH

Work completed under this grant involves close collaboration with Igor Mezic at the University of California Santa Barbara.

We study high-level path planning for under-powered vehicles in known, spatially complex, time-varying ocean currents. In particular, we focus on the problem of controlling the vehicle from its initial position to a (possibly time-varying) target set in minimum time. Difficulties arise due to both the abundance of locally optimal trajectories and the lack of local controllability of AUVs in the presence

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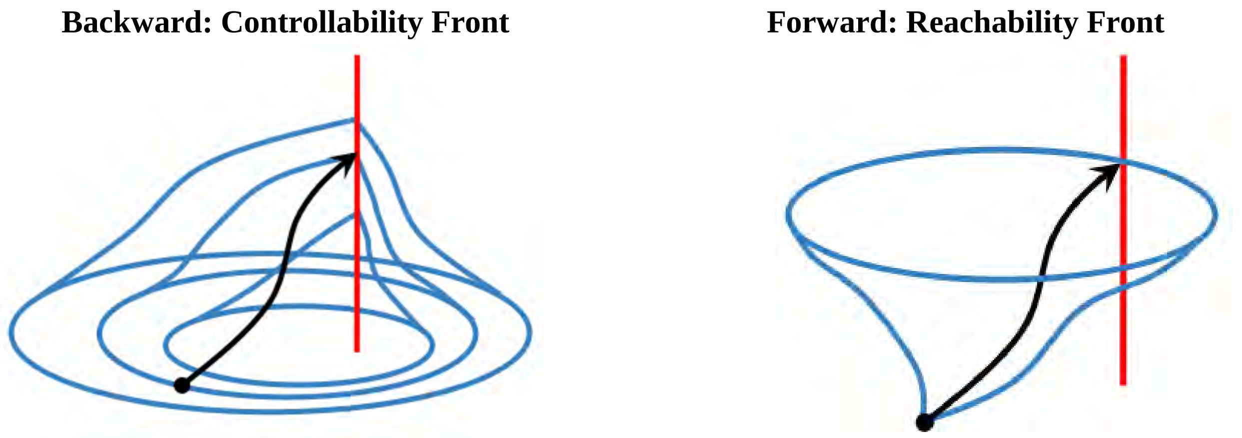
of strong currents. Globally optimal trajectories may, however, be obtained by solving a dynamic HJB (Hamilton Jacobi Bellman) PDE (partial differential equation) for the time-varying optimal time-to-go function. In prior work, a method of characteristics known as the ``extremal field" method, which is equivalent to tracking a 2D ``controllability front" backward in time and outward in space from the target set was demonstrated. Using this approach, several variants of the control problem were successfully solved in the context of time dependent, two-dimensional velocity fields produced by a data-assimilating coastal ocean model (NCOM in the Adriatic Sea).

In the present work, we exploit a special property of minimum time control to obtain the same globally optimal trajectories, far more efficiently. This is accomplished by tracking, for any potential initial AUV position, a one-dimensional ``reachability front" forward in time. The utility of the approach is demonstrated for the application of approximate AUV trajectory tracking in ocean flows. In this context, optimal trajectories computed using the newly developed forward in time, open-loop algorithm compare very favorably with those previously computed by the extremal field method.

Importantly, reformulation of the control problem to a forward-in-time approach allows for the efficient computation of globally optimal trajectories in current fields of the form u(x,y,t) := u(x,y,z(t),t). Given the disparity of vertical and horizontal current speeds in the ocean and the dive profiles of typical AUVs, the vertical speed of the AUV will dominate ambient vertical velocities almost everywhere. As such, the next level of investigation will concentrate on the role of vertical shear for AUV control in strong horizontal flow fields where the goal is optimal control in the horizontal for a vehicle with prescribed dive profile z(t). The reduction in dimensionality afforded by the *forward in time* approach allows feasible computations of globally optimal trajectories for AUV path planning in 3D, time-dependent horizontal flow fields.



Backward: Controllability Front

Forward: Reachability Front

Figure1: Schematic of the two approaches to computing optimal trajectories in time-varying flows. Left gives feedback control by computing, for each final state, the arrival time, T(t), backwards in time. Right panel shows the open loop approach of computing, for any position at the initial time, the reachability front forward in time.

The differences between the two approaches to computing globally optimal trajectories are shown succinctly in Figure 1. Previous funding resulted in reference [1] where we characterized solutions of minimum time HJB equations for which the value function is (a) discontinuous and (b) dynamic, using a novel implementation of the extremal field approach. In particular, we did this for minimum time *feedback* control in a time-varying flow fields produced by coastal ocean models (see Figure 2). Moreover, we re-interpreted the extremal field approach as a marker particle method of tracking the same fronts tracked by Ordered Upwind Methods, but which allows incremental refinement of the solution and is inherently easy to parallelize. To be precise, computing the extremals by integrating the Euler Lagrange equations backward in time is equivalent to tracking a 2D ``controllability front" backward in time and outward in space from the target set as shown in the left panel of Figure 1. Such computations provide, at any point in space-time, the 'time-to-go' function. Optimal trajectories move down the gradient of this function.

Given the relative simplicity of the cost functional for minimum time control, precisely the same globally optimal trajectories can be obtained more efficiently by tracking a 1D ``reachability front" forward in time. The resulting continuum in space time of fronts (or, equivalently, extremals) provides only a single slice of the solution of the HJB equation, and, in particular, a HJB equation for the arrival time function, not the time-to-go function. While this implies *open-loop* control, the optimal trajectories are identical and the reduction in dimensionality of the surface along with the ability to both efficiently re-mesh and trim globally suboptimal trajectories from this surface leads to large improvements in efficiency.

WORK COMPLETED

1) An efficient and accurate means of computing globally optimal trajectories using a *forward in time* approach has been developed. Results of the algorithm compare favorably with previously developed (and more computationally intensive) *backward in time* optimal control approaches. These comparisons have been done in the context of AUV control problems in time-varying flow fields produced by data-assimilating coastal ocean models. A manuscript detailing the application of the control algorithm to AUV/glider control in realistic ocean models is nearing completion for submission to IEEE journal of Ocean Engineering.

2) Preliminary work on the application of the *forward in time* control algorithm to horizontal vectors fields varying in 3 spatial dimensions has begun.

RESULTS

Results detailing the application and efficiency of the control algorithm for maneuvering autonomous sensor platforms for maximum loitering time in extended (*O*(20 x 20 km)) regions were considered previously. We showed, in the context of a well documented NRL coastal model of the Adriatic Sea, that the optimal control algorithm can provide an order of magnitude increase in the residence time of a glider in such a region when compared to simple, a-priori path planning strategies. Our goal here has been to develop the *forward in time* algorithm and demonstrate its efficiency and accuracy. In the following, we consider the simplest case of optimal path planning from an extended set of potential initial conditions to a single prescribed, fixed in time, final destination. More general situations such a station keeping, control to nominal (Lagrangian) trajectories, and way point tracking may be dealt with in an entirely analogous manner.

Comparison of Approaches: Figure 2 shows the path planning results computed using the full, *backward in time* extremal field approach in the context of the NCOM Adriatic flow field. The target destination is indicated by a fixed white dot located at (190,270) in model grid units. Each grid cell is approximately 1 x 1 km. Four sample globally optimal trajectories are shown along with the evolving time-to-go surface computed by backward in time integration, over all possible arrival times, from the final destination. The horizontal extent of the colored surface indicates the 24 hour controllability front of the target point – the set of all points in the domai which can be controlled to (190, 270) within 24 hours given that the maximum speed of the AUV is 0.9 km/h. The panel to the lower left gives the heading relative to the direction of the target. Interestingly, globally optimal trajectories rarely head directly to the target location but instead make use of the time dependent currents. The panel to the lower right shows the magnitude of the model currents along each trajectory indicating the ability of the algorithm to identify globally optimal trajectories and conduct path planning even in strong flows.

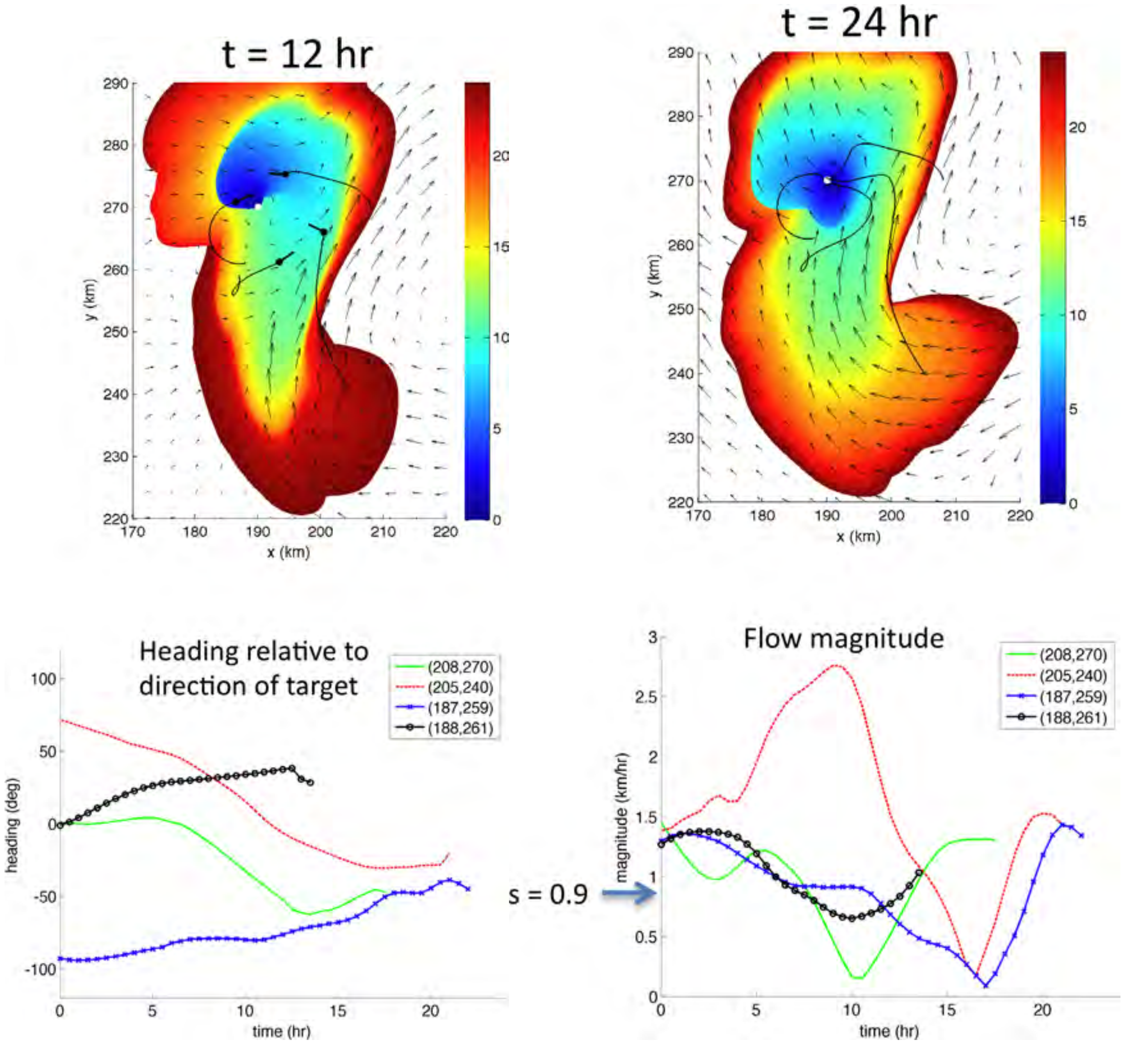


Figure 2: Top panel: Slices of time-to-go function, T(x,t), and four simulated closed loop trajectories for the time varying NCOM Adriatic flow . Left bottom panel: Heading relative to the target location of the four controlled AUV. Right bottom: Instantaneous magnitude of the flow speed along the four trajectories. The maximum AUV speed is s = 0.9.

Figure 2 shows a comparison of the forward and backward in time approaches to the same control problem. In each time slice, the reachability front (shown in thin black) of the trajectory initialized at (188, 261) is superimposed upon the previously computed time-to-go surface. Comparison of the optimal trajectory for the full algorithm (black dots) to the open-loop, *forward in time* optimal trajectory (black line) indicates that the two approaches produce identical final results.

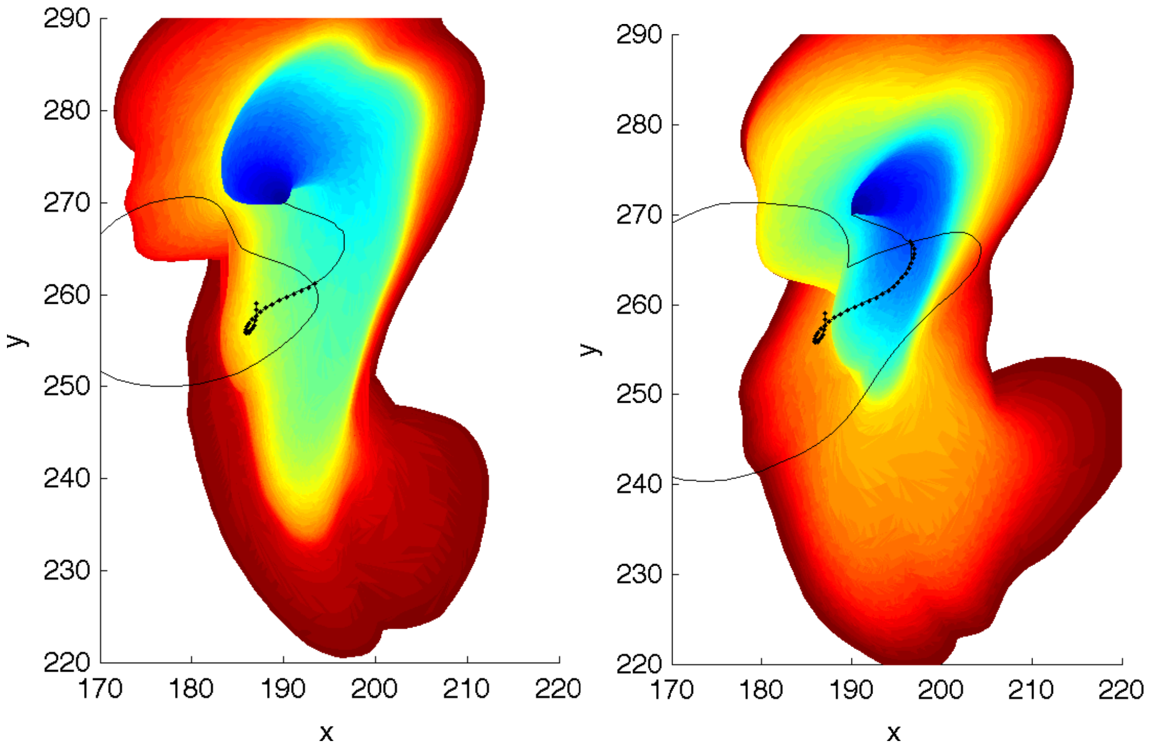


Figure 3: Snapshots of the computed forward in time reachability front (in thin black) superimposed on the previously computed time-to-go function for an NCOM Adriatic flow field. A single optimal trajectory computed using the open-loop, forward-in-time algorithm (solid line) is compared to an optimal trajectory computed using the closed-loop, backward in time approach (solid circles). The results of the two approaches are indistinguishable.

The efficiency of the *forward in time* approach for path planning from a given initial positions is shown in Figure 3 where a different time period of the NCOM data was used and the final position was fixed at (185, 255) in local grid units. This location is in a bathymetrically complex, eddying region near the model eastern (Croatian) coastline. The arrival time function is shown along with 6 sample globally optimal trajectories. The 45 hour reachability front is given by the last countour. The algorithmic efficiency derives from the ability to readily clip globally suboptimal trajectories from the computed one-dimensional reachability front. Such suboptimal trajectories necessarily occur at self-intersection points (cusps/shocks) in the front.

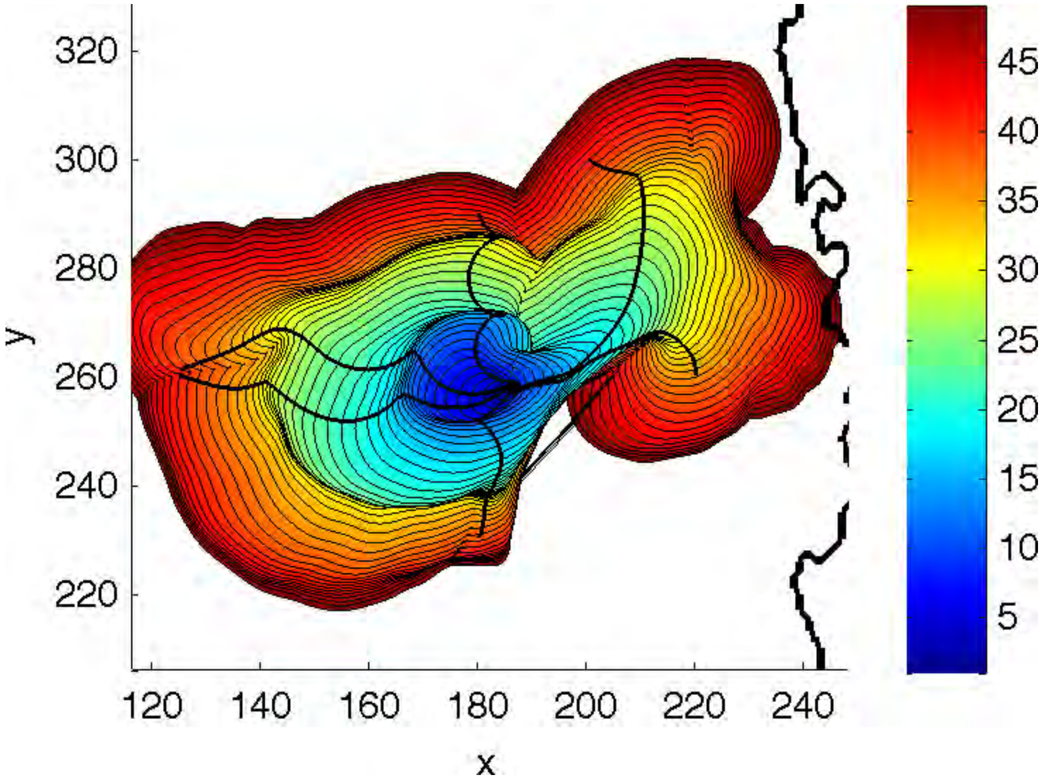


Figure 4: Example of globally optimally controlled trajectories computed in NCOM Adriatic model field. Shown are the arrival time function, the (discontinuous) reachability front and several optimal trajectories. Positions are relative to the model grid with the model Croatian coastline shown in black on the right.

IMPACT/APPLICATIONS

The research extends the application of rigorous and objective optimal control theory to a number of practical AUV/glider operational strategies. The algorithms developed are explicitly suited to computing optimal control in the context of highly nonlinear, time-dependent numerical vector fields derived from ocean model output. Gains in computational efficiency provided by a *forward in time*, open-loop approach will allow for direct investigation of path planning for AUV/gliders executing prescribed dive protocols (explicit prescription of z(t)) in complex, time dependent, 3-dimensional current fields given by model output.

RELATED PROJECTS

This work is collaboration under the ONR PLUS effort with Prof. I. Mezic´ and his group at the University of California Santa Barbara. The PI actively collaborates with Professors T. Ozgokmen and A. Griffa at the University of Miami on ONR funded research concerning Lagrangian data assimilation, optimal deployment strategies, model-data intercomparison and the sensitivity of Lagrangian Coherent Structure boundaries to model error and filtering. Similarly, the PI continues regular collaboration with Professors A.D. Kirwan and B. Lipphardt at the University of Delaware. All named collaborators are active participants in the ONR Multiple University Research Initiative: DYNAMICAL SYSTEMS THEORY IN 4D GEOPHYSICAL FLUID DYNAMICS funded in 2010.

PUBLICATIONS

1) B. Rhoads, I. Mezic' & A.C. Poje, *Minimum Time Control of Autonomous Underwater Vehicles*, Proceedings of the 49th IEEE Conference on Decision and Control, pp 5828-5834, 2010.

2) B. Rhoads, I. Mezic' & A.C. Poje, *Globally ptimal Control of AUVs in Strong, Time Varying Ocean Currents*, IEEE Journal of Ocean Engineering, 2011 (to be submitted).