Theory and Modeling of Internal Wave Generation in Straits

Karl R. Helfrich
Department of Physical Oceanography, MS-21
Woods Hole Oceanographic Institution
Woods Hole, MA 02543
phone: (508)-289-2870 fax: (508)-457-2181 email: khelfrich@whoi.edu

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LONG-TERM GOALS

The long-term goal is to improve understanding of the generation of nonlinear internal tides and waves by stratified flow over sills (e.g. straits) and shelves and the subsequent evolution of the radiated internal disturbance, with particular emphasis on the role of rotation.

OBJECTIVES

The primary objective will be on connecting the generation process with the dynamics of the disintegration of the radiated internal tide into shorter, nonlinear internal solitary-like waves and to predict the space and time scales for the emergence of waves and their properties (e.g. wave amplitudes, numbers, etc.). A central aspect of this work is to explore the role of rotation in the process. Rotation permits the presence of periodic, nonlinear inertia-gravity waves (i.e., the tide) that can act as attractors and arrest the steepening of the internal tide, and hence affect the production of the shorter solitary-like waves (Gerkema, 1996; Helfrich and Grimshaw, 2008). A related objective is the long-term effect of rotation on the emerging solitary waves, where it has been shown that these waves may decay to procude nonlinear wave packets (Helfrich, 1997; Grimshaw and Helfrich, 2008). Exploration of the role of propagation in two horizontal on these processes is also a goal.

APPROACH

The approach combines theoretical wave evolution models and numerical solutions of these models and solutions of the full Navier-Stokes equations. The theoretical models require some simplifications that, depending on the specific situation, may include restriction to two-layer flows, one-dimensional propagation, weak nonlinearity and either weakly non-hydrostatic or fully hydrostatic dynamics. The presence of rotation requires flow in the direction transverse to the propagation; however, variations of properties in this direction are ignored in models with propagation in one-dimensional propagation. The models used include those in the Korteweg-de Vries (KdV) family of equations modified to include higher-order nonlinearity, rotation, and transverse propagation. Fully nonlinear dynamics can be explored through an extension of the fully nonlinear, weakly non-hydrostatic internal wave theory of Miyata (1988) and Choi and Camassa (1999) to include rotation (Helfrich, 2007). In order to study the generation process, variable topography has been included in the model. These reduced wave equation models are complemented using 2.5- and 3-dimensional Navier-Stokes numerical models when

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appropriate. The use of reduced models is a drastic simplification in many situations, but the insight obtained from these models can guide the analysis of more complex models and observational data obtained as part of the IWISE field study.

WORK COMPLETED

Work during the last 12-18 months has focussed primarily on the steepening dynamics of the nonlinear internal tide and the evolution of internal solitary waves in the presence of rotation. The presence of "simple waves" in fully-nonlinear, continuously stratified systems was studied using numerical models (Ostrovsky and Helfrich (2011). The rotational decay of a single internal solitary wave into a nonlinear wave packet was explored in a set of laboratory experiments and comparison with theory and models (Grimshaw and Helfrich, 2011; Helfrich, Grimshaw and Johnson, in preparation). A previously proposed scaling criterion for the breaking of the internal tide, and the subsequent production of internal solitary-like waves, has been proven theoretically and further explored to demonstrate the significance and utility of the criterion in predicting the occurance of breaking (Grimshaw and Helfrich, in preparation). Theoretical work on large-amplitude internal solitary waves with trapped cores (Helfrich and White, 2010) has been extended to explore mass transport in the "leaky" cores and 3-dimensional stability of the waves. The PI has also acted as a co-editor for a special issue of *Nonlinear Processes in Geophysics* on internal solitary wave dynamics (see the overview by Grimshaw, Helfrich and Scotti, 2011).

RESULTS

Models for the evolution of internal solitary waves propagating in a single direction, such as the KdV equation, are possible because the nonlinear evolution of the waves can be described by "simple" wave dynamics. In a simple wave all information propagates in one direction and the wave phase speed can be written as a function of the amplitude. Hence the wave evolution can be described by a single first-order-in-time equation for the amplitude. In the KdV equation this is possible because the wave nonlinearity is restricted to be weak. If the wave amplitude is large, then it is known that simple waves can be found for two-layered, hydrostatic systems. In Ostrovsky and Helfrich (2011), it was shown through a numerical analysis that fully nonlinear, continuously-stratified, hydrostatic flows also posses simple waves. This suggests that fully-nonlinear, continuously stratified extensions of the KdV equation may be possible. Indeed, ad-hoc models of this type have been proposed and these new results place these models on firmer theoretical ground. Equations of this type would have great utility in modeling the space-time evolution of internal soliary waves in more realistic conditions of large amplitude and continuous stratification.

Is has been know for some time that rotation leads to the deacy of an initial solitary waves through a direct resonance with Poincare waves (Grimshaw, Ostrovsky and He, 1998; Melville, Tomasson and Renouard, 1989). Recent numerical and theoretical work has shown that the long-time result of this radiational decay is a nonlinear internal wave packet governed by an extended nonlinear Schrodinger equation (Helfrich, 2007; Grimshaw and Helfrich, 2008). In September 2009 a series of laboratory experiments were performed at the LEGI-Coriolis 13-meter rotating platform in Grenoble, France to test these ideas. Figure 1 shows two examples of the experimental results. Without rotation (left panel) the lead disturbance is a well-defined solitary wave with constant propagation speed. The introduction of rotation (right panel) leads to the decay of the initial solitary-like wave and the emergence of a

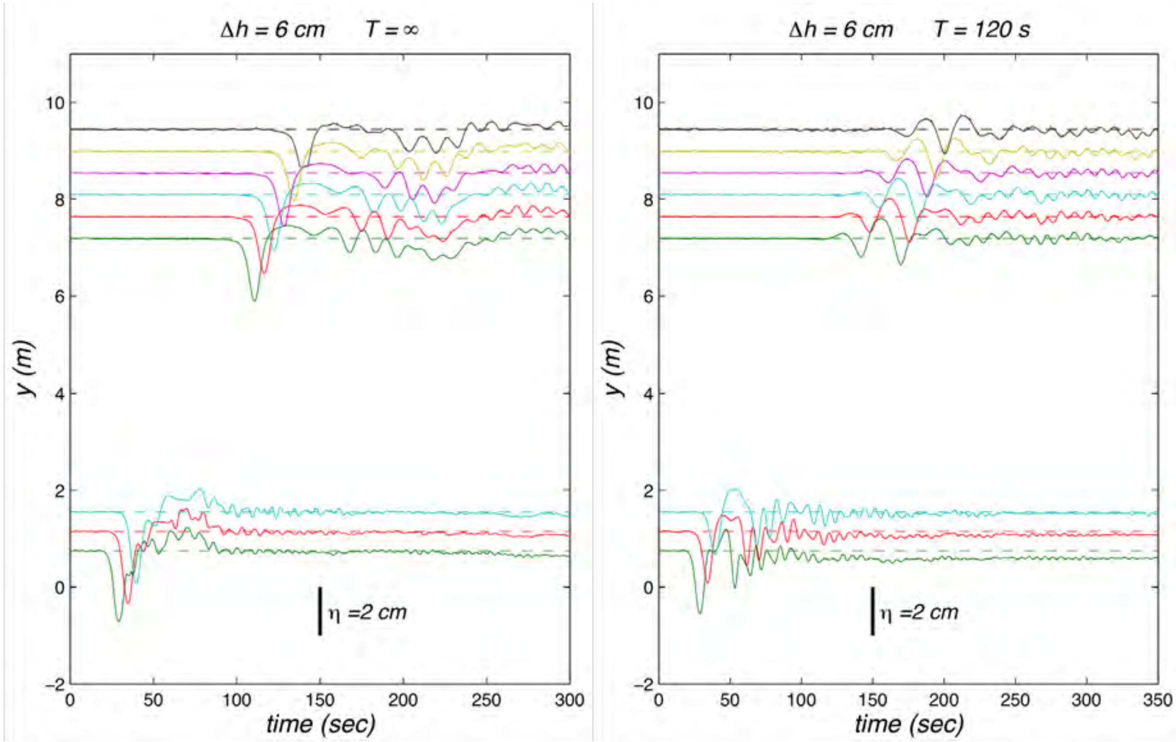

Figure 1: Interfacial displacement, η, as a function of time at several locations, y, downstream of a lock-type wave generator. The system has a two-layer stratification with undisturbed upper and lower layer depths of 6 cm and 30 cm, respectively. The density difference between the layers is 0.01 gm/cm^3. The panel on the left shows an example with an initial interface difference Δh = 6 cm between the lock and ambient in a non-rotating experiment. The right panel is a rotating run with platform rotation period T = 120 s. In the absence of rotation a solitary wave propagates across the 13 m platform at a constant speed. With rotation the initial wave evolves into a leading localized wave packet.

leading wave packet. The localized character of the packet is an indicator of nonlinear dynamics. The experiments show that the wave length and phase speed of the carrier waves, and the packet group speeds agree with the theoretical predictions. The experiments also show that more general initial conditions (e.g., geostrophic adjustment) will also produce the wave packets. This work is important since it broadens the type of wave groups that can be expected and may provide an explanation for observations that do not fit in the classic paradigm of a rank-ordered group of solitary-like waves. These results are presented in an article reviewing the theoretical analysis and some preliminary experimental data (Grimshaw and Helfrich, 2011). The complete experimental results are being finalized for publication (Helfrich, Grimshaw and Johnson, in preparation).

In many situations such as the South China Sea, stratified tidal flow over localized topography leads to the radiation of a low-mode internal tide. This internal tide many subsequently steepen due to nonlinearity and produce internal solitary-like waves. However, it is know that rotation acts to slow the steepening and may prevent the emergence of the shorter solitary-like waves (Gerkema, 1996; Helfrich and Grinshaw, 2008). The question of whether an internal tide will steepen was addressed in the using the Ostrovsky equation (KdV extended to include rotation)

$$\frac{\partial}{\partial x}\left(\frac{\partial \eta}{\partial t}+\alpha\eta\frac{\partial \eta}{\partial x}+\beta\frac{\partial^3 \eta}{\partial x^3}\right)=\gamma\eta \quad (1)$$

for the wave amplitude $\eta(x,t)$. The coefficients nonlinear, α, nonhydrostatic, β, and rotational terms, γ,

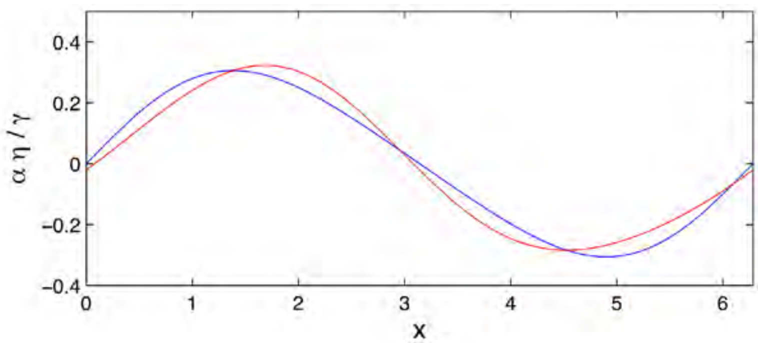

Figure 2: Two initial periodic waves as initial conditions for the reduced-Ostrovsky equation [(1) with $\beta = 0$]. The wave consist of a primary sinusoidal wave with wavelength 2π and the first harmonic with wavelength π. The amplitudes of the primary and harmonics are 0.3 and 0.03, respectively, for both initial conditions. The phase of the harmonic relative to the primary wave is 0 in the red wave and 1.24π in the red wave. The red wave has the larger initial slope while only the blue wave violates the breaking criterion $3\alpha\eta_{xx}/\gamma > 1$ (in the trough near x = 4.9). Numerical solutions of the reduce-Ostrovsky equation confirm that the red wave remains smooth, while the blue wave eventually breaks.

are functions of the background stratification, etc. In the hydrostatic limit ($\beta = 0$) the reduced-Ostrovsky equation has been used as a model for the nonlinear evolution of the internal tide. Boyd (2005) has shown from numerical calculations that the sinusoidal initial condition $\eta = a\sin(kx)$ will evolve to breaking if $3\alpha ak^2/\gamma > 1$. Farmer, Li and Park (2009) and Li and Farmer (2011) have used Boyd's breaking criterion to predict the conditions which lead to emergence of internal solitary-like waves in the South China Sea.

However, the general application of this criterion is limited by the fact that it is essentially a numerical, or scaling-based, criterion applicable only to sinusoidal initial conditions. Recently we have shown theoretically that breaking will occur in the reduced-Ostrovsky equation for any initial condition that possesses a region with $3\alpha\eta_{xx}/\gamma > 1$ (Grimshaw and Helfrich, in preparation). For the siunsoidal initial condition this reduces to Boyd's criterion. However, this new criterion is a more powerful statement since it is a local test applied at every *x* of the initial condition. Also it is clear that it is a condition on the maximum curvature of the initial condition, and not the slope as Farmer, Li and Park (2009) and Li and Farmer (2011) have interpreted Boyd's criterion. For example, Figure 2 showns two nearly similar initial waves composed of the superposition of a sinusoidal wave plus its first harmonic. The amplitudes of the primary and first harmonic components are identical in both initial condition. The only difference is the phase of the harmonic relative to the primary wave. The red wave has the largest (negative) slope. According to non-rotating theory this initial condition would break before the blue and be more likely to break with rotation according to Farmer, Li and Park (2009). A simple application of Boyd's criterion with $k = 2\pi/L = 1$, where $L = 2\pi$ is the length of the primary wave, one might expected both of the waves to either break or remain smooth since $3\alpha a k^2/\gamma$ is almost the same. However, only the blue wave, with the smaller slope, violates the breaking criterion $3\alpha\eta_{xx}/\gamma > 1$ (in the wave trough near $x = 5$). Numerical calculations with the reduced-Ostrovsky equation show that it does indeed goes to breaking, while the red wave does not. Numerical tests with other initial conditions indicate that the number of independent breaking events is given by the number of distinct regions in the initial condition that violate the criterion $3\alpha\eta_{xx}/\gamma > 1$. This is illustrated in Figure 3 for a Gaussian intial condition. Note that for this case it would be difficult to apply Boyd's criterion since the initial condition is far from a sinusoidal wave and the emergence of two breaking events would not be predicted.

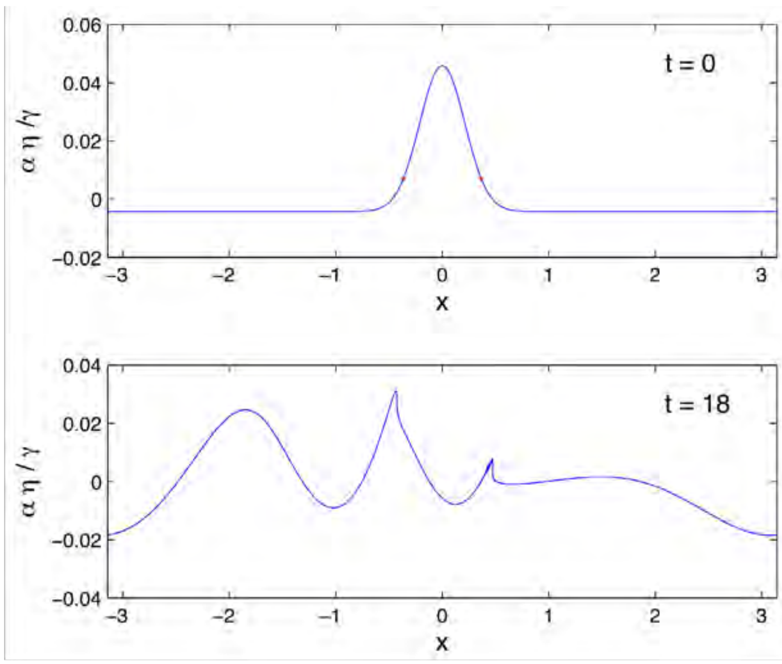

Figure 3: Solution of the reduced-Ostrovsky equation [(1) with $\beta = 0$] for a Gaussian initial condition (top panel) with two regions, centered on the red dots that violate the breaking criterion. The solution (in a periodic domain) at t = 18 is shown in the lower panel. Two distinct breaking fronts near x = -0.4 and 0.4 are evident. In a nonhydrostatic calculation (finite β in (1)) solitary-like waves would emerge from the breaking fronts.

The significance of this new result for oceanographic conditions is still being assessed. However, Boyd's numerically-based breaking criterion that Li and Farmer (2011) employed for the South China Sea observations has now been formally derived. Their success with Boyd's criterion probably stems from the near sinusoidal initial conditions. However, from the formal derivation of the new criterion comes clearer understanding of the conditions that result in breaking, and thus the emergence of solitary-like waves. It remains to be seen whether the condition is relevant to less restrictive models than (1) and if the new interpretation leads to improved predictions of the solitary wave emergence in the deep basin of the South China Sea.

IMPACT/APPLICATIONS

The ubiquitous nature of large amplitude internal solitary waves in the world's coastal oceans and marginal seas is clear from observations. These waves are can have significant effects on coastal mixing through breaking as they propagate and shoal, and they may also lead to substantial horizontal mass transport. Since the waves are frequently generated through the radiation of an internal tide by barotropic tidal flow over localized topography (as is apparently the case at in the Luzon Strait), this work will help understand what fraction of the energy put in at the tidal frequency ends up as internal solitary waves, the space and time scales for that transformation, and the characteristics of the resulting solitary-like waves.

RELATED PROJECTS

This project is part of the IWISE DRI. It is also closely related to the PI's collaboration on the ONR MURI on Integrated Oceanographic, Atmospheric, and Acoustic Physics titled: "Integrated Modeling and Analysis of Physical Oceanographic and Acoustic." In that project, the PI is responsible for efforts to integrate reduced-dynamics wave models into regional hydrostatic coastal circulation models for the estimation of internal wave conditions and their effect on acoustic propagation. Some of the basic wave dynamics work funded by this grant will have direct application to the MURI work.

REFERENCES

Boyd, J., 2005. Microbreaking and polycnoidal waves in the Ostrovsky-Hunter equation. *Phys. Lett. A* **338**, 36-43.

Choi W. and Camassa, R. 1999. Fully nonlinear internal waves in a two-fluid system. *J. Fluid Mech.* **396**, 1-36.

Farmer, D., Q. Li, and J-H Park, 2009. Internal wave observations in the South China Sea: The role of rotation and nonlinearity. *Atmos.-Ocean* **47**, 267-280.

Gerkema, T., 1996. A unified model for the generation and fission of internal tides in a rotating ocean. *J. Mar. Res.* **54**, 421-450

Grimshaw, R., J.-M. He, and L. Ostrovsky, 1998. Terminal damping of a solitary wave due to rotation in rotational systems. *Stud. Appl. Math.* **101**, 197-210.

Helfrich, K. R., 2007. The decay and return of rotating internal solitary waves. *Phys. Fluids* **19**, 026601.

Helfrich, K. R. and R. H. J. Grimshaw, 2008. Nonlinear disintegration of the internal tide. *J. Phys. Oceano.* **38**, 686-701.

Li, Q. and D. Farmer, 2011. The generation and evolution of nonlinear internal waves in the deep basin This of the South China Sea. *J. Phys. Oceano.* **41**, 1345-1363.

Melville, W. K., G. Tomasson, and D. Renouard, 1989. On the stability of Kelvin waves. *J. Fluid Mech.* **206**, 1-23.

Miyata M. 1988. Long internal waves of large amplitude. In *Nonlinear Water Waves*, IUTAM Symposium Tokyo 1987, ed. K. Horikawa and H. Maruo, pp. 399-406, Berlin:Springer Verlag.

PUBLICATIONS

Ostrovsky, L. and K. R. Helfrich, 2011. Strongly nonlinear, simple internal waves in continuously-stratified shallow fluids. *Nonlinear Processes in Geophys.* **18**, 91-102.

Grimshaw, R. and K. Helfrich, 2011. Effect of rotation on internal solitary waves. *IMA J. Appl. Maths.*, submitted.

Grimshaw, R. and K. Helfrich, Integrability and breaking in the reduced-Ostrovsky equation. in preparation for *Studies Appl. Math.*

Grimshaw, R., K. R. Helfrich, and A. Scotti, 2011. Large-amplitude internal waves in the coastal ocean. *Nonlinear Processes in Geophys.,* in press.

Helfrich, K. R., R. Grimshaw, and E. R. Johnson, Experiments on the effect of rotation on internal solitary waves. in preparation for *J. Fluid Mech.*

Helfrich, K. R. and B. L. White, 2010. A model for large-amplitude internal solitary waves with trapped cores. *Nonlinear Processes in Geophys.* **17**, 303-318.