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Stability and Tolerance to Optical Feedback of Quantum Dot Lasers

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14. ABSTRACT

This report results from a contract tasking Universite Libre de Bruxelles as follows: Quantum dot (QD) lasers have attracted a lot of attention because of their low threshold currents, low line width enhancement factors, and temperature insensitivity. Although the self-assembled dots have provided an enormous stimulus to work in this field, there remain a number of critical issues involving their growth and formation: greater uniformity of size, controllable achievement of higher quantum dot density, and closer dot-to-dot interaction range will further improve laser performance. Better understanding of carrier confinement dynamics and capture times, and better evaluation of loss mechanisms, will further improve device characteristics. Because of the large variety of QD lasers currently tested in laboratories, there are several obstacles in the characterization and modeling of these devices that need to be overcome. A delay-differential equation model of a passively mode-locked quantum-dot laser originally proposed by Vladimirov and Turaev [1] has been used recently by AFRL/RYDP to simulate asymmetric pulse intensities [2] which have been observed experimentally. The model equations for the passively mode-locked QD laser include delay and exponential nonlinearities and are too complicated for analytical studies. This has motivated very recently investigations of simpler experimental set-ups as turn-on experiments, the optically injected laser or the laser subject to a delayed optical feedback. In collaboration with N. Usechak and V. Kovanis at AFRL/RYDP and the experimental group of L.F. Lester (University of New Mexico), we propose a multi-disciplinary approach of the stability of QD lasers subject to optical feedback. The group of T. Erneux at the Universite Libre de Bruxelles (ULB) will be responsible for the modeling and analysis of rate equation models. Numerical simulations will be shared by the Brussels and AFRL groups.

The following publications acknowledge the support from this grant: 1. T. Piwonski, J. Pulka, G. Huyet, J. Houlihan, E. A. Viktorov, and T. Erneux, Mixed state effects in waveguide electro-absorbers based on quantum dots, Appl. Phys. Letters 99, 171103 (2011); 2. Christian Otto, Kathy Lüdge, Evgeniy Viktorov and Thomas Erneux, Quantum dot laser tolerance to optical feedback, In "Nonlinear Laser Dynamics", K. Lüdge (Ed), Wiley-VCH Weinheim, Germany (2012); 3. Optically injected quantum dot lasers, T. Erneux, E.A. Viktorov, B. Kelleher, D. Goulding, S.P. Hegarty, and G. Huyet, Optics Letters 35, 937-939 (2010); 4. Analytical approach to modulation properties of quantum dot lasers, K. Lüdge, E. Schöll, E. Viktorov, and T. Erneux, J. Appl. Physics, 109, 103112 (2011); 5. Dimensional signature on noise-induced excitable statistics in an optically injected semiconductor laser, B. Kelleher, D. Goulding, G. Huyet, E. A. Viktorov, T. Erneux, and S. P. Hegarty, Phys. Rev. E84, 026208 (2011)

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1. T. Piwonski, J. Pulka, G. Huyet, J. Houlihan, E. A. Viktorov, and T. Erneux, Mixed state effects in waveguide electro-absorbers based on quantum dots, Appl. Phys. Letters 99, 171103 (2011)
2. Christian Otto, Kathy Lüdge, Evgeniy Viktorov and Thomas Erneux, Quantum dot laser tolerance to optical feedback, In "Nonlinear Laser Dynamics", K. Lüdge (Ed), Wiley-VCH Weinheim, Germany (2012)

In (1), multi-pulse heterodyne pump-probe measurements are used to investigate the reverse bias dynamics of InAs/GaAs quantum dots in a waveguide structure. It is the last publication of a series of pump-probe experiments done in collaboration with the group of G. Huyet at Cork [1]-[4].

Publication (2) is a chapter of a book collecting different contributions on Laser Dynamics. It considers the stability condition derived by Mork et al. [5] given by

$$k < k_c \equiv \frac{\Gamma^{QW}}{\sqrt{1+\alpha^2}} \tag{1}$$

where α is the linewidth enhancement factor and Γ^{QW} is defined as the damping rate of the relaxation oscillations multiplied by the diode cavity round-trip time. Eq. (1) was previously suggested by Helms and Petermann [6] as a simple analytical criteria for tolerance with respect to optical feedback. Helms and Petermann [6] also proposed the empirical law given by

$$k_c = \Gamma^{QW} \frac{\sqrt{1+\alpha^2}}{\alpha^2}. \tag{2}$$

Both Eqs. (1) and (2) are used in current experimental studies of quantum dot (QD) lasers subject to optical feedback. This has motivated the analysis of two different rate equation models for QD lasers. In both cases, we derive an expression that has the same format as 1 but with a expression for the damping rate $\Gamma_{QD} > \Gamma_{QW}$.

A combined experimental-theoretical paper describing the response of QD lasers in turn-on experiments has been submitted. Another paper on optical feedback is in preparation.

References

[1] T. Piwonski, G. Huyet, J. Pulka, G. Madden, J. Houlihan, E.A. Viktorov, T. Erneux, and P. Mandel, Intradot dynamics of InAs quantum dot based electroabsorbers, Appl. Phys. Letters 94, 123504 (2009)

[2] T. Erneux, E.A. Viktorov, P. Mandel, T. Piwonski, G. Huyet, and J. Houlihan, The fast recovery dynamics of a quantum dot semiconductor optical amplifier, Appl. Phys. Letters 94, 113501 (2009)

[3] Evgenii Viktorov, Thomas Erneux, Paul Mandel, Tomasz ,Piwonski, Gillian Madden, Jaroslaw Pulka, Guillame Huyet, and John Houlihan, Recovery time scales in a quantum dot absorber, Appl. Phys. Letters 94, 263502 (2009) [Selected for the July 13, 2009 issue of Virtual Journal of Nanoscale Science & Technology]

[4] T. Piwonski, J. Pulka, E. A. Viktorov, G. Huyet, R. J. Manning, J. Houlihan, P. Mandel, and T. Erneux, Induced absorption dynamics in quantum dot based waveguide electroabsorbers, Appl. Phys. Lett. 97, 121103 (2010)

[5] J. Mork, B. Tromborg, and J. Mark, Chaos in semiconductor lasers with optical feedback: theory and experiment, IEEE J. Quant. Electr. 28, 93-108 (1992)

[6] J. Helms and K. Petermann, A simple analytical expression for the stable operation range of laser diodes with optical feedback, IEEE J. of Quant. Electronics 26, 833-836 (1990)

Third quartely report for the Grant No FA8655-09-1-3068, entitled Stability and Tolerance to Optical Feedback of Quantum Dot Lasers
PI: Thomas Erneux

The following papers acknowledges the support from this grant:

1. Optically injected quantum dot lasers, T. Erneux, E.A. Viktorov, B. Kelleher, D. Goulding, S.P. Hegarty, and G. Huyet, Optics Letters 35, 937-939 (2010)
2. Analytical approach to modulation properties of quantum dot lasers, K. Lüdge, E. Schöll, E. Viktorov, and T. Erneux, J. Appl. Physics, 109, 103112 (2011)
3. Dimensional signature on noise-induced excitable statistics in an optically injected semiconductor laser, B. Kelleher, D. Goulding,, G. Huyet, E. A. Viktorov, T. Erneux, and S. P. Hegarty, Phys. Rev. E84, 026208 (2011)

Paper (1) reports on an experimental and theoretical study of a single mode quantum dot (QD) laser subject to injection. The stability diagram considerably differs from the one known for the quantum well (QW) semiconductor. Experimentally, two features stand out; the first is an absence of instabilities resulting from relaxation oscillations, and the second is the observation of a region of bistability between two locked solutions. Using rate equations appropriate for quantum-dot lasers, we analytically determine the stability diagram in terms of the injection rate and frequency detuning.

At the TU-Berlin, a microscopically based rate equation model for QD lasers is studied mainly numerically. The model separately treats the dynamics of electrons and holes, and the carrier-carrier scattering rates depend nonlinearly on the wetting layer carrier densities. Paper (2) summarizes more than two year efforts to simplify the five rate equations in a form for which analytical expressions can be obtained for the relaxation oscillation frequency and damping rate. .

Paper (3) describes new experimental observations of noise-induced excitable pulses for an injected QD laser. The authors use a third order phase equation instead of Adler equation for the interpretation of their results.

Second quartely report for the Grant No FA8655-09-1-3068, entitled Stability and Tolerance to Optical Feedback of Quantum Dot Lasers
PI: Thomas Erneux

Experiments on a quantum dot (QD) DFB laser subject to optical injection have been realized. The stability diagram has been determined in terms of the slave-master frequency detuning and the optical injection rate. The domains of stability strongly differ from the same domains observed for the conventional quantum well laser. We have considered a three-variable model for the QD laser [1] in the case where it admits a strong damping of the relaxation oscillations. We explain numerically and analytically the unusual stability properties observed experimentally. A publication with the group of G. Huyet (Cork) in Optics Letters summarizes the results and acknowledge the support of the AF [2]. Another manuscript has been submitted for publication.

Parallel to the collaboration with Cork, work has been initiated with the group of E. Schöll (TU Berlin) in order to simplify a five variable model used by the group [3]. Our objective is to determine if there exist similar features between this model and the three variable model used in Cork. A manuscript summarizing the asymptotic analysis is in preparation.

By the end of 2009, the group in Brussels became partner of the French ANR project called TELDOT devoted to the development of quantum dot lasers for Telecom applications. To this end, work has been started with the group of P. Besnard (Lannion). This group is now performing injection experiments using a quantum dash DFB laser fabricated by Alcatel. This study could be particularly relevant for the AF which has recently supported experiments using a quantum dash laser in a Fabry-Perot cavity [4].

References

[1] T. Erneux, E.A. Viktorov, and P. Mandel, Phys. Rev. A76, 023819 (2007)

[2] T. Erneux, E.A. Viktorov, B. Kelleher, D. Goulding, S.P. Hegarty, and G. Huyet, Optics Letters 35, 937-939 (2010)

[3] K. Lüdge, M.J.P. Bormann, E. Malic, P. Hövel, M. Kuntz, D. Bimberg, A. Knorr, and E. Schöll, Phys. Rev. B78, 035316 (2008)

[4] M. Pochet, N.A. Naderi, N. Terry, V. Kovanis, L.F. Lester, submitted (2010)

First quartely report for the Grant No FA8655-09-1-3068, entitled Stability and Tolerance to Optical Feedback of Quantum Dot Lasers
PI: Thomas Erneux

Abstract

Recent experiments have explored the response of a single mode optically injected QD laser. The experiments identify a domain of bistability between two steady states at negative detuning and a Hopf bifurcation close to the locking threshold at positive detuning. Moreover, the laser is stable for all injection levels at zero detuning. These observations lead to a stability diagram injection amplitude versus detuning that contrasts to the diagram of a quantum well semiconductor laser. The bifurcations responsible for the QD laser stability diagram are explained analytically by using a model appropriate for QD lasers.

1 Introduction

Semiconductor lasers (SLs) have become the optical source of choice in many applications due to their high efficiency, simplicity of modulation, and small size. However, in some applications where low-intensity noise level are required, they suffer from wide-bandwidth intensity fluctuations that are enhanced by their inherent relaxation oscillations (ROs). The phenomenon of ROs is familiar in the laser physics community. When a laser is perturbed from its steady state operation, it does not immediately return to its original position. Either the laser quickly approaches equilibrium or it slowly decays to its stable steady state like a damped oscillator. These lasers are labelled as

Class A and Class B, respectively [1]. Class A lasers include the Ar, He-Ne, and dye lasers while Class B lasers include most of the lasers used today such as CO_2, solid-state, and semiconductor lasers. Subject to optical injection Class A and Class B lasers exhibit quite different stability properties. Class B lasers admit a rich number of sustained pulsating intensity regimes which have been studied systematically during the last decade for semiconductor and solid state lasers (see [2] for a recent review). Class A lasers free of ROs are much more stable [3]. Recent efforts have concentrated on increasing the photon lifetime above the carrier lifetime to suppress ROs in SLs. This can be achieved by either increasing the cavity length or the cavity finesse. The first technique has been successfully applied with a several-meter-long cavity in semiconductor laser [4]. The second alternative is technically easier and has been explored using half of a vertical cavity surface emitting laser ($\frac{1}{2}-$VCSEL) in a short external cavity [5, 6].

We consider optical injection of a single mode distributed feedback (DFB) quantum dot laser (QDL). These lasers have become increasingly pervasive in recent years and studies have already revealed some dynamical properties which demonstrate their superiority for applications [7]. A particular feature of these devices is an unusually high damping of the relaxation oscillations (ROs) [8] in comparison to their bulk and quantum well (QW) counterparts. Analytical studies have shown that the damping rate and the RO frequency can have the same timescale and may even be overdamped [9]. This high damping has been cited as the principal reason for the increased stability of such devices subject to optical feedback [10], optical injection [11], and mutual coupling [12] configurations. We determine an experimental stability diagram and note that it is considerably different to that of a conventional quantum well laser showing strong similarities to the stability diagram of a Class A laser. In particular, the injected QDL exhibits stability for arbitrary value of the injection rate provided the detuning is sufficiently low and bistabiliy between two coexisting steady states at negative detuning. These observations are substantiated analytically by studying rate equations appropriate for a QD laser.

2 Experiments

The slave DFB QDL used was a five layer structure grown by solid source MBE. It consisted of 2.4 InAs monolayers topped with 5 nm GaInAs, stacked in a 400 nm thick optical cavity. A 35nm GaAs spacer is used between the QD layers. Optical confinement was achieved using AlGaAs cladding layers. The experimental setup is similar to the one described in [13]. Of particular interest is the large area of stable locked operation. Regions of bistability between locked and periodic operation exist for both negative and positive detunings while for negative detuning, there is a domain of bistable operation in which two locked steady states coexist and the laser displays a noise induced switching. The transition between the two steady states is sharp and the relaxation dynamics includes only one spiking oscillation. It suggests that the decay of the relaxation dynamics occurs at the same time scale as that of the RO frequency which is typically the case for a Class A laser.

There are a number of fundamental differences between our stability diagram and the diagram reported in [14] for the conventional DFB QW laser (reviewed in detail in [2]). In the injected QW laser, there is only one Hopf bifurcation line that crosses twice the zero detuning line. Here we found two Hopf bifucation lines that never cross the sero detuning line. Moreover, the coexistence of two stable locking states for our QW laser is not possible for a QW laser except close to the laser threshold. These differences suggest a significant impact from the nonlinear capture dynamics in QDLs provided by the Pauli blocking factor which do not exist in the conventional formulation of a class B QW laser. Instead, we note a similarity between our stability diagram and the one for a class A laser [3]. It suggests that QDLs may exhibit both class A and class B dynamics depending on the carrier capture parameters as shown in [9] by exploring different asymptotic limits of a three variable rate equation model.

3 Theory

Our rate equations for a QD laser subject to an injected signal consist of three equations for the complex electric field E, the occupation probability in a dot ρ, and the carrier density n in the wetting layers, scaled to the QD density. They are given by [11]

$$\begin{aligned} E' &= \frac{1}{2}(1+i\alpha)\left[-1+g(2\rho-1)\right]E+\Gamma\exp(i\Delta t), && (1)\\ \rho' &= \eta\left[B\rho(1-\rho)-\rho-(2\rho-1)|E|^2\right], && (2)\end{aligned}$$

$$n' = \eta\left[J - n - 2Bn(1-\rho)\right]. \quad (3)$$

Prime means differentiation with respect to $t \equiv t'/\tau_{ph}$ where τ_{ph} is the photon lifetime. The factor $1-\rho$ is the Pauli blocking factor. The factor 2 in Eq. (3) accounts for the spin degeneracy in the quantum dot energy levels. J is the pump current per dot and α is the linewidth enhancement factor. The control parameters are the frequency detuning Δ defined as the frequency of the master laser minus that of the slave laser and the injection rate Γ. The fixed parameters B and η are ratio of basic time scales and are defined as $B \equiv \tau\tau_{cap}^{-1}$ and $\eta \equiv \tau_{ph}\tau^{-1}$ where τ and τ_{cap} denote the carrier recombination and capture times, respectively. Typical values are $\tau = 1$ ns, $\tau_{ph} = 2$ ps, and $\tau_{cap} = 10$ ps which imply $B = 10^2$ and $\eta = 2 \times 10^{-3}$.

As suggested by Goulding et al [11], we shall consider the value $g = 1.01$ for which a good agreement between theory and experiments is observed. In the case of the solitary laser, the product $B(g-1)$ appears in both the steady state expressions and in the characteristic equation [9]. Therefore, we need to take into account the relative values of B and $g-1$. Specifically, we propose an asymptotic analysis of Eqs. (1)-(3) valid in the limit $\varepsilon \equiv g-1$ small keeping $B\varepsilon$ as an $O(1)$ quantity. After introducing $g = 1+\varepsilon$ into Eq. (1), the expression in brackets becomes $[-2+2\rho+\varepsilon(2\rho-1)]$ and suggests to introduce $\rho = 1+\varepsilon u$ in order to balance all terms. The expression in brackets then is proportional to ε which motivates introducing the slow time scale $s \equiv \varepsilon t$. From Eqs. (1)-(3), we obtain the following equations for E, u, and n

$$E' = \frac{1}{2}(1+i\alpha)\left[1+2u(1+\varepsilon)\right]E + \gamma\exp(i\delta s), \quad (4)$$

$$u' = \varepsilon^{-2}\eta\left[-B\varepsilon nu - 1 - \varepsilon u - (1+2u\varepsilon)|E|^2\right], \quad (5)$$

$$n' = \varepsilon^{-1}\eta\left[J - n - 2B\varepsilon nu\right] \quad (6)$$

where prime now means differentiation with respect to s. The control parameters are $\gamma \equiv \varepsilon^{-1}\Gamma$ and $\delta \equiv \varepsilon^{-1}\Delta$. Since $\varepsilon^{-2} \gg \varepsilon^{-1}$ as $\varepsilon \to 0$, u is faster than n and we eliminate u adiabatically from Eq.(5). Specifically, we find $u = -(1+E^2)/(B\varepsilon n)$ as $\varepsilon \to 0$. Introducing the decomposition $E = R\exp(i\delta s + i\phi)$, Eqs. (4)-(6) reduce to the following equations for R, ϕ, and n

$$R' = \frac{1}{2}\left[1 - \frac{2(1+R^2)}{B\varepsilon n}\right]R + \gamma\cos(\phi), \quad (7)$$

$$
\begin{aligned}
R' &= -\delta + \frac{1}{2}\left[1 - \frac{2(1+R^2)}{B\varepsilon n}\right]\alpha - \frac{\gamma}{R}\sin(\phi), && (8)\\
n' &= \varepsilon^{-1}\eta\left[J - n - 2(1+R^2)\right]. && (9)
\end{aligned}
$$

If $\gamma = 0$, we find that the laser threshold of the solitary laser appears at $J = J_{th}$ where

$$J_{th} \equiv 2 + 2/(B\varepsilon). \tag{10}$$

We next assume that $J > J_{th}$. From the steady state equations, we determine $R^2 = R^2(\gamma)$ as (in implicit form)

$$\gamma^2 = R^2\left[F^2 + (-\delta + \alpha F)^2\right] \tag{11}$$

where

$$F \equiv \frac{1}{2}\left(1 - \frac{2(1+R^2)}{B\varepsilon\left(J - 2(1+R^2)\right)}\right). \tag{12}$$

From the linearized equations, we then obtain the characteristic equation for the growth rate λ. It is given by

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0, \tag{13}$$

where

$$
\begin{aligned}
a_1 &= -G - F + \varepsilon^{-1}\eta, && (14)\\
a_2 &= GF + (F-G)\alpha(\delta - \alpha F) + (\delta - \alpha F)^2 \\
&\quad -\varepsilon^{-1}\eta(G+F) + \varepsilon^{-1}\eta(F-G)(1-2F), && (15)\\
\varepsilon\eta^{-1}a_3 &= GF + (F-G)\alpha(\delta - \alpha F) + (\delta - \alpha F)^2 \\
&\quad +(F-G)(1-2F)\left(\alpha(\delta - \alpha F) - F\right) && (16)
\end{aligned}
$$

and

$$G = \frac{1}{2}\left(1 - \frac{2(1+3R^2)}{B\varepsilon\left(J - 2(1+R^2)\right)}\right). \tag{17}$$

The Routh-Hurtwitz stability conditions are $a_1a_2 - a_3 > 0$, $a_1 > 0$, and $a_3 > 0$. The saddle-node bifurcation point satisfies the condition $a_3 = 0$. The Hopf bifurcation point satisfies the condition $a_1a_2 - a_3 = 0$. Both conditions are quadratic equations in $(\delta - \alpha F)$. We first determine $\delta - \alpha F$ as a function of R^2 and then δ as a function of γ, using (11). The stability diagram is shown in Fig. 1. Only the Hopf bifurcation points from a stable steady state are shown ($a_1a_2 - a_3 = 0$, $a_1 > 0$, and $a_3 > 0$).

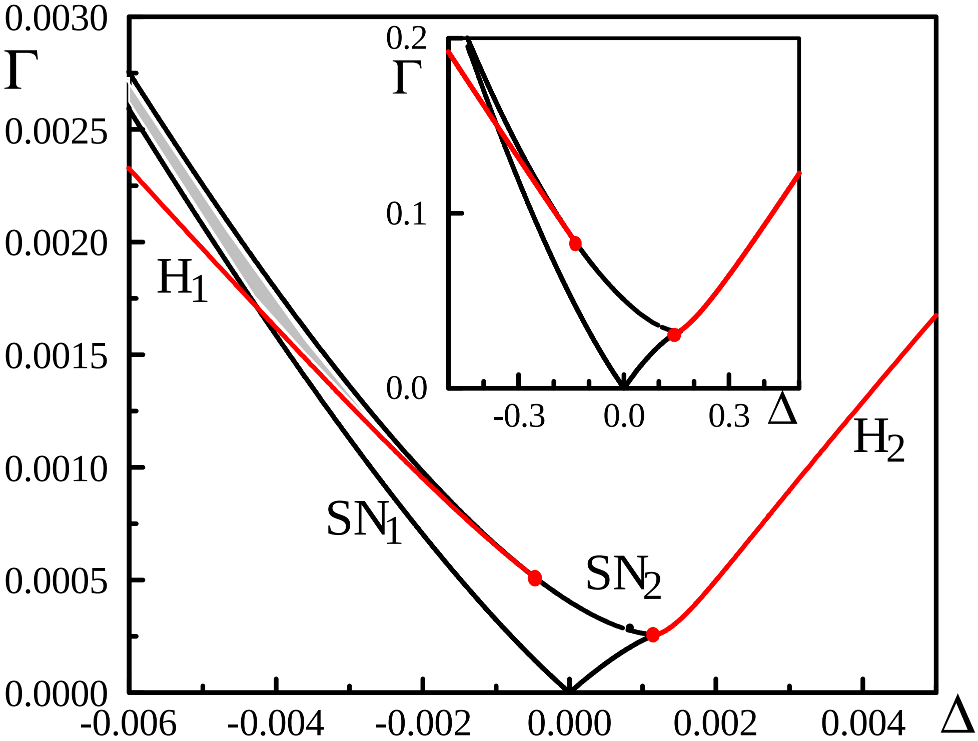

Figure 1: Stability diagram. SN and H denote the saddle-node and Hopf bifurcation points, respectively. The shaded region corresponds to the domain of steady state bistability. The values of the parameters are $g = 1.01$, $B = 10^2$, $\eta = 2 \times 10^{-3}$, $\alpha = 1.2$, and $J = 1.2 J_{th} = 4.8$ ($J_{th} = 4$). The dots are Fold-Hopf points where Hopf and SN bifurcation lines merge. Inset: stability diagram for a injected Class A laser (Eq. (1) in [3] with $\Gamma\kappa^{-1} = 1.2$, $\alpha = 1.2$, $\beta = 1$, $\Gamma \to \sigma S \kappa^{-1}$, $\Delta \to -\Delta\Omega\kappa^{-1}$, and $t \to \kappa t$).

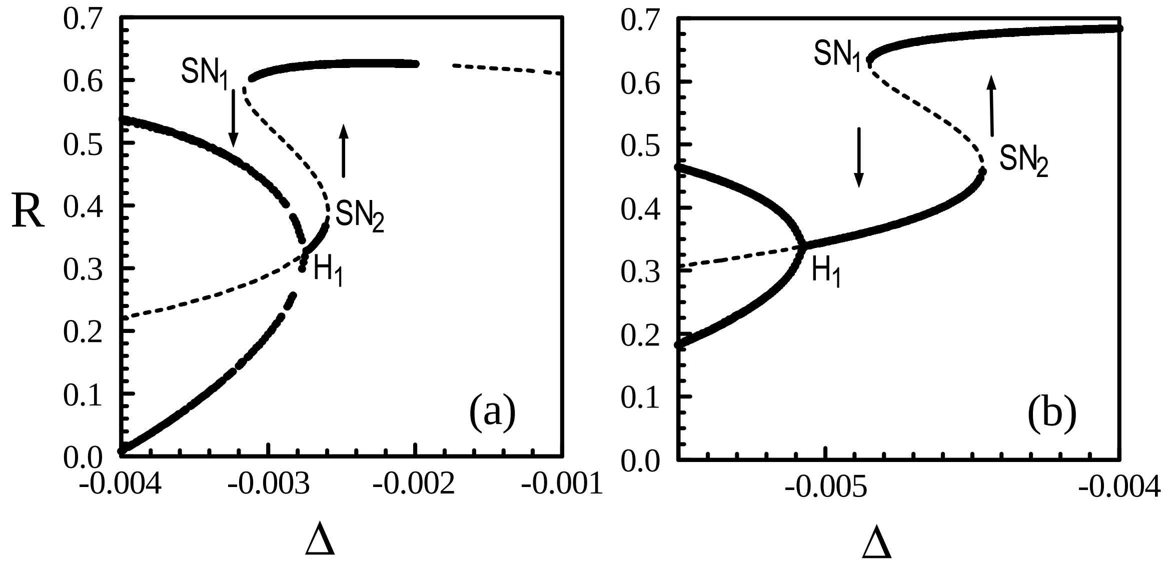

Figure 2: Bifurcation diagrams of the stable steady and time-periodic solutions. The extrema of R are shown as functions of the detuning Δ. The complete S-shaped branch of steady states is shown by a broken line. Same values of the parameters as in the previous figure. (a) partial bistability for $\Gamma = 0.0012$ and (b) full bistability for $\Gamma = 0.002$. The figures have ben obtained by scanning the detuning back and forth.

The stability diagram in Fig. 1 is qualitatively similar to the experimental mapping. Both the experimental and analytical stability diagrams predict stable locking for arbitrary values of the injection rate provided the detuning is sufficiently small. The domain of multiple steady state dominates for negative detunings (bounded by the lines SN_1 and SN_1 in Figure 1). At positive detuning, a single Hopf bifurcation (H_2 in Fig. 1) emerges from a single steady state. The absence of Hopf bifurcations at low injection levels has previously been noted allowing the observation of excitable pulses for both negative and positive detunings [13]. [A domain of bistability between locked states and periodic solutions for lower injections is predicted]. At negative detuning and for relatively high injection rates, there is a domain of bistability between two locked states. This phenomenon is not possible for QW SLs except if the pump current is very close to its threshold value[15]. The bistability phenomenon is here possible because of a Hopf bifurcation that stabilizes the lower intensity branch (H_1 in Fig. 1). Two bifurcation diagrams of the stable steady and periodic regimes are shown in Figure 2 illustrating the case of partial and full bistability. They have been determined numerically from Eqs. (7)-(9).

The Hopf bifurcation curves do not cross the zero detuning line as for QW SLs. Consequently, our QD SL exhibits higher stability properties which are important for some practical applications. We should however emphasize that our QD laser is equivalent to a Class A laser because it verifies the scaling law $(g-1)B = O(1)$. Other scalings are possible because of the large diversity of QD structures that are currently designed possibly leading to different conclusions concerning the RO damping and the effects of injections.

References

[1] F.T. Arecchi, G.L. Lippi, G.P. Puccioni,and J.R. Tredicce, Opt. Comm.51,308(1984)

[2] S. Wieczorek, B. Krauskopf, T. Simpson, and D. Lenstra, Phys. Rep. 416, 1 (2005)

[3] C. Mayol, R. Toral, C.R. Mirasso, and M.A. Natiello, Phys. Rev. A66, 013808 (2002)

[4] G. Baili, M. Alouini, C. Moronvalle, D. Dolfi, and F. Bretenaker, Opt. Lett. 31, 62 (2006)

[5] G. Baili, M. Alouini, D. Dolfi, F. Bretenaker, I. Sagnes, and A. Garnache, Opt. Lett. 32, 650 (2007)

[6] G. Baili, M. Alouini, T. Malherbe, D. Dolfi, I. Sagnes, and F. Bretanker, submitted, arXiv:0903.2827v1 (2009)

[7] D. Bimberg, M. Grundmann, and N.N. Ledentsov, "Quantum Dot Heterostructures", Wiley, NY 1999

[8] K. Lüdge, M.J.P. Bormann, E. Malic, P. Hövel, M. Kuntz, D. Bimberg, A. Knorr, and E. Schöll, Phys. Rev. B78, 035316 (2008)

[9] T. Erneux, E.A. Viktorov, and P. Mandel, Phys. Rev. A76, 023819 (2007)

[10] D. O'Brien, S.P. Hegarty, G. Huyet, and A.V. Uskov, Opt. Lett. 29, 1072 (2004)

[11] D. Goulding, S. P. Hegarty, O. Rasskazov, S. Melnik, M. Hartnett, G. Greene, J. G. McInerney, D. Rachinskii, and G. Huyet, Phys. Rev. Lett. 98, 153903 (2007)

[12] S. P. Hegarty, D. Goulding, B. Kelleher, G. Huyet, M.T. Todaro, A. Salhi, A. Passaseo and M. De Vittorio, Opt. Lett. 32, 3245 (2007)

[13] B. Kelleher, D. Goulding, S. P. Hegarty, G. Huyet, Ding-Yi Cong, A. Martinez, A. Lemaître, A. Ramdane, M. Fischer, F. Gerschütz, and J. Koeth, Opt. Lett. 34, 40 (2009)

[14] T.B. Simpson, Opt. Comm. 215, 135 (2003)

[15] A.Hohl, H. J. C. van der Linden, R. Roy, G. Goldsztein, F. Broner, and S.H. Strogatz, Phys. Rev. Lett. 74, 2220 (1995)